Strategic Beta and Style Investing: Implication of a (In)dependent Sorting

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Abstarct

We examine the performance of Strategic Beta on investment style portfolios instead of individual stocks. This method simplifies the allocation and reduces the errors in the covariance matrix of returns. We group stocks in categories on size, value and momentum characteristics according a traditional independent sort as in Fama and French (1993) and a dependent sort (Lambert and Hübner 2013). Because a dependent sort controls for correlated variables and better stratifies the stock universe in investment style portfolios, Strategic Beta on dependent portfolios delivers significant higher Sharpe ratios. We explain this outperformance with a decomposition of the diversification return from Booth and Fama (1992).

Passive investors have considered for more than fifty years capitalization-weighted indices as a good proxy for the tangency portfolio, that is the Maximum Sharpe Ratio (MSR). Although a natural way to represent stocks in a portfolio, traditional cap-weight indices constitute a poor proxy for the tangency portfolio because of their embedded momentum strategy (Hsu and Kalesnik 2014). Cap-weighted indices presents high concentration in large growing stocks with strong idiosyncratic risks and lead to high market reversals sensitivity with poor diversification properties (Arnott et al. 2016).

This evidence has incentivized the asset management industry (and personal investors) to find alternative ways for investing passively. Half way between active and passive investing, Strategic Beta investing relies on non-cap-weighted models to allocate stocks in portfolios to capture more efficiently systemic sources of market risk premium (i.e. beta). Often qualified as smart way of investing passively, recents debates emerged between the ones in favour (Amenc, Goltz, and Lodh 2016) and the ones against (Malkiel 2014, Podkaminer 2015) the marketing hype for judging the qualification of "smart" strategies.

Bhansali et al. (2012), Arnott et al. (2013), and, show that strategic beta strategies embed strong style exposures and might fail to produce steady positive excess returns in changing economic conditions. Baker, Bradley, and Wurgler (2011) emphasize the lack of risk-adjusted long-term outperformance of startegic beta portfolios compared to simple cap-weighted benchmarks. While Malkiel (2014) metaphorically associates the current marketing hype around strategic beta and the performance of these strategies to the random performance of a dart-throwing monkey, so-called "monkey business". However, more recent findings of Amenc, Goltz, and Lodh (2016) refute this hypothesis. In this paper, we do not intend to complement the arguments in favour (or against) the application of strategic beta strategies but rather to experiment under what circumstances portfolio construction methodologies can be of use to optimize its performance.

Whilst most of strategic beta strategies have been applied at individual stocks level, Boudt and Peeters (2013) show that investors might also find benefits in application at characteristic-sorted portfolios level. Froot and Teo (2008) confirm this practice for institutional investors who reallocate their funds across style groupings. We rationalize these assertions following the research of Barberis and Shleifer (2003) who demonstrate the natural tendency of investors to allocate funds according to asset categories and of Berk (2000) who explains that forming groups of stocks into style indices circumvent the burden of estimating large covariance-matrix of returns.

Our research contributions to the literature on strategic beta investing are twofold. First, we intend to reconstruct a proxy for the tangent market portfolio by applying strategic beta strategies on characteristic-sorted portfolios where firms have been sorted by size (market capitalization), book-to-market and momentum characteristics. This method simplifies the allocation and reduces the errors in the covariance matrix of return. Second, we motivate how portfolios (style indexing) should be constructed to improve the potential of strategic beta strategies. To this end, we constrast the empirical results of an independent sort

(Fama and French 1993) with the ones of dependent sorting methodology (Lambert and Hübner 2013, Lambert, Fays, and Hübner 2016).

Sorting out stocks with an independent sort on correlated variables (e.g. negative correlation between firms' market equity and book-to-market equity) might lead to suboptimal performance results because of a classification effect which exacerbates the correlation between the variables and leads to very unequal numbers of securities in portfolios (poor diversification). Controlling for the impact of correlated variables on the classifications assigned to firms' characteristics is done through a dependent sort (Daniel et al. 1997; Novy-Marx 2013; Wahal and Yavuz 2013). According to this simple but fundamental methodological change which allows for a proper split of the US investment universe and lead to a finer distinction of stock characteristics, we show that this methodology delivers stastically significant higher Sharpe ratio for Strategic Beta strategies.

In this empirical study, we decompose the source of performance of strategic beta strategies according to four value drivers: the choices of stock classifications (dependent vs independent), the rebalancing frequency, the amount of portfolios that stratifies the US equity market and the risk-oriented weighting function objectives to form a Strategic Beta strategy.

The rest of the paper is organized as follows. Section 1 presents a literature review. Section 2 describes the data used to construct the characteristic-based portfolios. Section 3 illustrates the implications of the sorting methodologies to form portfolios. Section 4 defines the different risk-oriented Strategic Beta used in this paper as well as the methodology to account for transaction costs. Section 5 performs mean-variance spanning tests to evaluate the efficiency of the Strategic Beta strategies conditional on the sorting methodology. Section 6 illustrates the role of diversification on strategic beta strategies. Section 7 concludes.

1 Literature review

The practice of weighting a firm in a portfolio according to its market capitalization has been a traditional structure to construct indices since the introduction of the CAPM (Sharpe 1964; Lintner 1965; Mossin 1966). A well-accepted illustration of this approach is applied for constructing risk factors portfolios (Fama and French 1993). But a recent plethora of papers have fueled the debate on the efficiency of a cap-weighted allocation, among them: Hsu and Kalesnik (2014) show that across four allocation strategies (e.g. equal-weight, fundamental weight, minimum variance and cap-weighted), the traditional cap-weighted index is the only allocation scheme to produce a negative measure of "skill". In theory, skill in portfolio management is related to alpha and by definition broad indices should not produce any form of abnormal return. But cap-weighted portfolios have by construction a drags down in their expected return because the strategy buys stocks when prices are high and sells stocks when prices are low. Although, traditional market weighted schemes suffer from poor diversification and from exposure to uncontrolled sources of risk (Amenc et al. 2014). This mechanical drawback have been recognized by DeMiguel, Garlappi, and Uppal (2009) to demonstrate the outperformance of equally weighted portfolios over alternative weighting schemes in terms of Sharpe ratio, a higher certainty equivalent value, and a lower turnover. Plyakha, Uppal, and Vilkov (2014) provide empirical evidence that the choice of weighting structure is not without consequences for asset-pricing tests based on portfolio constructions. Equal weighting stocks within portfolios helps to reduce stock concentration and exposure to idiosyncratic risk. In what resemble the most to a sequel, Plyakha, Uppal, and Vilkov (2015) decompose the sources of outperformance between cap-weighted and equally-weighted portfolios and suggest that a "1/N" strategy produces additional return from the rebalancing frequencies and an embedded reversal strategy it captures.

The seminal work on Modern Portfolio Theory (MPT) from Markowitz (1952) has pioneered the industry of portfolio management. Under several hypotheses[1], the theory describes how to reach the optimal asset allocation by minimizing the risk-return tradeoff and being tangent to the efficient frontier. This weighting scheme has for objective to maximize the Sharpe ratio (expected return and volatility of a portfolio) by attributing more weight to safer assets. Very close in its definition to the mean-variance portfolio, a minimum variance portfolio discards the estimation of the expected return and simply focus on finding the portfolio with the lowest risk. This reduce form strongly exploits the low beta anomaly (Baker, Bradley, and Taliaferro 2014; Frazzini and Pedersen 2014; Amenc, Goltz, and Martellini 2013). Aside from the obvious benefit of getting more return for less risk, the dynamic nature of the allocation often leads to portfolios with extreme positions in a limited number of assets (DeMiguel, Garlappi, and Uppal 2009). Yet, the advantage of the minimum variance optimization lies in the simplicity of parameters estimation. The objective function only requires the estimation on the assets covariance matrix to attribute weights to the portfolio constituents. But the estimation of the covariance matrix of a large number of securities requires sophisticated techniques because an estimation solely based on a sample period comes along with a lot of errors. (Berk 2000) explains that forming groups of stocks into style indices circumvent the burden of estimating large covariance-matrix of returns while Barberis and Shleifer (2003) demonstrate the natural tendency of investors to allocate funds according to asset categories. These conclusions also applies to institutional investors who tend to reallocate their funds across style groupings Froot and Teo (2008).

Amenc, Goltz, and Martellini (2013) categorize strategic beta approaches according to their level of risk optimality (deviation from the real MSR portfolio) and parameter estimation risk. Among the list of scientific diversification approaches who suffer from risk estimation in the covariance matrix appear the weighting schemes named risk-parity and maximum diversification. Both approaches avoid risk concentration and exploit the low beta anomaly (Clarke, de Silva, and Thorley 2013).

[1] The main assumptions refer to: unlimited risk-free borrowing and short selling, homogenous preferences, expectations and horizons, no frictions (taxes, transaction costs) and non-tradable assets (social security claims, housing, human capital). Thus, under real-world conditions the market portfolio may not be efficient (Sharpe (1991) and Markowitz (2005)).

On one side, a risk parity aims at equalizing the marginal contribution to portfolio volatility of each asset, so that each asset brings an equivalent contribution to portfolio risk (Maillard, Roncalli, and Teïletche 2010). On the other side, Choueifaty and Coignard (2008) use asset volatility (risk) to proxy for expected return and assume that expected return of an asset increases proportionally to its risks. Under this hypothesis, the maximum diversification portfolio is the portfolio tangent to the efficient frontier. Formally, the objective function of the maximum diversification aims at maximizing the ratio between the weighted average asset volatilities and the total portfolio volatility.

According to Malkiel (2014) smart indexing is not different from randomly constructed indexes in terms of performance as the main source of return seems to be related to style investing. Bhansali et al. (2012) show that such risk exposure concentration is implicit in most naïve smart beta indices.

Among the risk-based and heuristic ways of diversifying risk, risk parity has gained popularity. Asness, Frazzini, and Pedersen (2012) provide a theoretical foundation for risk parity portfolios. In the presence of leverage-average investors, safer assets should outperform riskier stocks on a risk-adjusted basis, the latter being overweighed into the portfolios of investors with restricted access to leverage. Risk parity especially overweighs safer assets to achieve equal risk contribution between asset classes. Risk parity on all universe of securities (stocks and bonds) tends to overweigh the allocation to bond. Such strategy therefore works well in decreasing interest rates (Chaves et al. 2011; Fisher, Maymin, and Maymin 2015). Risk parity portfolios try for both superior diversification and higher return (Qian 2011). Nevertheless, traditional risk parity strategies do not come without risk, especially the risk of underperformance in squeezed market conditions. Besides, risk parity does not imply low concentration in asset holdings but only low concentration in risk contributions (Steiner 2012). It can therefore be a low diversified portfolio in the MPT sense.

2 Data and Firm Characteristics

Fama and French's portfolios are constructed from the merge of the Center for Research in Security Prices (CRSP) and Compustat databases. The CRSP database contains historical prices information. Compustat delivers accounting information on all stocks listed on the major US stock exchanges. The sample period ranges from July 1963[2] to December 2015 and comprises all stocks listed on NYSE, AMEX, and NASDAQ. The analysis covers a total of 618 monthly observations. The benchmarks used in this analysis correspond to the cap-weighted market risk premium return on all US stocks directly provided by Wharton Research Data Services (WRDS) minus the one-month T-Bill rate (Treasury bill) from Ibbotson Associates. Following Fama and French (1993) to filter the database and construct cross-sectional portfolios, we keep stocks with a CRSP share code (SHRCD) of 10 or 11 at the beginning of month t, an

[2] Compustat and CRSP information are respectively available from January 1950 and January 1926. However, our sample starts, after correcting for survival and backfill biases, in July 1953 (FF rebalancing date). Moreover, 60 daily observations are required to estimate the covariance matrix. We decided to start the sample at the same date as in Fama and French (1993): July 1963.

exchange code (EXCHCD) of 1, 2 or 3, available shares (SHROUT) and price (PRC) data at the beginning of month *t*, available return (RET) data for month t, at least 2 years of listing on Compustat to avoid the survival bias (Fama and French, 1993) and a positive book-equity value at the end of December of year *y*-1.

The book value of equity is define as the Compustat book value of stockholders' equity (SEQ) plus the balance-sheet deferred taxes and investment tax credit (TXDITC). If available, we decrease this amount by the book value of preferred stock (PSTK). If the book value of stockholders' equity (SEQ) plus the balance-sheet deferred taxes and investment tax credit (TXDITC) is not available, we use the firm total asset (AT) minus the total liabilities (LT).

Book-to-market equity (B/M) is the ratio of the book value of equity for the fiscal year ending in calendar year *y*-1, divided by market equity. The market equity is defined as the price (PRC) of the stock times the number of shares outstanding (SHROUT) at the end of June *y* to construct the size characteristic and at the end of December of year *y*-1 to construct the B/M ratio.

Carhart (1997) extends the Fama and French's three-factor model with a momentum factor (i.e. a *t*-2 until *t*-12 cumulative prior-return) that reflects the return differential between the highest and the lowest prior-return portfolios.

3 Independent vs Dependent Sorts

3.1 Sorting Methodology on Correlated Variables

Fama and French (1993) portfolios are constructed using a 2x3 independent sorting procedure: a two-way sort (small and big) on the market capitalization and a three-way sort (low, medium, high) on the book-to-market equity ratio. These style classifications are defined according to NYSE[3] stocks exchange only. Six portfolios are constructed at the intersection of the 2x3 classifications and rebalanced on a yearly basis at the end of June. This construction methodology becomes a standard in asset pricing literature for constructing characteristic-sorted portfolios. However, the market equity and book-to-market equity of a firm are on average negatively correlated (-5%) as reported in Table 1.

Table I
Correlation between market equity and book-to-market equity

The table report the annual correlation between a firms market equity and book-to-market equity. The sample period is from July 1963 to July 2015.

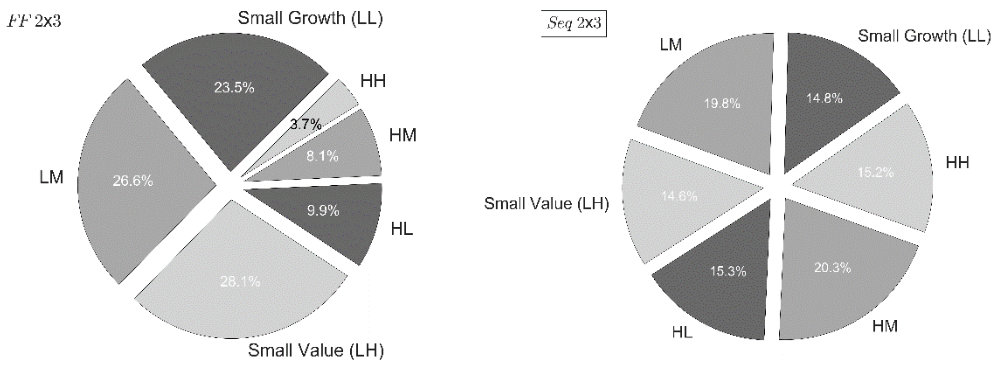
	(ME/BM)
Correlation	-0.05***
p-value	(0.0001)
obs	187096

[3] The NYSE is represented by stocks which account for the largest capitalization of the Center for Research in Security Prices (CRSP) database. The exchange code 1, 2 and 3 are respective for the NYSE, NASDAQ and AMEX.

Using an independent sort on negatively (or positively) correlated variabeles can induce, by design, a strong tilt towards the extreme categories of inverse ranks (low-high and high-low). Lambert, Fays, and Hübner (2016) show that the NYSE breakpoints exacerbate the imbalance in the number of stocks between small and large cap portfolios and that an independent sort leads to a higher number of stocks into small value portfolios. As a consequence, portfolios might not constitute a good strating point for estimating the size and value risk premiums. To allievate this potential sorting issues, we use a simple but fundamental change to Fama and French (1993) methodology as in Lambert, Fays and Hübner (2016) and form characteristics-sorted portfolios according to a depend sort. The authors to consider the whole sample rather than only the NYSE as breakpoints and motivate their choice to avoid Daniel et al. (1997, pp. 1057) remark "*the size breakpoints are designed so that there will be an equal number of NYSE firms in each of the [five] portfolios*". Put differently, it is not realistic to force (by construction) an equal number of NYSE firms to fall into each small size categories. In Figure 1, we illustrate the implication of these choices on the stocks repartition in portfolios. The dependent sort delivers a well-balanced stratification of US stocks into portfolios contrary to independent sorting methodology.

Figure I
Independent vs Dependent Sort Stock Distribution

The figure displays the stock distribution into the 2x3 characteristic-sorted portfolios on the size (low and high) and the book-to-market equity ratio (low, medium and high) for the independent (left) and the dependent (right) sorting methodologies. The independent sort uses the NYSE as breakpoints reference while the dependent sort uses all name breakpoints (NYSE, Nasdaq, Amex). The period ranges from July 1963 to December 2015.



3.2 Equally-weighted Portfolio Variance

Since we construct equally-weighted portfolios, the formula of the i^{th} portfolio variance (Gorton and Rouwenhorst 2006) can be written as follow,

$$\sigma_i^2 = \underbrace{\frac{1}{n}}_{\substack{\text{Weight on the assets} \\ \text{average variance}}} \overline{\text{var}} + \underbrace{\frac{n-1}{n}}_{\substack{\text{Weight on the assets} \\ \text{average covariance}}} \overline{\text{covar}} \tag{1}$$

Where n is the number of stocks in the i^{th} portfolio, $\overline{var}$ is the average variance of the n stocks and $\overline{covar}$ is the average covariance of these same n stocks.

As n (the number of stocks within a portfolio) becomes large, the variance of the portfolio (σ_i^2) converges to the average covariance of its constituents, $\sigma_i^2 \approx \overline{covar}$. We illustrate in Table 3 an extreme scenario where the stocks universe is composed of 100 stocks and 6 portfolios (2x3) are constructed based on the market equity and book-to-market equity ratio of a firm. From Figure 2, we know that the amount of stocks found in the portfolios sorted independetly are, on average, equal to 24, 26, 28, 10, 8, and 4 over our 60 years sample period for the portfolios denoted LL, LM, LH, HL, HM, and HH, respectively.

Using a dependent sort to classify stocks in style portfolios, the repartition of stocks gets close to $1/n$ (n is equal to 6) in the specific cas where the breakpoints are the 33$^{\text{th}}$ and 66$^{\text{th}}$ percentiles of the distribution on all breakpoints names (NYSE, Nasdaq, Amex). According to Panel A of Table 2, the independent sort assigns greater weights to the average variance of large capitalization stocks and more specifically to large value stocks (25%). As soon as the level of diversification in the portfolio decreases, higher weights will be given to the idiosyncratic risk of a stock. Panel B shows that even with a small amount of securities (100), the weight assigned to stocks' specific risk with a dependent sort remains fairly low (6.25%). Eventhough this issue may be immaterial for the construction of six portfolios with an average of 3,271 US stocks, the issue become important when there is a higher number of portfolios and/or in early stage of the sample.

Table II
The Source of Variance of Equally-Weighted Portfolios

The table displays the impact on equally-weighted portfolios' variance according to the amount of stocks that composed the portfolios. We illustrate the weights given on the average variance of assets with a stock universe composed of 100 stocks. Results for Fama and French (1993) methodology are presented based on Figure 2.

	Panel A: Independent Sort			Panel B: Dependent Sort		
Portfolios (2x3)	n stocks	Weights on $\overline{var}$	Weights on $\overline{covar}$	n stocks	Weights on $\overline{var}$	Weights on $\overline{covar}$
LL (Small Growth)	24	4.00%	96.00%	16	6.25%	93.75%
LM (Small Neutral)	26	4.00%	96.00%	16	6.25%	93.75%
LH (Small Value)	28	4.00%	96.00%	16	6.25%	93.75%
Large Growth	10	10.00%	90.00%	16	6.25%	93.75%
HM (Large Neutral)	8	13.00%	87.00%	16	6.25%	93.75%
HH (Large Value)	4	25.00%	75.00%	16	6.25%	93.75%

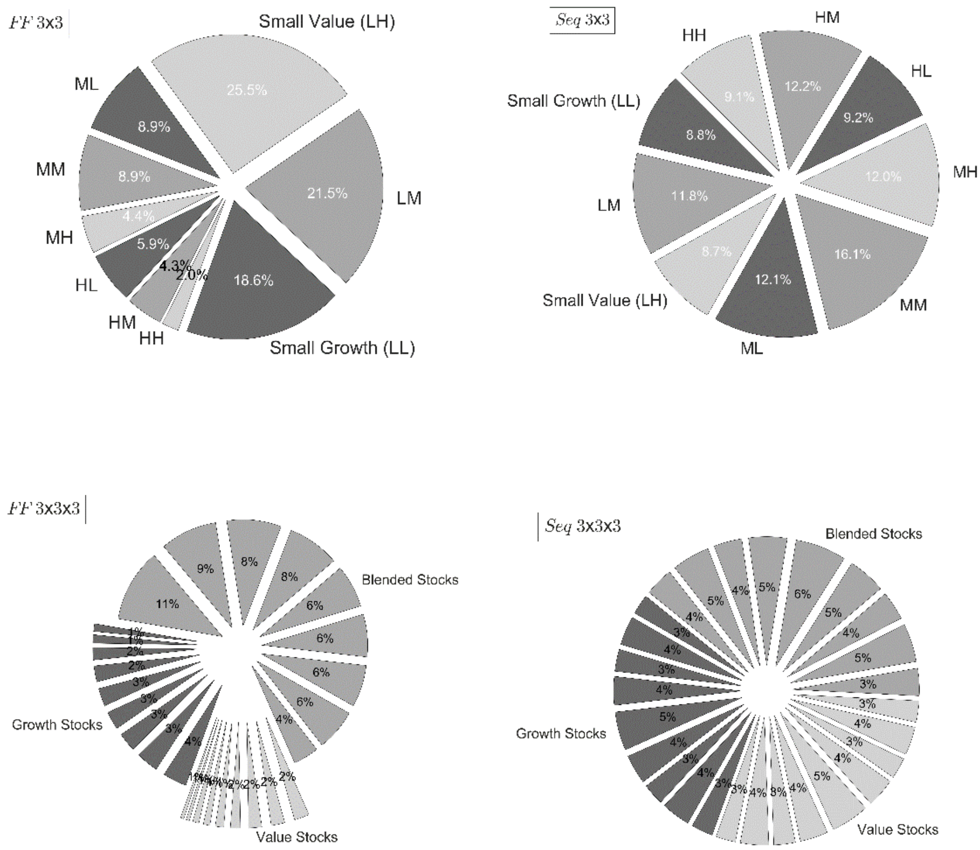
3.3 Style Portfolios Correlation

In this subsection, we illustrate the implication of the methodology on the cross-correlation between the portfolios when a greater amount of investment style portfolios are constructed according to the size, value and momentum chacteristics of a stock.

In Figure 2, we illustrate the stock repartation when the amount of portfolios is increased by either the amount of splits in the sample (from a 2x3 to a 3x3) or by adding a new characteristic, in this paper we choose the momentum following the approach of Lambert, Fays, and Hübner (2016). Clearly, the imbalanced of stocks becomes larger when stocks are sorted independently according to their characteristics.

Figure II
Independent vs Dependent Sort Stock Distribution

The figure displays the stock distribution among the 3x3 characteristic-sorted portfolios on the size (low, medium and high) and the book-to-market equity ratio (low, medium and high) for the independent and dependent sorting methodologies. We also report the average percentage of stock repartition among the 3x3x3 characteristic-sorted portfolios when the momentum is added as a third variable. For clarity, we group the 27 portfolios according to their value classifications. The period ranges from July 1963 to December 2015.



A traditional (rational) view on returns comovement states that comovement in prices reflects comovement in fundamental values (Barberis and Thaler 2003), if this remark remains true the correlation between the investment style portfolios should not only decreases as we augment the number of investment style portfolios but also if groups of stocks are categorized according to a finer methodology (dependent sort). Table 3 illustrates that both predictions are verified, the average correlation between the investment style portfolios are lower when the number of portfolios increases and stocks are sorted dependently. Results are reported for cap- and equal-weighted portfolios in Panel A and Panel B, respectively.

Table III
Correlation between Characteristic-Sorted Portfolios

The table reports the average correlation between the characteristic-sorted portfolios constructed according to a independent and dependent sorting methodology. The third column is the difference in average correlation between the independent and dependent sort. Correlations are estimated based on daily returns and the sample period ranges from 01/07/1963 to 31/12/2015.

	Independent Sort (1)	Dependent Sort (2)	Difference (1)-(2)
↓Number of portfolios			
	Panel A: Cap-weighted Portfolios		
2x3	84.99	78.00	6.99
3x3	84.99	75.81	9.18
3x3x3	78.38	66.8	11.58
	Panel B: Equally-weighted Portfolios		
2x3	87.13	82.64	4.49
3x3	85.62	78.01	7.61
3x3x3	78.63	69.12	9.51

4 Implementation of Strategic Beta Strategies

4.1 Inputs

Besides the choice of two methodologies to construct portfolios, i.e. independent and dependent sort, we control for three other parameters when construting our style indices: (1) the number of portfolios, from six (2x3), nine (3x3) or twenty-seven (3x3x3), (2) the allocation scheme, either cap- or equally-weighted and (3) the rebalancing frequency, i.e. monthly, quarterly, semi-annually, and annually. Each sorting methodology generates twenty-four combinations of portfolios to which we apply four different risk-oriented Strategic Beta, namely minimum variance, risk parity, maximum diversification and equally-weighted. Table 4 reports the objective function for the four Strategic Beta strategies. Because these portfolio optimizations rely on the covariance matrix, we review in the next subsection a *shrinkage* methodology to estimate the covariance with lower sampling errors.

Table IV
List of Smart Beta Strategies' Objective Functions

The table decomposes the smart beta strategies' objective function and the constraints applied on the constituents weights. The first column refers to the common name of the strategy and the second column report the abbreviation (Abb.) used in our paper. The third column makes references to main authors who analyze the strategy. The fourth column reports the objective function to minimize or maximize while the last column displays the unleveraged long-only constraint applied to constituents' weight.

Strategy	Referenced Authors	Objective function	Constraints
Minimum Variance (MV)	Clarke, de Silva and Thorley (2013)	$\min f(w) = \sum_{i}^{N}\sum_{j}^{N} w_i \hat{\sigma}_{ij} w_j$	
Maximum Diversification (MD)	Choueifaty and Coignard (2008)	$max\ f(w) = \frac{\sum_i^N w_i\ \hat{\sigma}_i}{\sqrt{\sum_i^N \sum_j^N w_i \hat{\sigma}_{ij} w_j}}$	$w_i \in [0,1]$ $\sum_{i=1}^{N} w_i = 1$
Risk parity (RP)	Maillard, Roncalli, and Teiletche (2010)	$min\ f(w) = \sum_{i}^{N}\sum_{j}^{N}(w_i \times (\Sigma_p w_i) - w_j \times (\Sigma_p w_j))^2$	
Equally-weighted (EW)	DeMiguel, Garlappi and Uppal (2009)	1/N	

4.2 Estimation of the Covariance Matrix

Talking about covariance matrix, Berk (2000, pp. 420) says *"Given a typical sample of 2000 stocks, this matrix has more than 2 million elements. With only 70 years of data, there is an obvious specification problem"*. He thus suggests to form groups of stocks to reduce this specification problem. However, only grouping stocks does not enterily resolve the issue because the sample covariance matrix has to estimate n(n-1)/2 pair-wise correlations. Increasing the amount of groups can lead to very noisy estimates and strong sample dependency[4]. This problem is also referred as to sampling error. To reduce the sampling error, Elton and Gruber (1973) use a constant correlation coefficient to shrink the assets' covariance towards a global average correlation estimator,

$$\hat{\rho} = \frac{1}{N(N-1)}\left(\sum_{i}^{N}\sum_{j}^{N}\hat{\rho}_{ij} - N\right) \tag{2}$$

[4] In our paper we form 6, 9, and 27 investment styles portfolios and use 60 daily returns to estimates the covariance matrix. In the most extreme case (27 portfolios), we are left with 0.17 data point per parameter what might represents a potential issue if we only consider the sample covariance matrix in our optimizations.

Ledoit and Wolf (2004) find an optimal structure for the covariance matrix and reduces the sampling error of a traditional sample covariance matrix (*S*) as described by equation (3),

$$\Sigma = \delta F + (1 - \delta) S \qquad (3)$$

where Σ is output covariance matrix from the *shrinkage* estimation, δ is the optimal shrinkage intensity[5]. S is the sample covariance matrix, F is the structured covariance matrix with assets' covariance estimated via the constant correlation estimator[6]. In this empirical study, the estimations of the sample and the structured covariance matrices are based on 60-day rolling windows to accommodate for gradual changes in the returns distribution and short term variations. On a practical basis, using monthly return to estimate the assets' covariance matrix might be cumbersome due to the short sample size of the Exchange-Traded Funds (ETFs) universe.

4.3 Transaction costs

In his paper on the taxonomy of market equity anomalies, Novy-marx and Velikov (2015) explains that the recent popularity for equal-weighted portfolios might be misleading after considering transacation costs because a naïve diversification allocation (1/n) puts more weights an small cap stocks, i.e. most illiquid and expensive stocks to trade. Because we form cap- and equally-weighted investment style portfolios to implement Strategic Beta strategies in this paper, but also because a dependent sort will also, by design, exarcebate the weight on small cap stocks. We have to assess the strategies performance on net return.

A simple approach is similar to Plyakha, Uppal, and Vilkov (2015) who implement a decreasing function of transaction costs from 1% in 1978 to 0.5% in 1993 for their S&P500 sample. But in our paper, we trade stocks on NYSE-Nasdaq-AMEX exchanges and consequently have to differentiate transactions costs for small and large cap stocks. We follow the approach similar to Novy-Marx and Velikov (2016) and use the individual stocks estimates from Hasbrouck (2009).

4.3.1 Gibbs estimates

Hasbrouck (2009) suggests to extend the Roll (1984) model with a Bayesian Gibbs sampling approach to model the price dynamics in a market with transaction costs. Hasbrouck model[7] is formalized as follow,

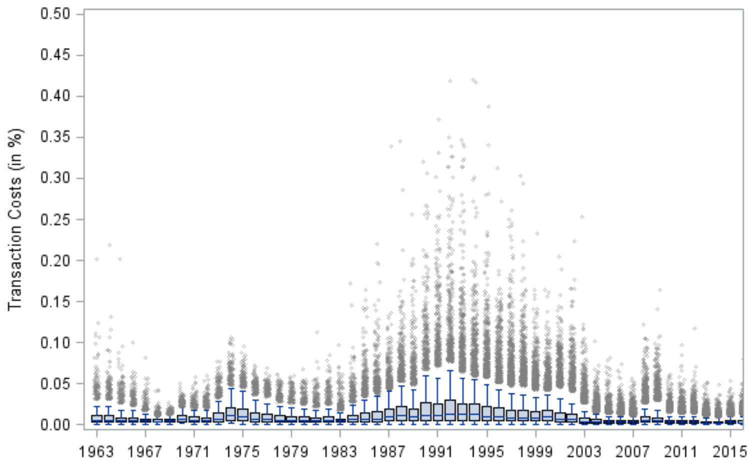
$$\Delta P_t = c \Delta Q_t + \beta_{rm} rm_t + \varepsilon_t \qquad (4)$$

where ΔP_t is the change in price, c is the effective cost of trading, Q_t indicates the direction of the trade (1 is for a buy and -1 is for a sale), rm_t is the market return on day t.

[5] Matlab code is made avaible at Prof. Wolf website.
[6] The covariance of the matrix F is given by $\sigma_{ij} = \hat{\rho}\sigma_i\sigma_j$.
[7] SAS code is made available on Prof. Hasbrouck website.

Hasbrouk's Bayesian methodology estimates the effective transaction costs (c) sequentially based on a strictly positive initial prior following a normal distribution with mean zero and variance equal to 0.05^2, denoted $N^+(\mu = 0, \sigma^2 = 0.05^2)$. For further details on the estimation technique, we refer to (Hasbrouck 2009; Marshall, Nguyen, and Visaltanachoti 2011; Novy-Marx and Velikov 2016). Figure 3 plots the distribution of the individual stocks' yearly estimates for transaction costs (variable c from equation 4) from 1963 to 2015.

Figure III
Variation of Transaction Costs Estimates as Hasbrouk (2009)
The figure illustrates a boxplot for the distribution of individual stocks transaction costs estimated as in Hasbrouk (2009). The sample period ranges from 1963 to 2015. The wiskers plots the distribution of the 5th to 95th percentile. Outliers are the grey dots.



Novy-Marx and Velikov (2016) uncovers a minor drawback to Hasbrouck's estimation technique with the requirement of relatively long series of daily prices to perform the estimation. This results in a number of missing observations for which Novy-Marx and Velikov (2016) performs a non-parametric matching method and attributes equivalent transacation costs to the stock with a missing value according to its closest match in size and idiosyncratic volatility. Since these missing observations only represents 4% of the total market capitalisation universe, we decided in this paper to naively replace the missing values with an arbitrary transaction costs of 0.50%. According to Figure 4, this heuristic choice will only exacerbate the costs to trade these (small size) stocks.

4.3.2 Transaction Costs on Strategic Beta

We report in Table 5 the transaction cots for our Strategic Beta strategies. We distinguish the transaction costs implications of rebalancing the investment styles portfolios on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. We also look at the number of constructed portfolios as presented in Figures 2 and 3. Results are displayed in annual terms (in %) and show that transaction costs have a linear relationship with the rebalancing frequency according to Patton and Timmermann (2010) test for decreasing monotonic relationship. The amount of transaction is also greater for portfolios sorted dependently which suggests that a small stocks have a higher weights under a dependent sort.

Table V
Strategic Beta and Transaction Costs

The table reports the annual transaction costs (in %) for the four different strategic beta strategies, equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). These strategies are applied on portfolios sorted independently or dependently. These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. Finally the number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). We also report the Patton and Timmermann (2010) test for decreasing monotonic relationship[8]. The sample period ranges from July 1963 to December 2015.

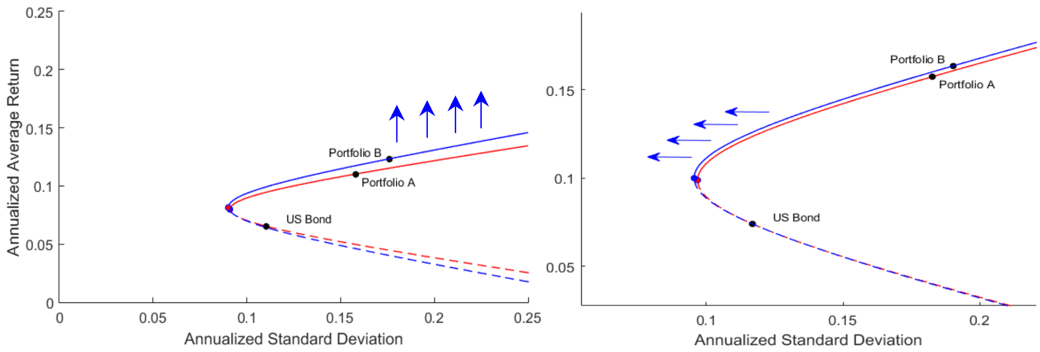
Rebalancing Frequency (in months)	1	3	6	12	Decreasing MR p-value	1	3	6	12	Decreasing MR p-value
	Independent Sort					Dependent Sort				
					Panel A: Cap-weighted					
EW (2x3)	0.05	0.02	0.02	0.01	**0.00**	0.10	0.07	0.06	0.05	**0.02**
EW (3x3)	0.05	0.04	0.02	0.01	**0.00**	0.13	0.09	0.06	0.05	**0.00**
EW (3x3x3)	0.06	0.04	0.04	0.02	**0.00**	0.13	0.09	0.06	0.05	**0.00**
MD (2x3)	0.07	0.04	0.02	0.02	**0.00**	0.22	0.13	0.09	0.07	**0.00**
MD (3x3)	0.13	0.06	0.04	0.02	**0.00**	0.39	0.21	0.13	0.09	**0.00**
MD (3x3x3)	0.22	0.11	0.07	0.04	**0.00**	0.56	0.30	0.17	0.10	**0.00**
MV (2x3)	0.30	0.14	0.09	0.05	**0.00**	0.56	0.27	0.17	0.10	**0.00**
MV (3x3)	0.51	0.18	0.10	0.05	**0.00**	1.01	0.33	0.18	0.11	**0.00**
MV (3x3x3)	0.55	0.20	0.10	0.05	**0.00**	1.01	0.34	0.18	0.09	**0.00**
RP (2x3)	0.06	0.04	0.02	0.01	**0.00**	0.13	0.09	0.06	0.06	**0.00**
RP (3x3)	0.06	0.04	0.02	0.02	**0.00**	0.15	0.10	0.07	0.06	**0.00**
RP (3x3x3)	0.07	0.05	0.04	0.02	**0.00**	0.17	0.11	0.07	0.05	**0.00**
					Panel B: Equally-weighted					
EW (2x3)	0.13	0.13	0.13	0.13	**0.00**	0.20	0.20	0.20	0.20	**0.00**
EW (3x3)	0.11	0.11	0.11	0.11	**0.00**	0.21	0.21	0.21	0.21	**0.00**
EW (3x3x3)	0.13	0.13	0.13	0.13	**0.00**	0.20	0.20	0.20	0.20	**0.00**
MD (2x3)	0.18	0.17	0.15	0.14	**0.00**	0.38	0.31	0.27	0.27	**0.01**
MD (3x3)	0.24	0.20	0.17	0.15	**0.00**	0.58	0.44	0.36	0.34	**0.00**
MD (3x3x3)	0.43	0.28	0.22	0.18	**0.00**	0.84	0.62	0.46	0.41	**0.00**
MV (2x3)	0.66	0.39	0.28	0.24	**0.00**	0.97	0.58	0.43	0.36	**0.00**
MV (3x3)	0.92	0.43	0.30	0.24	**0.00**	1.44	0.68	0.48	0.38	**0.00**
MV (3x3x3)	1.06	0.41	0.28	0.22	**0.00**	1.49	0.60	0.41	0.33	**0.00**
RP (2x3)	0.15	0.14	0.14	0.14	0.45	0.24	0.22	0.22	0.22	0.80
RP (3x3)	0.14	0.14	0.13	0.13	0.32	0.25	0.24	0.24	0.24	0.65
RP (3x3x3)	0.15	0.14	0.14	0.14	**0.00**	0.25	0.24	0.22	0.21	**0.00**

8 Matlab code is made available on Prof. Patton website.

5 Mean-Variance Spanning Test

Kan and Zhou (2012) establish a mean-variance spanning test based on a step-down approach. The approach increases the information on the improvement of the efficient frontier when N assets are added to a benchmark portfolio composed of K elements. The test identifies whether the improvement is at the level of the tangent or the global minimum-variance (GMV) portfolio. In Figure 4, we illustrate with the left (right) plot a significant improvement of the tangency (GMV) portfolio when a test asset (Portfolio B) is added to the benchmark assets (US Bond and Portfolio A).

Figure IV
Improving the Tangency (left) and GMV (right) Portfolio



The sophistication of Kan and Zhou (2012) procedure accounts for a better detection of the improvement in the investment set opportunities. Therefore, the step-down procedure suggest to first test $\alpha = 0_N$ using a OLS-regression. In this paper N is always equal to 1 (the number of test assets). Kan and Zhou (2012) present the test in matrix notations,

$$R = XB + E \tag{5}$$

Where R is the test asset (Portfolio B) returns and X is a K+1 matrix of the benchmark assets returns, in this paper K is equal to 2. The matrix X can be written as follow,

$$X = \begin{pmatrix} 1 & R_1^{\text{US bond}} & R_1^{\text{Portfolio A}} \\ \vdots & \vdots & \vdots \\ 1 & R_t^{\text{US bond}} & R_t^{\text{Portfolio A}} \end{pmatrix} \tag{6}$$

Finally B is a K+1 vector $[\alpha, \beta_{US\,bond}, \beta_{Portfolio\,A}]'$ and E is the error term vector $(\varepsilon_1, \dots, \varepsilon_{1t})'$.

Given that we jointly test if the N assets have an intercept equal to zero, Kan and Zhou (2012) identify the significance statistical test as similar to GRS F-test,

$$F_1 = \left(\frac{T - K - N}{N}\right)\left(\frac{\hat{a} - \hat{a}_1}{1 + \hat{a}_1}\right) \tag{7}$$

Where T is the number of observations, K is the number of benchmark assets, N is is the number of test assets, $\hat{a}_1 = \hat{\mu}_1{}'\hat{V}_{11}^{-1}\hat{\mu}_1$ with $\hat{V}_{11}$ is the variance of the benchmark assets, $\hat{a}$ take the same notation as $\hat{a}_1$ but refers to the benchmark assets plus the new test asset.

The rejection of this test implies that the test assets significantly improves the tangency portfolio of the benchmark assets.

The second test is conditional on the first constraint $\alpha = 0_N$, and verifies whether $\delta = 1_N - \beta 1_K = 0_N$. It is equivalent to estimating equation (5) without an intercept. The F-test for the second hypothesis is written as follow,

$$F_2 = \left(\frac{T - K - N + 1}{N}\right)\left[\left(\frac{\hat{c} + \hat{d}}{\hat{c}_1 + \hat{d}_1}\right)\left(\frac{1 + \hat{a}_1}{1 + \hat{a}}\right) - 1\right] \tag{8}$$

Where $\hat{c}_1 = 1_K{}'\hat{V}_{11}^{-1}1_K$, $\hat{d}_1 = \hat{a}_1\hat{c}_1 - \hat{b}_1^{\,2}$, are the efficient set (hyperbola) constants with $\hat{b}_1 = \hat{\mu}_1{}'\hat{V}_{11}^{-1}1_K$ for the benchmark assets. In equation (8), $\hat{a}$, $\hat{b}$, $\hat{c}$ and $\hat{d}$ are the equivalent notations for the benchmarket assets plus the new test asset. For a graphical illustration, we refer to Kan and Zhou (2012, pp. 158).

For the sake of brevity, we summarize the results for the step-down analysis[9] in Table 6. In Panel A, we report the results where the benchmark assets are the 30-Year US Treasury Bond and a Strategic Beta constructed on *independent* portfolios. The test assets is the same Strategic Beta constructed on *dependent* portfolios. Each Strategic Beta can be constructed according to twelve different combinations according to the rebalancing frequency and the amount of constructed portfolios. These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis while the number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). Results show that a Strategic Beta on *dependent* portfolios significantly improve the tangency portfolio of the benchmark assets for two risk-oriented strategies (max. div. and min. var.). The inverse is however not true, results in Panel B demonstrate that Strategic Beta on *independent* portfolios never improve the tangency portfolio of the benchmark assets with a confidence level of 95%. All results are net of transaction costs.

[9] The full analysis are reported in Appendices 1 to 4.

Table VI
Traditional Step-Down Approach Kan and Zhou (2012)

The table summarizes the results for the step-down regression-based mean-variance spanning test[10] from Kan and Zhou (2012). We display the results for the four different strategic beta strategies, equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). The Strategic Beta strategies are applied on portfolios sorted independently or dependently. These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. The number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). In total, each Strategic Beta can be constructed with 12 (denominator) ways. H1 test the null hypothesis that additional asset (Portfolio B) does not improve the *ex-post* tangency portfolio. H2 test the null hypothesis that additional asset (Portfolio B) do not improve the *ex-post* global-minimum-variance (GMV) portfolio. We report the frequency at which H1 and H2 are rejected with a confidence interval of 95%. The step-down Joint-p tests whether the efficient frontier is improved when adding Portfolio B to the benchmark assets. The sample period ranges from July 1963 to December 2015.

	H1: Tangency portfolio	H2: GMV portfolio	Step-down Joint-p	H1: Tangency portfolio	H2: GMV portfolio	Step-down Joint-p
	Panel A: Bench.=(US+Independent) Portfolio B = Dependent			Panel B: Bench.=(US+Dependent) Portfolio B = Independent		
	Cap-Weighted Portfolios					
EW	0/12	0/12	4/12	0/12	0/12	0/12
MD	5/12	12/12	12/12	0/12	0/12	1/12
MV	6/12	12/12	12/12	0/12	4/12	6/12
RP	0/12	11/12	12/12	0/12	0/12	0/12
	Equally-Weighted Portfolios					
EW	0/12	0/12	0/12	0/12	12/12	12/12
MD	9/12	5/12	11/12	0/12	9/12	12/12
MV	5/12	9/12	12/12	0/12	2/12	2/12
RP	0/12	0/12	7/12	0/12	4/12	7/12

In Table 7, we report the results of the previous spanning tests considered jointly. In this particular case, we are only interested to examine if the assumptions of a normal distribution for the returns of the strategies strongly impact our results. Kan and Zhou (2012) describe two extensions when returns are not distributed normally and have excess kurtosis. Using the moment conditions, they apply a GMM method to estimate the regression parameters of equation (5). The first test is a joint F-test based on the results in Table 6, in this specific case returns are still assumed to follow a normal distribution. In both Panels, the amount of rejection is high due to an improvement in the global minimum-variance. The second test is a GMM Wald test where returns are assumed to follow an elliptical distribution (controlling for heteroscedasticity and excess kurtosis). Rejections of spanning largely decrease in Panel B. The last test

[10] Matlab code is made available on Prof. Zhou website.

is a GMM Wald test for returns under general distributions[11]. Results indicate that there is over-rejection problems in Panel B of Table 6 because returns are assumed to follow a normal distribution. Yet, the rejections rate in Panel A appear to remain consistent.

Table VII
GMM Appraoch Kan and Zhou (2012)

The table summarizes the results for the GMM regression-based mean-variance spanning test from Kan and Zhou (2012). We display the results for the four different strategic beta strategies, equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). The Strategic Beta strategies are applied on portfolios sorted independently or dependently. These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. The number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). In total, each Strategic Beta can be constructed with 12 (denominator) ways. The step-down Joint-p tests whether the efficient frontier is improved when adding Portfolio B to the benchmark assets. W_a^e is the GMM Wald test when returns are assumed to have a multivariate ellicptical distribution. W_a is the GMM Wald and is valid under all distributions of returns. We report the frequency at which the tests are rejected with a confidence interval of 95%. The sample period ranges from July 1963 to December 2015.

	Step-down Joint-p	W_a^e	W_a	Step-down Joint-p	W_a^e	W_a
	Panel A: Bench.=(US+Independent) Portfolio B = Dependent			Panel B; Bench.=(US+Dependent) Portfolio B = Independent		
	Cap-Weighted Portfolios					
EW	4/12	0/12	0/12	0/12	0/12	0/12
MD	12/12	11/12	12/12	1/12	0/12	0/12
MV	12/12	5/12	12/12	6/12	0/12	1/12
RP	12/12	0/12	9/12	0/12	0/12	0/12
	Equally-Weighted Portfolios					
EW	0/12	0/12	0/12	12/12	0/12	4/12
MD	11/12	8/12	8/12	12/12	1/12	0/12
MV	12/12	5/12	8/12	2/12	1/12	0/12
RP	7/12	0/12	0/12	7/12	0/12	0/12

6 Diversification Return

Booth and Fama (1992) introduce the concept of the diversification return as a function of a portfolio geometric average return. A geometric average return is an important performance measure for portfolio management practices because it captures the variation of the portfolio's price. The measure represents the growth rate an investor would have earned if she held a portfolio since day one[12].

[11] For more details see Kan and Zhou (2012, pp. 171–177) and Chen et al. (2010). See also Chen, Ho, and Wu (2004) for GMM step-down resolution.

[12] Willenbrock (2011) notes the mathematical equation as $(1+g)^T$ with g the geometric average return and T holding periods.

Booth and Fama (1992) suggest that the geometric average returns (g) is strongly related to the portfolio expected arithmetic return (μ) and volatility (σ) and can be approximated by the following mathematical formula,

$$g_P = \mu_p - \frac{\sigma_p^2}{2} \tag{9}$$

From Plyakha, Uppal, and Vilkov (2015), we know that

$$\begin{aligned} \mu_p &= \sum_i^N E(w_i^t R_i^t) \\ &= \sum_i^N [E(w_i^t)E(R_i^t) + cov(w_i^t, R_i^t)] \end{aligned} \tag{10}$$

With $\mu_p = E(R_p)$

Erb and Harvey (2006) formalize the impact of not rebalancing as,

$$\mu_p - \sum_i^N E(w_i)\, \mu_i \tag{11}$$

However, we only know $E(w_i)$ *ex-post.* A more natural way to compare the impact of rebalancing *ex-ante* should be against an equal-weight allocation because it is the only allocation for which we know *ex-ante* $E(w_i)$, that is $(1/N)$ any else remaing equal. Equation (11) becomes,

$$\mu_p - \frac{1}{N}\sum_i^N \mu_i \tag{12}$$

Substituting (10) in (12), we have

$$\sum_i^N [E(w_i^t)E(R_i^t) + cov(w_i^t, R_i^t)] - \frac{1}{N}\sum_i^N E(R_i^t) \tag{13}$$

Rearranging the terms, the impact of not rebalancing takes the following form,

$$\underbrace{\sum_i^N cov(w_i^t, R_i^t)}_{\substack{\text{covariance} \\ \text{drag}}} + \underbrace{\sum_i^N \left(E(w_i^t) - \frac{1}{N}\right) E(R_i^t)}_{\substack{\text{Adjustment for} \\ \text{not being EW}}} \tag{14}$$

It is important to note that both terms vanish if we implement an equally-weighted strategy that rebalances at each period t (in this study on a monthly basis).

Finally Willenbrock (2011) formalizes the diversification return as,

$$DR = g_P - \sum_{i}^{N} E(w_i)\, g_i \tag{15}$$

In our paper, i stands for the i^{th} investment style portfolio that we construct using either an independent or dependent sorting methodology, p is the final Strategic Beta portfolio.

The concept of diversification in returns emphasizes that the geometric average return of a portfolio is greater than the sum of the geometric average return of the constituents it holds. Substituting (9) in (15) gives,

$$DR = \mu_p - \frac{\sigma_p^2}{2} - \frac{1}{N}\sum_{i}^{N}\left(\mu_i - \frac{\sigma_i^2}{2}\right) \tag{16}$$

Or by factorisation,

$$DR = \mu_p - \frac{\sigma_p^2}{2} - \frac{1}{N}\sum_{i}^{N}\mu_i + \frac{1}{N}\sum_{i}^{N}\frac{\sigma_i^2}{2} \tag{17}$$

And rearranging the terms,

$$DR = \underbrace{\mu_p - \frac{1}{N}\sum_{i}^{N}\mu_i}_{equation\ (12)} + \frac{1}{2}\left(\bar{\sigma}_i^2 - \sigma_p^2\right) \tag{18}$$

We can substitute the term from equation (12) with its decomposition from equation (14) and rewrite the diversification return as,

$$DR = \underbrace{\sum_{i}^{N}\mathrm{cov}(w_i^t, R_i^t)}_{\text{covariance drag}} + \underbrace{\sum_{i}^{N}\left(E(w_i^t) - \frac{1}{N}\right)E(R_i^t)}_{\text{Adjustment for not being EW}} + \underbrace{\frac{1}{2}\left(\bar{\sigma}_i^2 - \sigma_p^2\right)}_{\text{Variance Reduction Benefit}} \tag{19}$$

Equation (19) provides a benchmark[13] to compare a strategy using dynamics weights with the average constituents it holds. The benefits of applying risk-oriented Strategic Beta strategies (minimum variance, maximum diversification, and risk parity) can be compared to a naïve diversification allocation (equal-weight). We can isolate whether the incremental return benefit is earned by the weights of the strategy (first and second terms) or from "volatility harvesting"

[13] This test is implemented on gross returns. The decomposition do not hold when transaction costs are considered.

(last term). The full decomposition of the results can be found in Appendices 5 to 8.

To compare the benefits from diversication between the two sorting methodologies, we have to adjust the Diversification Return according to the *risk* of the benchmark sorting methodology - in this paper we take the independent sorted portfolios as benchmark. Equation (20) describe the adjustement for comparing the diversification return.

$$\pi_{\mathrm{DR}} = \frac{\sigma_{\mathrm{Independent}}}{\sigma_{\mathrm{dependent}}}\left(\mathrm{DR}_{\mathrm{dependent}}\right) - \left(\mathrm{DR}_{\mathrm{Independent}}\right) \quad (20)$$

In short, if the spread (π_{DR}) is positive, a dependent sort to construct portfolios delivers higher diversification benefits on risk-adjust basis. In Table 8, we report the results for four Strategic Beta strategies and the combination of rebalancing frequency, number of portfolios and portfolios allocation (cap- and equally-weighted). The spreads are strictly positive in 87 out of 96 strategies. Risk-oriented objective functions specifically designed to maximize the diversification benefits (max. div.) shows logically the best results.

Table VIII
Spread in Risk-adjusted Diversification Return

The table reports the diversification return spread from equation (16) for the four different strategic beta strategies, equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). These strategies are applied on portfolios sorted independently or dependently. These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. Finally the number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). The sample period ranges from July 1963 to December 2015.

Rebalancing →	Monthly	Quarterly	Semi-Annually	Annually
↓ Strategy	Panel A: Cap-Weighted Portfolios			
EW (2x3)	0.70%	1.00%	1.30%	1.20%
EW (3x3)	0.60%	1.00%	1.30%	1.10%
EW (3x3x3)	0.90%	1.50%	1.80%	1.30%
MD (2x3)	8.50%	8.30%	7.60%	6.80%
MD (3x3)	6.40%	6.70%	6.90%	6.60%
MD (3x3x3)	9.30%	9.20%	8.00%	4.60%
MV (2x3)	8.50%	2.80%	8.10%	9.80%
MV (3x3)	8.20%	4.20%	6.20%	7.70%
MV (3x3x3)	0.00%	-0.10%	2.40%	-1.10%
RP (2x3)	3.90%	4.30%	5.50%	4.70%
RP (3x3)	3.20%	3.70%	4.60%	4.10%
RP (3x3x3)	3.50%	3.90%	4.10%	3.40%
	Panel B: Equal-Weighted Portfolios			
EW (2x3)	-0.20%	-0.10%	-0.10%	0.10%
EW (3x3)	0.00%	0.30%	0.40%	0.30%
EW (3x3x3)	0.20%	0.80%	0.80%	0.50%
MD (2x3)	5.70%	5.30%	5.50%	7.50%
MD (3x3)	8.00%	7.10%	6.60%	10.80%
MD (3x3x3)	15.30%	12.70%	14.30%	12.90%
MV (2x3)	4.40%	3.90%	7.60%	5.90%
MV (3x3)	-0.80%	2.60%	3.50%	3.20%
MV (3x3x3)	-1.20%	1.40%	-0.90%	-5.70%
RP (2x3)	1.50%	1.70%	2.10%	2.40%
RP (3x3)	1.50%	1.80%	2.20%	2.70%
RP (3x3x3)	1.60%	1.90%	1.90%	2.40%

7 Conclusion

In this paper we follow the common practice of institutional investors and reallocate funds across style groupings (Froot and Teo 2008). This method appears more natural when dealing with a typical 2000 stocks universe and reduce the issues for estimating large covariance-matrix of returns. We show that the methodology to group stocks in different style buckets have strong implication on the performance of the final Strategic Beta strategy. To categories stocks in investment style portfolios, we stratify the universe along the dimensions of size, value and momentum characteristics. We implement two sorting methodologies to construct characteristic-based portfolios: a traditional *independent* sort (Fama and French 1993) and a *dependent* sort (Lambert, Fays, and Hübner 2016). To demonstrate the implications of the methodogies, we apply mean-variance spanning tests from Kan and Zhou (2012) on risk-oriented Strategic Beta strategies that use the characteristic-based portfolios as assets. Results show that a dependent sort to group stocks in portfolios provides significantly higher Sharpe ratios for risk-oriented Strategic Beta strategies. Results hold whether or not stocks are cap- or equally-weighted in portfolios, rebalanced at different frequencies and for returns net of transaction costs. Because a dependent sort controls for correlated variable and stratifies the stock universe in well-diversified portfolios (Lambert, Fays, and Hübner 2016), the sorting methodology delivers better diversification benefits for Strategic Beta strategies. To demonstrate this point, we provide a decomposition of the diversification return from Booth and Fama (1992). We uncover that the diversification return is, on risk-adjusted basis, higher for Strategic Beta implemented on dependent portfolios than on independent portfolios.

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Appendice 1

The table reports the results for the step-down regression-based mean-variance spanning test from Kan and Zhou (2012). We display the results when the benchmark assets are the 30-Year US Treasury Bond and the Strategic Beta on ***independent*** cap-weighted portfolios. The test assets is the same Strategic Beta on ***dependent*** cap-weighted portfolios. In total, there is four different Strategic Beta strategies: equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. The number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). The first column reports the correlation between a Strategic Beta on both sorting methodologies. F1 test the null hypothesis that additional test asset does not improve the *ex-post* tangency portfolio. F2 test the null hypothesis that additional test asset does not improve the *ex-post* global-minimum-variance (GMV) portfolio. We also report the step-down joint-*p* which tests whether the efficient frontier is improved when adding the test asset to the benchmark assets. W_a^e is the GMM Wald test when returns are assumed to have a multivariate ellicptical distribution, results are comparable to the step-down joint-*p*. W_a is the GMM Wald and is valid under all distributions of returns. The sample period ranges from July 1963 to December 2015.

						Step-Down Test					GMM Wald Test			
Strategy	Corr.	$\hat{\alpha}$	$\hat{\delta}$	*F*-test	*p*-value	F_1	*p*-value	F_2	*p*-value	Joint-*p*	W_a^e	*p*-value	W_a	*p*-value
						Monthly								
EW (2x3)	0.98	0.0000	0.0189	1.0130	0.3640	0.01	0.93	2.02	0.16	0.15	1.30	0.52	1.55	0.46
EW (3x3)	0.97	0.0001	0.0310	1.8230	0.1620	0.05	0.83	3.61	0.06	**0.05**	2.08	0.35	2.87	0.24
EW (3x3x3)	0.97	0.0003	0.0288	1.3960	0.2480	0.24	0.63	2.56	0.11	0.07	1.62	0.45	2.42	0.30
MD (2x3)	0.96	0.0012	0.0714	8.3880	0.0000	5.33	**0.02**	11.37	**0.00**	**0.00**	8.38	**0.02**	12.28	**0.00**
MD (3x3)	0.94	0.0013	0.1080	10.7860	0.0000	3.65	0.06	17.85	**0.00**	**0.00**	9.26	**0.01**	17.86	**0.00**
MD (3x3x3)	0.9	0.0022	0.1549	12.4000	0.0000	5.44	**0.02**	19.23	**0.00**	**0.00**	11.27	**0.00**	14.26	**0.00**
MV (2x3)	0.91	0.0019	0.1123	7.6330	0.0010	4.87	**0.03**	10.34	**0.00**	**0.00**	6.16	**0.05**	18.28	**0.00**
MV (3x3)	0.92	0.0018	0.1466	15.0730	0.0000	5.31	**0.02**	24.67	**0.00**	**0.00**	10.72	**0.01**	27.55	**0.00**
MV (3x3x3)	0.93	0.0005	0.0817	5.2970	0.0050	0.54	0.46	10.06	**0.00**	**0.00**	3.41	0.18	10.06	**0.01**
RP (2x3)	0.97	0.0006	0.0415	3.7240	0.0250	1.58	0.21	5.86	**0.02**	**0.00**	3.51	0.17	6.68	**0.04**
RP (3x3)	0.97	0.0006	0.0595	5.2740	0.0050	1.34	0.25	9.21	**0.00**	**0.00**	4.62	0.10	9.08	**0.01**
RP (3x3x3)	0.97	0.0008	0.0613	5.7450	0.0030	2.12	0.15	9.36	**0.00**	**0.00**	5.26	0.07	9.87	**0.01**

							Quarterly							
EW (2x3)	0.98	0.0001	0.0189	1.0060	0.3660	0.02	0.90	2.00	0.16	0.14	1.28	0.53	1.56	0.46
EW (3x3)	0.97	0.0001	0.0310	1.8130	0.1640	0.07	0.80	3.57	0.06	**0.05**	2.06	0.36	2.88	0.24
EW (3x3x3)	0.97	0.0003	0.0287	1.3900	0.2500	0.27	0.61	2.51	0.11	0.07	1.61	0.45	2.43	0.30
MD (2x3)	0.96	0.0012	0.0623	6.6670	0.0010	4.77	**0.03**	8.51	**0.00**	**0.00**	6.73	**0.04**	10.09	**0.01**
MD (3x3)	0.94	0.0013	0.1001	9.1350	0.0000	3.49	0.06	14.72	**0.00**	**0.00**	7.89	**0.02**	16.36	**0.00**
MD (3x3x3)	0.9	0.0023	0.1729	14.6390	0.0000	5.78	**0.02**	23.32	**0.00**	**0.00**	12.81	**0.00**	17.64	**0.00**
MV (2x3)	0.91	0.0015	0.1093	6.5930	0.0010	2.73	0.10	10.43	**0.00**	**0.00**	5.12	0.08	13.64	**0.00**
MV (3x3)	0.92	0.0015	0.1439	14.3150	0.0000	3.79	0.05	24.73	**0.00**	**0.00**	9.49	**0.01**	24.68	**0.00**
MV (3x3x3)	0.93	0.0007	0.0899	6.4010	0.0020	0.81	0.37	12.00	**0.00**	**0.00**	4.41	0.11	9.52	**0.01**
RP (2x3)	0.97	0.0006	0.0384	3.1790	0.0420	1.60	0.21	4.76	**0.03**	**0.01**	3.21	0.20	5.78	0.06
RP (3x3)	0.97	0.0007	0.0573	4.9510	0.0070	1.47	0.23	8.43	**0.00**	**0.00**	4.28	0.12	8.57	**0.01**
RP (3x3x3)	0.97	0.0008	0.0594	5.4180	0.0050	2.04	0.15	8.78	**0.00**	**0.00**	4.92	0.09	9.21	**0.01**
							Semi-Annually							
EW (2x3)	0.98	0.0001	0.0189	1.0040	0.3670	0.02	0.89	1.99	0.16	0.14	1.28	0.53	1.56	0.46
EW (3x3)	0.97	0.0001	0.0310	1.8110	0.1640	0.07	0.79	3.56	0.06	**0.05**	2.05	0.36	2.89	0.24
EW (3x3x3)	0.97	0.0003	0.0288	1.3960	0.2480	0.29	0.59	2.51	0.11	0.07	1.62	0.44	2.46	0.29
MD (2x3)	0.96	0.0011	0.0668	6.4900	0.0020	3.79	0.05	9.15	**0.00**	**0.00**	6.47	**0.04**	9.24	**0.01**
MD (3x3)	0.94	0.0014	0.1081	10.3410	0.0000	3.71	0.06	16.90	**0.00**	**0.00**	9.31	**0.01**	16.55	**0.00**
MD (3x3x3)	0.9	0.0021	0.1697	14.4240	0.0000	4.88	**0.03**	23.82	**0.00**	**0.00**	13.05	**0.00**	15.23	**0.00**
MV (2x3)	0.91	0.0019	0.0858	5.1340	0.0060	4.71	**0.03**	5.52	**0.02**	**0.00**	4.69	0.10	11.05	**0.00**
MV (3x3)	0.93	0.0016	0.1100	8.6880	0.0000	4.14	**0.04**	13.17	**0.00**	**0.00**	5.96	0.05	17.21	**0.00**
MV (3x3x3)	0.94	0.0010	0.0995	8.3200	0.0000	1.82	0.18	14.80	**0.00**	**0.00**	6.51	**0.04**	11.41	**0.00**
RP (2x3)	0.97	0.0006	0.0354	2.8010	0.0620	1.94	0.17	3.66	0.06	**0.01**	3.11	0.21	5.22	0.07
RP (3x3)	0.97	0.0007	0.0561	4.7940	0.0090	1.74	0.19	7.84	**0.01**	**0.00**	4.27	0.12	8.11	**0.02**
RP (3x3x3)	0.97	0.0007	0.0575	5.0630	0.0070	1.92	0.17	8.19	**0.00**	**0.00**	4.63	0.10	8.14	**0.02**
							Annually							
EW (2x3)	0.98	0.0001	0.0187	0.9860	0.3740	0.01	0.91	1.96	0.16	0.15	1.25	0.53	1.53	0.47
EW (3x3)	0.97	0.0001	0.0309	1.7940	0.1670	0.07	0.79	3.52	0.06	**0.05**	2.03	0.36	2.86	0.24
EW (3x3x3)	0.97	0.0003	0.0287	1.3910	0.2500	0.30	0.58	2.48	0.12	0.07	1.62	0.45	2.46	0.29
MD (2x3)	0.96	0.0010	0.0613	5.2740	0.0050	3.11	0.08	7.42	**0.01**	**0.00**	5.31	0.07	8.72	**0.01**
MD (3x3)	0.94	0.0014	0.1035	8.8440	0.0000	3.55	0.06	14.08	**0.00**	**0.00**	7.82	**0.02**	14.28	**0.00**
MD (3x3x3)	0.9	0.0018	0.1475	9.9280	0.0000	3.51	0.06	16.28	**0.00**	**0.00**	9.22	**0.01**	12.92	**0.00**
MV (2x3)	0.91	0.0021	0.1009	6.7670	0.0010	5.79	**0.02**	7.69	**0.01**	**0.00**	5.89	0.05	15.36	**0.00**
MV (3x3)	0.93	0.0017	0.0998	7.8050	0.0000	4.67	**0.03**	10.88	**0.00**	**0.00**	5.57	0.06	14.75	**0.00**
MV (3x3x3)	0.94	0.0009	0.1355	14.8070	0.0000	1.57	0.21	28.02	**0.00**	**0.00**	10.57	**0.01**	17.03	**0.00**
RP (2x3)	0.97	0.0006	0.0367	2.8470	0.0590	1.54	0.22	4.15	**0.04**	**0.01**	3.02	0.22	5.00	0.08
RP (3x3)	0.97	0.0007	0.0593	5.1100	0.0060	1.67	0.20	8.54	**0.00**	**0.00**	4.78	0.09	7.86	**0.02**
RP (3x3x3)	0.97	0.0008	0.0576	4.7380	0.0090	1.83	0.18	7.63	**0.01**	**0.00**	4.65	0.10	7.11	**0.03**

Appendice 2

The table reports the results for the step-down regression-based mean-variance spanning test from Kan and Zhou (2012). We display the results when the benchmark assets are the 30-Year US Treasury Bond and the Strategic Beta on ***independent*** equally-weighted portfolios. The test assets is the same Strategic Beta on ***dependent*** equally-weighted portfolios. In total, there is four different Strategic Beta strategies: equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. The number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). The first column reports the correlation between a Strategic Beta on both sorting methodologies. F1 test the null hypothesis that additional test asset does not improve the *ex-post* tangency portfolio. F2 test the null hypothesis that additional test asset does not improve the *ex-post* global-minimum-variance (GMV) portfolio. We also report the step-down joint-p which tests whether the efficient frontier is improved when adding the test asset to the benchmark assets. W_a^e is the GMM Wald test when returns are assumed to have a multivariate ellicptical distribution, results are comparable to the step-down joint-p. W_a is the GMM Wald and is valid under all distributions of returns. The sample period ranges from July 1963 to December 2015.

						Step-Down Test					GMM Wald Test			
Strategy	Corr.	$\hat{\alpha}$	$\hat{\delta}$	F-test	p-value	F_1	p-value	F_2	p-value	Joint-p	W_a^e	p-value	W_a	p-value
Monthly														
EW (2x3)	0.99	0.0000	-0.0107	0.3820	0.6830	0.00	0.98	0.77	0.38	0.38	0.46	0.79	0.62	0.74
EW (3x3)	0.98	0.0004	0.0054	0.3540	0.7020	0.69	0.41	0.02	0.90	0.37	0.66	0.72	0.94	0.62
EW (3x3x3)	0.98	0.0005	0.0079	0.4860	0.6160	0.93	0.34	0.05	0.83	0.28	0.88	0.65	1.31	0.52
MD (2x3)	0.97	0.0010	0.0229	2.1220	0.1210	3.56	0.06	0.68	0.41	**0.02**	3.55	0.17	5.73	0.06
MD (3x3)	0.96	0.0018	0.0625	5.9120	0.0030	7.25	**0.01**	4.53	**0.03**	**0.00**	8.49	**0.01**	13.96	**0.00**
MD (3x3x3)	0.93	0.0032	0.1101	10.4300	0.0000	13.08	**0.00**	7.63	**0.01**	**0.00**	13.61	**0.00**	18.55	**0.00**
MV (2x3)	0.96	0.0012	0.0504	3.7630	0.0240	3.64	0.06	3.87	0.05	**0.00**	5.26	0.07	4.86	0.09
MV (3x3)	0.96	0.0011	0.0935	8.5000	0.0000	2.85	0.09	14.11	**0.00**	**0.00**	6.14	**0.05**	14.27	**0.00**
MV (3x3x3)	0.96	0.0008	0.0654	5.4520	0.0040	1.87	0.17	9.02	**0.00**	**0.00**	4.02	0.13	6.82	**0.03**
RP (2x3)	0.99	0.0003	0.0006	0.3810	0.6830	0.73	0.40	0.04	0.85	0.34	0.76	0.68	0.94	0.63
RP (3x3)	0.98	0.0007	0.0219	1.5260	0.2180	2.16	0.14	0.89	0.35	**0.05**	2.32	0.31	3.68	0.16
RP (3x3x3)	0.98	0.0008	0.0273	2.1170	0.1210	2.74	0.10	1.49	0.22	**0.02**	2.92	0.23	4.87	0.09

							Quarterly							
EW (2x3)	0.99	0.0000	-0.0107	0.3820	0.6830	0.00	0.98	0.77	0.38	0.38	0.46	0.79	0.62	0.74
EW (3x3)	0.98	0.0004	0.0054	0.3540	0.7020	0.69	0.41	0.02	0.90	0.37	0.66	0.72	0.94	0.62
EW (3x3x3)	0.98	0.0005	0.0078	0.4860	0.6160	0.93	0.34	0.05	0.83	0.28	0.88	0.65	1.31	0.52
MD (2x3)	0.98	0.0009	0.0151	1.6350	0.1960	3.08	0.08	0.19	0.67	0.05	2.98	0.23	4.41	0.11
MD (3x3)	0.96	0.0017	0.0581	5.1810	0.0060	6.40	**0.01**	3.93	**0.05**	**0.00**	7.39	**0.03**	12.21	**0.00**
MD (3x3x3)	0.92	0.0029	0.1104	8.9160	0.0000	9.99	**0.00**	7.73	**0.01**	**0.00**	10.78	**0.01**	16.08	**0.00**
MV (2x3)	0.97	0.0012	0.0548	4.3650	0.0130	3.82	0.05	4.89	**0.03**	**0.00**	7.60	**0.02**	5.07	0.08
MV (3x3)	0.96	0.0014	0.0736	6.3090	0.0020	4.52	**0.03**	8.05	**0.01**	**0.00**	5.85	0.05	12.53	**0.00**
MV (3x3x3)	0.96	0.0011	0.0610	5.3830	0.0050	3.51	0.06	7.23	**0.01**	**0.00**	4.72	0.09	7.07	**0.03**
RP (2x3)	0.99	0.0003	0.0002	0.3940	0.6740	0.74	0.39	0.05	0.82	0.32	0.79	0.68	0.93	0.63
RP (3x3)	0.98	0.0007	0.0207	1.5060	0.2230	2.24	0.14	0.77	0.38	0.05	2.33	0.31	3.57	0.17
RP (3x3x3)	0.98	0.0008	0.0269	2.0870	0.1250	2.69	0.10	1.48	0.22	**0.02**	2.85	0.24	4.69	0.10
							Semi-Annually							
EW (2x3)	0.99	0.0000	-0.0107	0.3820	0.6820	0.00	0.98	0.77	0.38	0.38	0.46	0.79	0.62	0.74
EW (3x3)	0.98	0.0004	0.0054	0.3540	0.7020	0.69	0.41	0.02	0.90	0.37	0.66	0.72	0.94	0.62
EW (3x3x3)	0.98	0.0005	0.0078	0.4860	0.6160	0.93	0.34	0.05	0.83	0.28	0.88	0.65	1.31	0.52
MD (2x3)	0.98	0.0009	0.0176	1.8590	0.1570	3.43	0.07	0.29	0.59	**0.04**	3.36	0.19	4.71	0.10
MD (3x3)	0.96	0.0016	0.0529	4.6080	0.0100	5.93	**0.02**	3.26	0.07	**0.00**	6.62	**0.04**	10.26	**0.01**
MD (3x3x3)	0.93	0.0031	0.0983	9.1780	0.0000	12.35	**0.00**	5.90	**0.02**	**0.00**	12.41	**0.00**	15.93	**0.00**
MV (2x3)	0.96	0.0016	0.0468	4.1210	0.0170	6.17	**0.01**	2.06	0.15	**0.00**	9.03	**0.01**	6.74	**0.03**
MV (3x3)	0.96	0.0017	0.0960	10.4430	0.0000	6.93	**0.01**	13.82	**0.00**	**0.00**	9.04	**0.01**	20.12	**0.00**
MV (3x3x3)	0.96	0.0009	0.0701	6.3640	0.0020	2.47	0.12	10.23	**0.00**	**0.00**	5.61	0.06	7.98	**0.02**
RP (2x3)	0.99	0.0004	0.0006	0.5540	0.5750	1.05	0.31	0.06	0.81	0.25	1.11	0.58	1.32	0.52
RP (3x3)	0.98	0.0008	0.0208	1.6430	0.1940	2.56	0.11	0.73	0.40	**0.04**	2.60	0.27	3.85	0.15
RP (3x3x3)	0.98	0.0008	0.0274	2.1320	0.1190	2.70	0.10	1.56	0.21	**0.02**	2.88	0.24	4.64	0.10
							Annually							
EW (2x3)	0.99	0.0000	-0.0107	0.3820	0.6820	0.00	0.98	0.77	0.38	0.38	0.46	0.79	0.62	0.74
EW (3x3)	0.98	0.0004	0.0054	0.3540	0.7020	0.69	0.41	0.02	0.90	0.37	0.66	0.72	0.94	0.62
EW (3x3x3)	0.98	0.0005	0.0078	0.4860	0.6160	0.93	0.34	0.05	0.83	0.28	0.88	0.65	1.31	0.52
MD (2x3)	0.97	0.0011	0.0135	2.3480	0.0960	4.66	**0.03**	0.03	0.86	**0.03**	4.45	0.11	5.81	0.06
MD (3x3)	0.96	0.0021	0.0513	5.7170	0.0030	9.40	**0.00**	2.01	0.16	**0.00**	8.79	**0.01**	12.36	**0.00**
MD (3x3x3)	0.93	0.0029	0.0792	6.7670	0.0010	10.38	**0.00**	3.11	0.08	**0.00**	10.17	**0.01**	13.12	**0.00**
MV (2x3)	0.97	0.0013	0.0349	3.0340	0.0490	4.73	**0.03**	1.33	0.25	**0.01**	4.27	0.12	4.96	0.08
MV (3x3)	0.96	0.0015	0.0786	7.3760	0.0010	5.60	**0.02**	9.09	**0.00**	**0.00**	7.41	**0.03**	9.25	**0.01**
MV (3x3x3)	0.97	0.0003	0.0466	2.9930	0.0510	0.25	0.62	5.74	**0.02**	**0.01**	2.72	0.26	2.60	0.27
RP (2x3)	0.99	0.0004	-0.0017	0.5970	0.5510	1.02	0.31	0.18	0.67	0.21	1.18	0.55	1.30	0.52
RP (3x3)	0.98	0.0008	0.0203	1.7600	0.1730	2.91	0.09	0.61	0.44	**0.04**	2.81	0.25	3.68	0.16
RP (3x3x3)	0.98	0.0009	0.0230	1.9530	0.1430	3.06	0.08	0.85	0.36	**0.03**	2.96	0.23	4.06	0.13

Appendice 3

The table reports the results for the step-down regression-based mean-variance spanning test from Kan and Zhou (2012). We display the results when the benchmark assets are the 30-Year US Treasury Bond and the Strategic Beta on ***dependent*** cap-weighted portfolios. The test assets is the same Strategic Beta on ***independent*** cap-weighted portfolios. In total, there is four different Strategic Beta strategies: equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. The number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). The first column reports the correlation between a Strategic Beta on both sorting methodologies. F1 test the null hypothesis that additional test asset does not improve the *ex-post* tangency portfolio. F2 test the null hypothesis that additional test asset does not improve the *ex-post* global-minimum-variance (GMV) portfolio. We also report the step-down joint-*p* which tests whether the efficient frontier is improved when adding the test asset to the benchmark assets. W_a^e is the GMM Wald test when returns are assumed to have a multivariate ellicptical distribution, results are comparable to the step-down joint-*p*. W_a is the GMM Wald and is valid under all distributions of returns. The sample period ranges from July 1963 to December 2015.

						Step-Down Test					GMM Wald Test			
Strategy	Corr.	$\hat{\alpha}$	$\hat{\delta}$	F-test	*p*-value	F_1	*p*-value	F_2	*p*-value	Joint-*p*	W_a^e	*p*-value	W_a	*p*-value
						Monthly								
EW (2x3)	0.98	0.0004	0.0167	1.0170	0.3620	0.85	0.36	1.18	0.28	0.10	1.41	0.50	1.92	0.38
EW (3x3)	0.97	0.0004	0.0186	0.9470	0.3880	0.91	0.34	0.98	0.32	0.11	1.25	0.53	1.88	0.39
EW (3x3x3)	0.97	0.0003	0.0239	1.1610	0.3140	0.53	0.47	1.79	0.18	0.08	1.40	0.50	1.92	0.38
MD (2x3)	0.96	-0.0004	-0.0001	0.3440	0.7090	0.64	0.43	0.05	0.82	0.35	0.68	0.71	0.76	0.68
MD (3x3)	0.94	0.0000	0.0070	0.0520	0.9490	0.00	0.99	0.10	0.75	0.74	0.05	0.98	0.07	0.97
MD (3x3x3)	0.9	0.0001	0.0453	1.1970	0.3030	0.01	0.91	2.39	0.12	0.11	1.08	0.58	1.11	0.57
MV (2x3)	0.91	0.0003	0.0700	3.3080	0.0370	0.16	0.69	6.46	**0.01**	**0.01**	2.32	0.31	4.73	0.09
MV (3x3)	0.92	-0.0001	0.0064	0.0400	0.9610	0.00	0.95	0.08	0.78	0.74	0.04	0.98	0.05	0.98
MV (3x3x3)	0.93	0.0009	0.0480	2.4370	0.0880	1.78	0.18	3.09	0.08	**0.01**	1.99	0.37	4.88	0.09
RP (2x3)	0.97	0.0000	0.0077	0.1450	0.8650	0.00	0.98	0.29	0.59	0.58	0.14	0.93	0.19	0.91
RP (3x3)	0.97	0.0001	0.0067	0.0860	0.9180	0.06	0.80	0.11	0.74	0.60	0.09	0.96	0.17	0.92
RP (3x3x3)	0.97	0.0000	0.0045	0.0390	0.9620	0.00	0.97	0.08	0.78	0.76	0.04	0.98	0.05	0.98

							Quarterly							
EW (2x3)	0.98	0.0003	0.0167	0.9890	0.3720	0.78	0.38	1.19	0.28	0.10	1.36	0.51	1.84	0.40
EW (3x3)	0.97	0.0004	0.0186	0.9200	0.3990	0.84	0.36	1.00	0.32	0.11	1.20	0.55	1.80	0.41
EW (3x3x3)	0.97	0.0003	0.0239	1.1580	0.3150	0.49	0.48	1.83	0.18	0.09	1.38	0.50	1.88	0.39
MD (2x3)	0.96	-0.0004	0.0073	0.4270	0.6530	0.47	0.50	0.39	0.53	0.26	0.75	0.69	0.77	0.68
MD (3x3)	0.94	0.0000	0.0156	0.2420	0.7850	0.00	0.96	0.48	0.49	0.47	0.21	0.90	0.33	0.85
MD (3x3x3)	0.9	0.0001	0.0350	0.6750	0.5090	0.01	0.91	1.34	0.25	0.22	0.59	0.75	0.65	0.72
MV (2x3)	0.91	0.0008	0.0726	3.5680	0.0290	1.14	0.29	5.99	**0.02**	**0.00**	2.69	0.26	5.79	0.06
MV (3x3)	0.92	0.0003	0.0075	0.0890	0.9150	0.14	0.71	0.04	0.85	0.60	0.12	0.94	0.18	0.91
MV (3x3x3)	0.93	0.0008	0.0390	1.6810	0.1870	1.43	0.23	1.94	0.17	**0.04**	1.53	0.47	3.27	0.20
RP (2x3)	0.97	0.0000	0.0115	0.3180	0.7280	0.00	0.99	0.64	0.43	0.42	0.33	0.85	0.41	0.82
RP (3x3)	0.97	0.0001	0.0081	0.1110	0.8950	0.04	0.85	0.19	0.67	0.57	0.10	0.95	0.18	0.91
RP (3x3x3)	0.97	0.0000	0.0056	0.0580	0.9430	0.00	0.99	0.12	0.73	0.72	0.06	0.97	0.07	0.96
							Semi-Annually							
EW (2x3)	0.98	0.0003	0.0167	0.9820	0.3750	0.77	0.38	1.20	0.27	0.11	1.34	0.51	1.82	0.40
EW (3x3)	0.97	0.0004	0.0186	0.9090	0.4030	0.81	0.37	1.00	0.32	0.12	1.18	0.55	1.77	0.41
EW (3x3x3)	0.97	0.0003	0.0238	1.1470	0.3180	0.47	0.49	1.83	0.18	0.09	1.37	0.50	1.85	0.40
MD (2x3)	0.96	-0.0002	0.0112	0.3390	0.7120	0.15	0.70	0.53	0.47	0.33	0.48	0.79	0.47	0.79
MD (3x3)	0.94	0.0000	0.0104	0.1120	0.8940	0.00	0.98	0.22	0.64	0.63	0.11	0.95	0.14	0.93
MD (3x3x3)	0.9	0.0002	0.0292	0.4640	0.6290	0.06	0.81	0.87	0.35	0.29	0.41	0.82	0.44	0.80
MV (2x3)	0.91	0.0003	0.0823	4.9410	0.0070	0.18	0.68	9.72	**0.00**	**0.00**	3.41	0.18	6.71	**0.04**
MV (3x3)	0.93	0.0003	0.0367	1.0420	0.3530	0.11	0.74	1.98	0.16	0.12	0.65	0.72	1.33	0.52
MV (3x3x3)	0.94	0.0004	0.0214	0.5020	0.6060	0.38	0.54	0.63	0.43	0.23	0.48	0.79	0.95	0.62
RP (2x3)	0.97	-0.0001	0.0157	0.6140	0.5420	0.02	0.90	1.21	0.27	0.24	0.70	0.71	0.73	0.70
RP (3x3)	0.97	0.0000	0.0092	0.1430	0.8670	0.01	0.94	0.28	0.60	0.56	0.13	0.94	0.19	0.91
RP (3x3x3)	0.97	0.0000	0.0070	0.0860	0.9180	0.00	0.99	0.17	0.68	0.68	0.08	0.96	0.10	0.95
							Annually							
EW (2x3)	0.98	0.0003	0.0169	1.0100	0.3650	0.80	0.37	1.23	0.27	0.10	1.38	0.50	1.88	0.39
EW (3x3)	0.97	0.0004	0.0187	0.9190	0.3990	0.81	0.37	1.02	0.31	0.12	1.19	0.55	1.78	0.41
EW (3x3x3)	0.97	0.0003	0.0239	1.1510	0.3170	0.45	0.50	1.85	0.17	0.09	1.37	0.51	1.84	0.40
MD (2x3)	0.96	-0.0001	0.0184	0.5830	0.5580	0.04	0.85	1.13	0.29	0.25	0.65	0.72	0.70	0.71
MD (3x3)	0.94	0.0001	0.0236	0.5000	0.6070	0.01	0.92	0.99	0.32	0.30	0.44	0.80	0.61	0.74
MD (3x3x3)	0.9	0.0006	0.0623	2.1080	0.1220	0.48	0.49	3.74	0.05	**0.03**	1.91	0.39	2.72	0.26
MV (2x3)	0.91	0.0001	0.0748	4.0220	0.0180	0.02	0.88	8.04	**0.01**	**0.00**	2.85	0.24	4.84	0.09
MV (3x3)	0.93	0.0000	0.0393	1.3450	0.2610	0.00	0.98	2.69	0.10	0.10	0.92	0.63	1.65	0.44
MV (3x3x3)	0.94	0.0005	-0.0116	0.4870	0.6150	0.48	0.49	0.49	0.48	0.24	0.82	0.67	0.76	0.69
RP (2x3)	0.97	0.0000	0.0144	0.4750	0.6220	0.00	0.99	0.95	0.33	0.33	0.50	0.78	0.55	0.76
RP (3x3)	0.97	0.0001	0.0088	0.1220	0.8850	0.02	0.90	0.23	0.63	0.57	0.11	0.95	0.16	0.93
RP (3x3x3)	0.97	0.0000	0.0107	0.1810	0.8340	0.01	0.95	0.36	0.55	0.52	0.18	0.92	0.21	0.90

Appendice 4

The table reports the results for the step-down regression-based mean-variance spanning test from Kan and Zhou (2012). We display the results when the benchmark assets are the 30-Year US Treasury Bond and the Strategic Beta on ***dependent*** equally-weighted portfolios. The test assets is the same Strategic Beta on ***independent*** equally-weighted portfolios. In total, there is four different Strategic Beta strategies: equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. The number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). The first column reports the correlation between a Strategic Beta on both sorting methodologies. F1 test the null hypothesis that additional test asset does not improve the *ex-post* tangency portfolio. F2 test the null hypothesis that additional test asset does not improve the *ex-post* global-minimum-variance (GMV) portfolio. We also report the step-down joint-*p* which tests whether the efficient frontier is improved when adding the test asset to the benchmark assets. W_a^e is the GMM Wald test when returns are assumed to have a multivariate ellicptical distribution, results are comparable to the step-down joint-*p*. W_a is the GMM Wald and is valid under all distributions of returns. The sample period ranges from July 1963 to December 2015.

						Step-Down Test					GMM Wald Test			
Strategy	Corr.	$\hat{\alpha}$	$\hat{\delta}$	F-test	*p*-value	F_1	*p*-value	F_2	*p*-value	Joint-*p*	W_a^e	*p*-value	W_a	*p*-value
							Monthly							
EW (2x3)	0.99	0.0003	0.0340	4.3770	0.0130	0.76	0.39	8.00	**0.01**	**0.00**	5.06	0.08	7.67	**0.02**
EW (3x3)	0.98	0.0001	0.0335	2.7810	0.0630	0.06	0.81	5.51	**0.02**	**0.02**	3.21	0.20	3.89	0.14
EW (3x3x3)	0.98	0.0001	0.0363	2.8910	0.0560	0.04	0.85	5.75	**0.02**	**0.01**	3.35	0.19	3.88	0.14
MD (2x3)	0.97	-0.0003	0.0288	2.1720	0.1150	0.32	0.57	4.03	**0.05**	**0.03**	3.10	0.21	2.61	0.27
MD (3x3)	0.96	-0.0006	0.0259	1.6790	0.1870	0.88	0.35	2.48	0.12	**0.04**	2.69	0.26	2.40	0.30
MD (3x3x3)	0.93	-0.0010	0.0451	3.0070	0.0500	1.60	0.21	4.41	**0.04**	**0.01**	4.52	0.10	4.38	0.11
MV (2x3)	0.96	-0.0001	0.0280	1.1850	0.3060	0.05	0.83	2.33	0.13	0.11	1.62	0.45	1.28	0.53
MV (3x3)	0.96	0.0001	0.0044	0.0270	0.9730	0.03	0.86	0.02	0.88	0.76	0.03	0.99	0.06	0.97
MV (3x3x3)	0.96	0.0002	0.0150	0.3090	0.7340	0.09	0.76	0.53	0.47	0.36	0.22	0.89	0.37	0.83
RP (2x3)	0.99	0.0000	0.0266	2.5810	0.0770	0.01	0.93	5.16	**0.02**	**0.02**	3.17	0.21	3.36	0.19
RP (3x3)	0.98	-0.0001	0.0253	1.5920	0.2040	0.06	0.81	3.13	0.08	0.06	2.04	0.36	1.90	0.39
RP (3x3x3)	0.98	-0.0002	0.0213	1.2540	0.2860	0.15	0.70	2.36	0.13	0.09	1.58	0.45	1.53	0.47

							Quarterly							
EW (2x3)	0.99	0.0003	0.0340	4.3770	0.0130	0.76	0.39	8.00	**0.01**	**0.00**	5.06	0.08	7.67	**0.02**
EW (3x3)	0.98	0.0001	0.0335	2.7810	0.0630	0.06	0.81	5.51	**0.02**	**0.02**	3.21	0.20	3.89	0.14
EW (3x3x3)	0.98	0.0001	0.0363	2.8910	0.0560	0.04	0.85	5.75	**0.02**	**0.01**	3.35	0.19	3.88	0.14
MD (2x3)	0.98	-0.0002	0.0325	2.7500	0.0650	0.24	0.63	5.27	**0.02**	**0.01**	3.94	0.14	3.30	0.19
MD (3x3)	0.96	-0.0005	0.0287	1.7300	0.1780	0.63	0.43	2.83	0.09	**0.04**	2.61	0.27	2.34	0.31
MD (3x3x3)	0.92	-0.0006	0.0516	2.6190	0.0740	0.59	0.44	4.66	**0.03**	**0.01**	3.32	0.19	3.28	0.19
MV (2x3)	0.97	-0.0001	0.0214	0.7520	0.4720	0.06	0.82	1.45	0.23	0.19	1.32	0.52	0.78	0.68
MV (3x3)	0.96	-0.0002	0.0158	0.4160	0.6600	0.10	0.76	0.74	0.39	0.30	0.50	0.78	0.58	0.75
MV (3x3x3)	0.96	-0.0001	0.0154	0.4480	0.6390	0.05	0.83	0.85	0.36	0.30	0.48	0.79	0.57	0.75
RP (2x3)	0.99	0.0000	0.0262	2.5830	0.0760	0.01	0.94	5.17	**0.02**	**0.02**	3.15	0.21	3.42	0.18
RP (3x3)	0.98	-0.0001	0.0252	1.6500	0.1930	0.08	0.78	3.23	0.07	0.06	2.13	0.35	2.00	0.37
RP (3x3x3)	0.98	-0.0002	0.0205	1.1980	0.3030	0.16	0.69	2.24	0.14	0.09	1.51	0.47	1.50	0.47
							Semi-Annually							
EW (2x3)	0.99	0.0003	0.0340	4.3770	0.0130	0.76	0.39	8.00	**0.01**	**0.00**	5.06	0.08	7.67	**0.02**
EW (3x3)	0.98	0.0001	0.0335	2.7810	0.0630	0.06	0.81	5.51	**0.02**	**0.02**	3.21	0.20	3.89	0.14
EW (3x3x3)	0.98	0.0001	0.0363	2.8910	0.0560	0.04	0.85	5.75	**0.02**	**0.01**	3.35	0.19	3.88	0.14
MD (2x3)	0.98	-0.0003	0.0321	2.6780	0.0690	0.31	0.58	5.05	**0.03**	**0.01**	4.00	0.14	3.01	0.22
MD (3x3)	0.96	-0.0004	0.0306	1.8870	0.1520	0.55	0.46	3.23	0.07	**0.03**	2.75	0.25	2.38	0.31
MD (3x3x3)	0.93	-0.0010	0.0475	3.2010	0.0410	1.45	0.23	4.95	**0.03**	**0.01**	4.66	0.10	4.40	0.11
MV (2x3)	0.96	-0.0004	0.0435	2.8340	0.0600	0.31	0.58	5.36	**0.02**	**0.01**	6.39	**0.04**	2.94	0.23
MV (3x3)	0.96	-0.0005	-0.0037	0.2500	0.7790	0.50	0.48	0.00	0.97	0.46	0.47	0.79	0.61	0.74
MV (3x3x3)	0.96	0.0001	0.0080	0.0860	0.9180	0.01	0.93	0.16	0.69	0.64	0.07	0.96	0.09	0.95
RP (2x3)	0.99	0.0000	0.0259	2.6270	0.0730	0.01	0.93	5.25	**0.02**	**0.02**	3.28	0.19	3.27	0.20
RP (3x3)	0.98	-0.0002	0.0252	1.7380	0.1770	0.15	0.70	3.34	0.07	**0.05**	2.29	0.32	2.09	0.35
RP (3x3x3)	0.98	-0.0002	0.0197	1.1290	0.3240	0.17	0.69	2.10	0.15	0.10	1.43	0.49	1.40	0.50
							Annually							
EW (2x3)	0.99	0.0003	0.0340	4.3770	0.0130	0.76	0.39	8.00	**0.01**	**0.00**	5.06	0.08	7.67	**0.02**
EW (3x3)	0.98	0.0001	0.0335	2.7810	0.0630	0.06	0.81	5.51	**0.02**	**0.02**	3.21	0.20	3.89	0.14
EW (3x3x3)	0.98	0.0001	0.0363	2.8910	0.0560	0.04	0.85	5.75	**0.02**	**0.01**	3.35	0.19	3.88	0.14
MD (2x3)	0.97	-0.0004	0.0379	3.8610	0.0220	0.69	0.41	7.04	**0.01**	**0.00**	5.50	0.06	4.38	0.11
MD (3x3)	0.96	-0.0008	0.0394	3.5040	0.0310	1.61	0.21	5.39	**0.02**	**0.00**	5.12	0.08	4.92	0.09
MD (3x3x3)	0.93	-0.0007	0.0674	4.6050	0.0100	0.80	0.37	8.41	**0.00**	**0.00**	6.04	**0.05**	5.40	0.07
MV (2x3)	0.97	-0.0003	0.0365	2.4020	0.0910	0.22	0.64	4.59	**0.03**	**0.02**	2.68	0.26	2.94	0.23
MV (3x3)	0.96	-0.0003	0.0098	0.3240	0.7230	0.24	0.62	0.41	0.53	0.33	0.52	0.77	0.57	0.75
MV (3x3x3)	0.97	0.0006	0.0188	0.8570	0.4250	1.12	0.29	0.60	0.44	0.13	1.11	0.57	1.25	0.54
RP (2x3)	0.99	0.0000	0.0290	3.1390	0.0440	0.00	0.95	6.28	**0.01**	**0.01**	3.69	0.16	3.75	0.15
RP (3x3)	0.98	-0.0002	0.0263	1.9340	0.1450	0.23	0.64	3.65	0.06	**0.04**	2.49	0.29	2.30	0.32
RP (3x3x3)	0.98	-0.0002	0.0244	1.7060	0.1820	0.25	0.62	3.17	0.08	**0.05**	2.16	0.34	2.05	0.36

Appendice 5

The table reports the diversification return decomposition from equation (19) for the four different Strategic Beta strategies: equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). These strategies are applied on **cap-weighted** portfolios sorted **independently** on characteristics. These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. Finally the number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). The last four columns refer to terms from equation (19).The sample period ranges from July 1963 to December 2015.

	Strategy			Constituents Average						
Strategy	Arithmetic Return	Geometric Return	Variance	Geometric Return	Variance	Variance Reduction	Covariance Drag	Adjustment for not being EW	Variance Reduction Benefit	Diversification Return
Independent Sorted Portfolios										
Monthly										
EW (2x3)	1.084	0.970	0.230	0.947	0.275	0.045	0.000	0.000	0.023	0.0226
EW (3x3)	1.103	0.982	0.242	0.957	0.293	0.051	0.000	0.000	0.025	0.0253
EW (3x3x3)	1.125	0.999	0.250	0.960	0.328	0.078	0.000	0.000	0.039	0.0390
MD (2x3)	1.090	0.979	0.221	0.947	0.275	0.054	-0.005	0.011	0.027	0.0323
MD (3x3)	1.113	1.000	0.228	0.957	0.293	0.065	0.004	0.007	0.033	0.0428
MD (3x3x3)	1.090	0.972	0.236	0.960	0.328	0.093	-0.041	0.006	0.046	0.0120
MV (2x3)	1.167	1.062	0.210	0.947	0.275	0.065	0.013	0.069	0.033	0.1150
MV (3x3)	1.153	1.041	0.225	0.957	0.293	0.068	-0.001	0.051	0.034	0.0840
MV (3x3x3)	1.147	1.044	0.205	0.960	0.328	0.123	-0.018	0.040	0.061	0.0836
RP (2x3)	1.104	0.993	0.223	0.947	0.275	0.052	0.005	0.014	0.026	0.0457
RP (3x3)	1.122	1.005	0.235	0.957	0.293	0.058	0.005	0.014	0.029	0.0482
RP (3x3x3)	1.136	1.017	0.237	0.960	0.328	0.091	-0.001	0.012	0.046	0.0569

Quarterly										
EW (2x3)	1.088	0.973	0.229	0.947	0.275	0.045	0.003	0.000	0.023	0.0262
EW (3x3)	1.107	0.986	0.242	0.957	0.293	0.051	0.004	0.000	0.025	0.0295
EW (3x3x3)	1.130	1.005	0.250	0.960	0.328	0.078	0.005	0.000	0.039	0.0446
MD (2x3)	1.099	0.988	0.221	0.947	0.275	0.053	0.004	0.011	0.027	0.0413
MD (3x3)	1.122	1.007	0.229	0.957	0.293	0.064	0.012	0.006	0.032	0.0506
MD (3x3x3)	1.112	0.993	0.239	0.960	0.328	0.090	-0.017	0.005	0.045	0.0324
MV (2x3)	1.244	1.139	0.210	0.947	0.275	0.064	0.087	0.073	0.032	0.1918
MV (3x3)	1.202	1.090	0.224	0.957	0.293	0.069	0.049	0.050	0.035	0.1332
MV (3x3x3)	1.158	1.053	0.210	0.960	0.328	0.119	-0.002	0.036	0.059	0.0927
RP (2x3)	1.110	0.999	0.223	0.947	0.275	0.052	0.011	0.015	0.026	0.0519
RP (3x3)	1.130	1.012	0.235	0.957	0.293	0.058	0.012	0.015	0.029	0.0554
RP (3x3x3)	1.141	1.022	0.238	0.960	0.328	0.090	0.004	0.013	0.045	0.0619
Semi-Annually										
EW (2x3)	1.090	0.976	0.229	0.947	0.275	0.046	0.005	0.001	0.023	0.029
EW (3x3)	1.111	0.989	0.242	0.957	0.293	0.051	0.006	0.001	0.025	0.033
EW (3x3x3)	1.133	1.008	0.250	0.960	0.328	0.078	0.007	0.001	0.039	0.048
MD (2x3)	1.088	0.977	0.222	0.947	0.275	0.052	-0.009	0.012	0.026	0.030
MD (3x3)	1.106	0.990	0.232	0.957	0.293	0.061	-0.004	0.007	0.030	0.033
MD (3x3x3)	1.124	1.002	0.244	0.960	0.328	0.084	-0.004	0.004	0.042	0.041
MV (2x3)	1.201	1.095	0.213	0.947	0.275	0.062	0.047	0.070	0.031	0.148
MV (3x3)	1.230	1.117	0.227	0.957	0.293	0.066	0.081	0.046	0.033	0.160
MV (3x3x3)	1.134	1.024	0.220	0.960	0.328	0.109	-0.022	0.032	0.054	0.064
RP (2x3)	1.104	0.993	0.223	0.947	0.275	0.052	0.004	0.015	0.026	0.046
RP (3x3)	1.125	1.007	0.236	0.957	0.293	0.057	0.007	0.015	0.028	0.051
RP (3x3x3)	1.142	1.022	0.241	0.960	0.328	0.087	0.005	0.013	0.044	0.061
Annually										
EW (2x3)	1.093	0.978	0.229	0.947	0.275	0.045	0.007	0.002	0.023	0.031
EW (3x3)	1.112	0.991	0.243	0.957	0.293	0.050	0.008	0.002	0.025	0.034
EW (3x3x3)	1.133	1.008	0.250	0.960	0.328	0.078	0.006	0.002	0.039	0.048
MD (2x3)	1.091	0.980	0.222	0.947	0.275	0.053	-0.008	0.014	0.026	0.033
MD (3x3)	1.109	0.994	0.230	0.957	0.293	0.063	-0.005	0.010	0.032	0.037
MD (3x3x3)	1.145	1.027	0.236	0.960	0.328	0.093	0.014	0.006	0.046	0.066
MV (2x3)	1.148	1.041	0.213	0.947	0.275	0.061	0.001	0.063	0.031	0.094
MV (3x3)	1.148	1.035	0.224	0.957	0.293	0.068	0.007	0.037	0.034	0.079
MV (3x3x3)	1.168	1.049	0.238	0.960	0.328	0.090	0.011	0.033	0.045	0.089
RP (2x3)	1.102	0.990	0.224	0.947	0.275	0.051	0.002	0.016	0.025	0.043
RP (3x3)	1.119	1.001	0.236	0.957	0.293	0.057	0.001	0.015	0.029	0.045
RP (3x3x3)	1.137	1.016	0.243	0.960	0.328	0.086	-0.001	0.014	0.043	0.056

Appendice 6

The table reports the diversification return decomposition from equation (19) for the four different Strategic Beta strategies: equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). These strategies are applied on **equally-weighted** portfolios sorted **independently** on characteristics. These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. Finally the number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). The last four columns refer to terms from equation (19).The sample period ranges from July 1963 to December 2015.

	Strategy			Constituents Average						
Strategy	Arithmetic Return	Geometric Return	Variance	Geometric Return	Variance	Variance Reduction	Covariance Drag	Adjustment for not being EW	Variance Reduction Benefit	Diversification Return
				Independent Sorted Portfolios						
				Monthly						
EW (2x3)	1.209	1.068	0.283	1.044	0.330	0.047	0.000	0.000	0.024	0.024
EW (3x3)	1.192	1.054	0.276	1.026	0.331	0.054	0.000	0.000	0.027	0.027
EW (3x3x3)	1.208	1.068	0.280	1.027	0.362	0.082	0.000	0.000	0.041	0.041
MD (2x3)	1.255	1.119	0.272	1.044	0.330	0.058	0.001	0.044	0.029	0.075
MD (3x3)	1.251	1.119	0.266	1.026	0.331	0.065	0.009	0.051	0.033	0.092
MD (3x3x3)	1.251	1.114	0.273	1.027	0.362	0.089	-0.021	0.064	0.045	0.088
MV (2x3)	1.355	1.217	0.277	1.044	0.330	0.053	-0.066	0.212	0.027	0.172
MV (3x3)	1.369	1.230	0.278	1.026	0.331	0.052	0.004	0.174	0.026	0.204
MV (3x3x3)	1.255	1.138	0.233	1.027	0.362	0.129	-0.078	0.125	0.064	0.112
RP (2x3)	1.244	1.107	0.275	1.044	0.330	0.055	0.001	0.034	0.028	0.062
RP (3x3)	1.224	1.090	0.268	1.026	0.331	0.062	0.000	0.032	0.031	0.064
RP (3x3x3)	1.230	1.098	0.263	1.027	0.362	0.099	-0.007	0.029	0.050	0.072

					Quarterly					
EW (2x3)	1.216	1.074	0.283	1.044	0.330	0.047	0.006	0.001	0.024	0.030
EW (3x3)	1.198	1.060	0.276	1.026	0.331	0.054	0.006	0.001	0.027	0.034
EW (3x3x3)	1.215	1.076	0.280	1.027	0.362	0.082	0.007	0.001	0.041	0.049
MD (2x3)	1.262	1.125	0.273	1.044	0.330	0.057	0.009	0.044	0.028	0.081
MD (3x3)	1.258	1.125	0.267	1.026	0.331	0.064	0.017	0.050	0.032	0.099
MD (3x3x3)	1.268	1.132	0.273	1.027	0.362	0.089	-0.003	0.064	0.045	0.105
MV (2x3)	1.393	1.253	0.280	1.044	0.330	0.050	-0.033	0.217	0.025	0.209
MV (3x3)	1.366	1.224	0.285	1.026	0.331	0.046	-0.003	0.178	0.023	0.197
MV (3x3x3)	1.242	1.123	0.238	1.027	0.362	0.125	-0.089	0.123	0.062	0.096
RP (2x3)	1.253	1.115	0.275	1.044	0.330	0.055	0.009	0.035	0.027	0.071
RP (3x3)	1.233	1.098	0.268	1.026	0.331	0.062	0.008	0.033	0.031	0.072
RP (3x3x3)	1.239	1.107	0.264	1.027	0.362	0.098	0.001	0.030	0.049	0.081
					Semi-Annually					
EW (2x3)	1.221	1.079	0.282	1.044	0.330	0.047	0.009	0.003	0.024	0.035
EW (3x3)	1.203	1.065	0.276	1.026	0.331	0.054	0.009	0.003	0.027	0.038
EW (3x3x3)	1.219	1.080	0.279	1.027	0.362	0.083	0.009	0.003	0.041	0.053
MD (2x3)	1.262	1.125	0.273	1.044	0.330	0.057	0.005	0.048	0.028	0.081
MD (3x3)	1.256	1.123	0.266	1.026	0.331	0.065	0.008	0.056	0.032	0.097
MD (3x3x3)	1.290	1.151	0.277	1.027	0.362	0.085	0.015	0.068	0.042	0.125
MV (2x3)	1.389	1.252	0.275	1.044	0.330	0.055	-0.038	0.218	0.027	0.208
MV (3x3)	1.410	1.270	0.280	1.026	0.331	0.050	0.048	0.170	0.025	0.243
MV (3x3x3)	1.269	1.146	0.247	1.027	0.362	0.115	-0.059	0.121	0.058	0.119
RP (2x3)	1.256	1.119	0.275	1.044	0.330	0.055	0.010	0.037	0.027	0.075
RP (3x3)	1.237	1.103	0.268	1.026	0.331	0.062	0.011	0.035	0.031	0.077
RP (3x3x3)	1.247	1.113	0.266	1.027	0.362	0.096	0.007	0.032	0.048	0.087
					Annually					
EW (2x3)	1.218	1.077	0.280	1.044	0.330	0.049	0.006	0.003	0.025	0.033
EW (3x3)	1.200	1.063	0.274	1.026	0.331	0.056	0.006	0.003	0.028	0.037
EW (3x3x3)	1.216	1.077	0.277	1.027	0.362	0.085	0.005	0.003	0.043	0.051
MD (2x3)	1.266	1.128	0.276	1.044	0.330	0.054	0.010	0.047	0.027	0.084
MD (3x3)	1.255	1.122	0.267	1.026	0.331	0.063	0.007	0.057	0.032	0.095
MD (3x3x3)	1.291	1.154	0.275	1.027	0.362	0.087	0.010	0.073	0.044	0.127
MV (2x3)	1.407	1.268	0.279	1.044	0.330	0.051	-0.021	0.220	0.026	0.224
MV (3x3)	1.403	1.262	0.282	1.026	0.331	0.049	0.039	0.173	0.024	0.236
MV (3x3x3)	1.373	1.240	0.265	1.027	0.362	0.097	0.020	0.145	0.048	0.213
RP (2x3)	1.259	1.121	0.276	1.044	0.330	0.054	0.011	0.039	0.027	0.077
RP (3x3)	1.242	1.107	0.269	1.026	0.331	0.062	0.013	0.037	0.031	0.081
RP (3x3x3)	1.249	1.115	0.269	1.027	0.362	0.093	0.008	0.034	0.047	0.088

Appendice 7

The table reports the diversification return decomposition from equation (19) for the four different Strategic Beta strategies: equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). These strategies are applied on **cap-weighted** portfolios sorted **dependently** on characteristics. These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. Finally the number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). The last four columns refer to terms from equation (19).The sample period ranges from July 1963 to December 2015.

	Strategy			Constituents Average						
Strategy	Arithmetic Return	Geometric Return	Variance	Geometric Return	Variance	Variance Reduction	Covariance Drag	Adjustment for not being EW	Variance Reduction Benefit	Diversification Return
Dependent Sorted Portfolios										
Monthly										
EW (2x3)	1.092	0.967	0.250	0.935	0.314	0.064	0.000	0.000	0.032	0.032
EW (3x3)	1.120	0.982	0.276	0.946	0.347	0.071	0.000	0.000	0.036	0.036
EW (3x3x3)	1.160	1.015	0.289	0.960	0.400	0.111	0.000	0.000	0.055	0.055
MD (2x3)	1.181	1.062	0.238	0.935	0.314	0.076	-0.003	0.092	0.038	0.126
MD (3x3)	1.190	1.064	0.251	0.946	0.347	0.096	-0.032	0.101	0.048	0.118
MD (3x3x3)	1.223	1.084	0.278	0.960	0.400	0.122	-0.075	0.138	0.061	0.124
MV (2x3)	1.310	1.182	0.258	0.935	0.314	0.056	-0.025	0.243	0.028	0.246
MV (3x3)	1.263	1.135	0.256	0.946	0.347	0.091	-0.036	0.179	0.046	0.189
MV (3x3x3)	1.179	1.058	0.241	0.960	0.400	0.159	-0.115	0.134	0.080	0.098
RP (2x3)	1.148	1.028	0.241	0.935	0.314	0.073	0.005	0.051	0.036	0.092
RP (3x3)	1.168	1.036	0.264	0.946	0.347	0.083	0.000	0.048	0.042	0.090
RP (3x3x3)	1.194	1.062	0.264	0.960	0.400	0.135	-0.010	0.044	0.068	0.102

					Quarterly					
EW (2x3)	1.100	0.975	0.250	0.935	0.314	0.064	0.006	0.001	0.032	0.040
EW (3x3)	1.130	0.992	0.276	0.946	0.347	0.071	0.009	0.001	0.035	0.045
EW (3x3x3)	1.173	1.028	0.289	0.960	0.400	0.111	0.012	0.001	0.055	0.068
MD (2x3)	1.190	1.070	0.240	0.935	0.314	0.074	0.006	0.092	0.037	0.135
MD (3x3)	1.205	1.077	0.255	0.946	0.347	0.092	-0.019	0.104	0.046	0.131
MD (3x3x3)	1.250	1.108	0.284	0.960	0.400	0.116	-0.048	0.138	0.058	0.148
MV (2x3)	1.347	1.213	0.266	0.935	0.314	0.048	0.010	0.244	0.024	0.278
MV (3x3)	1.279	1.150	0.259	0.946	0.347	0.088	-0.014	0.173	0.044	0.203
MV (3x3x3)	1.190	1.067	0.246	0.960	0.400	0.153	-0.098	0.129	0.077	0.107
RP (2x3)	1.161	1.039	0.243	0.935	0.314	0.071	0.017	0.052	0.035	0.104
RP (3x3)	1.184	1.051	0.266	0.946	0.347	0.082	0.015	0.049	0.041	0.105
RP (3x3x3)	1.207	1.073	0.268	0.960	0.400	0.132	0.002	0.046	0.066	0.113
					Semi-Annually					
EW (2x3)	1.107	0.982	0.250	0.935	0.314	0.064	0.011	0.004	0.032	0.046
EW (3x3)	1.137	0.998	0.277	0.946	0.347	0.070	0.013	0.004	0.035	0.052
EW (3x3x3)	1.181	1.037	0.290	0.960	0.400	0.110	0.018	0.004	0.055	0.077
MD (2x3)	1.172	1.051	0.242	0.935	0.314	0.072	-0.015	0.094	0.036	0.116
MD (3x3)	1.190	1.061	0.260	0.946	0.347	0.088	-0.041	0.112	0.044	0.114
MD (3x3x3)	1.246	1.102	0.288	0.960	0.400	0.112	-0.053	0.139	0.056	0.142
MV (2x3)	1.363	1.227	0.271	0.935	0.314	0.043	0.044	0.226	0.021	0.292
MV (3x3)	1.336	1.204	0.263	0.946	0.347	0.084	0.042	0.174	0.042	0.258
MV (3x3x3)	1.188	1.062	0.253	0.960	0.400	0.146	-0.086	0.114	0.073	0.102
RP (2x3)	1.168	1.046	0.245	0.935	0.314	0.069	0.022	0.054	0.034	0.110
RP (3x3)	1.190	1.056	0.268	0.946	0.347	0.079	0.019	0.051	0.039	0.110
RP (3x3x3)	1.213	1.076	0.273	0.960	0.400	0.127	0.006	0.047	0.063	0.117
					Annually					
EW (2x3)	1.109	0.983	0.252	0.935	0.314	0.062	0.012	0.004	0.031	0.048
EW (3x3)	1.136	0.998	0.278	0.946	0.347	0.069	0.012	0.005	0.035	0.051
EW (3x3x3)	1.175	1.030	0.290	0.960	0.400	0.110	0.011	0.005	0.055	0.071
MD (2x3)	1.170	1.047	0.245	0.935	0.314	0.069	-0.014	0.092	0.034	0.112
MD (3x3)	1.193	1.063	0.260	0.946	0.347	0.087	-0.045	0.118	0.043	0.116
MD (3x3x3)	1.248	1.101	0.295	0.960	0.400	0.105	-0.047	0.136	0.052	0.141
MV (2x3)	1.311	1.177	0.269	0.935	0.314	0.045	0.010	0.209	0.023	0.242
MV (3x3)	1.259	1.128	0.262	0.946	0.347	0.085	-0.018	0.157	0.043	0.182
MV (3x3x3)	1.184	1.049	0.271	0.960	0.400	0.129	-0.088	0.113	0.065	0.089
RP (2x3)	1.159	1.035	0.248	0.935	0.314	0.066	0.016	0.051	0.033	0.100
RP (3x3)	1.178	1.044	0.269	0.946	0.347	0.078	0.009	0.049	0.039	0.097
RP (3x3x3)	1.201	1.063	0.278	0.960	0.400	0.122	-0.003	0.045	0.061	0.103

Appendice 8

The table reports the diversification return decomposition from equation (19) for the four different Strategic Beta strategies: equal-weighted (EW), maximum diversification (MD), minimum variance (MV), and risk-parity (RP). These strategies are applied on **equally-weighted** portfolios sorted **dependently** on characteristics. These portfolios can be rebalanced on a monthly (1), quarterly (3), semi-annually (6) or annual (12) basis. Finally the number of portfolios is either six (2x3), nine (3x3) or twenty-seven (3x3x3). The last four columns refer to terms from equation (19).The sample period ranges from July 1963 to December 2015.

	Strategy			Constituents Average						
Strategy	Arithmetic Return	Geometric Return	Variance	Geometric Return	Variance	Variance Reduction	Covariance Drag	Adjustment for not being EW	Variance Reduction Benefit	Diversification Return
				Dependent Sorted Portfolios						
				Monthly						
EW (2x3)	1.262	1.095	0.333	1.070	0.384	0.051	0.000	0.000	0.026	0.026
EW (3x3)	1.274	1.109	0.331	1.076	0.396	0.066	0.000	0.000	0.033	0.033
EW (3x3x3)	1.299	1.130	0.338	1.077	0.443	0.105	0.000	0.000	0.052	0.052
MD (2x3)	1.378	1.221	0.312	1.070	0.384	0.072	-0.014	0.130	0.036	0.152
MD (3x3)	1.432	1.277	0.310	1.076	0.396	0.086	-0.034	0.192	0.043	0.201
MD (3x3x3)	1.540	1.373	0.335	1.077	0.443	0.108	-0.050	0.291	0.054	0.295
MV (2x3)	1.456	1.306	0.302	1.070	0.384	0.083	-0.104	0.298	0.041	0.236
MV (3x3)	1.440	1.289	0.303	1.076	0.396	0.094	-0.101	0.267	0.047	0.213
MV (3x3x3)	1.319	1.189	0.260	1.077	0.443	0.183	-0.155	0.175	0.092	0.111
RP (2x3)	1.318	1.160	0.317	1.070	0.384	0.067	-0.009	0.065	0.034	0.090
RP (3x3)	1.326	1.168	0.315	1.076	0.396	0.081	-0.017	0.069	0.041	0.092
RP (3x3x3)	1.332	1.179	0.306	1.077	0.443	0.137	-0.029	0.063	0.069	0.102

					Quarterly					
EW (2x3)	1.271	1.104	0.333	1.070	0.384	0.051	0.007	0.002	0.026	0.034
EW (3x3)	1.285	1.120	0.331	1.076	0.396	0.065	0.009	0.002	0.033	0.044
EW (3x3x3)	1.314	1.145	0.338	1.077	0.443	0.105	0.014	0.002	0.052	0.068
MD (2x3)	1.381	1.224	0.314	1.070	0.384	0.070	-0.014	0.133	0.035	0.154
MD (3x3)	1.432	1.276	0.313	1.076	0.396	0.083	-0.036	0.194	0.042	0.200
MD (3x3x3)	1.544	1.371	0.346	1.077	0.443	0.097	-0.051	0.297	0.048	0.294
MV (2x3)	1.490	1.338	0.303	1.070	0.384	0.081	-0.079	0.307	0.041	0.268
MV (3x3)	1.468	1.315	0.305	1.076	0.396	0.091	-0.074	0.268	0.045	0.239
MV (3x3x3)	1.330	1.199	0.262	1.077	0.443	0.181	-0.146	0.177	0.090	0.122
RP (2x3)	1.330	1.171	0.318	1.070	0.384	0.067	0.000	0.068	0.033	0.101
RP (3x3)	1.340	1.182	0.316	1.076	0.396	0.080	-0.005	0.071	0.040	0.106
RP (3x3x3)	1.347	1.193	0.308	1.077	0.443	0.135	-0.016	0.065	0.067	0.116
					Semi-Annually					
EW (2x3)	1.276	1.110	0.333	1.070	0.384	0.051	0.009	0.005	0.026	0.040
EW (3x3)	1.292	1.126	0.331	1.076	0.396	0.065	0.011	0.006	0.033	0.051
EW (3x3x3)	1.320	1.151	0.338	1.077	0.443	0.105	0.015	0.007	0.052	0.074
MD (2x3)	1.384	1.227	0.315	1.070	0.384	0.070	-0.016	0.138	0.035	0.157
MD (3x3)	1.427	1.269	0.316	1.076	0.396	0.080	-0.052	0.206	0.040	0.194
MD (3x3x3)	1.592	1.416	0.351	1.077	0.443	0.092	0.000	0.294	0.046	0.339
MV (2x3)	1.533	1.382	0.302	1.070	0.384	0.082	-0.038	0.309	0.041	0.312
MV (3x3)	1.527	1.376	0.302	1.076	0.396	0.094	-0.023	0.276	0.047	0.300
MV (3x3x3)	1.333	1.198	0.270	1.077	0.443	0.173	-0.138	0.172	0.086	0.121
RP (2x3)	1.339	1.180	0.318	1.070	0.384	0.066	0.006	0.071	0.033	0.111
RP (3x3)	1.351	1.193	0.317	1.076	0.396	0.079	0.002	0.076	0.040	0.117
RP (3x3x3)	1.357	1.201	0.313	1.077	0.443	0.130	-0.010	0.068	0.065	0.124
					Annually					
EW (2x3)	1.276	1.110	0.331	1.070	0.384	0.053	0.007	0.006	0.027	0.040
EW (3x3)	1.288	1.124	0.329	1.076	0.396	0.068	0.007	0.007	0.034	0.048
EW (3x3x3)	1.312	1.145	0.335	1.077	0.443	0.108	0.006	0.007	0.054	0.068
MD (2x3)	1.416	1.255	0.321	1.070	0.384	0.063	0.009	0.145	0.031	0.185
MD (3x3)	1.485	1.323	0.325	1.076	0.396	0.071	-0.015	0.226	0.036	0.247
MD (3x3x3)	1.595	1.414	0.362	1.077	0.443	0.081	-0.006	0.303	0.040	0.337
MV (2x3)	1.537	1.383	0.308	1.070	0.384	0.076	-0.022	0.297	0.038	0.313
MV (3x3)	1.523	1.369	0.309	1.076	0.396	0.088	-0.018	0.267	0.044	0.293
MV (3x3x3)	1.405	1.254	0.301	1.077	0.443	0.142	-0.070	0.177	0.071	0.177
RP (2x3)	1.348	1.188	0.320	1.070	0.384	0.064	0.012	0.073	0.032	0.118
RP (3x3)	1.363	1.204	0.319	1.076	0.396	0.078	0.011	0.079	0.039	0.128
RP (3x3x3)	1.369	1.210	0.318	1.077	0.443	0.125	0.000	0.071	0.063	0.133