The Beta Neutral Model with Leverage E ect

Sebastien Valeyre* Denis Grebenkov† So ane Aboura‡

May 19, 2017

Abstract

We present a beta neutral model that includes the leverage e ect to allow hedge fund managers to target a near-zero beta for market neutral strategies. For this purpose, we derive a metric of correlation with leverage e ect to identify the ne relation between the market beta and volatility changes. An empirical test based on the most popular market neutral strategies is run from 2000 to 2015 with exhaustive data sets including 600 American stocks and 600 European stocks from the S&P 500, Nasdaq 100, and Euro Stoxx 600. Our ndings con rm the ability of the beta neutral model to withdraw an important part of the bias from the market neutral strategies.

Key words: Beta, Correlation, Volatility, Portfolio Management, Market Neutral Strategies.
JEL classi cation: C5, G01, G11, G12, G32.

*John Locke Investments, 38 Avenue Franklin Roosevelt, 77210 Fontainebleau-Avon, France, and Université de Paris XIII, Sorbonne Paris Cité, 93430 Villetaneuse, France

†Laboratoire de Physique de la Matière Condensée (UMR 7643), CNRS Ecole Polytechnique, University Paris-Saclay, 91128 Palaiseau, France

‡Université de Paris XIII, Sorbonne Paris Cité, 93430 Villetaneuse, France

1 Introduction

We propose a new measure of market beta that serves for portfolio hedging in market neutral strategies. The correct measurement of market betas is paramount for market neutral hedge fund managers who target a near-zero beta. Counter to common belief, perfect beta neutral strategies are difficult to achieve in practice, as the mortgage crisis in 2008 exemplified, when most market neutral funds remained correlated with stock markets and experienced considerable unexpected losses. This exposure to the stock index (Banz, 1981; Fama and French, 1992, 1993; Carhart, 1997; Ang et al., 2006) is even stronger during down market conditions (see, e.g., Mitchell and Pulvino (2001); Agarwal and Naik (2004); Bussière et al. (2015)). In such a period of market stress, hedge funds may even add no value (Asness et al., 2001). To address this complex issue, we first introduce the most popular beta neutral strategies, then we explain the nature of the problem with their practical implementation, and finally, we show how to resolve this problem.

In this paper, we test the quality of hedging for three popular strategies that have often been used by hedge funds. The first and most important strategy captures the low beta anomaly (Black, 1972; Black et al., 1972; Haugen and Heins, 1975; Haugen and Baker, 1991; Ang et al., 2006; Baker et al., 2013; Frazzini and Pedersen, 2014; Hong and Sraer, 2016) that defies the conventional wisdom on the risk and reward trade-off predicted by the CAPM (Sharpe, 1964). According to this anomaly, high beta stocks underperform low beta stocks. Similarly, stocks with high idiosyncratic volatility earn lower returns than stocks with low idiosyncratic volatility (Malkiel and Xu, 1997; Goyal and Santa-Clara, 2003; Ang et al., 2006, 2009). The related strategy consists in shorting high beta stocks and buying low beta stocks. The second important strategy captures the size effect (Banz, 1981; Reinganum, 1981; Fama and French, 1992), in which stocks of small firms tend to earn higher returns, on average, than stocks of larger firms. The related strategy consists in buying stocks with small market capitalization and shorting those with high market capitalization. The third strategy captures the momentum effect (Jegadeesh and Titman, 1993; Carhart, 1997; Grinblatt and Moskowitz, 2004; Fama and French, 2012), where past winners tend to continue to show high performance. This strategy consists in buying the past year's winning stocks and shorting the past year's losing ones.

The implementation of all these strategies requires a reliable estimation of the betas to maintain the hedge. First, Ordinary Least Squares (OLS) estimation remains the most frequently employed method, even though it is impaired in the presence of outliers, notably from small companies (Fama and French, 2008), illiquid companies (Amihud, 2002; Acharyaa and Pedersen, 2005; Ang et al., 2013), and business cycles (Ferson and Harvey, 1999). In these circumstances, the OLS beta estimator might be inconsistent. However, our approach focuses rather on normalizing the variables to make them closer to Gaussian and make the OLS estimator more consistent. Second, many papers report that betas are time varying (Blume, 1971; Fabozzi and Francis, 1978; Jagannathan and Wang, 1996; Fama and French,

1997; Bollerslevet al., 1998; Lettau and Ludvigson, 2001; Lewellen and Nagel, 2006; Ang and Chen, 2007). This can lead to measurement errors that could create serious bias in the cross-sectional asset pricing test (Shanken, 1992; Chan and Lakonishok, 1992; Meng et al., 2011). In fact, rms' stock betas do change over time for several reasons. A rm's assets tend to vary over time via acquiring or replacing new businesses that makes them more diversi ed. The betas also change for rms that change in dimension to be safer or riskier. For instance, nancial leverage may increase when rms become larger, as they can issue more debt. Moreover, rms with higher leverage are exposed to a more unstable beta (Galai and Masulis, 1976; DeJong and Collins, 1985). One way to account for the time dependency of beta is to consider regime changes when the return history used in beta estimation is long enough. Surprisingly, only one paper (Chen et al., 2005) suggests a solution to capture the time dependency and discusses regime changes for the beta using a multiple structural change methodology. The study shows that the risk related to beta regime changes is rewarded by higher returns. Another way is to examine the correlation dynamics. Francis (1979) nds that the correlation with the market is the primary cause of changing betas ... the standard deviations of individual assets are fairly stable . This nding calls for special attention to the correlation dynamics addressed in this paper but that are apparently insu ciently investigated in other works.

Despite the extended literature on this issue, little attention has been paid to the link between the leverage e ect[1] and the beta. The leverage e ect is de ned as the negative correlation between the securities' returns and their volatility changes. This correlation induces residual correlations between the stock overperformances and beta changes. In fact, earlier studies have heavily focused on the role of the leverage e ect on volatility (Black, 1976; Christie, 1982; Campbell and Hentchel, 1992; Bekaert and Wu, 2000; Bouchaud et al., 2001; Valeyre et al., 2013). Surprisingly, despite its theoretical and empirical underpinnings, the leverage e ect has not been considered so far in beta modeling, while it is a measure of risk. We aim to close this gap. Our paper starts by investigating the role of the leverage e ect in the correlation measure by extending the reactive volatility model (Valeyre et al., 2013), which e ciently tracks the implied volatility by capturing both the retarded e ect induced by the speci c risk and the panic e ect, which occurs whenever the systematic risk becomes the dominant factor. This allows us to set up a beta neutral model incorporating three independent components, all of them contributing to the reduction of the bias of the hedging. First, we take into account the leverage e ect on beta, where the beta of underperforming stocks tends to increase. Second, we consider a leverage e ect on correlation, in which a stock index decline induces an increase in correlations. Third, we model the relation between the relative volatility (de ned as the ratio of the stock's volatility to the index's volatility) and the beta. When the relative volatility increases, the beta increases as well. All three independent

[1] Note that we are not dealing with the restricted de nition of the leveraged beta that comes from the degree of leverage in the rm's capital structure.

components contribute to the reduction of biases in the naive regression estimation of the beta and therefore considerably improve hedging strategies.

The main contribution of this paper is the formulation of abeta neutral model with leverage e ect The economic intuition behind the beta neutral model is the derivation of a suitable beta measure allowing the implementation of the popular market neutral hedging strategies with reduced bias and smaller standard deviation. In contrast, portfolio managers who use naive beta measures remain exposed to systematic risk factors that create biases in their market neutral strategies. An empirical test is performed based on an exhaustive dataset that includes 600 American stocks and 600 European stocks from the S&P 500, Nasdaq 100, and Euro Stoxx 600 over the period 2000 to 2015, which includes several business cycles. This test validates the superiority of the beta neutral model over conventional methods.

The article is organized as follows. Section 2 outlines the methodology employed for the beta neutral model. Section 3 describes the data and discusses the empirical ndings, while Section 4 concludes.

2 The beta neutral model

In this section, we present a beta neutral model with three independent components. First, we take into account the speci c leverage e ect on beta. Second, we consider the systematic leverage e ect on correlation. Third, we model the relation between the relative volatility and the beta.

2.1 The leverage e ect on beta

We rst account for relations between returns, volatilities, and beta, which are characterized by the so-called leverage e ect. This component takes into account the phenomenon where a beta increases as soon as a stock underperforms the index. Such a phenomenon can be fairly well described by the leverage e ect captured in the reactive volatility model. We call the speci c leverage e ectthe negative relation between speci c returns and the risk (here, the beta), where the speci c return is the non-systematic part of the returns (a stock's overperformance). The speci c leverage e ect on beta follows the same dynamics as the speci c leverage e ect introduced in the reactive volatility model.

2.1.1 The reactive volatility model

This section aims at capturing the dependence of betas on stock overperformance (when a stock is overperforming, its beta tends to decrease). For this purpose, we rely on the methodology of the reactive volatility model (see Valeyre et al., 2013) to derive a stable measure of beta by using the renormalization factor that depends on the stock's overperformance.

We start by recalling the construction of the reactive volatility model, which explicitly accounts for the leverage e ect on volatility. Let $I(t)$ be a stock index at day t. It is well known that arithmetic returns, $r_I(t) = I(t)=I(t-1)$, are heteroscedastic, partly due to price-volatility correlations. Throughout the text, refers to a di erence between successive values, e.g., $I(t) = I(t) - I(t-1)$. The reactive volatility model aims at constructing an appropriate level of the stock index, $L(t)$, to substitute the original returns $I(t)=I(t-1)$ by less-heteroscedastic returns $I(t)=L(t-1)$.

For this purpose, we rst introduce two levels of the stock index as exponential moving averages (EMAs) with two time scales: a slow level $L_s(t)$ and a fast level $L_f(t)$. In addition, we denote by $L_{is}(t)$ the EMA (with the slow time scale) of the price $S_i(t)$ of the stock i at time t. These EMAs can be computed using standard linear relations:

$$L_s(t) = (1 - _s)L_s(t-1) + _s I(t); \quad (1)$$

$$L_f(t) = (1 - _f)L_f(t-1) + _f I(t); \quad (2)$$

$$L_{is}(t) = (1 - _s)L_{is}(t-1) + _s S_i(t); \quad (3)$$

where $_s$ and $_f$ are the weighting parameters of the EMAs that we set to $_s = 0{:}0241$ and $_f = 0{:}1484$ (see Bouchaud et al., 2001). The slow parameter corresponds to the relaxation time of the retarded e ect for the speci c risk, whereas the fast one corresponds to the relaxation time of the panic e ect for the systematic risk. These two relaxation times are found to be rather universal, as they are stable over the years and remain close to each other for di erent mature stock markets. The appropriate levels $L(t)$ and $L_i(t)$, accounting for the leverage e ect on the volatility, were introduced for the stock index and individual stocks, respectively:

$$L(t) = I(t) \quad 1 + F_\phi \quad \frac{L_s(t) - I(t)}{I(t)} \qquad 1 + ' \quad \frac{L_f(t) - I(t)}{L_f(t)} \quad ; \quad (4)$$

$$L_i(t) = S_i(t) \quad 1 + F_\phi \quad \frac{L_{is}(t) - S_i(t)}{S_i(t)} \qquad 1 + '_i \quad \frac{L_f(t) - I(t)}{L_f(t)} \quad ; \quad (5)$$

speci c risk systematic risk

with the parameters ' and $'_i$ quantifying the leverage and the ltering function $F_\phi(z) = \tanh(\ z\ )=$ with $= 3{:}3$. The ltering function was introduced to attenuate the contribution from eventual extreme events (in the limit $= 0$, there is no lter: $F_0(z) = z$). If $' = '_i$, the correlation between the stock index and the individual stock is not impacted by the leverage e ect. In turn, if $' > '_i$, the correlation increases when the stock index decreases. Although $'_i$ can generally be speci c to the considered stock, we ignore its possible dependence on and set $'_i = '$. Using the levels $L(t)$ and $L_i(t)$, we introduce the normalized returns:

$$\tilde{r}_I = \tilde{r}_I(t) = \frac{I(t)}{L(t-1)}; \qquad \tilde{r}_i = \tilde{r}_i(t) = \frac{S_i(t)}{L_i(t-1)} \quad (6)$$

and compute the renormalized variances $\tilde{s}_I^2$ and $\tilde{}_i^2$ through the EMAs as:

$$\tilde{}_I^2(t) = (1 - \ _\sigma)\tilde{}_I^2(t-1) + \ _\sigma \tilde{r}_I^2(t); \quad (7)$$

$$\tilde{}_i^2(t) = (1 - \ _\sigma)\tilde{}_i^2(t-1) + \ _\sigma \tilde{r}_i^2(t); \quad (8)$$

where $_\sigma$ is a weighting parameter that has to be chosen as a compromise between the accuracy of the estimated renormalized volatility and the reactivity of that estimation. Indeed, the renormalized returns are constructed to be homoscedastic only at short times because the renormalization based on the leverage effect with short relaxation times ($_s$, $_f$) cannot account for long periods of changing volatility related to economic cycles. Since economic uncertainty does not change significantly in a period of two months (40 trading days), we set $_\sigma$ to $1/40 = 0.025$. This sample length leads to a statistical uncertainty of approximately $\sqrt{1/40} \approx 16\%$. Finally, these renormalized variances can be converted into the reactive volatility $_I(t)$ of the stock index quantifying the systematic risk governed by the panic effect, and the reactive volatility $_i(t)$ of each individual stock quantifying the specific risk governed by the leverage effect:

$$_I(t) = \tilde{}_I(t)\frac{L(t)}{I(t)}; \quad (9)$$

$$_i(t) = \tilde{}_i(t)\frac{L_i(t)}{S_i(t)}. \quad (10)$$

2.1.2 The specific leverage effect on beta

The volatility estimation procedure naturally impacts the estimation of beta. Many financial instruments rely on the estimated beta, $_i$, which corresponds to the slope of a linear regression of stocks' arithmetic returns r_i on the index arithmetic return r_I:

$$r_i = \ _i r_I + \ _i; \qquad \text{with} \quad r_i = \frac{S_i(t)}{S_i(t-1)}; \quad r_I = \frac{I(t)}{I(t-1)}; \quad (11)$$

where $_i$ is the residual random component specific to stock i. We consider another beta estimate, $\tilde{}_i$, based on the reactive volatility model, in which the renormalized stock returns $\tilde{r}_i$ are regressed on the renormalized stock index returns $\tilde{r}_I$:

$$\tilde{r}_i = \tilde{}_i \tilde{r}_I + \tilde{}_i; \qquad \text{with} \quad \tilde{r}_i = \frac{S_i(t)}{L_i(t-1)}; \quad \tilde{r}_I = \frac{I(t)}{L(t-1)}. \quad (12)$$

We then obtain a reactive beta measure:

$$_i(t) = \tilde{}_i(t)\frac{_i(t)\ \tilde{}_I(t)}{_I(t)\ \tilde{}_i(t)} = \tilde{}_i\frac{L_{is}(t)I(t)}{L_s(t)S_i(t)}; \quad (13)$$

which includes two improvements:

- $\tilde{\ }_i$, which becomes less sensitive to price changes by accounting for the speci c leverage e ect:
- $_i\tilde{\ }_I = (\ _I\tilde{\ }_i)$, which changes instantaneously with price changes.

When taking into account the short-term leverage e ect in correlations, the reactive term is reduced to $\frac{L_{is}(t)I(t)}{L_s(t)S_i(t)}$. This term has a signi cant impact, as the beta of underperforming stocks should increase.

2.2 The systematic leverage e ect on correlation

We coin by systematic leverage e ectthe negative relation between systematic returns and the risk (here, the correlation), where the systematic returns are the non-speci c part of the returns (stock index performance). The systematic leverage e ect on correlation follows the same dynamics as the systematic leverage e ect introduced in the reactive volatility model (the phenomenon's duration is approximately 7 days for $a_s = 0{:}1484$). All correlations are impacted together in the same way by the systematic leverage e ect, and single stocks and their stock indexes should also shift in the same direction. This explains why the stock's beta will not change with respect to the index. The implication is that betas are not very sensitive to the systematic leverage e ect, in contrast to the speci c leverage e ect. We consider the impact of the short-term systematic leverage e ect on correlation. Assuming that the correlation between each individual stock and the stock index is the same for all stocks, one can de ne the implied correlation as[2]:

$$(t) = \frac{^2_I(t) - \sum_i w_i^2\ ^2_i(t)}{\sum_{i\neq j} w_i w_j\ _i(t)\ _j(t)}; \tag{14}$$

where w_i represents the weight of stock i in the index. Denoting:

$$e_I(t) = \frac{\hat{L}_s(t)}{I(t)} - 1; \qquad e_i(t) = \frac{\hat{L}_{is}(t)}{S_i(t)} - 1; \tag{15}$$

we use Eqs. (9, 10) to obtain:

$$= \frac{\tilde{\ }^2_I(1+e_I)^2\ \ 1 + \left(\frac{L_f - I}{L_f}\right)^2 - \ \ 1 + \left(\frac{L_f - I}{L_f}\right)^2 \sum_i w_i^2(1+e_i)^2\ ^2_i}{1 + \left(\frac{L_f - I}{L_f}\right)^2 \sum_{i\neq j} w_i w_j\ \tilde{\ }_i\tilde{\ }_j(1+e_i)(1+e_j)}: \tag{16}$$

[2] See http://www.cboe.com/micro/impliedcorrelation/impliedcorrelationindicator.pdf

If the weights w_i are small, we can ignore the second term; in addition, if the ϵ_i are small, then:

$$\sum_{i \neq j} w_i w_j \tilde{\sigma}_i \tilde{\sigma}_j (1+\epsilon_i)(1+\epsilon_j) \approx (1+\epsilon_I)^2 \tilde{\sigma}_0^2 ,$$

where $\tilde{\sigma}_0^2$ is an average of $\tilde{\sigma}_i^2$. Keeping only the leading terms of the expansion in terms of the small parameter $(L_f - l)/L_f$, one thus obtains:

$$\approx \frac{\tilde{\sigma}_I^2}{\tilde{\sigma}_0^2} \left[1 + 2(\zeta - \zeta') \frac{L_f - l}{L_f} \right] . \tag{17}$$

This relation shows the dynamics of the implied correlation induced by the leverage effect (accounted through the factor $(L_f - l)/L_f$). We assume that the same dynamics are applicable to correlations between individual stocks, i.e.,

$$\rho_{i,j} = \tilde{\rho}_{i,j} \left[1 + 2(\zeta - \zeta') \frac{L_f - l}{L_f} \right] ; \tag{18}$$

where $\tilde{\rho}_{i,j}$ are the parameters specific to each pair of stocks i and j. From this relation, we derive a measure of correlation accounting for the leverage effect between the single stock and the stock index:

$$\rho_i = \tilde{\rho}_i \left[1 + (\zeta - \zeta') \frac{L_f - l}{L_f} \right] ; \tag{19}$$

where $\tilde{\rho}_i$ are the parameters specific to each stock i. Note that there is no factor 2 in front of $(\zeta - \zeta')$ in Eq. (19) because we have a one-factor model here. We use Eq. (19) in the beta neutral model (see Eqs. (30, 36) below) to take into account the varying nature of the correlation in the regression. We rescale the measurement by the normalization factor $(1 + (\zeta - \zeta')(L_f - l)/L_f)$ and then recover the variation of the correlation through the denormalization factor $1/(1 + (\zeta - \zeta')(L_f - l)/L_f)$. We emphasize that the parameter ζ in Eq. (4) that quantifies the systematic leverage for the stock index is slightly different from the parameter ζ' in Eq. (5) that quantifies the systematic leverage for single stocks. According to Eq. (18), when the market decreases, correlations between stocks increase as $\zeta > \zeta'$, and therefore, the stock index volatility increases more than the single stocks volatility: $(\beta_i/\beta_I) < 0$. Once again, the beta is, in contrast to the correlation, weakly impacted by the systematic leverage effect, as all correlations increase in the same way. More precisely, it means that the impact of the increase of correlation in the beta measurement is compensated by a decrease of the relative volatility: $(\beta_i/\beta_I) < 0$, i.e., the single stock volatility increase is lower than that of the stock index volatility. For this reason, the beta neutral model in Eqs. (30, 36) is not very sensitive to the choice of ζ'. Nevertheless, we explain in this section how ζ' is calibrated using the implied volatility index. We measure the level of the systematic leverage effect ζ' for a single stock by regressing Eq. (17) with data from the market-implied

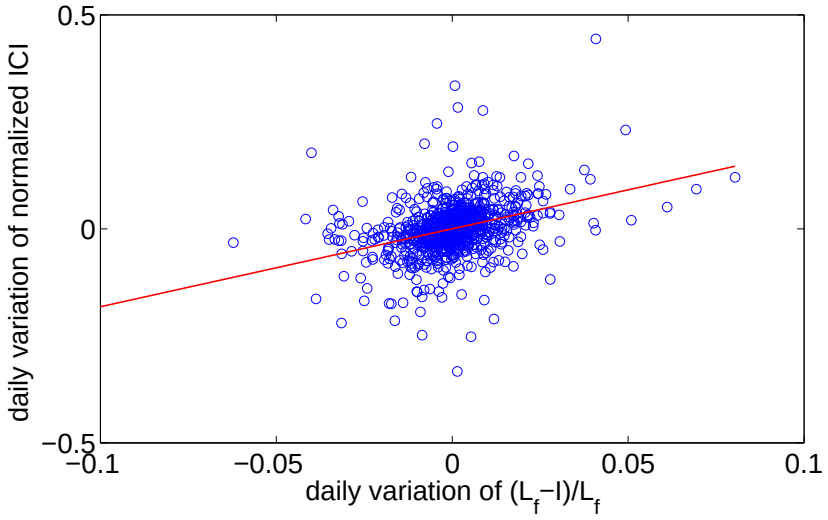

Figure 1: Daily variations of the CBOE S&P 500 Implied Correlation Indices (ICI) since their inception, divided by their mean, versus daily variations of the leverage factor $(L_f - I)/L_f$. A linear regression (solid line) yields the coefficient $1{:}82 \pm 0{:}16$ (i.e., $2(\text{‘} - \text{‘}') = 1{:}82$), with $R^2 = 0{:}13$ and t-statistics of $11{:}4$. Period: 2007-2015.

correlation S&P 500 index. Figure 1 illustrates the slope of this regression. By regressing $\frac{L_f - I}{L_f}$ against $\frac{\rho}{\tilde{\rho}_0}$, where $\tilde{}_0$ is the average of , we deduce that empirically we can set:

$$\text{‘} - \text{‘}' \approx 0{:}91 \pm 0{:}08; \tag{20}$$

with a t-statistics of $11{:}4$. Since $\text{‘} - \text{‘}' \ll \text{‘}\,(\approx 8)$, we deduce an important result, namely, that the systematic leverage impact on the correlation is more than 8 times smaller than the systematic leverage impact on volatility. The main consequence is that although statistically significant, the leverage effect is not a major component of the correlation.

2.3 The relation between the relative volatility and beta

In this part, we identify correlations between relative volatility and beta changes. We choose the relative volatility defined as the ratio $\tilde{}_i/\tilde{}_I$ as an explanatory variable of $\tilde{}_i$, because $\tilde{}_i$ is expected to be constant if the ratio $\tilde{}_i/\tilde{}_I$ is constant. However, empirically, the ratio $\tilde{}_i/\tilde{}_I$ can change dramatically between periods of high dispersion (i.e., when stocks are, on average, weakly correlated) and low systematic risk (i.e., when stock indexes are not stressed), and between periods of low dispersion and high systematic risk. Figure 2 illustrates, for both European and American markets, that the dispersion among stocks decreases, on average,

Figure 2: Normalized daily variations of σ_I, $\sigma_I/\sigma_I = \frac{\sigma_I(t) - \sigma_I(t-1)}{\sigma_I(t-1)}$, versus normalized daily variations of σ_I, $\sigma_I/\sigma_I = \frac{\sigma_I(t) - \sigma_I(t-1)}{\sigma_I(t-1)}$, for the European market (blue crosses) and the US market (red pluses). The two gray lines show the linear regression of both datasets, with regression coefficients of 0.40 ($R^2 = 0.60$) and 0.42 (with $R^2 = 0.59$) for the European and US markets, respectively. The time frame includes observations from the technology bubble burst and the U.S. subprime and Euro debt crises. Period: 1998-2015.

when markets become volatile. A linear regression of rescaled daily variations of yields:

$$\frac{\sigma_i(t)}{\sigma_i(t-1)} \quad 0.4\,\frac{\sigma_I(t)}{\sigma_I(t-1)} + \varepsilon_i; \tag{21}$$

where ε_i is the residual (specific) noise. Using the standard rules for infinitesimal increments, we find from this regression:

$$\frac{\sigma_i}{\sigma_I} \cdot \frac{\sigma_i}{\sigma_I} \quad \frac{\sigma_i\,\sigma_I}{\sigma_I^2} = \frac{\sigma_i}{\sigma_I} \quad \frac{\sigma_i}{\sigma_i} \quad \frac{\sigma_I}{\sigma_I} \cdot 0.6\frac{\sigma_i}{\sigma_I}\frac{\sigma_I}{\sigma_I}; \tag{22}$$

i.e., the relative volatility σ_i/σ_I is relatively stable but can change. This empirical relation shows that when there is a volatility shock in the market, the stock index volatility increases much faster than the average single stock volatility.

Because we want to take into account the impact of the relative volatility change on the beta measurement, we introduce the derivative of the beta with respect to the logarithm of

regression yields the value of f of that subset that corresponds to some average value of $\hat{\beta}_i$. Repeating this procedure over all subsets, we obtain the dependence of f on $\hat{\beta}_i$, which is plotted in Figure 4. We show that f increases with beta. For both European and US markets, we propose the following approximation of the function f:

$$f(\tilde{\beta}_i) = \begin{cases} 0; & \tilde{\beta}_i < 0.5; \\ 0.6(\tilde{\beta}_i - 0.5); & 0.5 < \tilde{\beta}_i < 1.6; \\ 0.6 & \tilde{\beta}_i > 1.6. \end{cases} \tag{29}$$

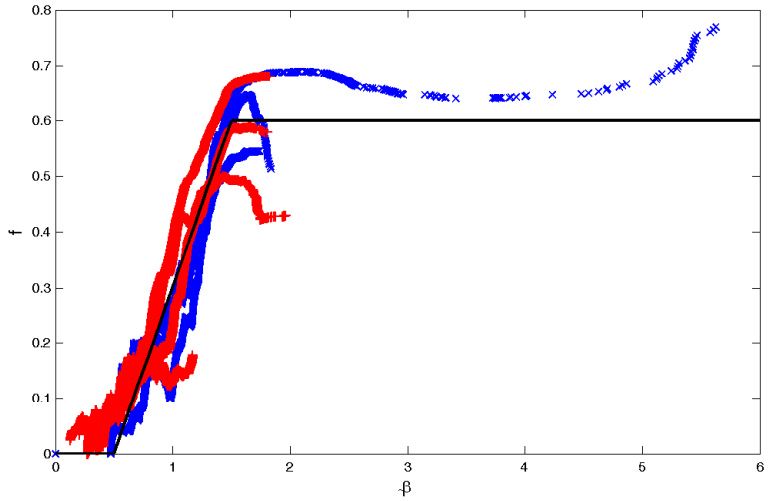

Figure 4: The function f from Eq. (23) versus beta for the European market (blue crosses) and the US market (red pluses). This function is estimated locally for 4 different time periods. The black solid line exhibits the function f as in Eq. (29). Period: 2000-2015.

2.4 Summary of the beta neutral model

In this section, we recapitulate the beta neutral model that combines the three independent components that we described in the previous sections: the specific leverage effect on beta, the systematic leverage effect on correlation, and the relation between the relative volatility and the beta. Starting with the time series $I(t)$ and $S_i(t)$ for the stock index and individual stocks, one computes the levels $S_f(t)$, $L(t)$, and $L_i(t)$ from Eqs. (2, 4, 5), the normalized stock index and individual stocks returns $\tilde{r}_I(t)$ and $\tilde{r}_i(t)$ from Eqs. (6), the normalized stock index volatility $\tilde{\sigma}_I(t)$ from Eq. (7), the renormalized stock index and individual stocks

returns $\hat{r}_I(t)$ and $\hat{r}_i(t)$ from Eq. (24), the associated volatilities $\hat{\sigma}_I(t)$ and $\hat{\sigma}_i(t)$ from Eqs. (26, 28), and the renormalized beta $\hat{\beta}_i(t)$ from Eq. (27). From these quantities, one re-evaluates the covariance between $\hat{r}_i$ and $\hat{r}_I$ by accounting for the leverage effects and excluding the other effects:

$$\hat{\Phi}_i(t) = (1 - \beta)\hat{\Phi}_i(t-1) + \beta \frac{\hat{r}_i(t)\,\hat{r}_I(t)}{\mathcal{L}(t)\,\mathcal{F}(t)}; \tag{30}$$

where

$$\mathcal{L}(t) = 1 + (\gamma - \gamma') \frac{L_f(t-1) - l(t-1)}{L_f(t-1)}; \tag{31}$$

$$\mathcal{F}(t) = 1 + \frac{2f'(\tilde{\sigma}_i(t))}{\tilde{\sigma}_i(t)} \Delta \frac{\tilde{\sigma}_i}{\tilde{\sigma}_I}; \tag{32}$$

are the two correction factors. The function f is approximated by Eq. (29), $\gamma - \gamma'$ is given by Eq. (20), and,

$$\Delta \frac{\tilde{\sigma}_i}{\tilde{\sigma}_I} = \frac{\tilde{\sigma}_i(t-1)/\tilde{\sigma}_I(t-1) - \sqrt{\sigma_i(t-1)}}{\sqrt{\sigma_i(t-1)}} \tag{33}$$

with,

$$\sigma_i(t) = (1 - \beta)\,\sigma_i(t-1) + \beta \frac{\tilde{\sigma}_i^2(t)}{\tilde{\sigma}_I^2(t)} \tag{34}$$

being the EMA of the squared relative volatility $\tilde{\sigma}_i^2/\tilde{\sigma}_I^2$.

Finally, the stable estimate of the normalized beta is:

$$\tilde{\beta}_i(t) = \frac{\hat{\Phi}_i(t)}{\hat{\sigma}_I(t)}; \tag{35}$$

from which the estimated reactive beta of stock i is deduced as:

$$\beta_i(t) = \tilde{\beta}_i(t) \frac{L_i(t)\,l(t)}{S_i(t)\,L(t)} \mathcal{L}(t)\,\mathcal{F}(t): \tag{36}$$

For this estimation, the normalized stable beta $\tilde{\beta}_i(t)$ is denormalized by the factor that combines the three main components: the specific leverage effect on beta $(L_i/S_i)(l/L)$, the systematic leverage effect $\mathcal{L}(t)$, and the sensitivity of beta to the relative volatility, $\mathcal{F}(t)$.

Every term impacts the hedging of a certain strategy:

- the term with $\gamma - \gamma'$ does not have significant impact on beta, as it is compensated in L_i/L, which models the short-term systematic leverage effect on correlation in Eqs. (30, 36) (introduced in Sec. 2.2), whereas the levels L_i and L were introduced in the reactive volatility model. However, it could impact the correlation by $+10\%$ if the market decreases by 10%.

- the term with $L_i I = (LS_i)$ that models the speci c leverage e ect on volatilities (introduced in Sec. 2.1.2) could impact beta by 10% if the stocks underperform by 10%. This term impacts the hedging of the short-term momentum strategy.

- the term with f that models the sensitivity of beta to the relative volatility (introduced in Sec. 2.3) could impact the beta by 10% if the relative volatility increases by 10%. This term impacts the hedging of the low volatility strategy.

3 Empirical ndings

3.1 Data description

For the empirical calibration of ' $-$ ', we chose the CBOE S&P 500 Implied Correlation Index (ICI), which is the rst widely disseminated market-based estimate of implied average correlation of the stocks that comprise the S&P 500 Index (SPX). This index begins in July 2009, with historical data back to 2007. We take the front-month correlation index data from 2007 and roll it to the next contract until the previous one expires. We also use the daily S&P 500 stock index. For the empirical calibration of the other parameters of the beta neutral model, we use the daily S&P 500 stock index and 600 American stocks from the S&P 500 index and the Nasdaq 100 index from January 1, 2000, to May 31, 2015. We also consider the 600 largest European stocks from the Euro Stoxx 600 index, over the same period. The same data are used for both calibration parameters and empirical tests.

3.2 Empirical tests

In this section, we show that exposure to the common risk factors can sometimes lead to a high exposure of market neutral funds to the stock market index if the betas are not correctly assessed. Indeed, although market neutral funds should be orthogonal to traditional asset classes, such is not always the case during extreme moves (Fung and Hsieh, 1997). For instance, Patton (2009) tests the zero correlation against non-zero correlation and nds that approximately 25% of the market neutral funds exhibit some signi cant non-neutrality, concluding that many market neutral hedge funds are in fact not market neutral, but overall they are, at least, more market neutral than other categories of hedge funds. The beta neutral model can help hedge funds be more market neutral than others. To demonstrate this, we empirically test the e ciency of our methodology in estimating the beta neutral model using the most popular market neutral strategies (low volatility, momentum and size):

- low volatility (beta) strategy: buying the stocks with the highest 30% beta and shorting those with the lowest 30% beta (estimated by the standard methodology);

- short-term momentum strategy: buying the stocks with the highest 15% one-month returns and shorting those with the lowest 15% one-month returns;
- long-term momentum strategy: buying the stocks with the highest 15% two-year returns and shorting those with the lowest 15% two-year returns;
- size strategy: buying the stocks with the highest 30% capitalization and shorting those with the lowest 30% capitalization.

The construction of the four most popular strategies is explained in Appendix B. For each strategy, we compare two di erent methods to estimate the beta: the standard method (with $L_i = S_i$, $L = I$, ' $= '' = 0$, and f $= 0$) and our improved method. We analyze the 90-day correlations of the strategy with the stock index. If the strategy was well hedged, the correlation would uctuate by approximately 0 within the theoretical 10% standard deviation (10% is obtained with uncorrelated Gaussian variables for 90-day correlations).
Table 1 summarizes the average correlation and its standard deviation of the four strategies with the stock index for the US and Europe markets. This average correlation re ects the existence of a bias in the hedging strategy. We see the highest bias for the low volatility strategy when hedged with the standard approach ($-25{:}5\%$ for USA and $-22{:}4\%$ for Europe). The standard deviation is approximately 20%, i.e., twice as high as expected if the volatility were stable, which means that the e ciency of the hedge is time-varying. This could represent an important risk for fund of funds managers, where hidden risk could accumulate and arise especially when the market is stressed. Indeed, the bias seems to be higher by approximately -60% for both the USA and Europe when the market was stressed in 2008. We see that the use of the beta neutral model reduces the bias in the low volatility factor, and that the residual bias comes from the selection bias (see Appendix A). When using the standard methodology, the possible loss in 2008 would have been $9{:}6\%$ ($= -60\% \times 40\% \times 8\%/20\%$) for a 40% stock decline with a fund invested entirely on a low volatility anomaly with a bias of -60% and a target annualized volatility 8% for the fund and 20% for the index.
We also see a signi cant bias for the short-term momentum strategy when hedged with the standard approach (approximately $-13{:}1\%$ in the USA and in Europe). The standard deviation is approximately 18%. The e ciency of the hedge depends on the recent past performance of the strategy. As soon as the strategy starts to lose, the e ciency will decline and risk will arise, as in 2009. Again, we see that the beta neutral model reduces the bias in the short-term momentum factor. The biases and standard deviations are lower for the long-term momentum strategy ($-6{:}3\%$ in the USA, with a $18{:}3\%$ standard deviation) and are of same magnitude for the size strategy ($7{:}6\%$ in the USA with a $17{:}0\%$ standard deviation). The beta neutral model further reduces the bias and the standard deviation. This is also valid for the European market.

We conclude that the beta neutral model reduces the bias of the low volatility factor when it is stressed by the market. The remaining residual is most likely explained by the

	Strategy \ Method	standard	improved
US	low volatility	-25.54% (21.73%)	-16.79% (21.43%)
	short-term momentum	-13.09% (18.96%)	6.06% (18.50%)
	long-term momentum	-6.27% (18.28%)	-2.95% (16.54%)
	size	-7.56% (17.00%)	-1.84% (17.26%)
Europe	low volatility	-22.39% (19.97%)	-14.68% (20.94%)
	short-term momentum	-13.05% (17.51%)	-0.64% (14.52%)
	long-term momentum	-4.42% (18.03%)	-1.55% (17.23%)
	size	3.12% (17.15%)	3.79% (15.63%)

Table 1: Correlation over the whole sample (and its standard deviation in parentheses) between the stock index and each of the standard and improved strategies for the US and Europe markets. The residual bias for the low volatility strategy in the improved method can be explained by the selection bias as demonstrated in Appendix A. Period: 2000-2015.

selection bias (see Appendix A for a formal proof). The improvement is more signi cant for the momentum factors and for the size factor in the U.S. only.

We also illustrate these findings by presenting the correlation between the stock index and the low volatility strategy (Figure 5) and the short-term momentum strategy (Figure 6), which are the strategies with the highest bias. A period surrounding the financial crisis was chosen (2007-2010). One can see that the beta, computed by the standard methodology, is highly negatively exposed to the stock index in 2008. In turn, the exposure is reduced within the beta neutral model. The improvement becomes even more impressive in extreme cases when the strategies are stressed by the market. We see that in some extreme cases (stress period with extreme strategies), the common approach could generate high biases (-50% for the short-term momentum strategies in 2008-2009 and -71% for the beta strategy in 2008). In each case, our methodology allows one to signi cantly reduce the bias.

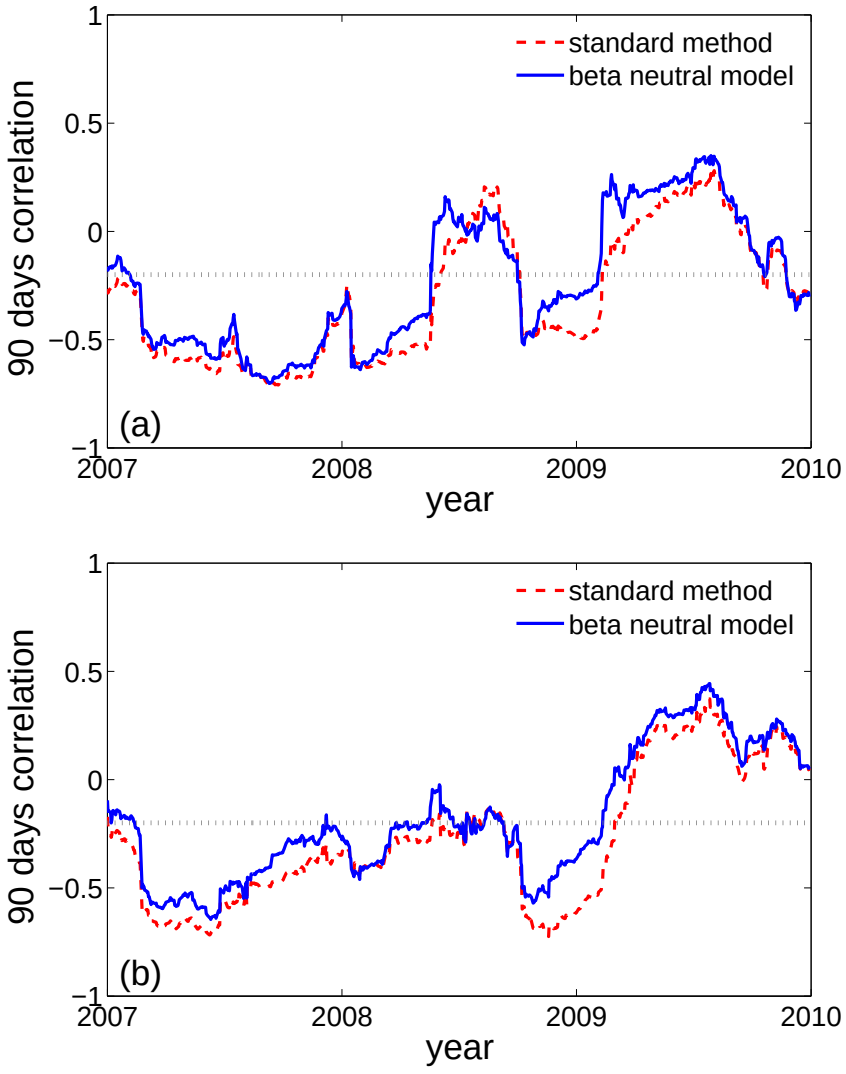

Figure 5: Ninety-day correlation of the low volatility factor with the stock index in the European market(a) and in the USA market(b) . Solid and dashed lines present the proposed beta neutral model and the standard methodology, respectively. The dotted horizontal line shows the selection bias of $-19{:}10\%$, as shown in Appendix A. A time frame surrounding the nancial crisis is chosen. Period: 2007-2010.

Figure 6: Ninety-day correlation of the short-term momentum factor with the stock index in the European market (a) and in the USA market (b) . Solid and dashed lines present the proposed beta neutral model and the standard methodology, respectively. A time frame surrounding the nancial crisis is chosen. Period: 2007-2010.