Pricing Kernels in International Markets: An Agnostic Study

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Abstract

An agnostic method to estimate pricing kernels, proposed in Pukthuantong and Roll (2015), is extended to an international framework. This estimation method strictly depends only on asset returns. Taking the method to the data, pricing kernels, using stock and bond prices denominated in local currency for China, the Eurozone, Japan, Russia, Switzerland, the United Kingdom, and the United States, are evaluated. Tests are performed to give evidence of a common pricing kernel. Empirical evidence is consistent with complete markets for the time period that spans from January 1999 to January 2017. Furthermore, we shed light on additional international financial market issues. High interest rate currencies tend to have low pricing kernel volatilities and market data are consistent with low international risk sharing indices, allaying suspicions for the international risk sharing puzzle. A confidence interval for the international risk sharing index is assessed and verified with Monte Carlo simulations.

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1 Introduction

The Stochastic Discount Factor or pricing kernel theory is a major foundation of asset pricing models. A stochastic discount factor captures how market agents discount future uncertain cash flows. It represents a complex measure of risk aversion and remains a delicate subject to apprehend, in particular empirically. The pricing kernel theory claims that, in complete markets, a unique pricing kernel prices all assets in a given time period. The paper investigates whether, as suggested by the pricing kernel theory, there exists a unique pricing kernel that prices any partition of assets, while considering assets from distinct economies. This has significant implications for both theoretical and empirical aspects of asset pricing theory. Indeed, such findings contribute to the literature, notably in international finance, to support or reject different features and assumptions of asset pricing models. In addition, it gives the empirical opportunity to use a pricing kernel estimated with a set of assets from one economy to price assets from another economy.

Building on a method proposed by Pukthuantong and Roll (2015) to estimate pricing kernels, we extend it to international assets and provide empirical and statistical evidence on how the pricing kernel theory stands in this context. In particular, both the uniqueness of the pricing kernel and its properties are investigated, and while the uniqueness of the pricing kernel follows from complete markets, the no-arbitrage condition holds on the positivity of the pricing kernel. The methodology to estimate the pricing kernel simply consists in a transformation of asset returns from two economies denominated in local currency and has the advantage neither to impose any restrictions regarding agents' preferences and consumptions nor to admit any pricing kernel specifications. Hence, no assumptions underpin the methodology.

Specifically, the paper considers assets from China, the Eurozone, Japan, Russia, Switzerland, the United Kingdom, and the United States. Pricing kernels are estimated with monthly stock and bond gross returns from January 1999 up to January

2017.

In the literature, several methods to estimate pricing kernels stand. A first technique uses aggregate consumption changes such as in Cochrane (1996) and Chapman (1997), among others. Nevertheless, it is commonly admitted that aggregate consumption changes are not volatile enough to match with asset prices and that consumption data are tainted with large imprecise measurements (Mehra and Prescott, 1985). Papers by Aït-Sahalia and Lo (1998) followed by Rosenberg and Engle (2002) constitute another attempt. Avoiding the use of aggregate consumption, Aït-Sahalia and Lo (1998) develop a non-parametric estimation of pricing kernels using Black and Scholes (1973) option prices. Rosenberg and Engle (2002) estimate time-varying pricing kernels specified as a power function of option data as well. In an international asset pricing model, Dumas and Solnik (1995) define pricing kernels as a linear function of world prices of exchange rate risk. While, Dittmar (2002) strongly advocates for the use of nonlinear pricing kernels and investigates a cubic form of these stemming from a Taylor expansion. In this spirit and in an international framework, Bansal et al. (1993) approximate a non-parametric pricing kernel model using polynomial series expansion with international weekly data from Germany, Japan, the United Kingdom and the United States. However, they do not investigate the pricing kernel theory across economy but they test their approach separately in each economy. All these papers focus on estimation techniques rather than on testing features of the pricing kernel theory and are not free from model specifications.

From this perspective, Pukthuantong and Roll (2015) are an exception. They derive an agnostic estimator of the pricing kernel, propose and implement tests for a common pricing kernel. Each test's features and power are examined in depth. Then, they apply the tests to returns on United States equities, bonds, currencies, commodities and real estate prices and find evidence of a common pricing kernel that prices all assets, independently of the asset class. Therefore, the work by Pukthuantong and Roll (2015) remains most closely related to this paper to which we contribute by adding an international perspective.

Indeed, once pricing kernels are estimated, four tests are conducted to statistically assess the uniqueness of the pricing kernel. To ensure the power of the tests, the number of assets should be large and at least twice the number of time periods (Pukthuantong and Roll, 2015). These four tests consist of, namely, the Kruskal and Wallis (1952), the Welch (1947), the Brown and Forsythe (1974), and the Kolmogorov-Smirnov tests. To compare with Pukthuantong and Roll (2015), we additionally use the non-parametric and distribution free Kolmogorov-Smirnov test as a complement to the Kruskal-Wallis test that mainly focuses on differences in medians. Empirical results show that these two tests do not always yield the same conclusion. These tests examine first and higher distribution moments. In particular, stocks and bonds are used in the estimation and pricing kernel testing is handled through varying combinations of the two asset classes. This offers an attempt to preclude the event of common characteristics in pricing kernels induced by the asset class and not the distinct economies. Finally, no strict statistical indication to reject the existence of a common stochastic discount factor, that is able to price any asset from any economy, has been found. Furthermore, empirical evidence is consistent, on average, with a positive stochastic discount factor. Besides, a rather comprehensive description of the estimated pricing kernels is provided and two international market issues related to pricing kernels are discussed.

In fact, a question, raised by Gavazzoni et al. (2013), concerns the validity of the assumption that pricing kernels are lognormally distributed (affine models (Cox et al., 1985) and asset pricing models (Bansal and Yaron, 2004) amongst others). A direct prediction of lognormal model is that high interest rate currencies are coupled with low pricing kernel volatilities. Gavazzoni et al. (2013) approximate conditional pricing kernel variances using interest rate conditional variances and find that log-normal models are inadequate for pricing kernels since the relation between interest rate currencies and pricing kernel volatilities is positive. Consequently, they argue that higher moments should be considered for future research. On the contrary, with pricing kernels being agnostically estimated, we model their conditional variances with a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model (Bollerslev, 1986) and we find that high interest rate currencies are associated with low volatility pricing kernels, which directly contradicts Gavazzoni et al. (2013).

Moreover, in the context of international markets, the question of capital market integration and risk sharing is of particular interest. A vast literature on market integration investigates its relation with risk sharing. It starts with Bekaert and Harvey (1995) who define as capital market integration the fact that similar risks traded in distinct markets should yield the same returns. A useful proxy of international risk sharing, expressed in terms of pricing kernels, is the international risk sharing index. This measure is defined in Brandt et al. (2006) as the ratio of how much risk is not shared over how much risk that there is to be shared between two economies. Brandt et al. (2006) base their argument on the idea that market integration lies in the correlation between stochastic discount factors. Along this line, Cochrane (2005) suggests that, in complete and perfectly integrated markets, marginal utility growth should be the same for all countries. Further, Brandt et al. (2006) document the so-called international risk sharing puzzle: the international risk sharing index is high when estimated with market data but low when estimated with consumption data. In this paper, findings demonstrate low international risk sharing indices that are consistent with indices obtained with consumption data (Obstfeld and Rogoff (2001), Brandt et al. (2006), Kose et al. (2009)), while market data are employed. In addition, Bakshi et al. (2015) also show that international risk sharing may be low with assets, especially for economies with high interest rate differentials and when market incompleteness is not precluded.

Another contribution of the paper is the inference of the international risk sharing index confidence interval. Indeed, to enrich the statistical quality of the index, a closed-form solution of its variance and its confidence interval is evaluated using the Fisher Z-transformation (Fisher, 1915, 1921). This transformation has been widely used in the literature to assess the statistical significance of the correlation coefficient because of its desired asymptotic normality properties (Hotelling (1953), Kowalski (1972), Meng et al. (1992), Bishara and Hittner (2016)). In addition, Lin (1989) develops a concordance coefficient and applies the transformation to evaluate its statistical properties for data from normal and non-normal distributions. Similar to the correlation and the concordance coefficients, the international risk sharing index lies in the interval (-1,1) and thus the inverse hyperbolic tangent transformation allows us to evaluate its variance and its confidence interval. The closed-form solution is verified by Monte Carlo simulations both with normal and non-normal data. Thereafter, we determine the statistical significance of the agnostically estimated international risk sharing indices.

Finally, a case study on the United States pricing kernels is conducted within a Fama and French (2016) risk factor-based approach. Predictability is assessed within an autoregressive-moving-average (ARMA) model.

The remainder of the paper is structured as follows. Section 2 provides a review of the theoretical framework. Section 3 provides data description. Section 4 presents test results and estimated pricing kernel statistical properties. Then, it discusses international financial market issues and ends with the case study on the United States pricing kernels. Finally, Section 5 concludes.

2 Methodology and Test Hypotheses

2.1 Fundamental Relation

The keystone relation of asset pricing theory, proposed by Harrison and Kreps (1979), Hansen and Richard (1987), and Hansen and Jagannathan (1991), posits that there exists a unique stochastic discount factor for a domestic investor, denoted by $\tilde{m}$ and defined as the Euler equation such that

$$\mathbb{E}_{t-1}(\tilde{m}_t \tilde{R}_{i,t}) \;=\; 1, \tag{1}$$

where $\tilde{R}_{i,t}$ is the gross return of an asset i at time t expressed in domestic currency.

Equally, in the foreign country with foreign traded assets expressed in the foreign currency, the foreign investor's pricing kernel satisfies

$$\mathbb{E}_{t-1}(\tilde{m}^*_t \tilde{R}^*_{i,t}) \;=\; 1, \tag{2}$$

where $\tilde{m}^*_t$ is the foreign stochastic discount factor and $\tilde{R}^*_{i,t}$ is the gross return of the foreign asset i at time t denoted in the foreign currency.

Another way to price foreign assets is to convert them into domestic currency using the expected spot exchange rate S_t and the spot exchange rate S_{t-1}. Exchange rates are given in direct quotation. In this case, we have

$$\mathbb{E}_{t-1}\left(\tilde{m}_t \frac{S_t}{S_{t-1}} \tilde{R}^*_{i,t}\right) \;=\; 1. \tag{3}$$

2.2 Estimation and Tests of Pricing Kernels with Foreign Assets

In this section, we build on Pukthuantong and Roll (2015). Under rational expectations, the realization of a random variable equals its expectation plus some noise. This means that for Equation (3), the realization of any asset i at time t is anticipated to be equal to

$$m_t \frac{S_t}{S_{t-1}} R^*_{i,t} = \mathbb{E}_{t-1}(\tilde{m}_t \frac{S_t}{S_{t-1}} \tilde{R}^*_{i,t}) + \epsilon^*_{i,t},$$

with $\epsilon^*_{i,t}$ being the surprise in the realization. Averaging over all T, we get

$$\frac{1}{T}\sum_{t=1}^{T} m_t \frac{S_t}{S_{t-1}} R^*_{i,t} = \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}_{t-1}(\tilde{m}_t \frac{S_t}{S_{t-1}} \tilde{R}^*_{i,t}) + \frac{1}{T}\sum_{t=1}^{T} \epsilon^*_{i,t} = 1 + \frac{1}{T}\sum_{t=1}^{T} \epsilon^*_{i,t} = 1 + \bar{\epsilon}_i{}^*.$$

In matrix form, one can write for any $i = 1, ..., N$ and $t = 1, ..., T$

$$\frac{1}{T}(\boldsymbol{R^* S})\boldsymbol{m} = 1 + \bar{\epsilon}^*,$$

where $\boldsymbol{R^* S}$ is a matrix of N rows and T columns, $\mathbf{m}$ is a T column vector and $\bar{\epsilon}^*$ is an N column vector.

By minimizing the sum of squared average surprises with respect to $\mathbf{m}$, we obtain the minimum squares estimator of $\mathbf{m}$,

$$\min_{m}((\bar{\epsilon}^*)'(\bar{\epsilon}^*)) = (\frac{1}{T}(\boldsymbol{R^* S})\boldsymbol{m} - 1)'(\frac{1}{T}(\boldsymbol{R^* S})\boldsymbol{m} - 1).$$

The estimate $\hat{\boldsymbol{m}}$ that satisfies the first-order condition is

$$\hat{\boldsymbol{m}} = T(((\boldsymbol{R^* S}))'(\boldsymbol{R^* S}))^{-1}(\boldsymbol{R^* S})'\mathbf{1}, \tag{4}$$

as long as $((\boldsymbol{R}^*\boldsymbol{S})'(\boldsymbol{R}^*\boldsymbol{S}))$ is non-singular.

With domestic assets, the estimate $\hat{\boldsymbol{m}}$ is simply given by

$$\hat{\boldsymbol{m}} = T(\boldsymbol{R}'\boldsymbol{R})^{-1}\boldsymbol{R}'\mathbf{1}, \tag{5}$$

assuming that $(\boldsymbol{R}'\boldsymbol{R})$ is invertible.

As suggested by the pricing kernel theory, we test whether there exists a unique pricing kernel that prices any partition of international assets.

Thereby, the null hypothesis of the Stochastic Discount Factor theory with international assets boils down to[1]

$$H_0 : \mathbb{E}\left(\hat{\boldsymbol{m}}(\text{domestic}) - \hat{\boldsymbol{m}}(\text{foreign})\right) = \mathbf{0}.$$

In particular, we consider the following set of hypotheses:

$$\begin{aligned}
H_{01} &: \mathbb{E}\left(\hat{\boldsymbol{m}}(\text{domestic stocks}) - \hat{\boldsymbol{m}}(\text{foreign stocks})\right) = \mathbf{0}, \\
H_{02} &: \mathbb{E}\left(\hat{\boldsymbol{m}}(\text{domestic bonds}) - \hat{\boldsymbol{m}}(\text{foreign bonds})\right) = \mathbf{0}, \\
H_{03} &: \mathbb{E}\left(\hat{\boldsymbol{m}}(\text{domestic stocks and bonds}) - \hat{\boldsymbol{m}}(\text{foreign stocks and bonds})\right) = \mathbf{0}, \\
H_{04} &: \mathbb{E}\left(\hat{\boldsymbol{m}}(\text{domestic bonds}) - \hat{\boldsymbol{m}}(\text{foreign stocks and bonds})\right) = \mathbf{0}.
\end{aligned}$$

A part of the paper consists in testing these hypotheses. For this purpose and to rule out the alternative hypotheses, we perform four statistical tests: the Kruskal and Wallis (1952), the Welch (1947), the Brown and Forsythe (1974), and the Kolmogorov-Smirnov tests.

Specifically, the Kruskal-Wallis test is a rank-based non-parametric test that investigates whether two samples originate from the same underlying distribution.

[1] Tests incorporate a joint hypothesis issue and if evidence are against the null hypothesis, rejection concerns either the pricing kernel theory or the estimation method or both.

It does so by determining if a sample stochastically dominates the other one. The null hypothesis is that independent samples are drawn from two distributions with no stochastic dominance, by comparing sums of ranks. It is associated with a test of equal medians.

The Welch test consists in an adaptation of the Student t-test of equal means for unequal variances and unequal sample sizes. The null hypothesis is that the two samples have equal means.

The Brown-Forsythe test establishes the equality of variances in two samples as the null hypothesis.

The two-sample Kolmogorov-Smirnov goodness-of-fit test assesses whether samples are drawn from the same distribution and compares the empirical distributions by testing the supremum of the distance between two distributions.

3 Data Description

We gather a panel of monthly equity prices denominated in local currency from several sectors: Electricity, Electronic and Electrical Equipment, Financial Services, Food Producers, Pharmaceutical, General Industrials, Real Estate Investment and Services[2] as well as bond prices denominated in local currency for China,[3] the Eurozone,[4] Japan, Russia, Switzerland, the United Kingdom, and the United States, from Thomson Reuters Datastream.[5] Furthermore, we collect one-month interbank interest rates and monthly closing exchange rates for each economy. Spot exchange rates are expressed as United States dollars per unit of foreign currency.

[2]We also include equities from Gas, Water and Multi-utilities, Industrial Engineering, Industrial Metal and Mining for Russia, Switzerland and the United Kingdom.

[3]Chinese bonds were not available during data collection.

[4]The Eurozone comprises Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain. It corresponds to the eleven countries that have created the Euro and Greece as it starts to adopt the Euro in January 2001 which is still early in our sample.

[5]Datastream prices are adjusted prices, which take stock splits and similar corporate actions into account, but not dividends.

The sample period spans from January 1999[6] to January 2017. As recommended by Pukthuantong and Roll (2015), to ensure the power of the tests and to satisfy the inversion property of the return matrix, we consider data subsamples between one year and two and a half years. Further, stock and bond gross returns are winsorized at the 10% level to reduce the impact of data errors.

With regard to data availability, the number of stocks varies among economies and per time period. We collect 906 stocks from China, 886 from the Eurozone, 456 from Japan, 147 from Russia, 152 from Switzerland, 469 from the United Kingdom and 1′330 from the United States. Panel (a) in **Table I** presents descriptive statistics of stock returns. All of them are expressed in U.S. dollars.[7] Stock returns from China and Russia are riskier than returns originating from the other economies. The average annualized volatility of Chinese and Russian stocks reaches more than 23% while, over the other economies, the average annualized volatility is about 15%. As expected, the majority of equity returns exhibit excess kurtosis and negative skewness.

We collect a set of 571 corporate bonds from the Eurozone, 711 from Japan, 395 from Russia, 303 from Switzerland, 70 from the United Kingdom and 1′067 from the United States. Bond returns descriptive statistics are reported in Panel (b) of **Table I**, expressed in U.S. dollars. Similarly, bond returns exhibit excess kurtosis except for Japanese, Swiss, and British bonds. Similar to equities, Russian bonds are found to be the most volatile. As a reference, bonds from the United States show a volatility of 5%.

Panel (c) of **Table I** reports exchange rate descriptive statistics. The most volatile currencies in the sample are, in descending order, the Russian ruble, the

[6] As in January 1999, the Euro is introduced.

[7] On average, returns on U.S. stocks are much higher than Eurozone stocks. A deeper investigation shows that the average returns on French and German stocks are of about 8% but only of about 0% for Spain.

Swiss franc and the Euro. The Chinese yuan is the least volatile currency.[8]

Finally, Panel (d) of **Table I** reports descriptive statistics of one-month interbank interest rate differentials in percentage with respect to the United States. Consistent with common knowledge, the interest rate differentials are negative for low interest rate currencies such as those of Japan, Switzerland and the Eurozone, while the opposite holds for high interest rate currencies.

4 Empirical Results

This section presents test results for a common pricing kernel. Initially, tests focus on national pricing kernels and then are extended to international assets. Second, we describe statistical properties of the estimated pricing kernels, denominated in local currency. Third, we discuss two international financial market puzzles and finally perform a case study on the United States pricing kernels.

4.1 Pricing Kernels under Tests

4.1.1 National Pricing Kernels

Following a univariate analysis, we replicate the work of Pukthuantong and Roll (2015). Again, while Pukthuantong and Roll (2015) focus solely on the United States, we additionally consider China, the Eurozone, Japan, Russia, Switzerland and the United Kingdom.

The test hypothesis writes as

$$H_{05} : \mathbb{E}\left(\hat{\boldsymbol{m}}(\text{stocks}) - \hat{\boldsymbol{m}}(\text{bonds})\right) = \mathbf{0},$$

for each economy. In order to ensure statistical power, we consider subsamples of one year that start from May 2011 as before that date the number of bonds for

[8]Since August 2015, the Chinese yuan has been more volatile, as China has devaluated it.

several economies are not large enough to conduct the estimation.[9] Beforehand, various lengths of time period have been tested but no differences in test results stand out. Results are robust to the size of the estimation window from Equation (5) as long as the number of assets is large enough.[10] The only exception concerns the results from the Brown and Forsythe (1974) test which strongly depends on the number of time periods relative to the number of assets. **Table II** reports results in the form of p-values.

Considering the findings pertaining to the United States, we obtain similar results to Pukthuantong and Roll (2015). This holds although we cover a different time period and different assets.[11] Pukthuantong and Roll (2015) collect monthly returns for July 2002 through December 2013. Mostly, we do not reject the null hypotheses of no stochastic dominance, equal means and no differences in distributions. It is only from June 2012 to June 2013, that we do reject the null of no differences in distributions at the 10% level of significance. We reject the null of similar variances for almost all subsamples. Yet Pukthuantong and Roll (2015) tend to reject the equality in variances as well.

For the remaining economies, p-values of the Kruskal and Wallis (1952), the Welch (1947) and the Kolmogorov-Smirnov tests indicate no statistical rejection of the null hypothesis at the 1% level, except for the United Kingdom that turns out to be associated with a very small number of bonds. Moreover, there is for most of the cases an indication of unequal variances. Similarly, the average number of available bonds in each economy is relatively small and much smaller than the number of stocks, which, again, as pointed out in Pukthuantong and Roll (2015), implies more sampling errors and might lead to difference in variances.

To conclude, there is no robust rejection of the null hypotheses except for Brown

[9] Indeed, for the majority of the economies, gross returns matrix is not invertible.

[10] Pukthuantong and Roll (2015) recommend a number of about $1'000$ assets.

[11] For instance, we consider stocks from different sectors than Pukthuantong and Roll (2015).

and Forsythe (1974) tests. Nonetheless, we conjecture that this stems from the relative small bonds sample. To verify the latter, we consider the estimation over six months[12] from July 2015 to January 2017 and we report results in **Table III**. By comparing results in **Table II** for the Eurozone, Japanese and United States assets, the evidence against equal variances diminishes as p-values increase. There is only weak sign of unequal variances which supports our conjecture.

Finally, results are in line with the findings of Pukthuantong and Roll (2015), which increases the validity of the agnostic method while supporting, in general, the hypothesis of complete markets in the gathered data. Moreover, conclusions are not economy-specific.

4.1.2 International Tests with Stocks Only

Starting with equities only, we add international complexity in the tests of the pricing kernel theory by considering a cross-section of economies. The domestic country is set to be the United States.

We estimate pricing kernels in U.S. dollars for each economy using gross returns on equities and exchange rates expressed in local currency together with Equation (4). Then, we perform the four tests on Hypothesis H_{01}. P-values are presented in **Table IV**. Again, we use subsamples of two years and a half to ensure better testing power.

Globally, results are similar to the univariate analysis conducted in the previous section. We find no indication of rejection of the Kruskal-Wallis and the Welch tests for any economy. Similarly, for the two-sample Kolmogorov-Smirnov goodness-of-fit test, p-values indicate no differences in distributions except for Russia and Switzerland where p-values indicate weak evidence against the null hypothesis.

However, we reject the null of equal variances for Japan, Russia, Switzerland

[12] The number of time periods is constraint by a lower bound in order to compute variances.

and for the United Kingdom in the bulk of the subsamples. The number of stocks for these economies is relatively small, especially compared to the number of United States stocks which can affect the power of the Brown and Forsythe (1974) test. We address a part of this issue by additionally accounting for bonds in Section 4.1.3. We also come to reject this null for China and for the Eurozone but in a minority of the subsamples.

Although p-values from the Brown and Forsythe (1974) test undermine our conclusions, ultimately, findings are consistent with Pukthuantong and Roll (2015) and in favor of the existence of a common pricing kernel for domestic and foreign economies. Notably, rejections do not increase when considering international assets compared with national assets.

4.1.3 Tests with Bonds

Turning to a second asset class to challenge our findings, we first repeat the exercise with bonds only, expressed in local currency. Together with exchange rates, we compute estimated foreign and domestic pricing kernels.[13] P-values for Hypothesis H_{02} are in **Table V.** Evidence in favor of a common pricing kernel still holds. We never reject the null of no stochastic dominance. Equivalently, we do not reject the null of equal means. Still, we do reject the null of equal variances and no differences in distributions in the minority of cases. Rejections do not allow us to unequivocally herald against a common pricing kernel.

Second, we incorporate both equities and bonds. In this case, we use the complete set of assets, which provides us with richer information and more tests power. Respectively, the total number of assets is 906 for China, $1'457$ for the Eurozone, $1'167$ for Japan, 542 for Russia, 455 for Switzerland, 539 for the United Kingdom and $2'397$ for the United States. We are able to estimate the complete time series of

[13] Again, except for China, for which we do not have data.

pricing kernels, except for Russia before June 2007. **Table VI** presents p-values for Hypothesis H_{03}. Evidence in favor of a common pricing kernel strengthens as rejections of Kolmogorov-Smirnov tests reduce. P-values tend to be higher for all tests, but we still tend to reject the hypothesis of equal variances, yet less frequently. This is especially the case for assets from Japan and from the United Kingdom which additionally consist of the smallest bond sample. Moreover, quantile-quantile plots of the foreign against the domestic pricing kernels are reported in **Figure 1**, they support test conclusions. In general, points fall about straight lines apart from outliers.

Third, we consider only domestic bonds in order to estimate domestic pricing kernels. On this basis, it increases the role played by the asset class. This partially allows us to mitigate the suspicion that assets from the same class, even if they come from distinct economies, behave in a similar manner. Specifically, we conduct tests of Hypothesis H_{04}. **Table VII** presents p-values. There is no evidence against equal means. The null of no stochastic dominance appears to be weakly rejected only in one subsample. We tend to reject the null of no differences in distributions more often, but not in the majority of subsamples. Given the difficulty of Hypothesis H_{04}, again, results tend not to be against the pricing kernel theory.

Finally, we are interested in results when one considers a longer sample period ($T = 217$). We do so only for the Eurozone and for the United States as, in total, the number of assets are large enough and respectively of $N = 1'457$ and $N = 2'397$. Results are reported in **Table VIII**. There is no evidence against the pricing kernel theory.

To conclude, whether we estimate domestic pricing kernels using domestic or foreign bonds, domestic or foreign equities, we find no evident statistical evidence against the Stochastic Discount Factor theory in international markets.

4.2 Estimated Pricing Kernels

In this section, we describe the pricing kernels estimated with local assets and denominated in local currency. First, **Table IX** presents pricing kernel descriptive statistics.[14] As expected, pricing kernel means are close to 1. The standard deviations range between 33% for the United States and 75% for the Swiss pricing kernels. As a comparison, Brandt et al. (2006) report volatilities between 63% and 69% at a monthly frequency for Germany, Japan, the United Kingdom, the United States from January 1975 through June 1998. The pricing kernel from Switzerland appears to be the most volatile, followed by that from the United Kingdom and from Japan. We conjecture that this is a first evidence towards the idea that low interest rate currencies tend to be associated with more volatile pricing kernels, as investigated in Section 4.3. Estimated pricing kernels exhibit a negative minimum which indicates possible arbitrage opportunities. Nevertheless, we reject the null hypothesis of zero means with test statistics that are all positive, at 26.9 (China), 34.7 (the Eurozone), 26.3.4 (Japan), 20.0 (Switzerland), 37.1 (the United Kingdom) and 37.1 (the United States). This is in line with Pukthuantong and Roll (2015) who report a t-statistics, for the United States low- and higher-leveraged equities of 20.2 and 16.8 respectively. These results demonstrate the absence of arbitrage opportunities on average.

For other moments, results are more heterogenous, although all pricing kernels demonstrate positive skewness, except for the Eurozone. P-values of the Jarque-Bera test indicate a normal distribution only for the Eurozone and the United States. Estimated pricing kernels exhibit serial correlation, except pricing kernels estimated with Eurozone that are serially independently distributed. Moreover, all pricing kernels are stationary given the Augmented Dickey-Fuller test and satisfy

[14] In the remainder of the paper, Russia is excluded from the analysis as its estimated pricing kernels are loaded with sampling errors, making estimations of poor quality as observed in Tables II, IV, V, VI and VII.

the HansenJagannathan bounds (Hansen and Jagannathan, 1991).

Second, **Figure 2** displays, for each economy, the time series of the estimated pricing kernels, in local currency, with smoothing spline methods used for curves fitting. Pricing kernels tend to be positive, which demonstrates an absence of arbitrage opportunities, except in the Swiss market. Notably, the estimated pricing kernels from the Eurozone, from Japan, the United Kingdom and the United States demonstrate a downturn at the beginning of the global financial crisis (which is thus associated with a higher discount rate). On the contrary, the opposite holds for Chinese pricing kernels.[15]

Third, **Figure 3** plots foreign-domestic quantile-quantile diagrams of pricing kernels expressed in local currency. They exhibit higher moment differences, including heavy tails and left skews. Results differ from **Figure 1** especially through higher levels of kurtosis and skewness. Therefore, in light of findings in Section 4.1 and **Figure 3**, it appears that currency risk embeds an important part of the differences in the cross-section of pricing kernels denominated in local currency. Further investigation, such as cross-sectional predictability or causal links, on economy-specific pricing kernels, is thus tainted by currency risk and is associated with findings of Hong (2001) and Kim and Roubini (2000).

Finally, thanks to the agnostically estimated pricing kernels, we shed light on two international market puzzles in the next two sections.

4.3 Currency Risk and the Lognormality Hypothesis

Gavazzoni et al. (2013) question the lognormality hypothesis of pricing kernels that is extensively used in the literature. Lognormal models imply that risky currencies are associated with low pricing kernel volatilities. Another empirical prediction of such models is that high interest rate currencies are associated with low pricing

[15] We conjecture that the appreciation of the Chinese Yuan relative to U.S. dollar during the financial crisis generates this effect.

kernel volatilities. However, Gavazzoni et al. (2013) document that high interest rate currencies demonstrate high pricing kernel volatilities.

Indeed, on the one hand, assuming conditional lognormal pricing kernels, the currency risk premium on the foreign currency is equal to[16]

$$\mathbb{E}_t s_{t+1} - f_t = \frac{1}{2}(Var_t \log(m_{t+1}) - Var_t \log(m^*_{t+1})), \tag{6}$$

where, the left-hand-side is the risk premium on the foreign currency[17] while the right-hand-side is the pricing kernel conditional variance differentials. The currency risk premium on the foreign currency is positive, if the foreign pricing kernel has a lower conditional variance. The economic rationale behind this is that the higher the variability in the marginal domestic utility, the lower the tolerance to exchange rate risk, and thus agents demand a risk premium on the foreign currency.

On the other hand, in international markets, the ratio of pricing kernels from two economies is equal to the exchange rate between these two economies:

$$m^*_{t+1} = m_{t+1}\frac{S_{t+1}}{S_t}. \tag{7}$$

As well, the covered interest rate parity implies that $f_t - s_t = i_t - i^*_t$, where f_t and s_t are the logarithm of the forward and the spot exchange rates in units of U.S. dollars per foreign currency. The interest rates in the domestic and foreign economies are respectively denoted by i_t and i^*_t. The forward premium, as referred by Fama (1984), can be decomposed such that $f_t - s_t = (f_t - \mathbb{E}_t s_{t+1}) + (\mathbb{E}_t s_{t+1} - s_t)$. The first term in parentheses is the currency risk premium. The second term in parentheses is the expected change in the exchange rate. In order to consistently capture the empirical forward premium anomaly, it is first shown in Fama (1984) and later in

[16] The reader may refer to the Appendix A.1 for derivations. This is also shown, for instance, in Backus et al. (2001), in Backus et al. (2013) and in Gavazzoni et al. (2013).

[17] The foreign currency risk premium is minus the currency risk premium (Backus et al., 2013).

Backus et al. (1995) or Backus et al. (2013), that the covariance between the currency risk premium and the change in exchange rates is negative. Furthermore, change in exchange rate tends to be associated negatively with interest rate differentials according to the uncovered interest rate parity (UIP) violations. Taken together, the currency risk premium and interest rate differentials are positively correlated.

Hence, Equation (6) together with the above-mentioned empirical evidence implies that interest rate differentials are negatively associated with pricing kernel volatilities. Stated differently, the lognormality hypothesis predicts that high interest rate currencies tend to have low pricing kernel volatilities. Yet Gavazzoni et al. (2013) find that pricing kernel volatilities and interest rates are strongly positively correlated. For instance, they report a correlation between Australian and the United States interest rate and volatility differentials of 0.52.

Using a GARCH model, we estimate the conditional variance of pricing kernels.[18] All correlations are negative and reported in **Table X**. They demonstrate a negative correlation of -0.15 on average. As a result, lognormal models appear to be consistent with empirical evidence, which contradicts Gavazzoni et al. (2013). Moreover, results demonstrate that the agnostic estimation methodology remains consistent with empirical evidence on the forward premium anomaly and on UIP violation.

Indeed, we conjecture that this contradictory finding is caused by the approximations taken in Gavazzoni et al. (2013) in estimating the conditional variance of the pricing kernels. To verify this conjecture, we replicate their procedure. In this case, we obtain very statistically positive correlations between the interest rate and the conditional volatility differentials. **Table XI** reports correlations using Gavazzoni et al. (2013) approximations. The average correlation is strongly positive and is equal to 0.92.

[18] The estimations with fat tail distributions in the GARCH processes yield similar results.

4.4 The International Risk Sharing Index

In this section, we contribute to the literature on the international risk sharing index in terms of pricing kernels, its attached puzzle, and, especially, its statistical inference.

Starting with Brandt et al. (2006), the international risk sharing index is defined as

$$\rho_{IRSI} = 1 - \frac{Var(\ln(m^*) - \ln(m))}{Var(\ln(m^*)) + Var(\ln(m))} = \frac{2Cov(\ln(m^*), \ln(m))}{Var(\ln(m^*) + Var(\ln(m))}.$$

This measure of dependence resembles a correlation but it accounts for the necessity of having completely equal pricing kernels and not only a linear relationship between them in order to have an international risk sharing index equals to unity (Brandt et al., 2006). It may be seen as a ratio between how different two countries pricing kernels are over how similar they are.

Bakshi et al. (2015) allow for incomplete markets and the international risk sharing index boils down to

$$\rho_{IRSI} = \frac{2Cov(m, m^*)}{Var(m) + Var(m^*)}.$$

Using the agnostic estimation, we measure the degree of international risk sharing for the sampled economies. Results are shown in **Table XII**. The risk sharing index between the Eurozone and the United Kingdom is the strongest, followed by the one of the United Kingdom and the United States and the one of the Eurozone and the United States. In fact, risk sharing index is negative only between Japan and Switzerland.

As in Bakshi et al. (2015), we find relative low international risk sharing indices, which are consistent with results obtained with consumption data. Indeed, for instance, Brandt et al. (2006) document a risk sharing index between the United States and the United Kingdom of 0.25 with consumption data and of 0.99 with market data. This supports our results as we report a risk sharing index of 0.29.[19] In this sample, the agnostic estimation provides evidence towards the absence of the international risk sharing puzzle.

In order to strengthen the statistical quality of the international risk sharing index, we contribute by evaluating a closed-form solution of the variance of ρ_{IRSI}. We apply Fisher Z-transformation as the international risk sharing index lies in the interval (-1,1) together with the Delta method. The transformation exhibits an asymptotic normal distribution which allows one to compute confidence intervals as shown by Monte Carlo simulations.

The Fisher Z-transformation uses the inverse of the hyperbolic tangent of $\hat{\rho}_{IRSI}$

$$\hat{Z} = \tanh^{-1}(\hat{\rho}_{IRSI}) = \frac{1}{2} \ln \frac{1 + \hat{\rho}_{IRSI}}{1 - \hat{\rho}_{IRSI}}.$$

The reader may refer to Appendix A.2 for derivations and one can show that $\hat{Z}$ is asymptotically normally distributed with mean $\frac{1}{2} \ln \frac{1+\rho_{IRSI}}{1-\rho_{IRSI}}$ and with variance

$$\sigma^2_{\hat{Z}} = \frac{1}{N} \left[\frac{(1 + \rho^2)}{\rho^2(1 - \rho^2_{IRSI})} - \frac{1 - \rho^4_{IRSI}}{(1 - \rho^2_{IRSI})^2} - \frac{1}{\rho^2} \right]$$

where ρ is the Pearson coefficient of correlation. Therefore, the confidence interval of $\hat{Z}$ is simply given by $[\hat{Z} \pm z_{1-\frac{\alpha}{2}} \sigma_{\hat{Z}}]$.

One can also show that $\hat{\rho}_{IRSI}$ is a consistent estimator of ρ_{IRSI} and has an asymptotic normal distribution with mean ρ_{IRSI} and variance

$$\sigma^2_{\hat{\rho}_{IRSI}} = \frac{1}{N - 2} \left[\frac{(1 - \rho^2_{IRSI})(\rho^2 + \rho^2_{IRSI})}{\rho^2} - (1 - \rho^4_{IRSI}) \right]. \tag{8}$$

[19] In Brandt et al. (2006), data span from January 1975 to June 1998.

To assess the confidence interval of the international risk sharing index, it suffices to use the confidence interval of $\hat{Z}$ that gives the lower and the upper limits, together with the following equation

$$\hat{\rho}_{IRSI} = \frac{e^{2\hat{Z}} - 1}{e^{2\hat{Z}} + 1}.$$

To evaluate the asymptotic normality of the inverse hyperbolic tangent transformation applied to the international risk sharing index, Monte Carlo simulations are performed for five values of ρ_{IRSI} with sample size of $N = 20$, $N = 50$, $N = 100$ and $N = 1'000$. We generate random numbers from a bivariate normal distribution, two correlated uniform and two correlated Gamma distributions. For each specification, $20'000$ paths are generated.

In **Table XIII**, means, standard deviations and Jarque-Bera p-values of $\hat{Z}$ and $\hat{\rho}_{IRSI}$ based on the $20'000$ runs are reported, as well as their standard deviation counterparts computed respectively with Equations (4.4) and (8). We reject the normality assumption for the distribution of $\hat{\rho}_{IRSI}$ for any specification and any sample size. However, the distribution of $\hat{Z}$ is normally distributed as long as $N = 50$, which corresponds to a rather small sample size. Clearly, the Fisher-Z transformation improves the asymptotic normality of the risk sharing index. In addition, Equations (4.4) and (8) of variances satisfactorily target the sample variances. They are accurate even when the sample size is small ($N = 20$) and are right on the target as long as $N > 100$ for every specification.

A Monte Carlo experiment from non-normal data constitutes a robust verification of the normality asymptotic distribution of $\hat{Z}$ and $\hat{\rho}_{IRSI}$. Pairs of random numbers are generated from uniform distributions that exhibit symmetry and thin tails. As well, we generate bivariate random numbers from a Gamma distribution with a shape parameter of 8 and a scale parameter of 4. The Gamma distribution exhibits asymmetry and fat tails in line with financial data. Moreover, the Gamma distribution has the advantage of being always positive which lends itself well to the stochastic discount factors. Results are provided in **Table XIV** and in **Table XV**. Both uniform and Gamma distributions yield similar conclusions.

Sample counterparts of variance are less accurate in the case of non-normal data and they always tend to be underestimated compared with variances obtained via Monte Carlo. Underestimation decreases as sample size increases. In the case of a fat tail distribution, the sample counterpart is almost right on the target when $N = 100$ and when the sample international risk sharing index is high. The normality hypothesis of $\hat{Z}$ holds only for large sample size for uniformly and Gamma distributed data. In particular, a sample size of $N = 200$ shows that this number of observations is sufficient not to reject the normal distribution hypothesis for $\hat{Z}$ in case of Gamma distributed data while we need $N > 400$ for uniformly distributed data.

Results from Monte Carlo simulations prove right the asymptotic normality of the Fisher-Z transformation applied to the international risk sharing index and validate Equations (4.4) and (8).

4.5 The United States Pricing Kernel: A Case Study

4.5.1 Risk Factor-based Approach

Investigating the risk factors inherent to the United States pricing kernels is the subject of this section. We focus on the Fama and French (1993) factor model as it remains the dominant risk factor-based model in the empirical asset pricing literature and account for latest development by Fama and French (2016). Additionally, in line with Cochrane (1996), the pricing kernel is defined as a linear function of a set of risk factor proxies. The section allows challenging the agnostic estimation method by confronting the estimated pricing kernels within a risk factor approach model.

First, **Table XVI** reports correlations between the Fama and French (2016) risk factors,[20] namely, the excess return market ($R_m - R_f$), the Small minus Big (SMB), the High minus Low (HML), the Robust minus Weak (RMW) and the Conservative minus Aggressive (CMA) portfolios and the estimated pricing kernels. The correlations are relatively strong and respectively equal to 0.25 ($R_m - R_f$), 0.10 (SMB), -0.07 (HML), -0.22 (RMW) and -0.06 (CMA).

As a comparison, Ghosh et al. (2016) extract pricing kernels under no arbitrage restrictions using an entropy minimization approach. They document substantial correlations ranging from 0.55 to 0.69 at a quarterly frequency, and, from 0.27 to 0.54 at an annual one, with ten Industry portfolios. However, to compute correlations, they use a different approach; they first perform a linear regression of their model-implied pricing kernels on the three Fama and French (1993) risk factors and then compute the correlation between their pricing kernels and their fitted values from the regression. Besides, they do not strictly use Fama and French (1993) portfolios, as we do, but form their own portfolios. Nevertheless, an identical procedure gives us a comparable correlation of 0.27.

Second, we add the so-called global foreign exchange volatility factor (VOL), as defined by Menkhoff et al. (2012), to account for the role of currency risk on local pricing kernels in light of the comparative results stemming from **Figure 1** and from **Figure 3**.[21] This also coincides with the idea of the international asset pricing model by Dumas and Solnik (1995).

The linear regression equation is given by

$$m_t = a + b' f_t + \epsilon_t, \tag{9}$$

where f_t consists of the five Fama and French (2016) risk factors and the VOL

[20] Fama and French (2016) portfolio returns are directly obtained from the Kenneth R. French website.

[21] A limitation is conveyed by the relatively small number of considered currencies. Results might strengthen with a more representative basket of currencies.

factor that corresponds to the innovations of an autoregressive model of order 1 on the time-varying volatility of the average exchange rate changes, referred to as the global FX volatility factor (Menkhoff et al., 2012).

Table XVII reports regression estimates.[22] Newey-West heteroskedasticity and autocorrelation-consistent weighting matrices with optimal lags are used. The market portfolio is the only significant risk factor that account for the variability of the estimated pricing kernels. The adjusted-R^2 is reasonable and is equal to 5%.

Nonetheless, based on the success of the Fama and French (2016) model, we expect stronger explanatory power. This suggests that the agnostic methodology tends to capture only a small part of the risk stemming from these risk factors in the sample or that a linear pricing kernel model is not the most appropriate model, as shown in Dittmar (2002).[23]

4.5.2 Forecasting

Predictability remains a central concern both in empirical and theoretical asset pricing. For instance, Campbell and Cochrane (1999) and Bansal and Yaron (2004) model pricing kernel components as AR(1) processes. In this line and as **Table IX** demonstrates the presence of serial correlation in the pricing kernels, we investigate pricing kernels forecastability within an ARMA(p,q) framework.

The estimated ARMA(p,q) model is given by

$$m_t = c + \sum_{i=1}^{p} \phi_{t-i} m_{t-i} + \sum_{j=1}^{q} \theta_{t-j} \epsilon_{t-j} + \epsilon_t.$$

In light of previous results, we additionally consider an ARMAX(p,q) that in-

[22]As an experiment, we also distinguish between the time periods before and after of the global financial crisis. Results differ in the sense that on the time period following the global financial crisis, the risk factors HML, RMW and CMA are additionally statistically significant at the 10% level only and the adjusted-R^2 is equal to 11%.

[23]Dittmar (2002) finds that highly nonlinear pricing kernels empirically outperform both linear and multifactor models.

cludes exogenous variables defined as

$$m_t = c + \beta^{'} f_t + \sum_{i=1}^{p} \phi_{t-i} m_{t-i} + \sum_{j=1}^{q} \theta_{t-j} \epsilon_{t-j} + \epsilon_t,$$

where f_t consists of the five Fama and French (2016) risk factors.

An ARMA(p,q) and an ARMAX(p,q) models are estimated in-sample from January 1999 to January 2013. From February 2013, we compute the out-of-sample one-step ahead forecast of the corresponding model. To measure forecasts accuracy, the sum of the mean square forecast errors is computed and reported in Table XVIII. Optimal lags length (p^*, q^*) are selected by the Akaike Information Criterion.

The minimum of the mean square errors is given by the ARMAX(p^*,q^*). Furthermore, the ARMA(p^*,q^*) outperforms the ARMA(1,1). Results underline the importance of long-lasting serial dependencies in modelling pricing kernels. Indeed, while Bansal and Yaron (2004) use an AR(1) process. Such number of lags is not sufficient as shown in Table XVIII, recent pricing kernel specifications such as Rosenberg and Engle (2002), serial correlation is not accounted for and this might lead to specification loss.

Figure 4 depicts pricing kernel time series together with the one-step ahead forecasts from an ARMAX(p^*,q^*) model. Predictions tend to resemble to the estimated pricing kernels, except at the beginning of the forecasting period where realized and forecasted go in the opposite direction.

To conclude, a possible pricing kernel specification as in Bansal and Yaron (2004) with more than one lag might be able to capture the serial dependency that we observe.

5 Conclusion

The method proposed by Pukthuantong and Roll (2015) is extended to international assets. It is straightforward and has the advantage of being completely agnostic. It has good prospects for estimating pricing kernels, anyhow, rapidly and without the use of any underlying assumptions.

To ensure power to the pricing kernel theory tests, as mentioned in Pukthuantong and Roll (2015), the number of assets should be large and significantly larger than the number of time periods. Considering stocks and bonds from distinct economies, there is no assertive evidence against the existence of a common pricing kernel. In particular, results are consistent with complete markets for the time period spanning from January 1999 to January 2017 for assets from China, the Eurozone, Japan, Russia, Switzerland, the United Kingdom and the United States. Again, these results suggest that with a large enough set of assets, it may also be possible to price any asset with a common pricing kernel on a similar time period in international financial markets.

Furthermore, with this methodology, we are able to consider various aspects of international pricing kernels. High interest rate currencies tend to have low pricing kernel volatilities consistently with lognormal models. International risk sharing indices are low even with market data which provides evidence against the existence of the international risk sharing puzzle.

Finally, the variance of the international risk sharing statistics is provided in a closed-form solution and verified by Monte Carlo simulations. Its Fisher-Z transformation is asymptotically normally distributed when data are normally distributed and is robust to uniform and Gamma distributions as long as the sample size is large enough.

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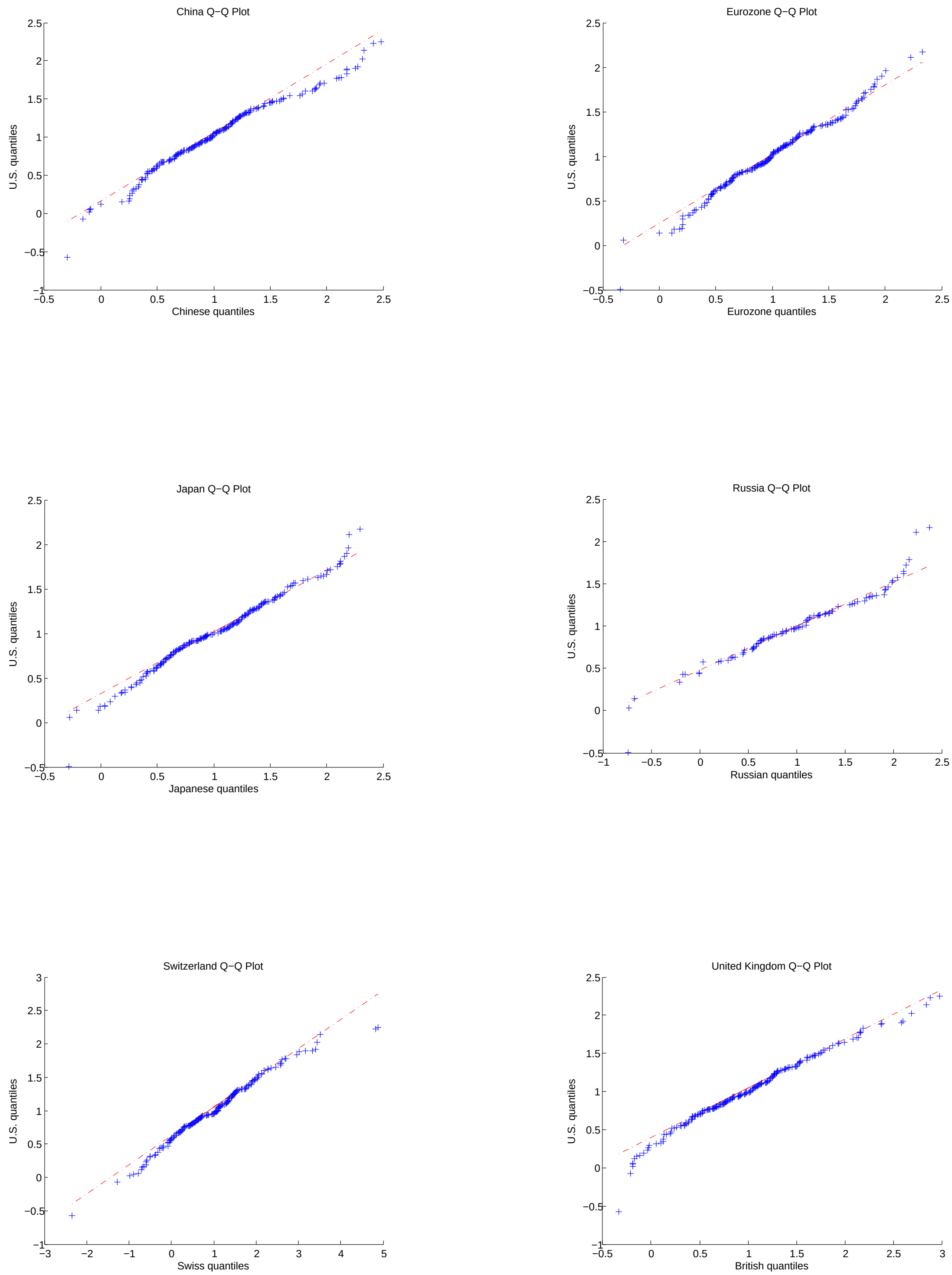

Figure 1: This figure shows quantile-quantile plots of the foreign against the United States estimated pricing kernels. All pricing kernels are expressed in U.S. dollars. The sample period is from January 1999 to January 2017.

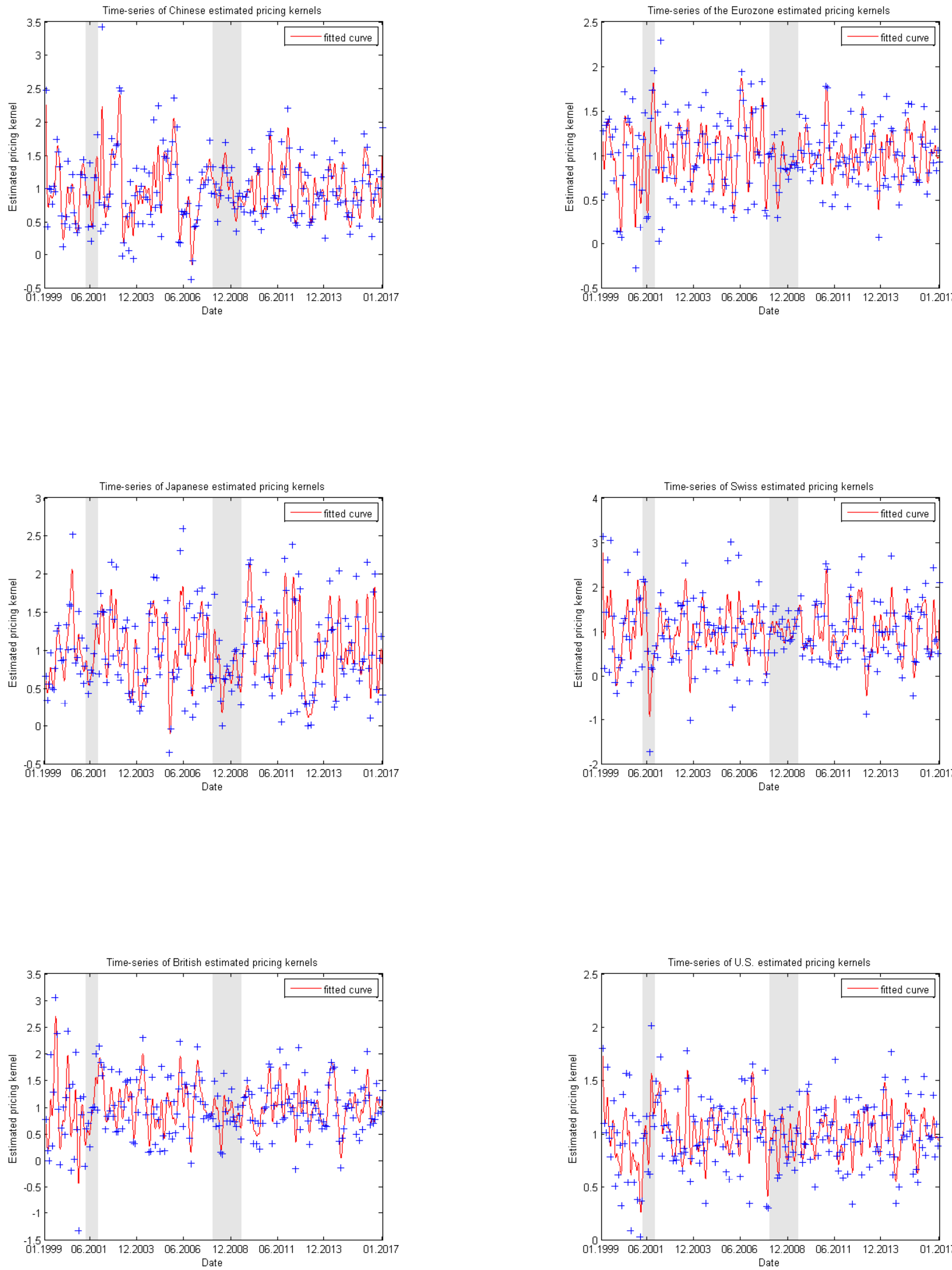

Figure 2: This figure depicts time series of estimated pricing kernels with smoothing splines. Overlay bands indicate United States recessions reported by the National Bureau of Economic Research. The sample period is from January 1999 to January 2017.

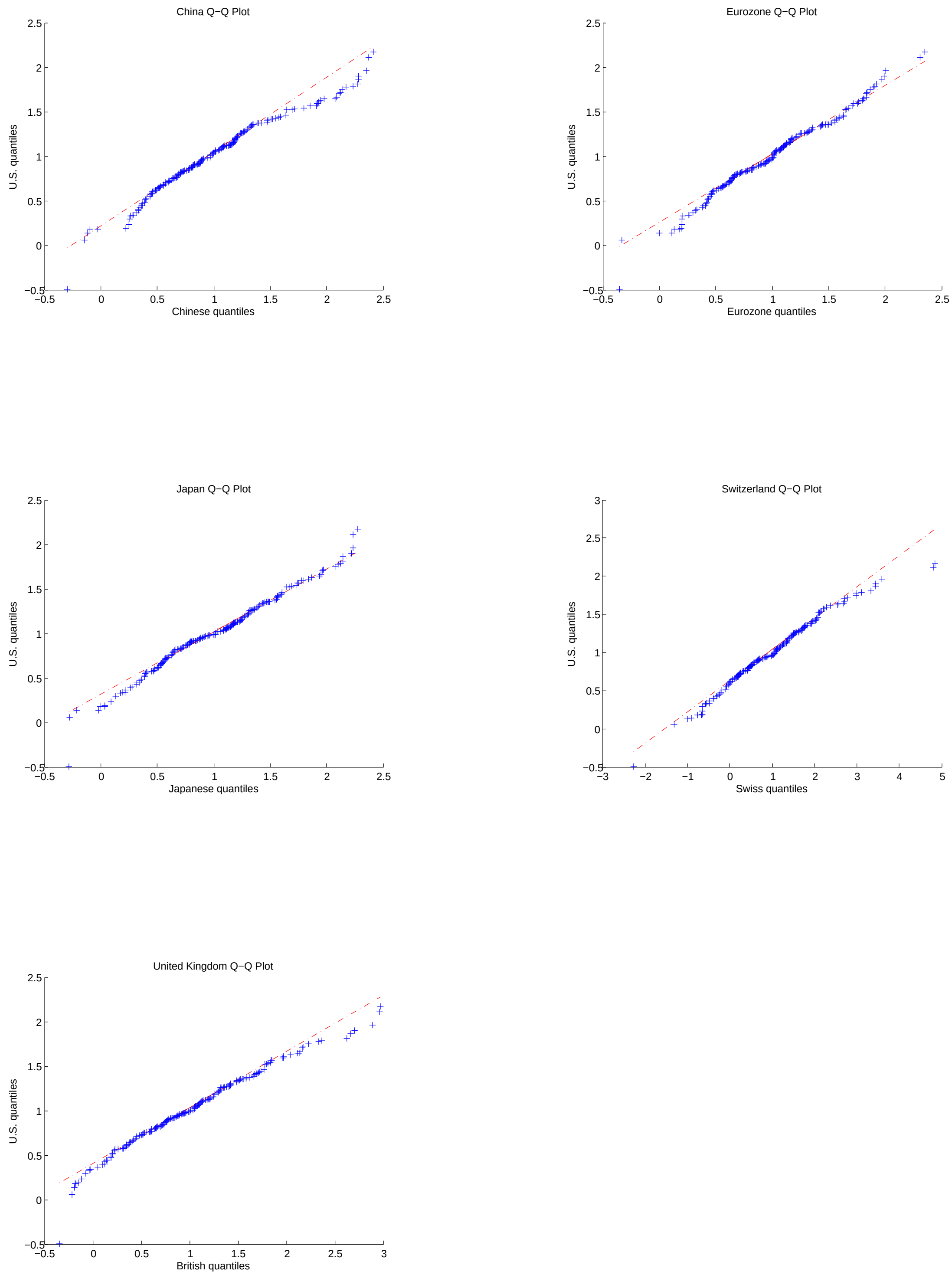

Figure 3: This figure displays quantile-quantile plots of foreign against the United States estimated pricing kernels. All pricing kernels are expressed in local currency. The sample period is from January 1999 to January 2017.

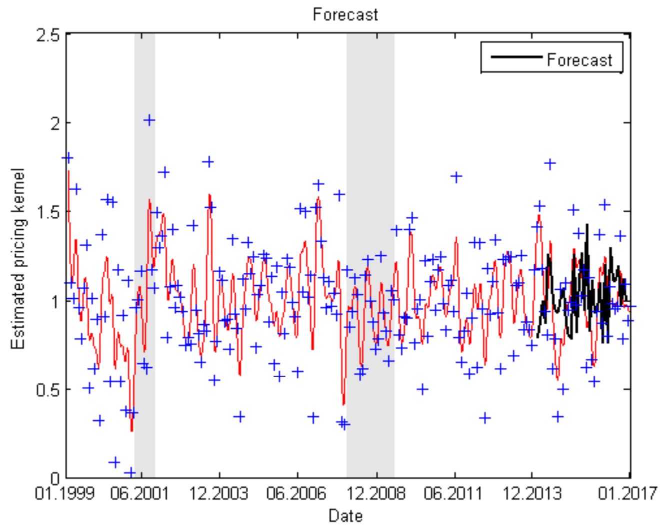

Figure 4: This figure displays the United States pricing kernels together with forecast responses (in black) of an ARMAX(p^*,q^*). The sample period spans from January 1999 to January 2017.

Table I: Descriptive statistics

	China	Eurozone	Japan	Russia	Switzerland	United Kingdom	United States
				Panel (a): Stocks			
Mean	17.35	2.66	6.35	12.43	7.10	1.31	12.24
Std. dev.	28.94	13.76	17.76	23.47	13.47	15.24	17.41
Min	-18.61	-14.65	-12.51	-19.74	-11.48	-16.52	-14.43
Max	19.92	10.77	15.64	19.19	9.64	13.95	21.91
Skewness	0.09	-0.65	0.04	-0.31	-0.34	-0.32	-0.28
Kurtosis	2.67	3.75	2.98	3.46	3.18	3.81	4.49
				Panel (b): Bonds			
Mean	-	-2.32	1.62	-2.75	2.61	-0.74	-1.40
Std. dev.	-	11.01	9.88	19.19	10.44	8.71	5.04
Min	-	-7.74	-5.75	-14.2	-7.50	-5.47	-5.28
Max	-	14.03	5.49	11.98	7.93	5.21	3.18
Skewness	-	0.29	-0.33	0.06	-0.38	-0.05	-0.66
Kurtosis	-	7.05	2.26	3.25	2.86	2.33	3.75
				Panel (c): Exchange rate returns with respect to U.S. dollar			
Mean	1.04	0.28	0.50	-3.79	2.39	-1.07	-
Std. dev.	1.90	10.50	9.90	13.80	11.10	9.20	-
Min	-2.43	-10.17	-8.70	-18.37	-13.60	-11.14	-
Max	2.10	11.00	6.85	10.92	13.03	10.20	-
Skewness	-0.05	-0.04	-0.19	-1.23	0.17	-0.30	-
Kurtosis	7.32	3.95	3.15	7.64	5.19	4.92	-
				Panel (d): Interest rate differentials relative to the U.S. interest rates			
Mean	1.73	-0.14	-1.93	6.20	-1.47	0.82	-
Std. dev.	0.70	0.36	0.61	1.34	0.40	0.31	-
Min	-2.57	-2.43	-5.48	-13.80	-37.80	-7.40	-
Max	5.63	1.84	1.20	2.140	1.00	27.90	-
Skewness	-0.26	-0.36	-0.71	0.67	-0.78	0.80	-
Kurtosis	2.36	2.25	2.00	3.86	2.17	2.46	-

The table reports descriptive statistics expressed in percentage: annualized means, annualized standard deviations, minimum, maximum, skewness and kurtosis in Panels (a), (b), (c) and (d) of respectively stock and bond returns, exchange and interest rates. Stock and bond returns are expressed in U.S. dollars. The sample period spans from January 1999 to January 2017. A dash (-) corresponds to unavailable and/or unused data.

Table II: P-values of univariate tests

	No stochastic dominance	Equal means	Equal variances	No differences in distributions
		p-values (05.11-05.12)		
China	0.98	0.98	0.57	0.83
Eurozone	0.46	0.96	0.02**	0.03**
Japan	0.94	0.99	0.01**	0.23
Russia	0.62	0.97	0.43	0.83
Switzerland	0.67	0.99	0.17	0.83
United Kingdom	0.12	0.98	0.17	0.03**
United States	0.86	0.96	0.01**	0.49
		p-values (06.12-06.13)		
China	0.98	0.96	0.75	0.99
Eurozone	0.66	0.99	0.01***	0.23
Japan	0.74	0.98	0.23	0.99
Russia	0.43	0.99	0.04**	0.03**
Switzerland	0.89	0.99	0.36	0.83
United Kingdom	0.46	0.98	0.02**	0.09*
United States	0.63	0.96	0.00***	0.09*
		p-values (07.13-07.14)		
China	0.66	0.96	0.41	0.49
Eurozone	0.49	0.98	0.03**	0.23
Japan	0.55	0.97	0.06**	0.23
Russia	0.49	0.99	0.67	0.49
Switzerland	0.70	0.99	0.27	0.83
United Kingdom	0.25	0.99	0.01**	0.01***
United States	0.43	0.97	0.03**	0.49
		p-values (08.14-08.15)		
China	0.89	0.92	0.14	0.82
Eurozone	0.66	0.99	0.66	0.49
Japan	0.86	0.99	0.02**	0.49
Russia	0.56	0.95	0.51	0.23
Switzerland	0.70	0.99	0.63	0.49
United Kingdom	0.08*	0.99	0.06*	0.01***
United States	0.46	0.95	0.01***	0.23
		p-values (09.15-09.16)		
China	0.82	0.99	0.92	0.83
Eurozone	0.98	0.99	0.02**	0.49
Japan	0.29	0.99	0.05**	0.03**
Russia	0.62	0.97	0.56	0.83
Switzerland	0.45	0.99	0.20	0.49
United Kingdom	0.32	0.99	0.01***	0.03**
United States	0.29	0.98	0.08*	0.49
		p-values (10.16-01.17)		
China	0.99	0.98	0.99	0.99
Eurozone	0.56	0.94	0.27	0.53
Japan	0.77	0.95	0.27	0.99
Russia	0.77	0.98	0.06**	0.53
Switzerland	0.77	0.94	0.51	0.99
United Kingdom	0.99	0.98	0.01***	0.53
United States	0.77	0.88	0.16	0.53

The table reports p-values for the Kruskal-Wallis (no stochastic dominance), Welch (equal means), Brown-Forsythe (equal variances) and Kolmogorov-Smirnov (no differences in distributions) tests on the pricing kernels estimated with equities and bonds denominated in local currency. The sample period spans from May 2011 to January 2017. We consider one year subsamples. The number of stocks is: 906 from China, 886 from the Eurozone, 456 from Japan, 147 from Russia, 152 from Switzerland, 469 from the United Kingdom and $1'330$ from the United States. We use 571 bonds from the Eurozone, 711 from Japan, 395 from Russia, 303 from Switzerland, 70 from the United Kingdom and $1'067$ from the United States. Note that China is a specific case in which only stock data are used in the tests due to the absence of bond data. A dash (-) indicates that matrices are singular and test results are not obtained. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***.

Table III: P-values of six months subsamples univariate tests

	No stochastic dominance	Equal means	Equal variances	No differences in distributions
		p-values (09.15-03.16)		
China	0.95	0.97	0.41	0.88
Eurozone	0.75	0.98	0.05*	0.42
Japan	0.34	0.99	0.17	0.13
Russia	0.57	0.97	0.47	0.88
Switzerland	0.56	0.97	0.07*	0.42
United Kingdom	0.22	0.99	0.19	0.13
United States	0.75	0.98	0.06*	0.42
		p-values (04.16-10.16)		
China	0.85	0.99	0.21	0.88
Eurozone	0.95	0.99	0.05**	0.42
Japan	0.48	0.99	0.36	0.42
Russia	0.85	0.99	0.44	0.88
Switzerland	0.95	0.96	0.16	0.42
United Kingdom	0.14	0.99	0.17	0.03*
United States	0.48	0.98	0.10	0.13

The table reports p-values for the Kruskal-Wallis (no stochastic dominance), Welch (equal means), Brown-Forsythe (equal variances) and Kolmogorov-Smirnov (no differences in distributions) tests on the pricing kernels estimated with both equities and bonds denominated in local currency. The sample period spans from September 2015 to Pctober 2016. The number of stocks varies among countries: 906 from China, 886 from the Eurozone, 456 from Japan, 147 from Russia, 152 from Switzerland, 469 from the United Kingdom and 1′330 from the United States. We also use 571 bonds from the Eurozone, 711 from Japan, 395 from Russia, 303 from Switzerland, 70 from the United Kingdom and 1′067 from the United States. Note that China is a specific case in which only stock data are used in the tests due to the absence of bond data. A dash (-) indicates that matrices are singular and test results are not obtained. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***.

Table IV: P-values of tests with stocks only

	China	Eurozone	Japan	Russia	Switzerland	United Kingdom
			No stochastic dominance			
p-value (01.99-07.01)	0.59	0.66	0.78	-	0.60	0.61
p-value (08.01-02.04)	0.97	0.96	0.78	-	0.72	0.80
p-value (03.04-09.06)	0.69	0.55	0.96	-	0.57	0.69
p-value (10.06-04.09)	0.46	0.84	0.82	-	0.94	0.85
p-value (05.09-11.11)	0.47	0.96	0.78	0.77	0.85	0.97
p-value (12.11-06.14)	0.96	0.82	0.67	0.45	0.42	0.52
p-value (07.14-01.17)	0.37	0.76	0.57	0.58	0.39	0.52
			Equal means			
p-value (01.99-07.01)	0.92	0.85	0.95	-	0.90	0.95
p-value (08.01-02.04)	0.93	0.92	0.97	-	0.90	0.83
p-value (03.04-09.06)	0.99	0.92	0.97	-	0.99	0.89
p-value (10.06-04.09)	0.67	0.91	0.98	-	0.95	0.97
p-value (05.09-11.11)	0.97	0.96	0.94	0.94	0.89	0.93
p-value (12.11-06.14)	0.90	0.98	0.96	0.92	0.97	0.96
p-value (07.14-01.17)	0.91	0.99	0.95	0.94	0.99	0.90
			Equal variances			
p-value (01.99-07.01)	0.06*	0.27	0.83	-	0.00***	0.00***
p-value (08.01-02.04)	0.07*	0.03**	0.01***	-	0.00***	0.04**
p-value (03.04-09.06)	0.05*	0.16	0.01***	-	0.00***	0.00***
p-value (10.06-04.09)	0.79	0.17	0.04**	-	0.00***	0.31
p-value (05.09-11.11)	0.09*	0.45	0.05**	0.00***	0.00***	0.02**
p-value (12.11-06.14)	0.30	0.04**	0.02**	0.00***	0.00***	0.00***
p-value (07.14-01.17)	0.00***	0.12	0.05**	0.00***	0.00***	0.03**
			No differences in distributions			
p-value (01.99-07.01)	0.56	0.78	0.78	-	0.03**	0.36
p-value (08.01-02.04)	0.78	0.22	0.12	-	0.03**	0.22
p-value (03.04-09.06)	0.78	0.36	0.36	-	0.12	0.56
p-value (10.06-04.09)	0.56	0.78	0.36	-	0.21	0.78
p-value (05.09-11.11)	0.12	0.78	0.12	0.12	0.03**	0.22
p-value (12.11-06.14)	0.94	0.36	0.78	0.01**	0.03**	0.22
p-value (07.14-01.17)	0.12	0.78	0.37	0.06*	0.03**	0.36

The table reports p-values for the Kruskal-Wallis (no stochastic dominance), Welch (equal means), Brown-Forsythe (equal variances) and Kolmogorov-Smirnov (no differences in distributions) tests on the pricing kernels estimated with equities only. The sample period spans from January 1999 to January 2017. The number of stocks vary among countries: 906 stocks are from China, 886 from the Eurozone, 456 from Japan, 147 from Russia, 152 from Switzerland, 469 from the United Kingdom and 1′330 from the United States. A dash (-) indicates that matrices are singular and test results are not obtained. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***.

Table V: P-values of tests with bonds only

	Eurozone	Japan	Russia	Switzerland	United Kingdom
			No stochastic dominance		
p-value (03.09-09.10)	0.65	0.83	-	0.53	0.34
p-value (10.10-04.12)	0.42	0.94	0.65	0.83	-
p-value (05.12-11.13)	0.90	0.67	0.78	0.97	0.31
p-value (12.13-06.15)	0.90	0.87	0.26	0.97	-
p-value (07.15-01.17)	0.56	0.34	0.59	0.76	0.11
			Equal means		
p-value (03.09-09.10)	0.96	0.99	-	0.99	0.98
p-value (10.10-04.12)	0.94	0.97	0.94	0.98	-
p-value (05.12-11.13)	0.99	0.92	0.90	0.97	0.96
p-value (12.13-06.15)	0.93	0.95	0.95	0.94	-
p-value (07.15-01.17)	0.98	0.94	0.63	0.98	0.95
			Equal variances		
p-value (03.09-09.10)	0.00***	0.04**	-	0.06*	0.49
p-value (10.10-04.12)	0.00***	0.00***	0.00***	0.06*	-
p-value (05.12-11.13)	0.78	0.96	0.76	0.40	0.02**
p-value (12.13-06.15)	0.22	0.00***	0.08*	0.07*	-
p-value (07.15-01.17)	0.66	0.00***	0.19	0.27	0.02**
			No differences in distributions		
p-value (03.09-09.10)	0.24	0.46	-	0.46	0.46
p-value (10.10-04.12)	0.12	0.46	0.25	0.74	-
p-value (05.12-11.13)	0.96	0.96	0.75	0.74	0.24
p-value (12.13-06.15)	0.74	0.25	0.05**	0.46	-
p-value (07.15-01.17)	0.46	0.02**	0.46	0.96	0.00***

The table reports p-values for the Kruskal-Wallis (no stochastic dominance), Welch (equal means), Brown-Forsythe (equal variances) and Kolmogorov-Smirnov (no differences in distributions) tests of the pricing kernels estimated with bonds only. The sample period spans from March 2009 to January 2017. We use, in total, 571, 711, 395, 303, 70 and $1'067$ bonds from respectively the Eurozone, Japan, Russia, Switzerland, the United Kingdom, and the United States. A dash (-) indicates that matrices are singular and test results are not obtained. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***.

Table VI: P-values of tests with stocks and bonds

	China	Eurozone	Japan	Russia	Switzerland	United Kingdom
			No stochastic dominance			
p-value (01.99-07.01)	0.58	0.70	0.73	-	0.60	0.62
p-value (08.01-02.04)	0.96	0.93	0.81	-	0.83	0.73
p-value (03.04-09.06)	0.71	0.63	0.99	-	0.37	0.58
p-value (10.06-04.09)	0.67	0.92	0.88	-	0.86	0.92
p-value (05.09-11.11)	0.43	0.82	0.74	0.76	0.86	0.92
p-value (12.11-06.14)	0.73	0.99	0.98	0.69	0.61	0.81
p-value (07.14-01.17)	0.61	0.73	0.65	0.963	0.71	0.65
			Equal means			
p-value (01.99-07.01)	0.91	0.87	0.98	-	0.93	0.95
p-value (08.01-02.04)	0.94	0.92	0.98	-	0.87	0.85
p-value (03.04-09.06)	0.99	0.93	0.96	-	0.98	0.91
p-value (10.06-04.09)	0.74	0.95	0.98	-	0.98	0.96
p-value (05.09-11.11)	0.98	0.95	0.94	0.93	0.93	0.93
p-value (12.11-06.14)	0.88	0.99	0.96	0.98	0.99	0.96
p-value (07.14-01.17)	0.93	0.99	0.98	0.81	0.99	0.89
			Equal variances			
p-value (01.99-07.01)	0.08*	0.35	0.94	-	0.00***	0.00***
p-value (08.01-02.04)	0.09*	0.04**	0.01***	-	0.00***	0.06*
p-value (03.04-09.06)	0.18	0.50	0.02**	-	0.00***	0.06*
p-value (10.06-04.09)	0.41	0.84	0.43	-	0.00***	0.67
p-value (05.09-11.11)	0.28	0.62	0.17	0.00***	0.00***	0.06*
p-value (12.11-06.14)	0.20	0.91	0.87	0.03**	0.00***	0.08*
p-value (07.14-01.17)	0.47	0.62	0.77	0.10	0.00***	0.61
			No differences in distributions			
p-value (01.99-07.01)	0.36	0.78	0.78	-	0.21	0.36
p-value (08.01-02.04)	0.77	0.36	0.22	-	0.06*	0.56
p-value (03.04-09.06)	0.94	0.56	0.36	-	0.06*	0.36
p-value (10.06-04.09)	0.78	0.78	0.78	-	0.36	0.78
p-value (05.09-11.11)	0.12	0.94	0.22	0.22	0.06*	0.36
p-value (12.11-06.14)	0.78	0.94	0.94	0.36	0.12	0.78
p-value (07.14-01.17)	0.78	0.78	0.56	0.78	0.36	0.78

The table reports p-values for the Kruskal-Wallis (no stochastic dominance), Welch (equal means), Brown-Forsythe (equal variances) and Kolmogorov-Smirnov (no differences in distributions) tests on the pricing kernels estimated with assets denominated in local currency. The sample period spans from January 1999 to January 2017. Respectively, the total number of assets is 906 for China, $1'457$ for the Eurozone, $1'167$ for Japan, 542 for Russia, 455 for Switzerland, 539 for the United Kingdom and $2'397$ for the United States. A dash (-) indicates that matrices are singular. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***.

Table VII: P-values of tests with foreign stocks and bonds, domestic bonds only

	China	Eurozone	Japan	Russia	Switzerland	United Kingdom
No stochastic dominance						
p-value (01.99-07.01)	0.58	0.51	0.41	-	0.89	0.62
p-value (08.01-02.04)	0.87	0.76	0.83	-	0.83	0.83
p-value (03.04-09.06)	0.90	0.86	0.98	-	0.49	0.81
p-value (10.06-04.09)	0.13	0.09*	0.12	0.75	0.25	0.11
p-value (05.09-11.11)	0.63	0.49	0.49	0.33	0.74	0.59
p-value (12.11-06.14)	0.74	0.81	0.69	0.63	0.95	0.87
p-value (07.14-01.17)	0.86	0.86	0.84	0.98	0.71	0.68
Equal means						
p-value (01.99-07.01)	0.95	0.94	0.99	-	0.95	0.97
p-value (08.01-02.04)	0.98	0.92	0.99	-	0.88	0.88
p-value (03.04-09.06)	0.69	0.95	0.98	-	0.98	0.93
p-value (10.06-04.09)	0.91	0.99	0.97	0.71	0.99	0.96
p-value (05.09-11.11)	0.97	0.98	0.95	0.99	0.93	0.96
p-value (12.11-06.14)	0.94	0.99	0.98	0.96	0.99	0.97
p-value (07.14-01.17)	0.96	0.97	0.99	0.81	0.98	0.89
Equal variances						
p-value (01.99-07.01)	0.00***	0.00***	0.00***	-	0.19	0.04**
p-value (08.01-02.04)	0.03**	0.04**	0.11	-	0.68	0.04**
p-value (03.04-09.06)	0.18	0.04**	0.46	-	0.16	0.38
p-value (10.06-04.09)	0.00***	0.00***	0.02**	0.68	0.22	0.02**
p-value (05.09-11.11)	0.01**	0.00***	0.02**	0.38	0.94	0.11
p-value (12.11-06.14)	0.00***	0.00***	0.00***	0.03**	0.78	0.03**
p-value (07.14-01.17)	0.49	0.06*	0.57	0.27	0.67	0.44
No differences in distributions						
p-value (01.99-07.01)	0.06*	0.01**	0.01**	-	0.56	0.56
p-value (08.01-02.04)	0.36	0.78	0.78	-	0.56	0.78
p-value (03.04-09.06)	0.78	0.36	0.78	-	0.36	0.94
p-value (10.06-04.09)	0.00***	0.01**	0.01**	0.22	0.22	0.00***
p-value (05.09-11.11)	0.12	0.22	0.78	0.56	0.94	0.56
p-value (12.11-06.14)	0.12	0.12	0.67	0.78	0.78	0.56
p-value (07.14-01.17)	0.78	0.36	0.57	0.78	0.36	0.22

The table reports p-values for the Kruskal-Wallis (no stochastic dominance), Welch (equal means), Brown-Forsythe (equal variances) and Kolmogorov-Smirnov (no differences in distributions) tests of the pricing kernels estimated with assets denominated in local currency. The pricing kernel for the domestic country is estimated using bonds only. The sample period spans from January 1999 to January 2017. In total, a number of 906 stocks from China, 886 from the Eurozone, 456 from Japan, 147 from Russia, 152 from Switzerland and 469 from the United Kingdom are used. We also gather 571 bonds from the Eurozone, 711 from Japan, 395 from Russia, 303 from Switzerland, 70 from the United Kingdom and 1′067 from the United States. A dash (-) indicates singular matrices. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***.

Table VIII: P-values of tests with Eurozone assets

	No stochastic dominance	Equal means	Equal variances	No differences in distributions
p-value (01-99-01.17)	0.66	0.98	0.27	0.58

The table reports p-values for the Kruskal-Wallis (no stochastic dominance), Welch (equal means), Brown-Forsythe (equal variances) and Kolmogorov-Smirnov (no differences in distributions) tests on the estimated pricing. The sample period spans from January 1999 to January 2017. We consider stocks and bonds from United States and from the Eurozone. In total, we have for the Eurozone and the United States, respectively, $1'457$ and $2'397$ assets. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***.

Table IX: Descriptive statistics

	China	Eurozone	Japan	Switzerland	United Kingdom	United States
Mean	0.990	0.999	1.000	0.998	1.000	1.000
Std. dev.	0.524	0.416	0.557	0.749	0.576	0.332
Min	-0.361	-0.272	-0.344	-1.718	-1.321	0.032
Max	3.414	2.283	2.586	3.125	3.042	2.011
Skewness	0.836	-0.050	0.437	0.150	0.077	0.028
Kurtosis	5.038	3.088	2.805	3.946	4.157	3.306
p-value Jarque-Bera	0.001	0.500	0.033	0.021	0.010	0.500
p-value Ljung-Box	0.064	0.230	0.032	0.011	0.085	0.001
p-value augmented Dickey-Fuller	0.001	0.001	0.001	0.001	0.001	0.001

The table reports descriptive statistics of the estimated pricing kernels; means, standard deviations, minimum, maximum, skewness, kurtosis, Jarque-Bera, Ljung-Box and augmented Dickey-Fuller p-values. Pricing kernels are expressed in local currency and are estimated with one year and a half subsamples. The sample period spans from January 1999 to January 2017.

Table X: Correlation between pricing kernel conditional volatility and interest rate differentials

China	Eurozone	Japan	Switzerland	United Kingdom
-0.15*	-0.00	-0.02	-0.34***	-0.22***

The table reports the correlation between pricing kernel conditional volatility and interest rate differentials. Conditional variances are estimated with a GARCH with Student's t-distribution. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***. The sample period is from January 1999 to January 2017.

Table XI: Correlation between pricing kernels approximated conditional variance (as in Gavazzoni et al. (2013)) and interest rate differentials

China	Eurozone	Japan	Switzerland	United Kingdom
0.91***	0.94***	0.95***	0.89***	0.92***

The table reports the correlation between pricing kernels conditional volatility and interest rate differentials. Conditional variances are estimated with a GARCH. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***. The sample period spans from January 1999 to January 2017.

Table XII: Risk Sharing Index

	China	Eurozone	Japan	Switzerland	United Kingdom	United States
China	1.00					
Eurozone	0.06**	1.00				
Japan	0.07**	0.15**	1.00			
Switzerland	0.19**	0.25**	-0.04**	1.00		
United Kingdom	0.08**	0.31**	0.12**	0.24**	1.00	
United States	0.06**	0.28**	0.08**	0.12**	0.29**	1.00

The table reports international risk sharing indexes. Pricing kernels are estimated using market data (equities and bonds). The sample period is from January 1999 to January 2017. The statistical significance at level 5% is obtained with Equation (9) and Equation (10) and is indicated by **.

Table XIII: Monte Carlo experiment from normal distributions

	N = 20			N = 50			N = 100			N = 1'000		
	Mean	Std. dev.	p-value	Mean	Std. dev.	p-value	Mean	Std. dev.	p-value	Mean	Std. dev.	p-value
						Specification n°1						
$\hat{\rho}_{IRSI}$	0.945	0.027	<0.01	0.948	0.015	<0.01	0.949	0.010	<0.01	0.950	0.003	<0.01
$S_{\hat{\rho}_{IRSI}}$		0.027			0.015			0.010			0.003	
$\hat{Z}$	1.831	0.236	0.06	1.832	0.144	>0.10	1.832	0.101	>0.10	1.832	0.032	>0.10
$S_{\hat{Z}}$		0.229			0.143			0.101			0.032	
						Specification n°2						
$\hat{\rho}_{IRSI}$	0.926	0.030	<0.01	0.929	0.017	<0.01	0.930	0.012	<0.01	0.931	0.004	<0.01
$S_{\hat{\rho}_{IRSI}}$		0.030			0.017			0.012			0.004	
$\hat{Z}$	1.667	0.200	<0.01	1.666	0.121	>0.10	1.667	0.085	>0.10	1.667	0.027	>0.10
$S_{\hat{Z}}$		0.196			0.121			0.085			0.027	
						Specification n°3						
$\hat{\rho}_{IRSI}$	0.770	0.092	<0.01	0.779	0.054	<0.01	0.781	0.037	<0.01	0.784	0.012	<0.01
$S_{\hat{\rho}_c}$		0.092			0.054			0.037			0.012	
$\hat{Z}$	1.059	0.224	0.03	1.056	0.136	>0.10	1.055	0.095	>0.10	1.056	0.030	>0.10
$S_{\hat{Z}}$		0.218			0.136			0.095			0.030	
						Specification n°4						
$\hat{\rho}_{IRSI}$	0.388	0.149	<0.01	0.396	0.091	<0.01	0.398	0.064	<0.01	0.400	0.020	0.02
$S_{\hat{\rho}_{IRSI}}$		0.148			0.091			0.064			0.020	
$\hat{Z}$	0.422	0.179	<0.01	0.424	0.109	>0.10	0.423	0.076	>0.10	0.423	0.024	>0.10
$S_{\hat{Z}}$		0.177			0.109			0.076			0.024	

The table reports mean, standard deviation and p-value for normality test for five specific values of the international risk sharing index. Results are based on $20'000$ runs generated from a bivariate normal distribution. The table also provides counterparts, $S_{\hat{Z}}$ and $S_{\hat{\rho}_{IRSI}}$, for the standard deviation of $\hat{Z}$ and $\hat{\rho}_{IRSI}$ computed respectively with Equation (4.4) and Equation (8).

Table XIV: Monte Carlo experiment from uniform distributions

	N = 20			N = 50			N = 100			N = 1'000		
	Mean	Std. dev.	p-value	Mean	Std. dev.	p-value	Mean	Std. dev.	p-value	Mean	Std. dev.	p-value
						Specification n°1						
$\hat{\rho}_{IRSI}$	0.947	0.027	<0.01	0.949	0.016	<0.01	0.949	0.011	<0.01	0.950	0.003	<0.01
$S_{\hat{\rho}_{IRSI}}$		0.027			0.015			0.010			0.003	
$\hat{Z}$	1.859	0.259	<0.01	1.843	0.157	<0.01	1.837	0.110	<0.01	1.832	0.035	>0.10
$S_{\hat{Z}}$		0.232			0.144			0.101			0.032	
						Specification n°2						
$\hat{\rho}_{IRSI}$	0.936	0.033	<0.01	0.939	0.019	<0.01	0.940	0.013	<0.01	0.940	0.004	<0.01
$S_{\hat{\rho}_{IRSI}}$		0.032			0.018			0.012			0.004	
$\hat{Z}$	1.767	0.259	<0.01	1.752	0.157	<0.01	1.749	0.110	<0.01	1.743	0.035	<0.01
$S_{\hat{Z}}$		0.232			0.144			0.101			0.032	
						Specification n°3						
$\hat{\rho}_{IRSI}$	0.783	0.096	<0.01	0.788	0.057	<0.01	0.791	0.040	<0.01	0.792	0.013	<0.01
$S_{\hat{\rho}_{IRSI}}$		0.094			0.056			0.038			0.012	
$\hat{Z}$	1.100	0.251	<0.01	1.084	0.153	<0.01	1.082	0.107	0.09	1.077	0.034	>0.10
$S_{\hat{Z}}$		0.232			0.144			0.101			0.032	
						Specification n°4						
$\hat{\rho}_{IRSI}$	0.435	0.188	<0.01	0.441	0.117	<0.01	0.441	0.081	<0.01	0.444	0.026	<0.01
$S_{\hat{\rho}_{IRSI}}$		0.184			0.115			0.081			0.025	
$\hat{Z}$	0.490	0.243	<0.01	0.483	0.148	<0.01	0.479	0.102	>0.05	0.478	0.032	>0.05
$S_{\hat{Z}}$		0.233			0.144			0.101			0.032	

The table reports mean, standard deviation and p-value for normality test for five specific values of the international risk sharing index. Results are based on $20'000$ runs generated from bivariate uniform distribution. The table also provides counterparts, $S_{\hat{Z}}$ and $S_{\hat{\rho}_{IRSI}}$, for the standard deviation of $\hat{Z}$ and $\hat{\rho}_{IRSI}$ computed with respectively Equation (4.4) and Equation (8).

Table XV: Monte Carlo experiment from Gamma distributions

	N = 20			N = 50			N = 100			N = 1'000		
	Mean	Std. dev.	p-value	Mean	Std. dev.	p-value	Mean	Std. dev.	p-value	Mean	Std. dev.	p-value
						Specification n°1						
$\hat{\rho}_{IRSI}$	0.943	0.029	<0.01	0.946	0.016	<0.01	0.948	0.011	<0.01	0.949	0.003	<0.01
$S_{\hat{\rho}_{IRSI}}$		0.028			0.015			0.010			0.003	
$\hat{Z}$	1.817	0.247	<0.01	1.818	0.154	<0.01	1.818	0.109	>0.01	1.819	0.034	>0.10
$S_{\hat{Z}}$		0.228			0.142			0.100			0.032	
						Specification n°2						
$\hat{\rho}_{IRSI}$	0.934	0.030	<0.01	0.937	0.017	<0.01	0.938	0.012	<0.01	0.939	0.004	<0.01
$S_{\hat{\rho}_{IRSI}}$		0.029			0.017			0.011			0.003	
$\hat{Z}$	1.737	0.230	<0.01	1.732	0.142	<0.01	1.732	0.100	<0.01	1.731	0.031	>0.10
$S_{\hat{Z}}$		0.212			0.132			0.092			0.029	
						Specification n°3						
$\hat{\rho}_{IRSI}$	0.749	0.099	<0.01	0.759	0.059	<0.01	0.762	0.041	<0.01	0.764	0.013	<0.01
$S_{\hat{\rho}_{IRSI}}$		0.094			0.056			0.038			0.012	
$\hat{Z}$	1.007	0.227	<0.01	1.008	0.140	<0.01	1.008	0.098	<0.01	1.008	0.031	>0.10
$S_{\hat{Z}}$		0.209			0.129			0.091			0.029	
						Specification n°4						
$\hat{\rho}_{IRSI}$	0.419	0.169	<0.01	0.428	0.107	<0.01	0.432	0.075	<0.01	0.436	0.024	<0.01
$S_{\hat{\rho}_{IRSI}}$		0.162			0.102			0.072			0.023	
$\hat{Z}$	0.465	0.214	<0.01	0.465	0.133	<0.01	0.466	0.092	<0.01	0.467	0.029	>0.10
$S_{\hat{Z}}$		0.201			0.126			0.089			0.028	

The table reports mean, standard deviation and p-value for normality test for five specific values of the international risk sharing index. Results are based on $20'000$ runs generated from a bivariate Gamma distribution. The table also provides counterparts,$S_{\hat{Z}}$ and $S_{\hat{\rho}_{IRSI}}$, for the standard deviation of $\hat{Z}$ and $\hat{\rho}_{IRSI}$ computed with respectively Equation (4.4) and Equation (8).

Table XVI: Risk factors correlation

Rm-Rf	SMB	HML	RMW	CMA
0.25***	0.10	-0.07	-0.22***	-0.06

The table reports the correlation between the United States estimated pricing kernels and the Fama and French (2016) factors. Starting with the market excess return ($R_m - R_f$), the Small minus Big (SMB), the High minus Low (HML), the Robust minus Weak (RMW) and the Conservative minus Aggressive (CMA) portfolios. The sample period spans from January 1999 to January 2017. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***.

Table XVII: Factor betas

	α	Rm-Rf	SMB	HML	RMW	CMA	VOL	R^2
$\hat{m}$	-0.10***	2.07**	-0.19	-0.10	-2.06	0.91	-2.68	
	(0.04)	(0.90)	(1.10)	(1.69)	(1.52)	(2.89)	(2.45)	0.05

The table reports results of Newey-West time-series regression of the logarithm of the United States pricing kernels on a constant (α), on the market excess return ($R_m - R_f$), the Small minus Big (SMB), the High minus Low (HML), the Robust minus Weak (RMW) and the Conservative minus Aggressive (CMA) portfolios and on the VOL factor. The latter is estimated by the innovations in global FX volatility obtained with an autoregressive model of order 1. Newey-West with optimal lags standard errors are reported in parentheses. The sample period spans from January 1999 to January 2017. Statistical significance at the 10%, 5% and 1% level is indicated by *, **, and ***. Reported R-squared are adjusted R-squared.

Table XVIII: Forecast errors

	$ARMA(1,1)$	ARMAX(1,1)	ARMA(p^*,q^*)	ARMAX(p^*,q^*)
MSE	4.48	4.09	3.09	2.63

The table reports sum of mean square errors of forecasts responses to the corresponding models. The mode for the ARMA(p^*,q^*) is an ARMA(7,1) while for the ARMAX(p^*,q^*), it is an ARMAX(9,3). The sample period spans from January 1999 to January 2017.

A Appendix

A.1 Derivations of the currency risk premium

In this section, we remind the reader how to derive the exchange risk premium. Assuming that the logarithm of the pricing kernels of the domestic and the foreign investors (m_{t+1}, m^*_{t+1}) are conditionally normally distributed with means (μ_t, μ^*_t) and variances (σ_t^2, σ_t^{2*}).[24] The first moments of the domestic and foreign pricing kernels are thus given by

$$\begin{aligned}\mathbb{E}_t(m_{t+1}) &= \exp(\mu_t + \frac{\sigma_t^2}{2}),\\ \mathbb{E}_t(m^*_{t+1}) &= \exp(\mu^*_t + \frac{\sigma_t^{*2}}{2}).\end{aligned}$$

Taking the ratio of two pricing kernels equals the exchange rate between these two economies:

$$m^*_{t+1} = m_{t+1}\frac{S_{t+1}}{S_t} \tag{10}$$

$$\log(m^*_{t+1}) = \log(m_{t+1}) + s_{t+1} - s_t. \tag{11}$$

The logarithm of the covered interest rate parity condition states that $f_t - s_t = i_t - i^*_t$ and by the Euler equation, one has that $i^*_t = -\log \mathbb{E}_t m^*_{t+1}$ and $i_t = -\log \mathbb{E}_t m_{t+1}$. Therefore,

$$f_t - s_t = -\log \mathbb{E}_t m_{t+1} + \log \mathbb{E}_t m^*_{t+1}.$$

Take the time-t conditional expectation of Equation (11) and subtract from it $f_t - s_t$, together with the first and second moments of the lognormal distribution of

[24]See as well Backus et al. (2013).

pricing kernels, we have

$$\begin{aligned}\mathbb{E}_t s_{t+1} - f_t &= (\log \mathbb{E}_t m_{t+1} - \mathbb{E}_t \log m_{t+1}) - (\log \mathbb{E}_t m^*_{t+1} - \mathbb{E}_t \log m^*_{t+1}) \\ \mathbb{E}_t s_{t+1} - f_t &= \frac{\sigma_t^2 - \sigma_t^{*2}}{2}.\end{aligned}$$

Hence, the currency risk premium is equal to

$$\mathbb{E}_t s_{t+1} - f_t = \frac{1}{2}(Var_t \log(m_{t+1}) - Var_t \log(m^*_{t+1})).$$

A.2 International risk sharing index: Confidence interval

In this section, we present derivations for the closed-form solution of the international risk sharing variance.

Let Y_1 and Y_2 be two jointly normal random distributions of dimension N. The sample international risk sharing index $\hat{\rho}_{IRSI}$ is given by

$$\hat{\rho}_{IRSI} = \frac{2S_{12}}{S_1^2 + S_2^2},$$

with $-1 \leq \rho_{IRSI} \leq 1$, $\rho_{IRSI} = 0$ if $\rho = 0$ and $\rho_{IRSI} = \rho$ if $\sigma_1 = \sigma_2$, where ρ is Pearson coefficient of correlation. The Fisher-Z transformation of $\hat{\rho}_{IRSI}$ is equal to

$$\hat{Z} = \tanh^{-1}(\hat{\rho}_{IRSI}) = \frac{1}{2}\ln\frac{1+\hat{\rho}_{IRSI}}{1-\hat{\rho}_{IRSI}},$$

which can be expressed in terms of $\nu = (v_1, v_2, v_3, v_4, v_5) = (Y_1, Y_2, Y_1^2, Y_2^2, Y_1Y_2)$, and thus rewritten as

$$\hat{Z} = g(\nu) = \frac{1}{2}\ln\left[\frac{v_3 - v_1^2 + v_4 - v_2^2 + 2v_5 - 2v_1v_2}{v_3 - v_1^2 + v_4 - v_2^2 - 2v_5 + 2v_1v_2}\right].$$

As the vector ν is a function of sample moments, its mean is $\mathbb{E}(\nu) = (\mu_1, \mu_2, \sigma_1^2 +$

$\mu_1^2, \sigma_2^2 + \mu_2^2, \sigma_{12} + \mu_1\mu_2)$ and its variance is $N^{-1}\Sigma$, where $\Sigma = [W_{ij}]_{5\times 5}$ with

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & 2\mu_1\sigma_1^2 & 2\mu_2\sigma_{12} & \mu_2\sigma_1^2 + \mu_1\sigma_{12} \\ \sigma_{12} & \sigma_2^2 & 2\mu_1\sigma_{12} & 2\mu_2\sigma_2^2 & \mu_1\sigma_2^2 + \mu_2\sigma_{12} \\ 2\mu_1\sigma_1^2 & 2\mu_1\sigma_{12} & 2\sigma_1^4 + 4\sigma_1^2\mu_1^2 & 2\sigma_{12}^2 + 4\mu_1\mu_2\sigma_{12} & 2\sigma_{12}\mu_1^2 + 2\sigma_{12}\sigma_1^2 + 2\mu_1\mu_2\sigma_1^2 \\ 2\mu_2\sigma_{12} & 2\mu_2\sigma_2^2 & 2\sigma_{12}^2 + 4\mu_1\mu_2\sigma_{12} & 2\sigma_2^2 + 2\sigma_2^2\mu_2^2 & 2\sigma_{12}\mu_2^2 + 2\sigma_{12}\sigma_2^2 + 2\mu_1\mu_2\sigma_2^2 \\ \mu_2\sigma_1^2 + \mu_1\sigma_{12} & \mu_1\sigma_2^2 + \mu_2\sigma_{12} & 2\sigma_{12}\mu_1^2 + 2\sigma_{12}\sigma_1^2 + 2\mu_1\mu_2\sigma_1^2 & 2\sigma_{12}\mu_2^2 + 2\sigma_{12}\sigma_2^2 + 2\mu_1\mu_2\sigma_2^2 & \sigma_1^2\sigma_2^2 + \mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2 + \sigma_{12}^2 + 2\mu_1\mu_2\sigma_{12} \end{pmatrix}.$$

Following the Delta method, $g(\nu)$ is asymptotically normally distributed

$$\sqrt{N}\left(g(\nu) - Z\right) \sim \mathcal{N}\left(0, \nabla g(\nu)^T \Sigma \nabla g(\nu)\right)$$

where $\nabla g(\nu) = \begin{pmatrix} \frac{\partial g}{\partial v_1} \\ \frac{\partial g}{\partial v_2} \\ \frac{\partial g}{\partial v_3} \\ \frac{\partial g}{\partial v_4} \\ \frac{\partial g}{\partial v_5} \end{pmatrix}$ are nonzero elements which after calculation are equal to

$$\begin{aligned} \left.\frac{\partial g}{\partial v_1}\right|_{\nu=\mathbb{E}(\nu)} &= \frac{-2\mu_2(\sigma_1^2+\sigma_2^2) + 4\sigma_{12}\mu_1}{(\sigma_1^2+\sigma_2^2+2\sigma_{12})(\sigma_1^2+\sigma_2^2-2\sigma_{12})} \\ \left.\frac{\partial g}{\partial v_2}\right|_{\nu=\mathbb{E}(\nu)} &= \frac{-2\mu_1(\sigma_1^2+\sigma_2^2) + 4\sigma_{12}\mu_2}{(\sigma_1^2+\sigma_2^2+2\sigma_{12})(\sigma_1^2+\sigma_2^2-2\sigma_{12})} \\ \left.\frac{\partial g}{\partial v_3}\right|_{\nu=\mathbb{E}(\nu)} &= \frac{-2\sigma_{12}}{(\sigma_1^2+\sigma_2^2+2\sigma_{12})(\sigma_1^2+\sigma_2^2-2\sigma_{12})} \\ \left.\frac{\partial g}{\partial v_4}\right|_{\nu=\mathbb{E}(\nu)} &= \frac{-2\sigma_{12}}{(\sigma_1^2+\sigma_2^2+2\sigma_{12})(\sigma_1^2+\sigma_2^2-2\sigma_{12})} \\ \left.\frac{\partial g}{\partial v_5}\right|_{\nu=\mathbb{E}(\nu)} &= \frac{2(\sigma_1^2+\sigma_2^2)}{(\sigma_1^2+\sigma_2^2+2\sigma_{12})(\sigma_1^2+\sigma_2^2-2\sigma_{12})}. \end{aligned}$$

Therefore, the variance of $\sigma^2_{\hat{Z}} = \nabla g(\nu)^T \Sigma \nabla g(\nu)$ equals

$$\sigma^2_{\hat{Z}} = \frac{1}{N}\left[\frac{(1+\rho^2)}{\rho^2(1-\rho^2_{IRSI})} - \frac{1-\rho^4_{IRSI}}{(1-\rho^2_{IRSI})^2} - \frac{1}{\rho^2}\right].$$

The asymptotic variance of $\hat{\rho}_{IRSI}$ given by the Delta method is equal to

$$
\begin{aligned}
\sigma^2_{\hat{\rho}_{IRSI}} &= \sigma^2_{\hat{Z}}(\tanh(\hat{Z})')^2 \\
&= \sigma^2_{\hat{Z}}(1-\rho^2_{IRSI})^2 \\
&= \frac{1}{N}\left[\frac{(1-\rho^2_{IRSI})(\rho^2+\rho^2_{IRSI})}{\rho^2} - (1-\rho^4_{IRSI})\right].
\end{aligned}
$$

Note that it is possible to replace N by $N-2$ to reduce finite-sample bias.

In order to verify previous derivations, we use Monte Carlo simulations for different parameter specifications. They consist of five values of ρ_{IRSI} with sample size: $N = 20$, $N = 50$, $N = 100$ and $N = 1'000$. For each specification, $20'000$ simulations are generated.

Specification n°1. Distributions of mean $(0,0)$ and covariance $\begin{pmatrix} 1 & 0.95 \\ 0.95 & 1 \end{pmatrix}$

with no difference in location and scale parameters and high Pearson correlation coefficient.

Specification n°2. Distributions of mean $(-\frac{\sqrt{0.1}}{2}, \frac{\sqrt{0.1}}{2})$ and covariance $\begin{pmatrix} 1.21 & 0.94 \\ 0.94 & 0.81 \end{pmatrix}$

with different location and scale parameters but high Pearson correlation coefficient.

Specification n°3. Distributions of mean $(-\frac{\sqrt{0.1}}{2}, \frac{\sqrt{0.1}}{2})$ and covariance $\begin{pmatrix} 0.81 & 0.79 \\ 0.79 & 1.21 \end{pmatrix}$

with different location and scale parameters and lower correlation.

Specification n°4. Distributions of mean $(-\frac{\sqrt{0.25}}{2}, \frac{\sqrt{0.25}}{2})$ and covariance $\begin{pmatrix} 1.78 & 0.44 \\ 0.44 & 0.44 \end{pmatrix}$

with an additional change: the Pearson correlation coefficient is set to 0.