Information, learning and High–Frequency Trading in electronic call auction markets

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Abstract

This paper investigates the impact of High Frequency quoting on the efficiency of prices in order driven call auction markets. Based on the framework of noisy rational expectations equilibria, we first provide a theoretical model where HFT and non–HFT traders coexist in a transparent order–driven call market. As the pre–call order batching procedure evolves in time, HFTs improve the precision of their signal by collecting public information through various electronic networks. To this extent, price efficiency is accelerated significantly as additional information is impounded into prices. Moreover, the model predicts that price efficiency is positively related to the number of HFTs in the market. To test empirically the prediction of our model, we utilize a unique set of intraday data that includes HFT flagged messages, enabling us to distinguish between computer and human trading. Our empirical analysis provides evidence that HFTs contribute significantly to price efficiency, corroborating our theoretical analysis.

Keywords: Market microstructure, High Frequency Trading, Call auction

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1. Introduction

In the past decade there has been a burst of algorithmic and high–frequency trading activity in financial markets, attributed to the increased computational power and the improvement of electronic network communication systems in terms of speed, accessibility and quality. Nowadays, a large proportion of total volume of transactions in electronic markets (e.g., stock currency and commodity markets) is triggered by High–Frequency Trading programs (hereafter referred to, also, as HFTs) that are designed to operate at ultra–high speeds during the intraday trading process (e.g., millisecond accuracy). In particular, HFTs receive market information, analyse it and, subsequently, submit, adjust or cancel orders into the electronic system, aiming for capital gain opportunities that arise from temporal and/or longer term asset price movements. Furthermore, algorithmic trading is often used by investors and fund managers as a means of reducing the cost of trading, via the application of 'slice and dice' techniques, as well as by market makers for various order handling and liquidity supply services (Brogaard, 2010; Jovanovic and Menkveld, 2011; Hendershott et al., 2011; Chlistalla, 2012; Foucault et al., 2015; Serbera and Paumard, 2016).

The common presence of HFTs and non–high frequency traders (hereafter referred to, also, as nHFTs) has raised several questions regarding the efficient operation and the stability of financial markets. On one hand, it is argued that HFTs reduce transaction costs and improve information speed and price efficiency, while enhancing liquidity through their ability to provide simultaneous access to interlinked electronic trading platforms (Hasbrouck and Saar, 2013; Conrad et al., 2015). On the other hand, the presence of HFTs may increase the possibility of abnormal price variations, through the rapid dissemination of quotes, or, even worse, of market failures (e.g., flash crashes) (Biais and Woolley, 2011; Huang and Wang, 2009; Madhavan, 2012; Hasbrouck and Saar, 2013; Chordia et al., 2013; Kirilenko et al., 2015).

Market regulators, financial practitioners and researchers have already turned their attention on high-frequency trading and its impact on market quality (e.g., liquidity, volatility and information speed). To this extent, there is an ongoing attempt to adjust investors trading strategies to the presence of HFTs, to invent new trading platforms and rules that will encapsulate the properties of high frequency and algorithmic trading, and to reduce the probability of market failures within the existing trading framework (Biais and Woolley, 2011; Madhavan, 2012; OHara, 2015; Foucault et al., 2015).

The present study contributes to the current understanding of the effect of HFTs on the price formation process in modern electronic markets. We examine, both analytically and empirically, the efficiency of prices generated by the call auction trading mechanism within a transparent order–driven setting. More specifically, based on the notion of noisy rational expectations equilibria, we develop a theoretical framework where nHFTs coexist in the market with HFTs (Grossman and Stiglitz, 1976; Grossman, 1976; Grossman and Stiglitz, 1980; Admati, 1985). Due to the nature of call auction mechanisms, one may argue that HFTs may not have an incentive to enter such markets. Indeed, HFTs are designed to trade continuously at ultra–high speeds, whereas trading in call auctions is conducted at a predetermined point in time. Nonetheless, HFTs are able to collect and process information fast and, thus, to form signals that are superior compared to the information set available to the rest of the (human) trading public. Therefore, HFTs may participate in call auction markets not because of their speed advantage but due to their informational advantage.

Herein, we first develop a static competitive equilibrium model where human insiders possess a private signal about the asset value. On the other hand, HFTs view the same signal as human insiders but with greater precision, due to their ability to collect and process the available public information over multiple data streams and electronic trading networks. We show that price efficiency improves with the number of machines in the market increased. We also extend the static model to incorporate strategic trading by including a price elasticity term into the standard Grossman and Stiglitz (1980) framework, as in Rindi (2008). We show that price efficiency is inversely correlated with the price elasticity factor. Subsequently, we build a dynamic (multi–period) model where competitive traders consider past equilibrium prices to adjust their bids. In this respect, we assume that as the order batching process evolves HFTs improve their precision by assimilating more information. Hence, their orders become more informative, accelerating the price discovery process.

To investigate empirically the prediction of our model, we employ a unique set of high–frequency data from the Euronext Paris market that includes the entire trading and order placement history for the CAC 40 stocks in year 2013. A significant advantage of the database used in the present study is that it includes flagged messages that pertain to HFT activity, proxied by the ratio of individual order lifetime over the average order lifetime for modifications and cancellations.[1] Therefore, we are able to distinguish directly between HFTs and nHFTs.

For the purposes of our analysis we re–construct preopening indicative prices and opening prices in terms of both HFT and nHFT data (e.g., Biais et al., 1999; Madhavan and Panchapagesan, 2000; Barclay and Hendershott, 2003; Anagnostidis et al., 2015). Subsequently, we examine empirically the information content of auction prices using the Weighted Price Contribution (WPC) statistic of Barclay and Warner (1993) together with the unbiasedness regression methodology of Biais et al. (1999). We find evidence that HFTs contribute significantly to price efficiency as the opening time approaches.

The paper is organized as follows: Section 2 provides a review of the literature. Section 3 develops the analytical framework. Section 4 presents an empirical application on the Paris market. Finally, Section 5 concludes the paper.

[1] The HFT classification is provided by the AMF (*Autorité des Marchés Financiers*).

2. Literature review

2.1. High frequency and algorithmic trading

The increasing interest on the effect of HFTs on the quality of financial markets has attracted considerable attention and has thus resulted in a large and ever-growing body of microstructure literature. Although HFTs are frequently labelled as a potential source of abnormal price movements, there is overwhelming evidence that trading by means of machine learning methods may actually improve market quality. For instance, Brogaard (2010), Carrion (2013) and Brogaard et al. (2014) study the NASDAQ market and indicate, collectively, that the presence of HFTs accelerates price discovery and price efficiency, while enhancing the provision of liquidity. Hasbrouck and Saar (2013) investigate, also, the 'low latency' trading environment in the NASDAQ market and provide significant evidence of positive correlation between high-frequency trading activity and market quality. In particular, the authors show that HFTs tend to reduce the spreads (i.e., the cost of trading) and short–term volatility, while increasing market depth (i.e., liquidity).

Hendershott et al. (2011) examine empirically the NYSE market and find that the rapid increase of algorithmic trading during the past decade has narrowed the spreads and reduced adverse selection costs for traders. Moreover, the authors find that algorithmic trading has improved market liquidity. In the order–driven setting, Hendershott et al. (2009) find that algorithmic trading in the DAX market improves price efficiency. Similarly, Boehmer et al. (2014) show that algorithmic trading has, on average, a positive effect on price efficiency and liquidity in 42 international equity markets around the world (order and quoted driven systems).

At this point it is pertinent to note that observers often refer to algorithmic trading and high-frequency trading as two equal entities. Nonetheless, HFTs are designed to act, exclusively, within the domain of millisecond or microsecond accuracy, whereas algorithmic trading machines (also known as ATs) aim, mostly, to carry out various order handling and liquidity providing strategies on behalf of investors and market makers, without necessarily being ultra fast. Essentially, high frequency trading is a subset of algorithmic trading, with emphasis placed both on order handling and execution speed (Chlistalla, 2012).

Although there is ample evidence of the positive effect of HFTs on traditional aspects of market quality, like liquidity and price efficiency, there are studies suggesting that, under circumstances, HFTs may contribute to the occurrence of market failures, such as flash crashes, the sharpening of price variations (e.g., widening of spreads) and the rise of systematic risk. Kirilenko et al. (2015), for example, investigate the NASDAQ flash crash on May, 2010. The authors show that the emergence of the sharp volatility spike at 2:45 is significantly related to the fast reaction of HFTs on the rapid depletion of liquidity. Boehmer et al. (2014) also find that algorithmic trading intensity is positively correlated with price volatility. In a theoretical paper, Biais et al. (2015) show that high speed connections (i.e., HFTs) enable investors to profit from trading by searching for desirable quotes at the spot and, by doing so, they impose additional adverse selection costs on slow traders. Chaboud et al. (2014) study algorithmic data from the EBS electronic trading system for currency pairs (euro–dollar, dollar–yen and euro–yen). The results of this study suggest that machine trades are often correlated, giving rise to systematic market risk. Similarly, Brogaard (2010) investigates data from the Nasdaq OMX and finds that order flows generated from HFTs are cross–correlated, possibly imposing additional non–diversifiable (il)liquidity risk on investors' portfolio selection strategies.

Thus far, the majority of related empirical studies have focused on the US Exchanges, mainly the NASDAQ dealers' market and the NYSE specialists' market, whereas order–driven systems have attracted less attention. From this point of view, the present study aims to shade further light on the impact of high frequency trading on the quality of automated electronic markets, focusing on the call auction mechanism.

2.2. Call auction trading

Call auction venues differ significantly from continuous systems in that orders are aggregated without trading, leading to the emergence of crossed supply–demand schedules. Subsequently, at a pre–specified point in time, buy and sell orders are matched and executed at a single equilibrium price. Due to their ability to aggregate disperse information about fundamental values into one single price, call auctions are typically adopted by Exchanges as ideal trading mechanisms during periods of increased market stress, such as the opening and the closing.

Because bidding is only theoretical during the batching period, pre–call communication games may emerge between investors. In particular, the pre–trade period in transparent call mechanisms may offer a learning environment where investors submit, adjust or cancel their orders by observing the flow of indicative clearing prices and the dynamics of the prevailing bids and asks. This is the case in the models of Jordan (1982), Vives (1995) and Biais et al. (1999), where competitive agents (i.e., traders, brokers and/or market makers) react to the disclosed information and drive the asset price to its equilibrium value. Moreover, Medrano and Vives (2001) and Biais et al. (2009) show that the presence of privately informed strategic traders, who attempt to manipulate the market, may add noise to the price discovery process, thus slowing down the speed of information. Lastly, Madhavan and Panchapagesan (2000) show that the presence of liquidity traders may also add noise to the price discovery process, through the submission of aggressive price–inelastic orders (e.g., market orders).

Information revelation and price discovery in the call auction mechanism have been studied empirically in several studies in the past. Ciccotello and Hatheway (2000), Cao et al. (2000), Barclay and Hendershott (2003), Barclay and Hendershott (2008) and Pagano et al. (2013) investigate the NASDAQ dealers' market opening. Madhavan and Panchapagesan (2000) examine the NYSE opening procedure and the role of specialists in price discovery and price efficiency. In the order–driven trading framework, Biais et al. (1999) and Pagano and Schwartz (2003) and Hillion and Suominen (2004) analyze the opening and the closing auctions of the EURONEXT Paris stock market, respectively. Comerton-Forde and Rydge (2006) and Moshirian et al. (2012) study the opening and closing auctions in the Australian order–driven stock market. Hauser et al. (2012) examine the behaviour of opening prices in the Tel Aviv Stock Exchange after the introduction of a random opening time, while Kalay et al. (2004) investigate, in the same market, the elasticity of clearing prices during the preopening period. Lastly, Anagnostidis et al. (2015) analyze the opening price discovery process in the Greek order–driven market.

The aforementioned studies indicate, altogether, the importance of market stability at periods of increased uncertainty, such as the opening or the closing. Additionally, they hint that the call auction is probably, until now, the most efficient mechanism for revealing prices after periods of no trade. The present study examines the case where informed and liquidity traders compete with HFTs within an order–driven transparent call system. Therefore, our results contribute to the current understanding of the impact of HFTs on the price formation process in automated call auction trading, complementing the existing literature. Our analysis is closely related to the recent study of Boussetta et al. (2016) who investigate the role of HTFs in the preopening period of the Paris market. The authors utilize a 2 year sample period, extending from 2012 to 2013, for the 120 SBF Index stocks. The authors report that HFTs play a significant role in the price discovery process through their quoting activity.

3. Analytical backround

3.1. Price efficiency in a static market with competitive traders and asymmetric information

Following previous literature (e.g., Vives, 1995), we consider an automated transparent call market where a single risky asset with random ex–post liquidation value, $u \sim N(0, \sigma_u^2)$,[2] and a risk–less asset are traded. Furthermore, we

[2]For simplicity we assume that $E(u) = 0$.

normalize the return of the risk free asset to zero and, for the sake of simplicity, we assume that market agents do not hold an initial endowment. The trading public consists of K fast HFT machines, M slow–human informed (nHFT), L uninformed and Z liquidity/noise traders. HFTs enter the market to exploit profitable opportunities using information that is collected through various electronic networks. Therefore, HFTs in our analysis are considered as privately informed traders, while information acquisition for HFTs is based on costly resources (Rindi, 2008). Regarding slow (human) informed traders, each one holds a small piece of private information. They view the asset price, however, with less precision compared to HFTs that have access to superior information and increased processing power. Uninformed traders arrive in the market to exploit profitable opportunities, like HFTs, whereas they do not hold private information. Noise traders arrive in the market for liquidity purposes (e.g., to close a short position).

All agents act as price takers; i.e., they do not influence strategically the clearing price with their order placement activity.[3] Further, informed and uninformed traders are assumed to be risk averse and, therefore, they have negative exponential expected utility functions of the form $U(W) = -\exp^{-AW}$, with A and W being the risk aversion coefficient and the terminal wealth, respectively; for simplicity, we assume that all agents have the same risk–aversion parameter (A). Also, HFTs, slow nHFT informed investors, uninformed investors and noise traders have, respectively, the demand functions Q_F (F indicates Fast), Q_S (S indicates Slow), Q_U (U indicates Uninformed) and Q, with $Q \sim N(0, \sigma_Q^2)$.[4]

[3]In transparent order–driven systems strategic insiders may submit manipulative large orders to affect the clearing price. Thus, in this case it is expected that additional noise is incorporated into prices (Medrano and Vives, 2001; Biais et al., 2009). Also, large liquidity investors may often influence the clearing price by submitting price inelastic market orders (Madhavan and Panchapagesan, 2000). In highly traded environments, however, where liquidity is plentiful, the effect of such orders on equilibrium prices is less pronounced (Madhavan and Panchapagesan, 2000).

[4]Our model includes a finite number of traders, whereas several studies focus on markets with a "continuum of traders" (e.g., Aumann, 1964; Vives, 1995; Foucault et al., 2011; Biais et al., 2015). With an infinite number of agents in the market, a conventional assumption is that errors in expectations cancel out and, therefore, signals are perfect in the aggregate. However, call auctions are usually less popular than continuous markets and, thus, in our

Because traders have continuous access to the evolution of the indicative price–volume pair, we assume that they behave rationally in the sense that, before posting their quotes, they update their expectations by observing the disseminated information; risk averse traders submit limit orders whereas liquidity/noise traders submit market orders.[5] After bidding, the market clears and trading takes place.

Uninformed traders do not hold private information and, thus, they use their prior beliefs conditional on the observed indicative price, p, to decide about their future investments. In particular, they maximize their expected utility

$$E[U(W)] = E[-e^{-AW}] \tag{1}$$

where $W = Q_U(u - p^*)$ is the terminal wealth and p^* is the clearing price. Note that since u is normally distributed, W is also normal

$$W \sim N(Q_U(E(u|p) - p^*), Q_U^2 Var(u|p)). \tag{2}$$

Using the properties of normal distribution, it is straightforward to show that maximizing the expected utility is equivalent to maximizing the following quantity (certainty equivalent)

$$E(W) - \frac{A}{2}\sigma_W^2. \tag{3}$$

Taking first order conditions, the following demand function is derived for uninformed traders

$$Q_U = \frac{E(u|p) - p}{A\,Var(u|p)}. \tag{4}$$

In contrast to uninformed traders, slow (human) informed investors hold a

analysis we prefer to assume a finite number of market participants. From this point of view, our model is closer to reality.

[5] We assume that the demand of liquidity trading is exogenous.

private signal s with $s \sim N(0, \sigma_s^2)$, such that

$$s = u + e \tag{5}$$

where $e \sim N(0, \sigma_e^2)$ is a residual term and $e \perp u$ ($\sigma_e^2 > 0$).[6] On the other hand, because HFTs have i) fast access to multiple data streams and electronic trading netwroks and ii) increased computational and analytical capabilities, they form a unique private signal that improves their position against human traders (informed and uninformed).[7] Therefore, it is reasonable to assume that HFTs observe the same signal s, as slow privately informed investors, but with greater precision, so that $E_F(e) = E_S(e) = 0$ and $Var_S(e) = \sigma_e^2 = \sigma_S^2 \geq \sigma_F^2$ (S and F are for Slow and Fast respectively), where σ_F^2 is the variance of HFTs' signal with $\sigma_S^2 \in [\sigma_F^2, +\infty)$ ($\sigma_{S,F}^2 > 0$). It, also, follows that $1/\sigma_F^2 \geq 1/\sigma_S^2$.[8] Similar to equation (4), we derive the demand function for informed traders

$$Q_{F,S} = \frac{E(u|s,p) - p}{A\,Var(u|s,p)} = \frac{E(u|s) - p}{A\,Var(u|s)} \tag{6}$$

where F and S correspond to HFTs and slow nHFT informed traders. Notice that equation (6) implies that informed traders' signal s contains all information about the asset price. Thus, informed traders are indifferent to observing the indicative price p.

Proposition 3.1. *There exists a unique static linear equilibrium such that price efficiency is given by the following relation*

$$Var(u|p = \alpha s + \beta Q)^{-1} = \frac{1}{\sigma_u^2} + \frac{1}{\sigma_e^2 + (\beta/\alpha)^2 \sigma_Q^2}, \tag{7}$$

[6] For simplicity we have assumed that all insiders observe the same private signal, s.

[7] In a similar manner, Biais et al. (2015) assume that the signal of HFTs is perfect in the aggregate, within a market consisting of infinite traders. Our model, however, differs in that we consider a finite amount of traders.

[8] Our model is similar to that of Hirshleifer and Luo (2001) who consider a competitive market with rational, overconfident and liquidity traders. In their framework, rational traders perceive the true distribution of e, whereas overconfident traders believe, falsely, that the variance of e is smaller than the true variance.

where α and β are (uniquely defined) real numbers such that

$$\frac{\beta}{\alpha} = \frac{ZA}{K/\sigma_F^2 + M/\sigma_S^2}. \tag{8}$$

Proof. We begin by assuming that uninformed traders view the asset price as a linear function of the private signal s and the demand of noise traders (Rindi, 2008; De Jong and Rindi, 2009):

$$\hat{p} = \alpha s + \beta Q, \tag{9}$$

for some real numbers α and β. Uninformed traders consider the above–mentioned conjecture to form the conditional expectation and variance of u. In view of the theorem of projection for normal variables, the following conditional moments are obtained:

$$E(u|p = \alpha s + \beta Q) = \frac{\alpha\sigma_u^2}{\alpha^2\sigma_u^2 + \alpha^2\sigma_e^2 + \beta^2\sigma_Q^2}p \tag{10}$$

and

$$Var(u|p = \alpha s + \beta Q) = \sigma_u^2\left(\frac{\alpha^2\sigma_e^2 + \beta^2\sigma_Q^2}{\alpha^2\sigma_u^2 + \alpha^2\sigma_e^2 + \beta^2\sigma_Q^2}\right). \tag{11}$$

Accordingly, given that u and e are jointly normal, we use the projection theorem to derive the conditional expectations for informed traders:

$$E_{F,S}(u|s) = s\frac{\sigma_u^2}{\sigma_u^2 + \sigma_{F,S}^2} = \gamma s \tag{12}$$

and

$$Var_{F,S}(u|s) = \sigma_{F,S}^2\frac{\sigma_u^2}{\sigma_u^2 + \sigma_{F,S}^2} = \gamma\sigma_{F,S}^2 \tag{13}$$

where $\gamma = \sigma_u^2/(\sigma_u^2 + \sigma_{F,S}^2)$ and, as before, F and S correspond to fast HFT and slow nHFT informed investors.

The market condition to calculate the clearing price is that the excess demand is zero:

$$KQ_F + MQ_S + LQ_U + ZQ = 0 \tag{14}$$

Using the conditional expectations derived earlier, equation (14) becomes

$$K\frac{\gamma s - p}{\gamma A\, \sigma_F^2} + M\frac{\gamma s - p}{\gamma A\, \sigma_S^2} + L\frac{E(u|p) - p}{A\, Var(u|p)} + ZQ = 0. \tag{15}$$

Next, we rearrange equation (15) to acquire

$$\begin{aligned} s\left(\frac{K}{A\sigma_F^2} + \frac{M}{A\,\sigma_S^2}\right) + ZQ = -L\frac{E(u|p)}{A\,Var(u|p)} + \\ p\frac{L}{A\,Var(u|p)} + p\left(\frac{K}{\gamma A\sigma_F^2} + \frac{M}{\gamma A\,\sigma_S^2}\right), \end{aligned} \tag{16}$$

which is equivalent to

$$s\left(\frac{K}{A\sigma_F^2} + \frac{M}{A\,\sigma_S^2}\right) + ZQ = Rp = R(\alpha s + \beta Q) = R\alpha s + R\beta Q, \tag{17}$$

for some constant R.

At the final step of our analysis, we impose rational expectations by matching the coefficients on the left and the right hand sides of equation (17).[9] By doing so, we obtain the following conditions that uniquely determined the unknown constants: $R = Z/\beta$, $R\alpha = \dfrac{K}{A\sigma_F^2} + \dfrac{M}{A\,\sigma_S^2}$ and $\dfrac{\alpha}{\beta} = \dfrac{K}{ZA\sigma_F^2} + \dfrac{M}{ZA\,\sigma_S^2}$.

The proof is concluded upon rewriting equation (11) to obtain price-efficiency as follows,

$$Var(u|p = \alpha s + \beta Q)^{-1} = \frac{1}{\sigma_u^2} + \frac{1}{\sigma_e^2 + (\beta/\alpha)^2\sigma_Q^2}, \tag{18}$$

$$\frac{\beta}{\alpha} = \frac{ZA}{K/\sigma_F^2 + M/\sigma_S^2}, \tag{19}$$

where, as before, $\sigma_S^2 \in [\sigma_F^2, +\infty)$, $\sigma_S^2 > 0$. Equation (18) represents the informativeness of p about the liquidation value, u, of the asset (Vives, 1995). □

We observe that the risk aversion coefficient A is inversely correlated with

[9] Realizations coincide with expectations in equilibrium.

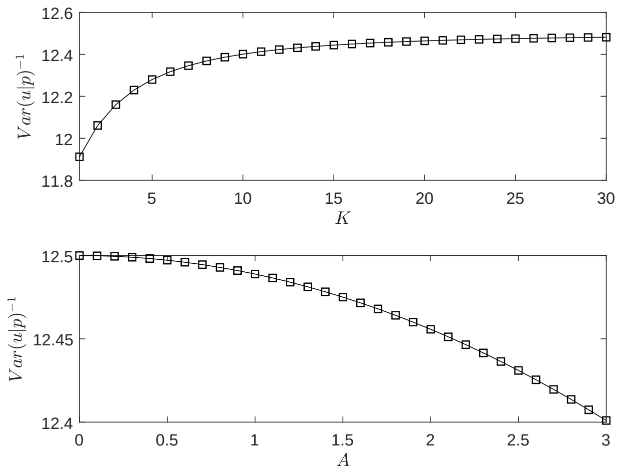

Figure 1: Top: $Var(u|p)^{-1}$ as a function of the number of HFTs (K), $A = 3, Z = 10, M = 10, \sigma_u^2 = 0.4, \sigma_s^2 = 0.5, \sigma_F^2 = 0.05, \sigma_S^2 = 0.1$ and $\sigma_Q^2 = 0.1$. Bottom: $Var(u|p)^{-1}$ as a function of the risk aversion coefficient (A), $K = 10, Z = 10, M = 10, \sigma_u^2 = 0.4, \sigma_s^2 = 0.5, \sigma_F^2 = 0.05, \sigma_S^2 = 0.1$ and $\sigma_Q^2 = 0.1$.

market efficiency; when traders are more/less risk averse (that is, the risk aversion coefficient A increases/decreases) market efficiency is lower/higher. This result is natural in the sense that bids become less/more aggressive and, therefore, information is impounded into prices with lower/higher speed (see Figure 1). Also, noise trading (i.e., Z) is inversely correlated with price efficiency (Grossman and Stiglitz, 1976, 1980).

Regarding high frequency trading, it is clear that as the number of machines, K, increases, price efficiency improves (see Figure 1). The same result holds for the accuracy of HFTs signal. Indeed, as σ_F^2 decreases, price informativeness increases and vice versa. Moreover, as the number of human informed traders, M, increases, clearing prices reflect more information. They are never fully revealing, however, due to the presence of noise in the market.

3.2. *Price efficiency in a static market with strategic traders and asymmetric information*

In this Section, following Rindi (2008), we consider a transparent call auction market where informed traders act strategically; that is, they take into account the impact of their own orders on the clearing price. As before, to maximize expected utility, informed traders use the following first order condition:

$$\gamma s - p - \frac{\theta p}{\theta Q_S} Q_S - A Q_S \gamma \sigma_S^2 = 0. \tag{20}$$

In contrast to the competitive equilibrium, the price p in equation (20) is a function of informed traders' demand. Therefore, price impact (i.e., $\frac{\theta p}{\theta Q_S}$) is now incorporated into the model. Solving for Q_S, we acquire

$$Q_S = \frac{\gamma s - p}{\omega_S + A\, \gamma \sigma_S^2}, \tag{21}$$

where $\omega_S = \dfrac{\theta p}{\theta Q_S}$ is the price elasticity term. Similarly, we can derive the demand functions for HFTs and uninformed traders:

$$Q_F = \frac{\gamma s - p}{\omega_F + A\, \gamma \sigma_F^2}, \tag{22}$$

$$Q_U = \frac{E(u|p) - p}{\omega_U + A\, Var(u|p)}, \tag{23}$$

where ω_F and ω_U are the corresponding price impact rates. We can now formulate and prove the analogue of proposition 3.1 for the strategic equilibrium.

Proposition 3.2. *There exists a unique strategic linear equilibrium such that price efficiency is given by the following relation*

$$Var(u|p = \alpha s + \beta Q)^{-1} = \frac{1}{\sigma_u^2} + \frac{1}{\sigma_e^2 + (\beta/\alpha)^2 \sigma_Q^2}, \tag{24}$$

where α and β are (uniquely defined) real numbers with

$$\frac{\beta}{\alpha} = \frac{Z}{K\gamma/(\omega_F + \gamma A\ \sigma_F^2) + M\gamma/(\omega_S + \gamma A\ \sigma_S^2)}. \tag{25}$$

Proof. As in the proof of Proposition 3.1, we postulate that uninformed traders view the asset price as a linear combination of the private signal s and the demand of noise traders,

$$\hat{p} = \alpha s + \beta Q, \tag{26}$$

for some real numbers α and β, to be determined. Next, we substitute the expressions for the demand functions into the market clearing condition (14), which yields,

$$K\frac{\gamma s - p}{\omega_F + \gamma A\ \sigma_F^2} + M\frac{\gamma s - p}{\omega_S + \gamma A\ \sigma_S^2} + L\frac{E(u|p) - p}{\omega_U + A\ Var(u|p)} + ZQ = 0. \tag{27}$$

Upon rearrangement, equation (27) becomes

$$\begin{aligned} s\left(\frac{K\gamma}{\omega_F + \gamma A\sigma_F^2} + \frac{M\gamma}{\omega_S + \gamma A\ \sigma_S^2}\right) + ZQ = \\ -L\frac{E(u|p)}{A\ Var(u|p)} + p\frac{L}{A\ Var(u|p)} + \\ p\left(\frac{K}{\omega_F + \gamma A\sigma_F^2} + \frac{M}{\omega_S + \gamma A\ \sigma_S^2}\right), \end{aligned} \tag{28}$$

that is equivalent to

$$s\left(\frac{K\gamma}{\omega_F + \gamma A\sigma_F^2} + \frac{M\gamma}{\omega_S + \gamma A\ \sigma_S^2}\right) + ZQ = Rp = R(\alpha s + \beta Q) = R\alpha s + R\beta Q, \tag{29}$$

for some constant R. Matching the coefficients yields the following three conditions for the unique determination of the parameters,

$$R = Z/\beta, \tag{30}$$

$$R\alpha = \frac{K\gamma}{(\omega_F + \gamma A\sigma_F^2)} + \frac{M\gamma}{(\omega_S + \gamma A\ \sigma_S^2)} \tag{31}$$

and

$$\frac{\alpha}{\beta} = \frac{K\gamma(\omega_S + \gamma A\,\sigma_S^2) + M\gamma(\omega_F + \gamma A\,\sigma_F^2)}{Z(\omega_F + \gamma A\sigma_F^2)(\omega_S + \gamma A\,\sigma_S^2)}. \tag{32}$$

Further, by manipulating equation (32), we acquire

$$\frac{\beta}{\alpha} = \frac{Z}{K\gamma/(\omega_F + \gamma A\,\sigma_F^2) + M\gamma/(\omega_S + \gamma A\,\sigma_S^2)}. \tag{33}$$

We can now utilize equations (18) and (33) to compute price efficiency for the strategic equilibrium and thus complete the proof. □

Notice that the risk aversion coefficient is inversely correlated with price efficiency, as in the competitive market model. Similarly, the increase of noise trading, Z, has a negative effect on price efficiency. The amounts of insider (M) and HF (K) trading are both positively correlated with price efficiency, as in the competitive equilibrium. Also, as the precision of HFTs increases, the information content of equilibrium prices improves. More importantly, notice that as the price impact of informed, uninformed and/or HF traders increases, price efficiency diminishes. In other words, when traders consider the effect of their demands on the price formation process, they are willing to trade less aggressively and, therefore, information is incorporated into prices with a smaller rate. By contrast, when traders increase competition ($\omega_{F,S}$ decreases), information is incorporated into prices faster (Rindi, 2008). In the extreme case where ω is equal to zero, we obtain the pure competitive equilibrium.

3.3. Price efficiency in a dynamic market with competitive traders and asymmetric information

3.3.1. Derivation of the equilibrium

In this section, we extend the derived static model to a time-dependent call-auction mechanism. We consider that the order batching takes place during a predefined time period from t_{start} to t_{end} that can, without loss of generality, be normalized so that $t_{start} = 0$ and $t_{end} = 1$. Bidding can occur at discrete time increments t_n and in order to simplify the algebra we assume that the sequence

$\{t_n\}$ is infinite and $\lim t \to \infty = 1$. With this premise all variables in the static model are *a priori* functions of time.

At time instance t_n we have,

$$E[U(W_n)] = E[-e^{-AW_n}] \tag{34}$$

where $W_n = Q_{U_n}(u - p_n)$ is the terminal wealth and p_n is the clearing price. Similarly to the static case, maximization of the expected utility is equivalent to the maximization of,

$$E(W_n) - \frac{A}{2}\sigma^2_{W_n}. \tag{35}$$

For the case of uniformed traders, first order conditions yield the following demand function

$$Q_{U_n} = \frac{E(u|p^{n-1}) - p_n}{A\,Var(u|p^{n-1})}, \tag{36}$$

where $p^{n-1} = \{p_0, p_1, ..., p_{n-1}\}$ is the set of past equilibrium prices that all traders observe.

As before, nHFT informed investors receive a private signal s with $s \sim N(0, \sigma^2_S)$, such that

$$s = u + e \tag{37}$$

where $e \sim N(0, \sigma^2_e)$ and $e \perp u$ ($\sigma^2_e > 0$). On the other hand, HFTs observe the same signal s but with greater precision than human insiders, so that

$$s = u + e \tag{38}$$

with $E_F(e) = 0$ and $Var_F(e) = \sigma^2_F \leq \sigma^2_e = \sigma^2_S$ ($\sigma^2_S > 0$). Moreover, we assume that as the call auction procedure evolves, HFTs aggregate information and, in turn, their position improves relative to that of nHFT insiders; i.e. we assume that $\sigma_F = \sigma_{F_n}$, the function $|\sigma^2_{F_n} - \sigma^2_S|$ is increasing with n and that $\lim_{n\to\infty} \sigma_{F_n} = \sigma_{F_\infty}$.

By employing the same first order conditions as in (36) we can derive the demand function for informed traders

$$Q_{F_n,S_n} = \frac{E(u|s,p^{n-1}) - p_n}{A\,Var(u|s,p^{n-1})} = \frac{E(u|s) - p_n}{A\,Var(u|s)} \tag{39}$$

where F and S correspond to HFTs and slow nHFT informed traders. We are now ready to prove the existence and uniqueness of a linear equilibrium for the dynamic call auction.

Proposition 3.3. *At each time instance, t_n, there exists a unique linear dynamic equilibrium such that price efficiency is given by the following relation*

$$Var(u|p_n)^{-1} = \left(\sigma_u^2 - \frac{\sigma_S^4}{\sigma_s^2 + A^2 Z_n^2 \frac{\sigma_{Q_n}^2}{\frac{K_n^2}{\sigma_{F_n}^4} + \frac{M_n^2}{\sigma_S^4}}} \right)^{-1}. \tag{40}$$

where Z_n, K_n and M_n denote the magnitude of noise trading, the number of HFTs and the number of nHFTs, correspondingly.

Proof. For the derivation of the equilibrium, we assume that uninformed traders view the asset price at time instance t_n as a linear function of the private signal $s_n = s$, the demand of noise traders Q_n and the set of past prices p^{n-1}. In other words, we assume that, at all times, the following (linear) flow rule holds,

$$p_n = \alpha_n s_n + \beta_n Q_n + \phi(p^{n-1}), \tag{41}$$

with α_n, β_n denoting components of sequences of real numbers, to be determined, and ϕ being a linear functional.[10] The flow of available information is illustrated in Figure 2. Uniformed traders utilize the above flow rule to form the conditional expectation and variance of u, $E(u|p^{n-1})$ and $Var(u|p^{n-1})$, re-

[10]We assume that the private signal s and the public signal p^{n-1} are independent, conditionally on the liquidation value of the asset.

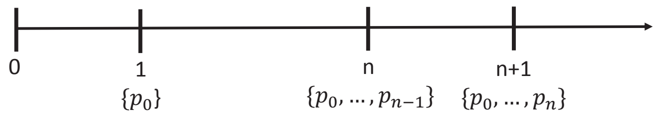

Figure 2: Evolution of the order batching process. In each round traders observe the sequence of past prices to form their expectations.

spectively. These can be easily derived using the projection theorem for normal variables, as shown in the static version of the model presented earlier. For informed traders, the following conditional expectations are obtained:

$$E_{F,S}(u|s) = s\frac{\sigma_u^2}{\sigma_u^2 + \sigma_{F_n,S}^2} = \gamma_{F,S}s \tag{42}$$

and

$$Var_{F,S}(u|s) = \sigma_{F_n,S}^2\frac{\sigma_u^2}{\sigma_u^2 + \sigma_{F_n,S}^2} = \gamma_{F,S}\sigma_{F_n,S}^2 \tag{43}$$

where $\gamma_{F,S} = \sigma_u^2/(\sigma_u^2 + \sigma_{F_n,S}^2)$.

Now, in order to calculate the clearing price we set the excess demand to zero:

$$K_nQ_{F_n} + M_nQ_{S_n} + L_nQ_{U_n} + Z_nQ = 0 \tag{44}$$

Using the conditional expectations derived earlier, equation (44) becomes

$$K_n\frac{\gamma_F s - p_n}{\gamma_F A\ \sigma_{F_n}^2} + M_n\frac{\gamma_S s - p_n}{\gamma_S A\ \sigma_S^2} + L_n\frac{E(u|p^{n-1}) - p_n}{A\ Var(u|p^{n-1})} + Z_nQ = 0. \tag{45}$$

Solving for the equilibrium price yields

$$p_n = s\left(\frac{K_n}{A\ \sigma_{F_n}^2} + \frac{M_n}{A\ \sigma_S^2}\right)B^{-1} + L_n\frac{E(u|p^{n-1})}{A\ Var(u|p^{n-1})}B^{-1} + Z_nQ_nB^{-1}, \tag{46}$$

with

$$B = \frac{K_n}{\gamma_F A\ \sigma_{F_n}^2} + \frac{M_n}{\gamma_S A\ \sigma_S^2} + \frac{L_n}{A\ Var(u|p^{n-1})}. \tag{47}$$

In equilibrium, realizations must coincide with expectations and, therefore, by matching the coefficients we acquire the following conditions,

$$\alpha_n = \left(\frac{K_n}{A\,\sigma^2_{F_n}} + \frac{M_n}{A\,\sigma^2_S} \right) B^{-1} \tag{48}$$

$$\beta_n = Z_n B^{-1} \tag{49}$$

$$\phi(p^{n-1}) = L_n \frac{E(u|p^{n-1})}{A\,Var(u|p^{n-1})} B^{-1}. \tag{50}$$

To infer on price efficiency, we first re–write equation (46) as follows:

$$\Gamma^{-1} B p_n - \Gamma^{-1} L_n Q_{U_n} = s + \Gamma^{-1} Z_n Q_n, \tag{51}$$

where

$$\Gamma = \frac{K_n}{A\,\sigma^2_{F_n}} + \frac{M_n}{A\,\sigma^2_S}. \tag{52}$$

Uninformed traders know Q_{U_n} (and p_n that is observed) and, thus, they can extract from the clearing price, using equation (51), the following (noisy) signal

$$\Theta = s + \Gamma^{-1} Z_n Q_n = s + \frac{A\,\sigma^2_{F_n}\sigma^2_S}{K_n \sigma^2_S + M_n \sigma^2_{F_n}} Z_n Q_n, \tag{53}$$

which is a linear transformation of p_n and, therefore, observationally equivalent with p_n; that is, $E(u|p_n) = E(u|\Theta)$ and $Var(u|p_n) = Var(u|\Theta)$ (Rindi, 2008;

De Jong and Rindi, 2009). Finally, we calculate price efficiency,

$$Var(u|p_n)^{-1} = Var(u|\Theta)^{-1} = \tag{54}$$

$$\left(\sigma_u^2 - \frac{\sigma_S^4}{\sigma_s^2 + A^2 Z_n^2 \dfrac{\sigma_S^4 \sigma_{F_n}^4 \sigma_{Q_n}^2}{K_n^2 \sigma_S^4 + M_n^2 \sigma_{F_n}^4}} \right)^{-1} = \tag{55}$$

$$\left(\sigma_u^2 - \frac{\sigma_S^4}{\sigma_s^2 + A^2 Z_n^2 \dfrac{\sigma_{Q_n}^2}{\dfrac{K_n^2}{\sigma_{F_n}^4} + \dfrac{M_n^2}{\sigma_S^4}}} \right)^{-1} . \tag{56}$$

and thus complete the proof. □

It is easy to observe that as the order batching procedure evolves and the precision of HFTs, $1/\sigma_{F_n}^2$, increases, price efficiency improves ceteris paribus. The same argument holds for the number of HFT machines, K_n; price efficiency improves with K_n increased and vice versa. The effect of HFTs and nHFTs appears to be symmetric; however, this is not the case in our model since the temporal relaxation of the accuracy of HFTs $\sigma_{F_n}^2$ affects price efficiency in a significant way.

3.3.2. On the speed of information revelation

Having derived the equilibrium for the dynamic case, we advert to the study of the speed of information revelation. More specifically, we examine how the inclusion of HFTs affects price efficiency $Var(u|p_n)^{-1}$ and the rate at which $Var(u|p_n)^{-1}$ approaches its equilibrium value as $n \to \infty$. The key point in our asymptotic analysis is our assumption that the accuracy of the signal of HFTs is a function of time and, in particular, a sequence $\sigma_{F_n}^2$ that converges to $\sigma_{F_\infty}^2$ as $n \to \infty$.

Consider, first, equation (54) and fix the number of machines (K_n), insiders (M_n) and noise traders (Z_n, Q_n). Evidently, (54) is now free from memory–

related terms with the exception of $\sigma^2_{F_n}$. As n increases, the only term that changes and thus drives price efficiency towards its equilibrium value is $\sigma^2_{F_n}$. In this case, the global maximum of price efficiency is

$$\lim_{n\to\infty} Var(u|p_n)^{-1} = \tag{57}$$

$$\left(\sigma_u^2 - \frac{\sigma_S^4}{\sigma_s^2 + A^2 Z_n^2 \dfrac{\sigma^2_{Q_n}}{\dfrac{K_n^2}{(\sigma^2_{F_\infty})^2} + \dfrac{M_n^2}{\sigma_S^4}}} \right)^{-1}, \tag{58}$$

where $\sigma^2_{F_\infty} \leq \sigma^2_S$ is the limit of $\sigma^2_{F_n}$ as n approaches infinity.

Next, assume that the variance of the signal of HFTs $\sigma^2_{F_n}$ converges to $\sigma^2_{F_\infty}$ with a rate of convergence n^{-r}, $r > 0$; note that the rate can be sub–linear or super–linear depending on whether r is smaller or larger than unity.[11] With this rate fixed, we wish to determine the rate of convergence of $Var(u|p_n)^{-1}$. It is easy to show, after a straightforward series of calculations, that the term $\sigma^4_{F_n} = (\sigma^2_{F_n})^2$ in equation (54) is present both in the nominator and the denominator,

$$Var(u|p_n)^{-1} = \left(\frac{(\sigma_u^2 M_n^2 \sigma_s^2 + \sigma_u^2 A^2 Z_n^2 \sigma^2_{Q_n} \sigma_S^4 - \sigma_S^4 M_n^2)\sigma^4_{F_n} + (\sigma_u^2 \sigma_s^2 \sigma_S^4 - \sigma_S^8) K_n^2}{\sigma_s^2 \sigma_S^4 K_n^2 + (M_n^2 \sigma_s^2 + A^2 Z_n^2 \sigma^2_{Q_n} \sigma_S^4)\sigma^4_{F_n}} \right)^{-1}. \tag{59}$$

Therefore, using standard calculus for sequences of numbers, we can easily deduce that the rate of convergence of $Var(u|p_n)^{-1}$ is n^{-2r}. This result asserts that as the order batching process evolves in time (i.e., as consecutive rounds yield indicative clearing prices) and HFTs improve the precision of their signal, the rate of information revelation is accelerated drastically. To illustrate this finding, Figure 3 plots the evolution of $Var(u|p_n)$ after $n = 10$ rounds for two

[11] Recall that in our model the precision of HFTs increases with time, relative to that of human insiders. Therefore, it is expected that it will converge towards its limit value, at some rate, as time evolves.

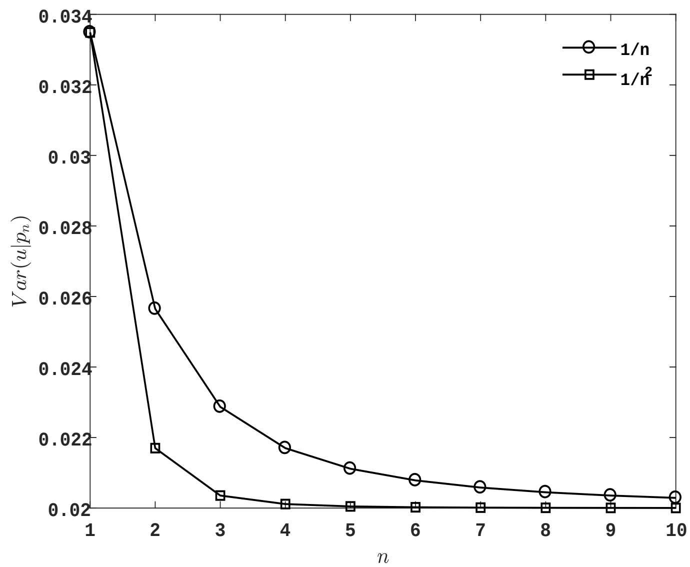

Figure 3: The evolution of $Var(u|p_n)$ after $n = 10$ rounds for two cases: i) $\sigma^2_{F_n} = \sigma^2_e(1/n)$ and ii) $\sigma^2_{F_n} = \sigma^2_e(1/n^2)$. We also fix the following parameters: $A = 3, Z_n = 10, K_n = 10, M_n = 10, \sigma^2_u = 0.2, \sigma^2_s = 0.5, \sigma^2_S = \sigma^2_e = 0.3$ and $\sigma^2_{Q_n} = 0.1$.

cases: i) $\sigma^2_{F_n} = \sigma^2_e(1/n)$ and ii) $\sigma^2_{F_n} = \sigma^2_e(1/n^2)$.[12] We also fix the following parameters: $A = 3, Z_n = 10, K_n = 10, M_n = 10, \sigma^2_u = 0.2, \sigma^2_s = 0.5, \sigma^2_S = \sigma^2_e = 0.3$ and $\sigma^2_{Q_n} = 0.1$. Evidently, the error decays after each round while converging to its limit. Notice, also, the effect of the rate of convergence of $\sigma^2_{F_n}$, as for case (ii) the residual noise decays considerably faster; in fact, only after a few rounds.

3.4. Discussion

In the present Section we presented an analytical background that describes the bidding activity in a transparent order–driven single call market, where HFTs coexist with human insiders and uninformed traders. Our theoretical framework contributes to the relative microstructure literature on rational ex-

[12]In round $n = 1$ we assume that $\sigma^2_{F_n} = \sigma^2_e = \sigma^2_S$

pectations models (e.g., Vives, 1995), incorporating the effect of HF quoting on the asset equilibrium price. The main findings from the model, regarding the activity of HFTs in the auction market, are as follows:

- Price efficiency in the call auction market increases (decreases) with the number of HFTs increased (decreased). In other words, as the amount of HF quoting increases (decreases), clearing prices become less (more) noisy.

- Price efficiency increases with the precision of the signal of HFTs increased. HFTs gather information from multiple data sources and faster than any other human (slow) trader, forming superior signals about the true value of the asset. Thus, as the order batching procedure evolves, the informational content of HF quotes submitted into the system increases and, in turn, equilibrium prices become less noisy.

In addition to these findings, we provide some basic features of the speed of convergence of the asset price to its equilibrium value. In specific, we show that the asset price incorporates information at a rate of (n^{-2r}), where n^{-r} is the speed with which HFTs aggregate public information. Thus, HFTs contribute significantly to the price discovery process.

In the subsequent analysis we examine empirically the predictions of our model using an intraday data set from the order driven Paris stock market opening call. The Paris preopening period is transparent and, therefore, market participants are able to observe the evolution of the indicative price–volume pair and, in turn, to submit or adjust their orders. Thus, the order batching process can be viewed as a sequence of theoretical market clearings where rational agents drive the price to its equilibrium value as time evolves. Consequently, the pre–opening period constitutes a natural laboratory to examine our model findings.

Our study is closely related to that of Biais et al. (1999), who find that preopening indicative prices in the Paris Bourse become more informative as the clearing time approaches; the authors refer to this feature as a "learning pattern". In this respect, they argue that investors learn from each other by observing the preopening order flow, thus submitting more informative orders as the opening time approaches. Herein, we are interested to examine the presence of such learning patterns and, more important, to investigate the magnitude of the effect of HF quotes on the informativeness of preopening and opening prices in the Paris market. As predicted by our model, we expect that HFTs accelerate price discovery and price efficiency due to the information content of their quotes. Thus, we expect that prices predict better the true value of the asset when machines are active in the market.

4. An empirical application

4.1. The Paris Euronext market

Stock trading at the Euronext Paris Platform is conducted in two main ways: a) the order–driven market model and b) the LP quote driven market model. The former is the one examined in the present study, whereas the latter concerns securities which are traded continuously via the quotation of designated Liquidity Providers (LPs), much like the operation of the NASDAQ dealers' market.

The order–driven system includes either continuous or periodic auction trading. The first mechanism concerns the more liquid securities, like those comprising the CAC 40 Index, whereas the second is for the less liquid securities. The continuous double auction mechanism, examined herein, is operated under the following daily time schedule:[13]

1. 07:15–09:00 Preopening phase – Order accumulation period
2. 09:00 Opening auction
3. 09:00–17:30 Main trading session: Continuous session
4. 17:30–17:35 Pre–closing phase – Order accumulation period
5. 17:35 Closing auction

[13] The trading day schedule can be found in the 'Euronext Notice 4-01 Universal Trading Platform Trading manual', available at: `https://www.euronext.com/en/regulation/harmonised-rules`

6. 17:35–17:40 Trading at the last phase (at the close)
7. 17:40–07:15 After hours trading

The opening call auction procedure lasts 1 hour and 45 minutes. During this time period investors are allowed to submit, modify, or cancel orders, while observing the disclosed information on the evolution of the indicative clearing price–volume pair and the prevailing bid–ask quotes. Since trading is absent, all orders are stored into the central limit order book with price–time execution priority. Three main types of orders are allowed during the preopening period: a) market on opening orders, b) pure market orders and c) limit orders.[14] Also, market on opening orders and pure market orders have priority against limit orders at the time of the auction. Figure 4 illustrates the formulation of crossing supply and demand lines, due to the absence of trade, for a hypothetical set of limit prices and quantities. Notice that because stock prices are discrete, it is possible that more than one equilibrium values are present.

After the end of the accumulation period, the electronic system considers the supply–demand schedule formed by the queuing orders, seeking for the price that maximizes the trading volume; that is, the equilibrium value. If the maximum volume principle suggests more than one equilibrium prices, then the opening price is set according to the minimum volume surplus principle. Lastly, if more than one prices satisfy the minimum surplus principle, then the system fixes as the opening price the one that is closer to the reference price; the latter is usually the price of the last trade before the preopening period. After the opening price is set, buy and sell orders are matched and executed in a single trade and at a single opening price. Unexecuted market or limit orders are sent forth to the main session with the original price and time priority; market orders are stored as limit orders at the opening price.[15]

[14] Because they do not include price preference, market and on open sell (buy) orders are aggregated at the best ask (bid).

[15] For additional details on the EURONEXT Paris stock market opening and main sessions, see, also, Biais et al. (1999) and Biais et al. (1995), respectively.

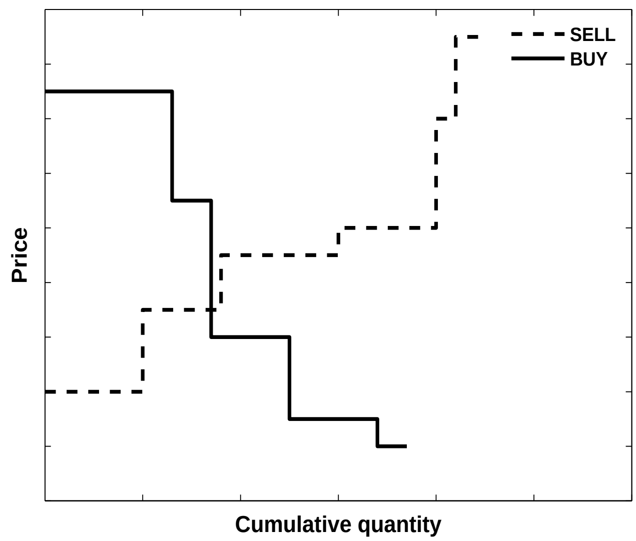

Figure 4: A hypothetical supply–demand schedule at the opening.

4.2. Data sample

The data sample used in the present study is retrieved from the EUROFIDAI–BEDOFIH high frequency database and includes 32 stocks from the CAC 40 Index in 2013 (251 trading days).[16] The sample encompasses two main files: i) trades and ii) orders. The first contains information about the trading history in the EURONEXT Paris market. More specifically, the data set includes information about the time (accurate to the microsecond), the price and the quantity of negotiations. The second includes information about the order placement history; time of submission, price, size, side, duration, type, validity and time of release from the system (either because of execution or because of cancellation).

An important aspect of the data set, crucial to our work, is a unique HFT flag that accompanies order and trade messages. In particular, and in line with

[16]For the sake of consistency, we select to work on stocks that are continuously listed on the CAC 40 index from 2010, while we have excluded stocks that exhibit missing values within the sample period (2013). The Paris market data included in the BEDOFIH database are provided by the AMF. Further Information on the BEDOFIH database can be found at: `https://www.eurofidai.org/en/Intraday_Bedofih_Equipex_en.html`

the AMF documentation, each message is categorized according to the following list:

a) HFT: High Frequency trader
b) nHFT: Non–High Frequency trader
c) MIXED: Mixed trader (Bank account applying HFT)

The particular classification is based on the average lifetime of total order cancellations, compared to the average lifetime of individual traders' cancellations and modifications. For instance, a trader who cancels or modifies orders too fast, compared to the average speed of cancellations or adjustments, is classified as HFT. The nFHT and MIXED flags are similarly applied. It is important to mention that once a trader is classified as HFT, nHFT or MIXED, this flag is immutable.

4.3. Preliminary analysis

This Section provides a preliminary analysis of the preopening and opening trading activity in the Paris market that motivates the empirical analysis of the present study. Table 1 reports the average, across days and stocks, number of submissions, modifications and cancellations, during the preopening period, in 15 minute frequency and for each trader type; HFT, MIXED and nHFT. The first thing to notice is that HFT activity is practically absent until 8:00, whereas it rapidly increases prior to open. The average number of submissions in the last 15 minutes is 158.3 and 190.6 for HFT and MIXED traders, respectively, whereas the corresponding values are less than 2 before 8:00. On the other hand, nHFT submissions are more prevalent at the start and at the end of the preopening session.[17] The average number of nHFT submissions is 61.9 between 7:15 and 7:30, whereas it decreases to 8.8 until 8:30. Subsequently, it increases to 108.5 until 8:45 to reach 51.4 prior to open. As far as modifications and

[17]Boussetta et al. (2016) also report that nHFT traders submit their quotes at the start at at the end of the preopening session for the 120 SBF stocks in the Paris market and for the years 2012 and 2013.

cancellations is concerned, they are almost negligible for all types of traders during most of the preopening period, whereas they increase significantly prior to opening.

Turning into the order book volume, Table 1 reports the volume (i.e., standing shares) percentage relative to total order book volume in 15 minute frequency for the preopening period and for each trader type. The total number of nHFT shares is consistently higher during the entire preopening period, compared to the other types of trading; the average nHFT preopening volume percentage is approximately equal to 80.6%. Notice, also, that as the preopening time approaches the volume percentage associated with nHFTs is relatively reduced, whereas for HFT and MIXED traders it increases. At the opening, nHFT volume accounts for, approximately, 77.3% of totally submitted shares, whereas the corresponding percentage for HFT and MIXED traders is 4% and 18.8%, respectively. Interestingly, by comparing the average number of order submissions with the volume percentage, we infer that HFT related orders are smaller than nHFT orders prior to open. This finding is consistent with the argument that HFT algorithms are frequently employed by investors for the application of 'slice and dice' techniques, to reduce price impact or to camouflage their informational advantage (Barclay and Warner, 1993; Chakraborty et al., 2012).

Table 2 reports means for the opening volume, number of trades, value of transactions, percentage of value of transactions relative to total daily value of transactions, as well as HFT, nHFT and MIXED trading activity, for the 32 stocks in our sample. Evidently, the opening trade accounts for, approximately, 1.5% of total daily traded volume.[18] This percentage is rather low compared to the 10% reported by Biais et al. (1999) for the opening auction of the CAC 40 stocks in 1993. Observe, however, that the percentage of HFT and MIXED trades at the opening is significantly increased; almost 67.5%, on average, of

[18]Similar to our findings, Boussetta et al. (2016) report an average opening volume of 1.3% to 2.6% for the 120 SBF stocks in the Paris market and for the years 2012 and 2013.

total trading activity at the opening involves HFT or MIXED flags.

Overall, the results reported in Tables 1 and 2 suggest that only a small HFT and MIXED fraction ($\approx 22\%$) of total order book volume, attributed to orders with small size, is responsible for, approximately, 70% of total traded volume at the opening. We infer, therefore, that as the opening time approaches HFTs become more aggressive compared to nHFTs, contributing significantly to the determination of the clearing price.

Table 1: Preopening order flow for the CAC 40 sample.

Time	7:15–7:30	7:30–7:45	7:45–8:00	8:00–8:15	8:15–8:30	8:30–8:45	8:45–Open
Submissions							
HFT	0.0	0.0	1.8	0.4	0.3	228.1	158.3
MIXED	0.0	0.0	0.6	29.1	3.7	12.6	190.6
nHFT	61.9	54.0	22.8	16.7	8.8	108.5	51.4
Modifications							
HFT	0.0	0.0	0.0	0.0	0.6	0.1	40.1
MIXED	0.0	0.0	0.0	0.0	0.2	0.7	59.0
nHFT	0.0	0.0	0.0	0.0	0.0	0.3	18.3
Cancellations							
HFT	0.0	0.0	0.0	0.0	0.1	0.4	33.3
MIXED	0.0	0.0	0.0	0.0	1.6	1.1	34.2
nHFT	6.2	9.5	9.6	2.3	2.1	2.1	5.8
Total volume (%)							
HFT	0.1	0.1	1.0	1.0	1.0	5.6	4.0
MIXED	14.8	14.6	14.6	20.1	20.1	19.9	18.8
nHFT	85.1	85.2	84.4	78.9	78.8	74.5	77.3

This Table reports the average, across days and stocks, number of submissions, revisions and cancellations during the preopening and in 15 minute frequency, along with the corresponding volume percentage relative to total order book volume, for each type of trader; HFT, MIXED and nHFT. Zero values (0.0) represent very small averages.

Table 2: Opening statistics for the CAC 40 sample.

Company	Volume	Number of trades	Value of transactions	Trades value (%)	HFT (%)	MIXED (%)	nHFT (%)	HFT & MIXED (%)
Credit Agricole	126,450.4	190.0	961,768.0	1.5	11.5	32.5	55.9	44.1
Airbus Group	13,825.0	124.3	1,345,870.8	2.2	8.5	51.9	39.7	60.3
Air Liquide	30,873.8	89.4	711,201.7	1.3	9.4	62.3	28.3	71.7
Carrefour	82,812.9	189.7	3,338,181.9	2.0	6.8	55.8	37.3	62.7
Sanofi	9,838.0	80.6	1,209,136.6	1.7	9.6	63.8	26.6	73.4
Total	8,684.8	67.9	365,504.3	1.3	12.5	53.6	33.9	66.1
LOreal	8,979.2	47.0	263,187.7	1.0	7.9	71.8	20.3	79.7
Vallourec	8,681.8	84.8	433,528.2	1.1	11.8	50.2	37.9	62.1
Accor	52,697.2	147.2	4,042,631.4	2.1	7.1	63.7	29.2	70.8
Lafarge	96,080.6	145.3	1,533,643.2	1.5	8.2	55.6	36.2	63.8
Axa	27,964.4	92.5	1,530,292.3	1.7	5.5	75.3	19.3	80.7
Danone	7,230.0	61.9	653,103.8	1.4	11.1	67.7	21.2	78.8
Pernod Ricard	13,625.6	106.8	1,838,979.8	1.8	8.6	62.2	29.2	70.8
Lvmh	10,728.2	104.0	776,669.9	1.6	9.6	58.7	31.8	68.2
Michelin	2,999.2	47.8	496,045.7	1.1	9.5	65.3	25.2	74.8
Vivendi	8,587.2	75.8	690,507.0	1.6	8.2	65.5	26.4	73.6
Kering	19,881.3	78.9	1,177,242.7	1.5	8.3	70.5	21.2	78.8
Schneider Electric	34,432.6	79.4	375,577.1	1.1	11.2	50.4	38.3	61.7
Veolia Environn.	22,453.5	79.3	748,770.5	1.3	10.1	64.2	25.8	74.2
Unibail-Rodamco	8,941.0	58.6	357,072.3	1.1	12.5	59.1	28.4	71.6
Saint Gobain	25,555.6	115.9	1,017,887.4	1.7	7.5	55.8	36.7	63.3
Cap Gemini	73,411.7	120.8	1,211,221.9	1.7	9.3	59.5	31.2	68.8
Vinci	6,762.4	44.1	390,563.3	1.0	10.9	71.9	17.2	82.8
Publicis Groupe	63,557.5	170.5	2,067,061.8	1.3	7.4	49.9	42.7	57.3
Societe Generale	57,987.2	179.4	2,730,520.4	1.5	6.9	55.8	37.3	62.7
BNP Paribas	8,402.9	82.5	665,356.5	1.6	10.9	61.3	27.7	72.3
Technip	15,950.1	78.0	890,811.0	1.5	9.7	62.7	27.6	72.4
Renault	128,075.6	168.1	1,071,560.2	1.3	7.4	49.0	43.6	56.4
Orange	71,155.7	169.9	1,164,808.0	1.6	7.0	55.5	37.5	62.5
GDF Suez	21,409.4	103.1	600,561.3	1.4	10.5	50.2	39.3	60.7
EDF	24,168.1	117.5	466,494.5	1.4	6.2	49.4	44.4	55.6
Alstom	41,076.0	194.6	1,777,128.8	1.7	4.8	50.1	45.1	54.9
Mean	35,415.0	109.2	1,153,215.3	1.5	8.9	58.5	32.6	67.4

Average values at the opening: volume, number of trades, value of transactions, percentage of value of transactions relative to total daily value of transactions and percentages of HFT, nHFT and MIXED trading activity, for the 32 CAC 40 sample stocks.

4.4. Price discovery

Before we examine price efficiency in the preopening, we measure the contribution of each 15 minute preopening interval to total price discovery. To do so, we use the Weighted Price Contribution (WPC) statistic of Barclay and Warner (1993), which is defined as follows:[19]

$$WPC_k = \sum_{i=1}^{S} \left(\frac{|\Delta P_i|}{\sum_{i=1}^{S} |\Delta P_i|} \right) \times \left(\frac{\Delta P_{k,i}}{\Delta P_i} \right), \tag{60}$$

where ΔP_i is the total logarithmic price change in preopening for stock i and $\Delta P_{k,i}$ is the logarithmic price change for interval k, $k = 1, ..., 8$. The first parenthesis in equation (60) is the weighting factor for each stock to control for potential heteroscedasticity in preopening returns, whereas the second parenthesis is the contribution of interval k to total preopen price adjustment (Barclay and Hendershott, 2003). Note that the WPC statistic, as defined above, is the weighted average price contribution across stocks. Subsequently, this quantity is averaged across days to obtain the overall WPC estimate. Similar results can be acquired by averaging first across days, within equation (60), and then across stocks. In this case, however, it is likely that the common market component in stock returns will add bias to the average estimate of the WPC statistic. Therefore, following Barclay and Hendershott (2003), we select to calculate the WPC statistic day by day and then to average across days. Lastly, Table 3 summarizes the stock–day sample distribution for each 15 minute intraday preopening return used in the WPC analysis.

[19]The WPC statistic has also been applied by Cao et al. (2000), Ciccotello and Hatheway (2000) and Barclay and Hendershott (2003) for the NASDAQ market, Ellul et al. (2005) for the London Stock Exchange, Moshirian et al. (2012) for the Australian Stock Exchange and Anagnostidis et al. (2015) for the Athens Exchange.

Table 3: Distribution of stock returns.

	Mean	Median	Std	Min	Max
Overnight returns over all orders					
close to 7:15	0.005	0.004	0.005	-0.061	0.047
close to 7:30	0.000	0.000	0.006	-0.059	0.086
close to 7:45	-0.001	0.000	0.008	-0.074	0.047
close to 8:00	-0.001	0.000	0.008	-0.074	0.047
close to 8:15	-0.001	0.000	0.009	-0.074	0.063
close to 8:30	-0.001	0.000	0.009	-0.078	0.055
close to 8:45	-0.001	0.000	0.010	-0.127	0.049
close to open	0.000	0.000	0.008	-0.094	0.086
close to close	0.001	0.001	0.016	-0.126	0.116
Overnight returns over nHFT orders					
close to 7:15	0.005	0.004	0.005	-0.061	0.047
close to 7:30	0.000	0.000	0.006	-0.059	0.086
close to 7:45	-0.001	0.000	0.008	-0.074	0.047
close to 8:00	-0.001	0.000	0.008	-0.079	0.047
close to 8:15	-0.001	0.000	0.009	-0.078	0.055
close to 8:30	-0.001	0.000	0.009	-0.081	0.060
close to 8:45	-0.002	0.000	0.009	-0.076	0.057
close to open	0.000	0.000	0.009	-0.094	0.060
Intraday returns					
close to 7:15	0.005	0.004	0.005	-0.061	0.047
7:15-7:30	-0.005	-0.004	0.007	-0.048	0.077
7:30-7:45	-0.001	0.000	0.006	-0.077	0.053
7:45-8:00	0.000	0.000	0.004	-0.064	0.040
8:00-8:15	0.000	0.000	0.005	-0.049	0.058
8:15-8:30	0.000	0.000	0.005	-0.076	0.073
8:30-8:45	0.000	0.000	0.007	-0.102	0.067
8:45 to open	0.001	0.001	0.009	-0.092	0.115

This Table reports summary statistics for the distribution of return variables utilized in the present study; mean, median, standard deviation, minimum and maximum.

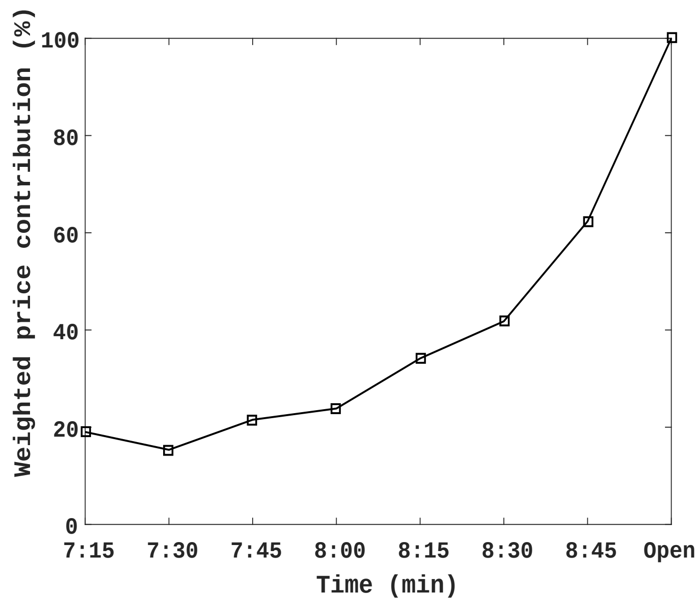

Figure 5: Cumulative Weighted Price Contribution statistic during the preopening period.

Figure 5 illustrates the evolution of the cumulative Weighted Price Contribution statistic within the preopening period. It is interesting to observe that in the first minutes of the preopening, price discovery is very low (20%), reflecting the lower rate of price adjustment. By contrast, in the last 15 minutes WPC is rapidly accelerated; the corresponding WPC statistic is approximately 38%. We, therefore, infer that bidding is more aggressive prior to open, indicating that traders submit orders with the intention of being filled. This result, in conjunction with the preliminary analysis presented above, suggests that the price formation process in the preopening is closely related to the amount of HFT.

4.5. Price efficiency

To test the hypothesis that accurately informed HFTs contribute to price efficiency in call auction trading, we employ the unbiasedness regression technique of Biais et al. (1999).[20] To do so, we use the current day closing price as a representation of the fair value of the stock, V. This proxy is based on the assumption that at the end of the trading day prices reflect all market information.[21] Now, consider the information set I_0 at the start of the preopening period (i.e., the start of the trading day). Then, the previous day closing price represents the expected value of the stock at time zero ($t = 0$) conditional on I_0; denote this expectation by $E(V|I_0)$. If traders consider the disclosed information on the indicative prices and the order book dynamics, then as time progresses equilibrium prices P_t should become more informative; that is, $P_t = E(V|I_t)$ with I_t being the information set at time $t > 0$. On the other hand, if equilibrium prices incorporate noise, then they should reflect the information set I_0 plus a noise term e_t. Thus, $P_t = E(V|I_0) + e_t$ with $e_t \perp V$.

To conduct the econometric test, we initially compute equilibrium prices over a 15 min frequency in the preopening period. Subsequently, we utilize the following overnight logarithmic returns: 1) previous day close to current day preopening time t, R_{ct} and 2) previous day close to current day close, R_{cc}. The first type proxies the difference between the preopening price P_t and the equilibrium value at the start of the day $E(V|I_0)$, $[P_t - E(V|I_0)]$, whereas the second type proxies the change of the equilibrium price of the stock, $[V - E(V|I_0)]$, after the end of the trading day. The idea is to examine the correlation between the two types of returns over the sample trading days, at each preopening time stamp, to infer on price efficiency. Summary statistics on the distribution of overnight returns are reported in Table 2.

We estimate the following linear regression at each preopening time:

$$R_{cc} = a + bR_{ct} + \epsilon. \tag{61}$$

[20] The unbiasedness regression methodology has been used in several empirical studies in the past; see, for example, Ciccotello and Hatheway (2000), Barclay and Hendershott (2003), Madhavan and Panchapagesan (2000) Comerton-Forde and Rydge (2006), Moshirian et al. (2012) and Anagnostidis et al. (2015).

[21] Note that other prices during the day can also be employed as fair value proxies; mid–day prices for example (Madhavan and Panchapagesan, 2000).

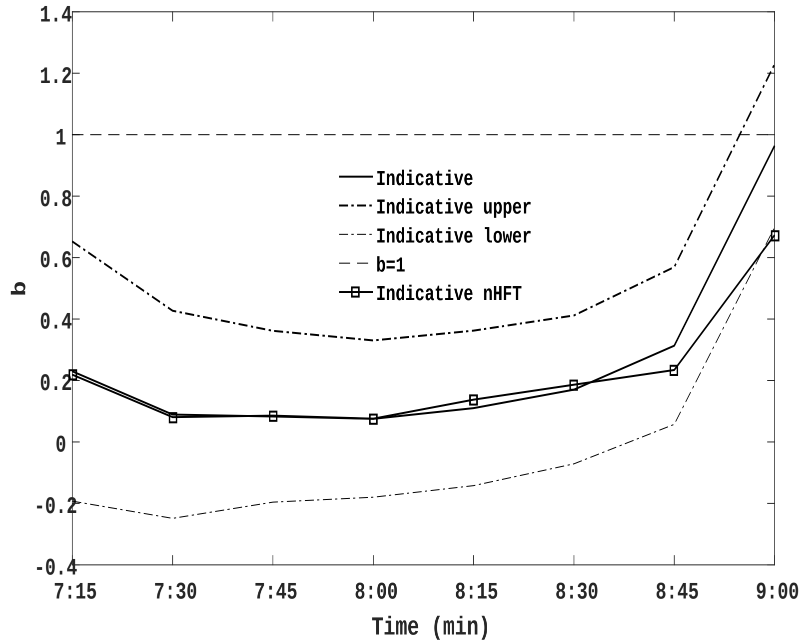

Figure 6: Average, across stocks, b coefficients obtained by estimating equation (61) (solid line) along with average 95% confidence intervals (upper/lower), using actual indicative and opening prices. Together are plotted the average estimated b coefficients obtained by using nHFT equilibrium prices (squared line).

If the slope coefficient b is equal to unity, then stock preopening equilibrium prices are efficient as they reflect all market information at time t (I_t). By contrast, if b is different from unity, then indicative prices reflect information plus some noise.[22] Notice that the variance of the residual term in equation (61) is comparable to the residual variance obtained in equation (18), after conditioning on the linear rule $p = \alpha s + \beta Q$.

[22]Note that stock price efficiency at the opening can also be examined using variance ratio statistics over daily data (e.g., opening and closing prices). The unbiasedness regression technique, however, is advantageous, as it enables us to examine the dynamics of stock prices within the trading day. Further, by running a separate regression for each consecutive time interval within the preopening, we avoid nonstationarity issues that arise due to price adjustments as the price discovery evolves.

4.5.1. Individual regression results

Here we present the results acquired from the unbiasedness regression methodology by running a separate OLS regression for each stock, in line with most empirical studies in the relative literature (Biais et al., 1999; Comerton-Forde and Rydge, 2006; Moshirian et al., 2012). Figure 6 plots the estimated average, across stocks, b coefficient during the preopening period for 15 minute frequency, along with the estimated average 95% confidence intervals.[23] The first thing to notice is that early preopening indicative prices are rather noisy. This result reflects the fact that, at that time of the day, investors' bids are still not adjusted to incorporate early news announcements and/or overnight information. On the other hand, there is evidence that as the opening time approaches, prices reflect more information, whereas they become efficient at the open; the average b is equal to 0.9640 and statistically equal to unity at the 5% probability level. This 'learning' pattern is similar to that reported by Biais et al. (1999).[24] Evidently, traders adjust their quotes according to the publicly available market information and, therefore, prices become more informative as the preopening process evolves.

To examine the effect of HFTs on opening prices, we evaluate the unbiasedness regressions using indicative equilibrium prices calculated exclusively on nHFT orders. According to the model described earlier, we expect that prices incorporate less information when generated solely by slow nHFT investors. Figure 6 juxtaposes the average, across stocks, b coefficients obtained from equation (61) using the actual indicative prices against the mean b coefficients obtained using nHFT indicative prices. The two patterns of b estimates are similar, suggesting that investors learn about true values by observing the available information. At the opening, however, nHFT prices are still noisy; the

[23] To infer on the statistical significance of b estimates, we use NeweyWest (N-W) SEs that control for potential heteroscedasticity and serial correlation in the residual time series.

[24] Learning patterns in the preopening are also reported in Ciccotello and Hatheway (2000) and Barclay and Hendershott (2003) for the NASDAQ, Comerton-Forde and Rydge (2006) and Moshirian et al. (2012) for the Australian Exchange and Anagnostidis et al. (2015) for the Athens Exchange.

average b coefficient is equal to 0.6731 while the corresponding 95 % confidence interval, not illustrated in Figure 6, is (0.6471, 0.7130). Thus, b is statistically different from unity at the 5 % probability level. This finding suggests that HFT activity contributes significantly to the price discovery process by enhancing the information content of clearing prices.

4.5.2. Panel data regression results

The average b coefficient results described above are obtained under the presumption that the residual innovations obtained by the unbiasedness regressions are cross–sectionally independent. Financial panel data, however, exhibit often cross–sectional dependence. Thus, for purposes of robustness, we evaluate the unbiasedness regressions by conducting panel data analysis with the use of two–way clustered errors; that is, standard errors that are robust with respect to time (within-group) and firm (between-group) correlations (Thompson, 2011).

To investigate for cross–correlation in the data, we conduct the statistical test proposed by Pesaran (2004) over the residuals from the panel regressions. In particular, under the null hypothesis of cross–correlation, the following statistic is asymptotically standard normal distributed:

$$CD = \sqrt{\frac{2m}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{i,j} \right) \sim N(0,1), \tag{62}$$

where N is the total number of stocks, m the number of time observations and $\hat{\rho}_{i,j}$ the estimated pairwise correlation for securities i and j. Note that the particular statistic improves on the well–known Breusch and Pagan (1980) test that is accurate only for $m >> N$. In particular, Pesaran (2004) shows that the CD statistic performs equally well for small samples with respect to m and/or N. Moreover, it is robust against non–stationarity and/or structural breaks in the time series at hand. The estimated CD statistic for the preopening interval panel regressions ranges from 6.165 to 6.272 (> 1.96). Hence, the null hypothesis of cross–sectional independence is rejected in all cases, confirming the choice to conduct corroborating panel data analysis.

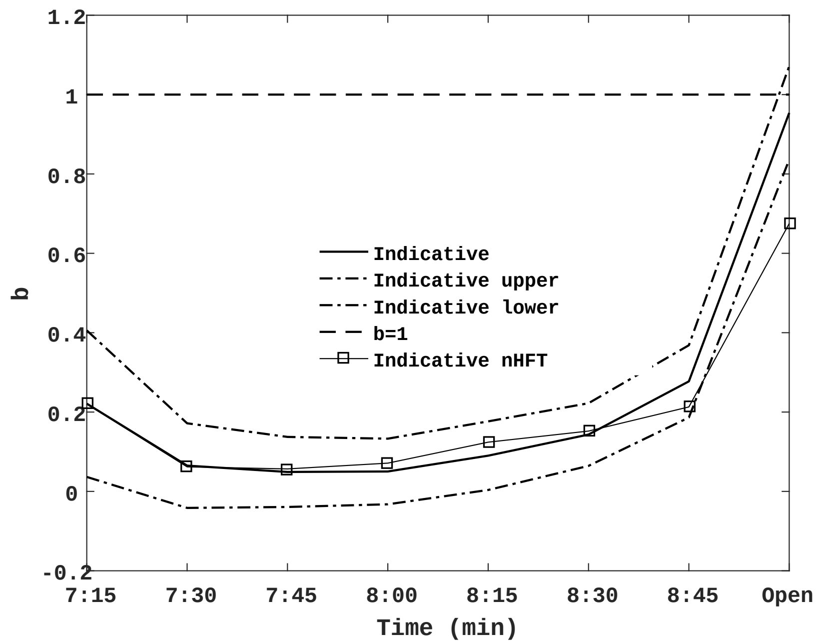

Figure 7: Unbiasedness panel regression b coefficients (solid line) along with the corresponding 95% confidence intervals (upper/lower), obtained by using actual indicative and opening prices in equation (61). Together are plotted panel b coefficient estimates obtained by using nHFT equilibrium prices (squared line).

Figure 7 plots the results from the estimated panel regressions. The first thing to notice is that, due to the use of two–way clustered errors, the confidence intervals are narrower compared to those presented in Figure 6. Thus, we are able to accurately detect deviations from unity for the b coefficient. Consistent with the findings from the individual analysis, the pattern of b coefficients suggests that investors ‘learn’ from the available information and, in turn, adjust their orders accordingly; the b coefficient at the opening is equal to 0.9539 and is statistically equal to unity at the 5% level. Notice, also, that for the nHFT related regressions the estimated b coefficient is 0.6749 and statistically different from unity at the 5% level; the corresponding confidence interval is $(0.572, 0.784)$. Overall, the panel data analysis results are in line with the individual equation analysis in that HFTs improve price efficiency in the Paris market opening auction.

5. Conclusions

We examine the effect of HFTs on stock prices in transparent order–driven call auction markets. To do so, we develop an analytical framework based on the notion of noisy rational expectations equilibria, where HFTs coexist with nHFTs in the call market. The key assumption in our model is that machines have access to a semi–strong form efficient signal, the precision of which improves as the order batching procedure evolves. Based on this assumption we deduce that the informativeness of preopening indicative prices is positively correlated with HFT activity; that is, the number of HFTs in the market as well as the information content of the signal of HFTs.

To test empirically the prediction of our theoretical framework, we use a unique set of intraday data from the Paris Euronext stock market for the CAC 40 stocks in year 2013. In particular, we construct equilibrium prices on the basis of both HFT and nHFT data. Subsequently, we employ the Weighted Price Contribution (WPC) statistic of Barclay and Warner (1993) together with the unbiasedness regression methodology of Biais et al. (1999), to investigate the price formation process in the preopening. Our findings hint that HFTs accelerate price discovery and price efficiency, especially during the last minutes of the order batching period. Thus, we infer that the presence of HFTs in order–driven call auctions enhances the quality of clearing prices.

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