TESTING THE CONDITIONAL CAPM WITH THE OPTIMAL INFORMED INVESTORS' PORTFOLIO

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Abstract

We test the CAPM conditional on agents' information by replacing a value-weighted stock index traditionally used as proxy for the market portfolio by the optimal informed investors' portfolio derived in Burlacu, Guéniche and Jimenez-Garcès (2016). Indeed, in asset-pricing tests, realized stock returns are used as proxy for expected stock returns. However, stock returns being realized on the basis of *all* the investors' information, the conditional CAPM can only be tested with a portfolio conditional on public as well as private information. We provide a test free of theoretical critics, whether on a misspecification of the market portfolio or the inability of testing conditional models. We show that conditioning on informed investors' information allows estimating the real beta. Moreover, the optimal informed investors' portfolio being not subject to the informational risk, we find empirical evidence that it allows estimating more precisely the market risk premium, while the use of a stock index does not disentangle it from the information risk premium.

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1. Introduction

More than three hundred, that is the number of anomaly variables counted by Harvey, Liu and Zhu (2015). These variables that help to forecast returns are called anomalies since, according to the Sharpe (1964) – Lintner (1965a, b) – Mossin (1966) capital asset pricing model (CAPM), only the systematic risk captured by beta should be awarded, as specific risk can be eliminated through diversification. Note that there is nothing wrong in principle that stocks with for example high book-to-market ratios provide higher average returns, so long as their betas are also higher. That is precisely here that the serious problem of the CAPM arises.

As discussed by Roll (1977), the only testable hypothesis is that the market portfolio is mean-variance efficient. This implies (1) a linear relationship between a stock's expected return and its covariance with the market portfolio, and that (2) the market beta suffices to describe the cross-section of expected returns. Black, Jensen and Scholes (1972), and Fama-MacBeth (1973), provided the first tests for these two elements. If these empirical tests were positive in the pre-1969 period, the following investigations, notably by Fama-French (1992), acknowledged a flat relation between beta and average stock returns, and even worse, that betas are lower for higher return stocks.

Hence the general consensus is that the unconditional CAPM is unable to explain satisfactorily the cross-section of average returns on stocks. So, whether this model can explain stock returns by their covariance with the market portfolio in time-series regressions, i.e. there is a market risk premium, it is however not a good description of returns, i.e. it does not capture much risk. The puzzle is thus not with patterns in average returns, such as this value effect reported above, that have been at a center of a race for their discovery these last decades, but that market beta goes the wrong way. But, has market beta been correctly measured? This is this central question that we explore.

In 1977, Roll formulated his well-known critic that "the theory is not testable unless the exact composition of the true market portfolio is known and used in the tests". Stambaugh (1982) challenges this statement by testing whether conclusions about the model's validity are *sensitive* to the choice of the market proxy. Including returns from bonds, real estate, and consumer durables in addition to common stocks, he concluded that "even when stocks represent only 10% of the portfolio's value, inferences about the CAPM are virtually identical to those obtained with a stocks-only portfolio". However, Jagannathan and Wang (1996) showed that the inclusion of the return on human capital[2] as an extra adhoc factor does help explaining the cross-section of average stock returns, especially if it is scaled by a conditioning variable.

The unconditional CAPM traditionally used empirically is indeed just a special case, static, of a more general version *conditional on investors' information sets at one time*. Moreover, market portfolio's *conditional* mean-variance efficiency predicted by the model does not necessarily also imply *unconditional* mean-variance efficiency, as demonstrated by Hansen and Richard (1987). Hence, for Jagannathan and Wang (1996), the poor ability of the static CAPM to explain the cross-sectional variations in average stock returns comes from the fact that beta varies over time. However, testing a dynamic version of the CAPM (i.e. with betas and expected returns allowed to vary over time) is challenging. Jagannathan and Wang (1996) kind of circumvent this technical difficulty by deriving, from the conditional CAPM, an unconditional two-factor model. They introduce a conditioning factor, the default premium (the yield spread between BAA- and AAA-rated bonds), measuring the instability of an asset's beta over a business cycle. Their results support the fact that the dynamic CAPM provides a well better description of average stock returns than the static CAPM, and that contrary to the former, it cannot be rejected by the data. The size and book-to-market effects, documented by Fama-French (1992) as being robust, show in this case little ability to explain what is left unexplained.

But for Lewellen and Nagel (2006), the failure of the unconditional CAPM cannot only be explained by time variations in beta. The observed pricing errors with the unconditional CAPM and asset-pricing anomalies such as the value effect are simply too large for that. Dybvig and Ross (1985) raise another point, that "an uninformed observer using the tools of mean variance and security market line analysis to measure the performance of a portfolio manager who has superior information is unlikely to be able to make any reliable inferences". More generally, a *conditional* linear factor model encompasses two broad elements: time variations of the parameters, and as function of information possessed by investors. Hence, if

[2] See also Campbell (1996); Santos and Veronesi (2006); and Lettau and Ludvigson (2001a, 2001b) with the consumption based capital asset pricing model (CCAPM).

Jagannathan and Wang (1996) overcame empirically the first difficulty, the latter has still never been addressed. In this paper, we will thus focus on this second aspect, i.e. conditioning moments on information. How to test the CAPM conditioning on investors' information available at the time of trading, knowing that there are informational asymmetries?

The only way would be to use as proxy for the market portfolio, a hypothetical portfolio held by a perfectly informed investor about every asset, i.e. conditional on publicly available information but also on private information. Such a portfolio is unobservable, hence Cochrane's (2005, p. 143) conclusion, based on what he calls the "Hansen-Richard critique[3]", that, even if the Roll critique could be overcome, the conditional linear factor model is not testable. However, another trend in the literature has made considerable progress these last decades, the one on rational expectations equilibrium (REE) models, which consider information asymmetries among investors. More specifically, REE models collapse into a conditional CAPM while adopting an investor's point of view. Hence, Burlacu, Guéniche and Jimenez-Garcès (2016) adopt an informed agent's perspective, and reconstitute the unobservable private signals by extracting information from prices and volumes, with a methodology derived from REE models. This allows them to build empirically the optimal informed investors' portfolio which meets all the above prerequisites. This portfolio, conditional on *all* the investors' information, is the only stock market portfolio usable empirically since the realization of returns is influenced by *all* the investors' information, including private signals. Whether the conditional CAPM works theoretically for every investor, in presence of information asymmetry this model can only be used in practice from the point of view of a perfectly informed investor about every asset. Therefore, we use the optimal informed investors' portfolio as proxy for testing the conditional CAPM. Working on the French sample, we however cannot include returns from human capital in this preliminary paper, proxied by the growth rate of wages, which were proved to influence the model's validity inference, the INSEE not providing monthly data for national accounts[4].

[3] See Hansen and Richard (1987)

[4] The INSEE publishes on a quarterly basis, with no derogations possible, "Comptes de revenu des ménages – Total des ressources" and "Dividendes reçus par les ménages", which are the equivalent of "1. Personal income" and "15. Personal dividend income" in Table *2.6. Personal Income and Its Disposition, Monthly* published by the Bureau of Economic Analysis, used by Jagannathan and Wang (1996) for the U.S.

Hence, we perform a test of the conditional CAPM using the optimal informed investors' portfolio as proxy for the market portfolio. We employ Fama-MacBeth (1973) test methodology, the most widely used in the literature, on a single period. As this procedure does not take into account the time variations in the risk premiums, we will only capture the effect of conditioning on information in this first approach. Performing it on several periods, as in the original article of Fama-MacBeth (1973), would allow some gradual time-related changes in betas. However, these rolling betas would not completely capture time-variation in true betas, and a real dynamic approach such as Adrian, Crump and Moench (2015) would be far better. We keep this for future research.

The paper is organized as follows. The next section presents the conditional CAPM. Section 3 describes the data and the Fama-MacBeth methodology with and our theoretical hypotheses. In section 4, we discuss the empirical results. Section 5 concludes.

2. Theoretical background

The traditional CAPM derives the linear relationship between expected stock returns and their covariance with the return on all invested wealth, also known as "market portfolio". Indeed, in the seminal papers, investors are hypothesized to hold the same information set at the same time and have identical investment opportunities. In equilibrium, the market itself is thus the single optimal mean–variance efficient portfolio. The degree of risk aversion only intervenes on the allocation decision between a borrowing or lending at the risk-free rate and the market portfolio which will guarantee the higher return for the level of risk exposure that investors are willing to bear. This is the Tobin (1958) separation theorem.

However, in a context of information asymmetry, Jimenez-Garcès (2004, p. 66) showed that informed investors' demand is not only function of the asset supply but also of their information. Informed investors' portfolios being sensitive to their private information, holding the market portfolio is then no longer optimal. Informed traders are agents who hold private information. They can be board or management members, or even agents analyzing more closely available public information. Burlacu, Fontaine and Jimenez-Garcès (2005) define "private information" as information: (1) being relevant for assessing the firms' value, (2) possessed by only a limited number of investors, and (3) not reflected by stock prices.

Rational expectations equilibrium models provide the expression of the equilibrium price under these circumstances. Biais, Bossaerts and Spatt (2010) showed that their overlapping generations REE model collapses into a conditional CAPM while adopting the view of a representative agent. Burlacu, Fontaine and Jimenez-Garcès (2015) have derived, from a multi-asset version of the Grossman and Stiglitz (1980) model, the conditional CAPM of the uninformed investor. Finally, Burlacu, Guéniche and Jimenez-Garcès (2016) have reproduced this demonstration from an informed agent's perspective, based on the Admati (1985) model.

The conditional model from an agent a's perspective in a context of information asymmetry writes:

$$E(\tilde{r}_i^{t+1}|I_a^t) - r_f = \beta_i^t\left(E(\tilde{r}_m^{t+1}|I_a^t) - r_f\right) \tag{1}$$

where β_i^t is the conditional beta defined by:

$$\beta_i^t = \frac{Cov(\tilde{r}_i^{t+1}, \tilde{r}_m^{t+1}|I_a^t)}{Var(\tilde{r}_m^{t+1}|I_a^t)} \tag{2}$$

with $E(\tilde{r}_i^t|I_a^t)$ the agent a's expectation *conditional* on his time t information about the asset i return.

Hence, contrary to the unconditional Sharpe–Lintner–Mossin CAPM, where investors have homogenous return expectations, moments (mean, variance, covariance) are with respect to agent a's information set at time t, I_a^t. Expectations about a stock's future payoff are then different across investors. There are informed investors who receive a signal generally[5] composed of the following dividend plus a common and an idiosyncratic error terms. To simplify, let's consider henceforth a single signal of the form $\tilde{F}^{t+1} = \tilde{\theta}^t + \tilde{\varepsilon}^t$, with $\tilde{F}^{t+1}$ the future dividend, $\tilde{\theta}^t$ the private information received by informed investors and $\tilde{\varepsilon}^t$ a common

[5] REE models do not all have the same information structure. Basically we distinguish two main categories. In aggregated information (AI) frameworks (initiated by Hellwig, 1980), investors receive signals with different precisions. i.e. with different individual error terms, but there is no residual uncertainty (i.e. no common error term). While in information transmission (IT) frameworks (initiated by Grossman and Stiglitz, 1980), informed investors receive all the same signal but there remains a residual uncertainty. Some variations can be found, such as differential information (e.g. He and Wang, 1995) where each investor receives a different signal, or more recently, with dispersed information (initiated by Jimenez-Garcès, 2004) combining specifications of AI and IT.

error term. Following Burlacu *et al*. 2016, adapted to this information structure, the asset-pricing model from an informed investor's perspective writes:

$$E_I^t(\tilde{r}_i^{t+1}) - r_f = \beta_{I,i}^t\left(E_I^t\left(\tilde{r}_{opt.I}^{t+1}\right) - r_f\right) \tag{3}$$

where the subscript I indicates that the variables are conditioned on the informed investor's information set, i.e. $I_a^t = \{\theta^t, P^t\}$, since an informed agent observes privately the value θ taken by the random variable $\tilde{\theta}$ as well as the public equilibrium price of assets P^t, and the subscript $opt.I$ denotes the optimal informed investor's portfolio.

The beta conditional on informed investors' information at time t, $\beta_{I,i}^t$, is considered as the real beta as returns are realized on the basis of public but also private information. This beta perceived by informed traders for a stock i, is obtained by regressing stock i's return onto the optimal informed investors' portfolio, while the traditional beta comes through the use of a value-weighted stock index. Jimenez-Garcès (2004, equation (4-2) p.137) develops the real beta:

$$\beta_{I,i}^t = \frac{Cov\left(\tilde{r}_i^{t+1}, \tilde{r}_m^{t+1} \middle| \tilde{\theta}^t\right)}{Var\left(\tilde{r}_m^{t+1} \middle| \tilde{\theta}^t\right)} \tag{4}$$

$$= \frac{Cov(\tilde{r}_i^{t+1}, \tilde{r}_m^{t+1}) - Cov\left(\tilde{r}_{\theta_i}^{t+1}, \tilde{r}_{\theta_m}^{t+1}\right)}{Var(\tilde{r}_m^{t+1}) - Var\left(\tilde{r}_{\theta_m}^{t+1}\right)}$$

$$= \beta_i^t + Var\left(\tilde{r}_{\theta_m}^{t+1}\right)\frac{\beta_i^t - \beta_{\theta_i}^t}{Var(\tilde{r}_m^{t+1}) - Var\left(\tilde{r}_{\theta_m}^{t+1}\right)}$$

where $\tilde{r}_{\theta_i}^{t+1} = \frac{\tilde{\theta}_i^t - P_i^t}{P_i^t}$ and $\tilde{r}_{\theta_m}^{t+1} = \frac{\tilde{\theta}_m^t - P_i^t}{\theta_m^t}$, respectively the part of asset's i return and the part of the market return known by an informed investor,

with $P_m^t = \sum_{i=1}^n P_i^t z_i^t$ the price of the market portfolio relative to the number of investors, and $\tilde{\theta}_m^t = \sum_{j=1}^n \tilde{\theta}_j^t z_j^t$ the private information on the market portfolio's future payoff, in a model with n risky assets, investors being either uninformed (UI) or informed (I), i.e. agent $j \in \{UI, I\}$, and z the supply per capita,

and

$$\beta_{\theta_i}^{t} = \frac{Cov\left(\tilde{r}_{\theta_i}^{t+1}, \tilde{r}_{\theta_m}^{t+1}\right)}{Var\left(\tilde{r}_{\theta_m}^{t+1}\right)} \tag{5}$$

called "information beta", which concerns exclusively the part of the return known by informed investors. It characterizes the sensitivity of the private information on stock i to the private information on the market portfolio.

In this paper, we focus on conditioning moments on information, leaving time variations for future research. In this particular case, β^t is thus considered as being constant, and we can hereafter drop the superscript t to simplify the exposition.

Hence, with this development, Jimenez-Garcès (2004) demonstrates that, if the real beta, $\beta_{I,i}$, and the traditional (unconditional) beta, β_i, are equal in a symmetric information context, in presence of asymmetric information an information term arises. If the information beta, β_{θ_i}, is higher than the unconditional beta, the real beta can be lower than the unconditional beta. This occurs for stocks sensitive to the market portfolio in an informational point of view. It can be the case of large companies since they weight more heavily in the market portfolio, and they are likely to be sensitive to market common factors, causing a correlation between the information of their stock with the one of the market. The opposite happens when $\beta_{\theta_i} < \beta_i$. This may be the case of stocks for which informational asymmetry is more related to the specific factor. The information of these securities appears to be little sensitive to the market one. Small firms could meet this characteristic. In the special case where the information beta equals the traditional beta, notably when information asymmetry is homogenous[6] across securities, then information asymmetry does not affect beta estimation and the real beta equals the unconditional beta.

Consequently, in presence of asymmetric information, investors require an "information risk premium" (or "IRP") in addition to the systematic risk premium. The systematic risk premium, also called "market risk premium", is the product of the traditional beta and the expected return of the market portfolio in excess of the risk-free rate, while the information risk

[6] when the precision of uninformed and informed investors' expectations is proportional. The informational advantage of informed investors over uninformed investors is then identical for all risky assets (please see Jimenez-Garcès, 2004, p. 58). In this case, the real beta and the traditional beta are equal to the beta perceived by uninformed investors, conditional on prices, and to the beta of a representative agent.

premium is the return required by uninformed investors to compensate the risk of being adversely selected. For Covrig, Fontaine, Jimenez-Garcès and Seasholes (2007), "the IRP represents the amount an asset's price at date 0 is below its expected future value solely due to agents not having full information about future payoffs". Indeed, the price is, ceteris paribus, lower when there are informational asymmetries. Covrig et *al*. (2007, expression (17) p. 12) express the information risk premium and define it as the difference between the risk premium when not all agents are fully informed and the risk premium when all agents are fully informed. They show that, in a multi-asset framework with uncorrelated residual uncertainties and no factor structure, the IRP is proportional to what they call "signal-to-noise measure", equal to the difference between the market's average uncertainty about future payoffs and residual uncertainty about these payoffs. When this measure is small, investors have a lot of information about future payoffs, the IRP is low, and prices are high. Now, while taking into account correlated residual uncertainties and/or a factor structure, the IRP is affected by the covariance terms but this interpretation still holds.

We calculate the IRP as the difference between average realized returns and average returns required by informed investors. Realized returns are first time-averaged for each asset and then averaged across all assets. At a date t, the required return by informed investors for an asset i is:

$$E_I^t(\tilde{r}_i^t) = r_f + \beta_{I,i}\left(\tilde{r}_{opt.I}^t - r_f\right) \tag{6}$$

In summary, in a symmetric information context, the traditional beta, obtained by using a value-weighted stock index as proxy for the market portfolio, is the real beta and captures the systematic risk. By contrast, in presence of informational asymmetries, the traditional beta is not the real beta since an information beta comes into play. The traditional beta obtained with a stock index is then a combination of the real beta and the information beta. We thus capture indiscriminately the market risk premium and the information risk premium. To disentangle these two elements, we must use the optimal informed investors' portfolio, not subject to the informational risk. It will allow estimating solely the market risk premium and in this way the real beta.

3. Empirical Test

3.1. Data

The dataset is the same as in Burlacu *et al*. (2016). This sample contains French common stock listed on Euronext.liffe Paris from August 1996 to December 2014. Monthly returns are extracted from Datastream. The risk-free rate is the France Treasury bill 1-month. To test whether the CAPM holds for this sample, we retain only firms with a complete data history over the 4 years of the portfolio formation period and the 5 years of the estimation period. Over the 1,615 firms, only 251 meet this requirement. Finally, as proxies for the market portfolio, we use Burlacu *et al*. (2016) monthly returns of the optimal informed investors' portfolio, and monthly returns of the corresponding value-weighted index. Please refer to their paper for the methodology of the optimal informed investors' portfolio, which consists in extracting information from prices and volumes.

3.2. Methodology

We use the two-stage cross-sectional regression approach of Fama and MacBeth (1973), (FM). First, market betas are estimated for each security, then we examine their explanatory power on stock returns. Using estimates instead of real market risk premiums inevitably induce an error-in-variables (EIV) bias. FM procedure implies working with a large number of individual assets grouped in portfolios, rather than individual assets, to reduce this EIV problem. Securities are assigned to portfolios according to their beta, in order to minimize within-portfolio variation in betas. Using Fama-French (1992) wording, we refer to these betas as pre-ranking betas, and estimate them in a preliminary step.

More specifically, following Fama and MacBeth (1973), we form 20 portfolios[7] from pre-ranking individual betas, obtained by regressing for each stock the first 4 years of monthly excess returns (with dividends reinvested) over the risk-free rate against the market portfolio excess return. Please note that we limit the portfolio formation period to 4 years, compared to 7 years in FM methodology, in order to keep a sufficiently large sample. Then, post-ranking individual betas, $\hat{\beta}_i$, are calculated by using the next 5 years of monthly asset excess returns,

[7] Please refer to Table 1 for details.

and updated yearly by extending the time-series length with one to three additional years. Portfolios are equally-weighted. Portfolio betas, $\hat{\beta}_{p,t}$, and portfolio returns, $R_{p,t}$, are monthly adjusted for delisted stocks or stocks with missing returns. Finally, for each of the following 48 months, the following cross-sectional regression is performed:

$$R_{p,t} = \hat{\gamma}_{0,t} + \hat{\gamma}_{1,t} \cdot \hat{\beta}_{p,t-1} + \hat{\gamma}_{2,t} \cdot \hat{\beta}^2_{p,t-1} + \hat{\gamma}_{3,t} \cdot \bar{s}_{p,t-1}(\hat{e}_i) + \hat{\eta}_{p,t} \tag{7}$$

The values of these coefficients will provide fundamental indications about the validity of the CAPM, as demonstrated by Fama and MacBeth (1973). They derive three testable conditions (pp. 609-11) common to Sharpe-Lintner and Black versions[8], plus one condition specific to the S-L version:

- S-L hypothesis: with risk-free borrowing and lending, the expected returns on zero-beta assets[9] equal the risk-free rate[10], and thus the intercept should equal the risk-free rate.
- C1: higher risk should be associated with higher expected return.
- C2: the relationship between the expected return of a security and the covariance with the market portfolio is linear.
- C3: there is no other measure of risk than the market beta, β.

The intercept $\hat{\gamma}_{0,t}$ represents the pricing error, i.e. the cross-section of average stock returns left unexplained by the model. If $E(\tilde{\gamma}_{0,t}) = 0$ then the S-L hypothesis is upheld by the data[11]. $\hat{\gamma}_{1,t}$, the market risk premium, reflects the ability of the market beta to explain the cross-section of average returns. We expect its value to be non-statistically different from zero while using a traditional index as market portfolio or the optimal uninformed investors' portfolio,

[8] The Sharpe–Lintner version of the CAPM allows unrestricted borrowing and lending at a risk-free rate. In comparison, the Black version (1972), more general, releases this assumption. Hence, in the absence of a riskless asset, the only requirement is that the expected market return be greater than the expected return on zero-beta assets.

[9] Fama-French (2004) give the following definition: "A risky asset's return is uncorrelated with the market return – its beta is zero – when the average of the asset's covariances with the returns on other assets just offsets the variance of the asset's return. Such a risky asset is riskless in the market portfolio in the sense that it contributes nothing to the variance of the market return".

[10] This leads to the familiar Sharpe-Lintner CAPM equation, equal to the minimum variance condition of the market portfolio but with the risk-free rate instead of the expected return on zero-beta assets.

[11] In the Sharpe–Lintner version of the CAPM, the intercept should equal the risk-free rate, denoted r_f, i.e., using stock returns, $E(\tilde{\gamma}_{0,t}) = r_f$. Equivalently, we test $E(\tilde{\gamma}_{0,t}) = 0$ using excess stock returns.

and significantly positive while using the optimal informed investors' portfolio (C1). We could have expected a higher premium with the stock index as it captures both the systematic and informational risks, but using it is not appropriate. Indeed, in an asymmetric information context, whether the conditional CAPM works in theory just as well for informed investors as for uninformed investors or from the point of view of a representative agent, the fact that realized returns are used in practice as proxy for expected returns makes the conditional CAPM only usable by perfectly informed investors. The linear relationship between an asset i and the optimal uninformed portfolio, or with a value-weighted stock index, is not guarantee. To compare results, we will nonetheless test the three possibilities. $\hat{\gamma}_{2,t}$ tests whether the relationship between expected return and β is linear (C2). If $E(\tilde{\gamma}_{2,t}) = 0$, then the linearity hypothesis is not rejected. Finally, $\hat{\gamma}_{3,t}$ indicates whether there are other measures of risk, in addition to β, that contribute systematically to observed average returns (C3). We expect its value to be non-statistically different from zero while using a traditional index as proxy for the market portfolio or the optimal uninformed investors' portfolio, and significantly different from zero while using the optimal informed investors' portfolio. Indeed, we argue that using a market index not only estimates the systematic risk but also the information risk, while using the optimal informed investors' portfolio, not subject to the information risk, allows estimating solely the systematic risk. In the first case, the systematic risks and information risks are thus incorporated into the model, while in the second situation only the systematic risk is present, leaving the information risk outside the model.

To further correct for EIV bias, the coefficients from the cross-sectional regressions are averaged over time using the Litzenberger and Ramaswamy (1979) method (LR correction hereafter). Coefficients are weighted by the inverse of their standard error when summing across the cross-sectional regressions, in order to place more (resp. less) weight on parameters that are estimated more (resp. less) precisely.

4. Results

From August 1996 to December 2014, the average observed monthly return is 0.97% (12.31% annualized) for the 251 French common stocks listed on Euronext.liffe Paris. Informed investors required 0.40% (4.95% annually) in average for holding these stocks. Hence, we find an information risk premium of 0.57% (7.04% annually). By way of comparison, uninformed investors require 0.35% (4.23% annualized). We would expect a higher return requirement from uninformed investors, subject to adverse selection, but remember that the CAPM *cannot* be applied in practice in their case (cf. section 3.2). In presence of information asymmetry, the price is, ceteris paribus, lower compared to a context with symmetric information. Uninformed investors' required return is thus underestimated. Finally, in Table 1 we can see that betas are in average higher for the value-weighted stock index than for the informed portfolio, with 0.654 compared to 0.109.

[Insert Table 1, Panel C about here]

Main results of the FM procedure are presented in Table 2, Panel A. Consistent with our expectations, using the value-weighted index as proxy for the aggregate wealth portfolio, the coefficient γ_1 associated to the market beta β is not significantly different from zero. That is also the case of γ_3. That indicates that there is no other risk measure, in addition to β, that systematically contributes to average returns.

[Insert Table 2 about here]

Using instead the optimal informed investors' portfolio as proxy for the market portfolio, γ_1 is much higher and close to be significant with a t-stat[12] at 1.543. γ_3 is also statistically different from zero, which indicates that a risk premium is missing in the model. The cross-sectional[13] R^2, showing the fraction of the cross-sectional variation of average returns that can be explained by the model, is also mechanically lower (5.4 percent with the conditional portfolio compared with 9.5 with the value-weighted stock index). Finally, γ_0 and γ_2 being not

[12] t-statistics are computed as:

$$t\left(\bar{\hat{\gamma}}_j\right) = \frac{\bar{\hat{\gamma}}_j}{s\left(\bar{\hat{\gamma}}_j\right)/\sqrt{n}}$$

[13] Cross-sectional R^2 is calculated as:

$$R^2 = 1 - \frac{Var_c\left(\bar{\hat{\eta}}_p\right)}{Var_c\left(\bar{R}_p\right)}$$

where Var_c denotes a cross-sectional variance, and variables with bars over them denote time-series averages.

significantly different from zero in the two cases, the S-L and linearity hypotheses are not rejected.

Furthermore, to reassure the reader about the consistency of our results, we perform the FM test on the optimal uninformed investors' portfolio. In Table 2, Panel B, we observe that C3 hypothesis also cannot be rejected, so no other factor explaining systematically stock returns is missing. Consequently, this portfolio is subject to the information risk. We also notice that the R^2 is only 0.1%.

Finally, as we used a value-weighted index, we also test that our results are not sensitive to this weighting scheme. To this end, we perform the FM methodology with an equally-weighted index, just like Black, Jensen and Scholes (1972) or Fama and MacBeth (1973). Table 3 compares the use of these two indices as proxies for the market portfolio. Our observations remain unchanged. The beta on the equal-weighted return is even of the wrong sign, but not significantly. This shows that the beta on the value- or equal- weighted return is not a statistically significant determinant of the cross-section of average returns. In presence of informational asymmetries both are inappropriate.

[Insert Table 3 about here]

4.1. Interpretation

From its outset, relying on the CAPM literature, the market risk premium of a firm has been estimated empirically by using a simple value- or equal-weighed stock index. However, the CAPM was built in a symmetric information context, condition that has been proven not held in practice. With this proxy, two risk premiums are actually captured empirically: the market risk premium as intended, but combined with an information risk premium. In this sense, the estimated β is thus a complete measure of the risk of a security or a portfolio since no other risk measure appears (C3), but it does not reflect solely the systematic risk. It captures both the systematic risk and the informational risk. Using the optimal informed investors' portfolio, which is conditional on *all* information, public but also private, and thus free of the informational risk, the systematic risk is then more precisely captured, leaving aside the informational risk.

Moreover, as shown in section 2, in presence of informational asymmetries the traditional beta, obtained by regressing a stock's return onto a value-weighted stock index, is a combination of the real beta and the information beta. This explains why we find a higher beta by using the stock index. Using the market portfolio to estimate the systematic risk would only be appropriate in a world without adverse selection, as designed by the traditional CAPM. In practice, because of information asymmetry, the conditional CAPM fits more with reality. As realized returns are used as proxy for expected returns, only the use the optimal informed investors' portfolio is correct. Using a stock index provides a noisy measure of the market beta and is inappropriate.

4.2. Robustess tests

We address in this section the important question of whether conditioning the market portfolio on information allows the conditional CAPM to explain asset-pricing anomalies. To this end, we group stocks on a firm characteristic known to be correlated with expected returns and we test the conditional CAPM on the stocks within each group.

First, we form 100 portfolios on the basis of size and market betas (10×10), well known to lead to a CAPM rejection (Fama-French, 1992). More specifically, stocks are sorted into size deciles, subdivided into beta deciles, estimated by using the value-weighted stock index over 24 to 60 months of past returns. Portfolios are formed each year, and stock returns are equally-weighted.

Time-series averages of portfolio returns are given in Panel A of Table 4 where Si/Bj denotes the portfolio whose size is in the ith decile and market beta is in the jth decile. The rates of return range from a low of 0.16 percent to a high of 2.35 percent per month. The dispersion is thus higher than reported in Fama-French (1992, Table I[14]) and Jagannathan and Wang (1996, Table I). Statistical tests are more effective with dispersed returns. We also find the negative small firm premium documented in Burlacu *et al*. (2016). When we compute the average return for the five first size deciles across the betas, we find 0.88%, compared to 0.91% for the five last deciles, i.e. a size premium of –.03%. This is equivalent to calculating

[14] FF (1992) and JW (1996) only use returns from nonfinancial firms, while we also use returns from financial firms to be consistent with the optimal informed investors' portfolio, built from all firms.

$SMB = (S1/B1 + S1/B2 + \cdots + S5/B10)/50 - (\mathrm{S6/B1\ +\ S6/B2\ +\ \ldots\ +\ S10/B10})/50$. Note that the sample is here not exactly the same than in Burlacu *et al*. (2016), the requirements for data availability being different. We have 653 stocks/year in average for the 6 FF portfolios formed on size and book-to-market, compared with 566 stocks/year in average for this 100 FF portfolios formed on size and beta. The betas of the portfolios are presented in Panel B. They range from a low of 0.17 to a high of 1.81. We calculate the size of a portfolio as the equally-weighted average of the logarithm of market value of stocks (in million dollars). The time-series averages of portfolio size are presented in Panel C. They range from a low of 1.46 to a high of 9.71.

[Insert Table 4 about here]

Results of the FM tests are found in Table 5. The average slopes allow determining which explanatory variables on average have non-zero expected premiums. Controlling for size, λ_1 becomes negative, even if not significantly. Therefore, sorting stocks on size, the market does not help explain average stock returns, whether for the unconditional CAPM, as in Fama-French (1992), or conditioning on information. This strong size effect suggests that the CAPM solely conditional on information is inconsistent with the data. However, several studies, such as Jagannathan and Wang (1996) or Lettau and Ludvigson (2001b), found that allowing time variations, the size effect almost disappear.

[Insert Table 5 about here]

Then, we group stocks in 25 portfolios with a two-pass sort on size and book-to-market ratio (5×5). Table 6 provides their summary statistics. Using returns on this 25 size and book-to-market sorted portfolios, we now examine the power of various beta representations to explain the cross-section of average returns. On the first line, we start with solely the beta on the return of the optimal informed investors' portfolio. Then, we introduce the size and book-to-market factors (2×3 sorts) documented by Fama-French (1993)[15], which has since been

[15] In accordance, with Fama-French procedure, Market Equity (ME) is defined as the unadjusted share price multiplied by the number of ordinary shares issue. Book common equity (BE) is defined as the book value of stockholders' equity, plus balance-sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. We use ME and BE at the end of December of year $t-1$ to compute the book-to-market ratio (BtM), and ME at the end of June of year t to measure firms' size. To be included a year, a firm must have

widely studied. Finally we enrich the three-factor model with the recent profitability[16] and investment[17] factors ($2 \times 2 \times 2 \times 2$ sorts[18]). This Fama-French (2015) five-factor model has the merit of attempting to put order back, once again, in what Cochrane likes to call "the factor zoo". FM results are reported in Table 7. The R^2 of the cross-sectional regressions show the fraction of the cross-sectional variation in average returns that is explained by each model. Lettau and Ludvigson (2001b) note that, "although the cross-sectional R^2 is not a formal test of model specification, it is an informative summary statistic of how well each model fits the data, and it neatly illustrates the anomaly emphasized by Fama and French (1992) that the classic CAPM explains virtually none of the cross-sectional variation in returns on these portfolios".

[Insert Table 6 about here]

[Insert Table 7 about here]

We indeed observe that the t-statistic associated to the market premium λ_1 on the first line is significantly negative, which contradicts the CAPM theory. This negative sign on the market portfolio is pervasive in the literature. The R^2 for this regression summarizes this failure, only 1.1 percent of the cross-sectional variation in average returns can be explained by the beta for the market return. By way of comparison using a value-weighted index instead of the

a ME data available at these two dates, and a positive BE as well (although there are several possible explanations for a negative BE, these firms are likely to be financially distressed). To avoid survivorship bias, we also do not include firms unless they have two years of BE. Six FF portfolios are formed at the end of June of year t, to be sure that the BE will be known, even for firms whose the fiscal year ends at the end of March of year t. They are obtained by crossing the size and BtM criteria (2×3). We compute the median size to split the sample into two groups (small and big) at the end of June of year t. The ranking on BtM is realized at the end of December of year t by computing two quantiles (30 and 70%) to break the sample into three groups (low, medium and high). The 6 FF portfolios are then monthly value-weighted from July of year t to the end of June of year $t+1$. Please refer to Fama-French (1993) for more details.

[16] Operating profitability, in the sort for June of year t, is measured with accounting data for the fiscal year ending in year $t-1$ and is defined as annual revenues (sales) minus cost of goods sold, interest expense, and minus selling, general, and administrative expenses, all divided by book equity.

[17] Investment, in the sort for June of year t, is the growth of total assets for the fiscal year ending in $t-1$ divided by total assets at the end of $t-2$.

[18] Breakpoints for the size, value, profitability and investment factors is the median of the sample.

optimal informed investors' portfolio, we find a R^2 of 2.9 percent[19]. The Fama-French three- and five-factor models perform much better, explaining respectively 44.1 and 57.2 percent of the cross-sectional variation in returns. λ_2 is negative is both cases (lines 2 and 3), even if not significantly, meaning that the size premium is negative. Shares of firms with small market values have thus had smaller returns, on average, than large firms. The value premium is significantly positive in both cases. This is consistent with results in Burlacu *et al*. (2016), where they acknowledged a negative small firm effect and a positive value premium on the French market. λ_4 and λ_5, corresponding to the profitability and investment risk premiums, on the third line are both significantly positive. Finally λ_0 is significantly negative in the three cases, so we reject the null hypothesis that the pricing error is zero.

Nonetheless, we conduct further tests for the *joint* significance of the pricing errors. To this end, we calculate the average pricing errors for the set of 25 portfolios associated with the different specifications tested above. More specifically, we perform a Wald test[20] of the null hypothesis that all the pricing errors are *jointly* zero. The asymptotic χ^2 joint distribution of the intercepts gives the model test statistic, with error terms being independent and identically distributed over time, homoscedastic and independent of the factors. Although the small sample properties of an asymptotically efficient estimator are poor (see Hansen, Heaton and Yaron, 1995), strong results still allow to draw conclusions. This tests suits for single factor models, such as the conditional CAPM, and we calculate its multivariate counterpart, the Gibbons, Ross and Shanken (1989) statistic[21], for the Fama-French three- and five- factor models.

[19] This result is consistent with our first findings, but since at this stage we are interested on whether conditioning on information allows explaining anomalies, we chose to not clutter up with tables presenting the results with the value-weighted index.

[20] We test the null hypothesis that the regression intercepts are *jointly* equal to zero, against the alternative hypothesis that they are *jointly* superior to zero:

$$T\left[1+\left(\frac{E_T(f)}{\hat{\sigma}(f)}\right)^2\right]\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\sim\chi_N^2$$

with N the number of assets over $t=1,2,\cdots,T$, $E_T(f)$ the sample mean of the factor, and $\hat{\sigma}(f)$ its sample standard deviation. $\hat{\alpha}=[\hat{\alpha}_1 \quad \hat{\alpha}_2 \quad \cdots \quad \hat{\alpha}_N]'$ is the vector of intercepts and $\hat{\Sigma}$ the residual covariance matrix. The reader can also consult Cochrane (2005, chap 12) for an excellent textbook on the chi-square and GRS tests.

[21] Gibbons-Ross-Shanken (1989) test:

Table 8 details the average pricing errors, where Si/Vj denotes the portfolio whose size is in the ith quintile and book-to-market is in the jth quintile. For example, it can be seen in the first column which details the average pricing errors associated with the conditional CAPM, that the small growth portfolio (S1/B1) has a negative average pricing error (–.5130), whereas the small value portfolio (S1/B5) has a positive one (1.0906). The same pattern can be observed in the rest of the sizes, except the largest one. This is the value-spread puzzle. At the bottom of the table, we report the results of a chi-square test for the conditional CAPM and GRS test for the three models. The *p*-values being lower than 5 percent, we reject the null hypothesis and accept the alternative hypothesis that the true intercepts are *all* higher than zero. All these models are incomplete descriptions of expected returns, even if the inclusion of the size, value, profitability and investment factors improves the description of average returns. Surprisingly, that's the three-factor model which produces the lowest GRS statistics, but the five-factor presents the lowest mean squared error (MSE) and the highest R^2. The MSE is the sum of squared errors standardized by the number of degrees of freedom. The advantage of this measure over a simple average of the pricing errors, is thus to take into account the precision of the estimates of the intercepts and the number of factors. To use Fama-French (2015) wording, the five-factor model is thus "the model that is the best (but imperfect) story for average returns on portfolios".

[Insert Table 8 about here]

$$\frac{T-N-K}{N}\left(1+E_T(f)'\hat{\Omega}^{-1}E_T(f)\right)^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\sim F_{N,T-N-K}$$

where

$$\hat{\Omega}=\frac{1}{T}\sum_{t=1}^{T}[f_t-E_T(f)][f_t-E_T(f)]'$$

with K the number of factors.

5. Conclusion

The systematic risk is captured by the market beta, that we called the "real beta". In the traditional CAPM, which considers symmetric information, this beta is obtained by regressing a stock's return onto a value-weighted stock index serving as proxy for the market portfolio. The systematic risk premium, also called "market risk premium", is the product of the market beta and the expected return of the market portfolio in excess of the risk-free rate. In practice, realized stock returns are used as proxy for expected stock returns.

However, in presence of informational asymmetries, stock returns are realized on the basis of public but also private information. The "real beta" is thus the ratio of the covariance between the realized stock return of a security and the portfolio conditional on all the investors' information (publicly available but also private information), divided by the variance of this portfolio return. This portfolio is the optimal informed investors' portfolio.

Hence, in an asymmetric information context, an information beta comes into play and the traditional beta obtained by using a value-weighted stock index is not the real beta. The traditional beta is then a combination of the real beta and the information beta. By using a stock index, we thus capture indiscriminately the market risk and the information risk. To disentangle these two elements, we must use the optimal informed investors' portfolio, not subject to the informational risk. It allows estimating solely the real beta and obtaining the market risk premium.

We provide empirical evidence for these theoretical hypotheses. We compared results using two different proxies for the market portfolio: (i) a value-weighted stock index, which is inappropriate but widely used; (ii) the optimal informed investors' portfolio, the only appropriate portfolio given that it is conditional on private and public information and that returns are realized on the basis of private and public information. The use of the stock index provides a higher beta as it captures both the market and the information risk. The R^2 is also higher and the Fama-MacBeth procedure rejects the existence of other factors explaining systematically stock returns. By contrast, the informed portfolio delivers a higher risk premium and the FM test indicates that at least another factor explaining systematically stock returns is missing in the model. This supports the fact that a stock index, which reflects the view of a representative investor, i.e. a fictitious agent whose beliefs are a weighted average of the informed and uninformed agents' beliefs, is subject to the informational risk, while the portfolio conditional on private and public information does not suffer from adverse selection.

Hence, conditioning on information brings us closer to a positive relation between the market beta and average returns as predicted by the Sharpe–Lintner–Black model, relation which was obscured so far by an improper use of the CAPM. A stock index *cannot* be used as proxy for the market portfolio. We are convinced that, once we will have taken into account time variations, we will find a significant positive linear relationship between the market beta and the cross-section of average returns by using the optimal informed investors' portfolio. We also find that conditioning on information does not allow the CAPM to explain asset-pricing anomalies, i.e. why for example firms with high book-to-market ratios outperform those with low book-to-market ratios. More specifically, we tested for the well documented size and value effects. In view of Jagannathan and Wang (1996) results, we also strongly believe that the effect of these anomalies will be greatly reduced once we will have incorporated the effects of time and the return on human capital as part of the returns on aggregate wealth. Lettau and Ludvigson (2001b) confirmed and wrote: "Fama-French factors are mimicking portfolios for risk factors associated with time variation in risk premia. Once the (C)CAPM is modified to account for such time variation, it performs about as well as the Fama-French model in explaining the cross-sectional variation in average returns".

These preliminary results already constitute a major contribution, both for academics and practitioners. In addition of misleading empirical tests of the conditional CAPM, the use of a "noisy" proxy led to a mismeasurement of mutual funds' performance, with systematically negative alphas. As acknowledged by Burlacu, Fontaine and Jimenez-Garcès (2013), it comes from the fact that the information risk premium is applied twice: mutual funds' realized return *net of expenses*, while managers charge their clients for the information acquisition costs, is compared with an expected return from a model which includes an information risk premium.

If mutual funds would consistently underperform, their very existence would be crippled, while in practice the deposits size has dramatically increased over the last decades[22].

There is still room for improvements in the testing procedure. Indeed, we are now aware of the limitations of the Fama-MacBeth procedure, which are the price of its simplicity. To correct the errors-in-variable bias, we use Litzenberger and Ramaswamy (1979), but there are many other possibilities (see for example Shanken, 1992; Hansen and Jagannathan, 1997; Jagannathan and Wang, 1998; Kan, Robotti and Shanken, 2013). More importantly, these corrections apply to time-invariant coefficients, within the FM method which implies that risk premiums are constant, while there is strong evidence that prices of risk vary over time (Campbell and Shiller, 1988; Cochrane, 2011). We leave for future research the use of Adrian, Crump and Moench (2015)'s dynamic version of the Fama-MacBeth estimator, as well as including the return on human capital in accordance with Jagannathan and Wang (1996).

Fama-French (1992) wrote: "Resuscitation of the SLB model requires that a better proxy for the market portfolio (a) overturns our evidence that the simple relation between beta and average stock returns is flat and (b) leaves beta as the only variable relevant for explaining average returns".

We have great hope that this challenge will be soon faced by extending this work.

[22] Deposits in undertakings for collective investment (UCI) governed by French law has for example increased from 888 billion of euros in 2001 (1,473 with mandates and UCI organized under foreign laws) to 1,683 in 2015 (respectively 3,591). Source: AMF.

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Tables

Table 1: Descriptive statistics

Panel A: Overview

	Subperiod	Duration
1. Portfolio formation	1996–2000	48 months
2. Estimation	2000–2005	60 months
3. Testing	2005–2009	48 months

Yearly periods are from the end of August t to the end of July $t+1$. After 2005, the initial 5-year estimation period is extended annually up to 8 years (2000–2008).

Panel B: Number of stocks allocated to portfolios

Portfolio	1	2–19	20	Total
Nb of stocks	17	12	18	251

The middle 18 portfolios each has $int(N/20)$ stocks, and the first and last portfolios each receives:

$$int\left(\frac{N}{20}\right)+\frac{1}{2}\left[N-20int\left(\frac{N}{20}\right)\right]$$

with N the total number of securities to be allocated to portfolio and $int(N/20)$ be the largest integer equal to or less than $N/20$. When N is odd like here, the last portfolio (highest $\hat{\beta}$) gets an additional security.

Panel C: Average betas

Period	Updates	Average β_i Informed	Uninformed	Index
1996–1999		.109	.110	.654
2000–2005		.149	.257	.750
	2006	.167	.257	.777
2005–2008	2007	.163	.246	.773
	2008	.265	.349	.812
Time-average		.171	.244	.753

Individual betas are computed for each security against: (i) the optimal informed investors' portfolio; (ii) the optimal uninformed investors' portfolio; (iii) and the value-weighted stock index. They are then averaged across securities. 1. We calculate pre-ranking firms' betas with time-series from 8/1996 to 7/2000. 2. We calculate post-ranking firms' betas with time-series from 8/2000 to 7/2005. 3. Firms' betas are yearly updated with time-series from 8/2000 to 7/2006, from 8/2000 to 7/2007 and from 8/2000 to 7/2008

Table 2: Test of the CAPM using Fama-MacBeth (1973)

Panel A

Value-weighted index

	$\bar{\gamma}$	$s(\gamma)$	$t(\gamma)$
γ_0	–.049	1.021	–.33
γ_1	.084	1.434	.41
γ_2	–.117	1.294	–.63
γ_3	.010	1.159	.06

R^2 = .095

Optimal informed investors' portfolio

	$\bar{\gamma}$	$s(\gamma)$	$t(\gamma)$
γ_0	–.037	1.227	–.21
γ_1	.222	.997	1.54
γ_2	–.137	1.068	–.89
γ_3	–.473	1.220	–2.69

R^2 = .054

Panel B

Optimal Uninformed investors' portfolio

	$\bar{\gamma}$	$s(\gamma)$	$t(\gamma)$
γ_0	.037	1.046	.25
γ_1	–.114	.947	–.84
γ_2	.107	.940	.79
γ_3	–.075	1.140	–.46

R^2 = .001

(i) 20 portfolios are formed according to individual security betas, estimated with a times-series from 30/08/1996 to 31/07/2000.

(ii) Portfolio betas and portfolio non-beta risk are calculated by equal-weighting respectively individual security betas and standard deviations of the residual returns for individual securities, estimated with time-series from 31/08/2000 to 29/07/2005. Firms' betas and the standard deviation of the residuals are then recalculated annually with time-series from 31/08/2000 to 31/07/2006, 31/08/2000 to 31/07/2007, and from 31/08/2000 to 31/07/2008.

(iii) Cross-sectional OLS regressions are run each month from 31/08/2005 to 31/07/2009, adjusting monthly portfolio betas and portfolio non-beta risk by removing delisted firms or having missing data.

In the last column are reported the t-statistics for each coefficient estimate, corrected with Litzenberger and Ramaswamy (1979) methodology.

Table 3: Comparing value- and equal-weighting

Value-weighted index

	$\bar{\gamma}$	*s(γ)*	*t(γ)*
γ_0	–.049	1.021	–.33
γ_1	.084	1.434	.41
γ_2	–.117	1.294	–.63
γ_3	.010	1.159	.06

R^2 = .095

Equally-weighted index

	$\bar{\gamma}$	*s(γ)*	*t(γ)*
γ_0	.175	1.301	.93
γ_1	–.084	1.485	–.39
γ_2	.057	1.176	.34
γ_3	–.149	1.185	–.87

R^2 = .157

(i) 20 portfolios are formed according to individual security betas, estimated with a times-series from 30/08/1996 to 31/07/2000. The return on a value-weighted stock index is used as proxy for the market return in the left table, compared with the return on an equally-weighted index in the right table.

(ii) Portfolio betas and portfolio non-beta risk are calculated by equal-weighting respectively individual security betas and standard deviations of the residual returns for individual securities, estimated with time-series from 31/08/2000 to 29/07/2005. Firms' betas and the standard deviation of the residuals are then recalculated annually with time-series from 31/08/2000 to 31/07/2006, 31/08/2000 to 31/07/2007, and from 31/08/2000 to 31/07/2008.

(iii) Cross-sectional OLS regressions are run each month from 31/08/2005 to 31/07/2009, adjusting monthly portfolio betas and portfolio non-beta risk by removing delisted firms or having missing data.

In the last column are reported the t-statistics for each coefficient estimate, corrected with Litzenberger and Ramaswamy (1979) methodology.

Table 4: Summary statistics for 100 FF portfolios sorted on size and beta, 7/1999 to 12/2014

Panel A: Time-Series Averages of Returns (in %)

	β1	β2	β3	β4	β5	β6	β7	β8	β9	β10
S1	.94	2.02	1.52	1.61	2.35	1.84	2.16	1.55	.83	.82
S2	.66	.18	1.35	1.01	1.46	.61	.84	.21	.78	.92
S3	.45	.50	.26	.83	1.51	.16	.98	.77	1.39	.31
S4	.46	1.01	1.31	1.14	.27	.43	.45	.74	.44	.45
S5	.79	.99	1.21	.28	.52	.60	.72	.75	.34	.35
S6	.67	.94	1.02	1.22	.82	1.04	1.02	1.09	.76	.71
S7	1.18	.69	1.44	.98	.80	.82	1.79	.87	.73	.28
S8	1.04	1.00	1.24	1.24	1.01	1.01	.93	1.07	.58	.32
S9	1.29	1.04	1.15	.59	1.00	1.43	1.45	.61	.56	.62
S10	1.27	.50	1.04	.75	.62	.99	.66	.61	.30	.63

Panel B: The Estimated Betas

	β1	β2	β3	β4	β5	β6	β7	β8	β9	β10
S1	.63	.17	.39	.42	.54	.68	.58	.95	.94	1.22
S2	.35	.38	.21	.44	.46	.75	1.02	.76	1.08	1.26
S3	.30	.41	.33	.56	.75	.68	.84	.88	1.26	1.44
S4	.33	.37	.42	.49	.63	.55	.73	.90	1.15	1.51
S5	.60	.26	.54	.67	.66	.64	.79	.86	1.32	1.79
S6	.43	.46	.48	.42	.53	.81	.76	.95	1.19	1.60
S7	.41	.36	.46	.69	.59	.78	.82	1.06	1.34	1.46
S8	.33	.40	.66	.67	.84	.75	1.00	1.21	1.62	1.81
S9	.45	.59	.68	.63	.81	.83	1.09	.95	1.40	1.66
S10	.52	.78	.77	.93	.87	1.21	1.26	1.40	1.47	1.81

Panel C: The Time-Series Averages of Size (log million €)

	β1	β2	β3	β4	β5	β6	β7	β8	β9	β10
S1	1.46	1.63	1.63	1.67	1.69	1.72	1.75	1.67	1.64	1.60
S2	2.54	2.58	2.62	2.63	2.66	2.59	2.60	2.63	2.69	2.64
S3	3.29	3.27	3.21	3.29	3.24	3.25	3.23	3.27	3.23	3.24
S4	3.76	3.79	3.83	3.85	3.75	3.82	3.84	3.78	3.76	3.79
S5	4.32	4.35	4.38	4.32	4.29	4.35	4.30	4.30	4.30	4.30
S6	4.88	4.93	4.90	4.91	4.88	4.92	4.95	4.88	4.87	4.88
S7	5.49	5.51	5.52	5.53	5.52	5.50	5.53	5.53	5.51	5.50
S8	6.30	6.30	6.35	6.33	6.30	6.31	6.27	6.28	6.29	6.22
S9	7.31	7.39	7.24	7.46	7.48	7.48	7.55	7.45	7.50	7.51
S10	9.31	9.71	9.48	9.33	9.29	9.41	9.47	9.31	9.30	9.40

The size of a stock is defined as the logarithm of the market value of the asset.

Table 5: Fama-MacBeth two-step procedure applied to the CAPM using 100 FF portfolios, Monthly returns, 7/1999 to 12/2014

Value-weighted index

	$\bar{\gamma}$	$s(\lambda)$	$t(\lambda)$
λ_0	.836	2.385	4.78
λ_1	−.489	3.884	−1.72

R^2 = .118

Optimal informed investors' portfolio

	$\bar{\gamma}$	$s(\lambda)$	$t(\lambda)$
λ_0	.642	1.821	4.81
λ_1	−.315	2.698	−1.59

R^2 = .019

Fama-MacBeth (1973) two-stage test procedure is applied to monthly returns from July 1999 to December 2014 for the 100 Fama-French portfolios sorted by size and beta:

(i) Portfolio returns are regressed against variables hypothesized to explain expected returns:

$$R_i^t = \beta_{0,i} + \beta_{1,i} \cdot MKT^t + \varepsilon_i^t$$

The return on a value-weighted stock index is used as proxy for the market return in the left table, compared with the return on the optimal informed investors' portfolio in the right table. These time-series regressions, from 30/08/1996 to 31/12/2014, deliver estimates of the portfolio beta.

(ii) Each month, from 30/08/1996 to 31/12/2014, the cross-section of returns on portfolios is regressed on these factor exposures:

$$R_i^t = \lambda_{0,i} + \lambda_1^t \cdot \hat{\beta}_{1,i} + \eta_i^t$$

Coefficients are then weighted by the inverse of their standard error, and time series averaged to obtain the risk premium of each factor.

In the last column are reported the t-statistics for each coefficient estimate, corrected with Litzenberger and Ramaswamy (1979) methodology.

Table 6: Summary statistics for 25 FF portfolios sorted on size and value, in percentage points, 8/1996 to 12/2014

	S1/V1	S1/V2	S1/V3	S1/V4	S1/V5	S2/V1	S2/V2	S2/V3	S2/V4	S2/V5
Min	−17.40	−16.82	−38.53	−13.79	−13.48	−19.61	−20.25	−20.03	−18.02	−35.47
1st Quartile	−4.17	−2.65	−3.15	−1.65	−1.31	−2.66	−2.12	−1.84	−2.27	−.97
Mean	.003	.53	.28	1.00	1.70	.34	.51	.69	.59	1.60
Mediane	.15	−.24	−.65	.68	.97	−.35	.51	.76	.52	1.61
3rd Quartile	3.39	4.11	2.71	3.55	4.30	3.03	2.98	3.59	3.46	3.81
Max	27.12	24.14	37.33	26.31	34.73	35.90	50.34	13.55	44.30	42.45

	S3/V1	S3/V2	S3/V3	S3/V4	S3/V5	S4/V1	S4/V2	S4/V3	S4/V4	S4/V5
Min	−27.42	−18.70	−17.01	−14.94	−17.74	−26.76	−20.39	−17.02	−17.51	−18.15
1st Quartile	−3.29	−1.93	−1.58	−.83	−.95	−3.12	−2.41	−1.21	−0.97	−1.00
Mean	.81	.84	.80	1.13	1.12	.25	.76	1.16	1.44	1.27
Mediane	.65	.90	1.25	1.53	1.41	.73	1.16	1.58	1.93	1.37
3rd Quartile	3.56	3.87	3.84	4.03	3.28	3.87	3.70	4.08	4.03	4.07
Max	50.38	21.14	11.09	13.56	13.83	45.50	18.41	10.98	18.81	25.75

	S5/V1	S5/V2	S5/V3	S5/V4	S5/V5
Min	−18.72	−18.57	−19.12	−22.08	−21.37
1st Quartile	−2.21	−2.33	−1.38	−2.30	−3.97
Mean	.54	1.01	1.22	.95	.86
Mediane	1.09	1.46	1.66	1.64	1.02
3rd Quartile	3.54	4.53	4.68	4.55	5.90
Max	18.85	15.18	16.20	27.42	29.55

These 25 value-weight size-B/M portfolios are the intersections of the independent 5×5 market equity and book-to-market equity (B/M) sorts. Please refer to Fama-French (1993) for details. The portfolio S1/V1 contains stocks belonging to both the 1st size quintile and the 1st B/M quintile.

Table 7: Fama-MacBeth regressions using 25 FF portfolios, Monthly returns, 8/1996 to 12/2014

		Factors					
	Constant	MKT	SMB	HML	RMW	CMA	R^2
(1)	.339	−.180					.011
	(2.68)	(−1.94)					
(2)	.248	−.093	−.227	.437			.441
	(2.55)	(−1.04)	(−1.37)	(3.30)			
(3)	.333	−.157	−.127	.291	.199	.191	.572
	(3.79)	(−2.01)	(−.78)	(3.24)	(3.01)	(2.37)	

Fama-MacBeth (1973) two-stage test procedure is applied to monthly returns from August 1996 to December 2014 for the 25 portfolios sorted by size and book-to-market ratios (see Table 6):

(i) Portfolio returns are regressed against variables hypothesized to explain expected returns:

(1) $R_i^t = \beta_{0,i} + \beta_{1,i} \cdot MKT^t + \varepsilon_i^t$

(2) $R_i^t = \beta_{0,i} + \beta_{1,i} \cdot MKT^t + \beta_{2,i} \cdot SMB^t + \beta_{3,i} \cdot HML^t + \varepsilon_i^t$

(3) $R_i^t = \beta_{0,i} + \beta_{1,i} \cdot MKT^t + \beta_{2,i} \cdot SMB^t + \beta_{3,i} \cdot HML^t + \beta_{4,i} \cdot RMW^t + \beta_{5,i} \cdot CMA^t + \varepsilon_i^t$

with SMB the zero-investment size portfolio; HML the zero-investment value portfolio; RMW the zero-investment profitability portfolio which is long position on stocks with robust profitability, i.e. firms which profitability is above the median, and short on stocks with weak profitability, i.e. firms which profitability is below the median; and CMA the zero-investment investment portfolio which is long position on conservative stocks, i.e. firms with the lowest 50 percent investment, and short on aggressive stocks, i.e. firms with the highest 50 percent investment.

These time-series regressions, from 30/08/1996 to 31/12/2014, deliver estimates of the betas.

(ii) Each month, from 30/08/1996 to 31/12/2014, the cross-section of returns on portfolios is regressed on these factor exposures:

(1) $R_i^t = \lambda_{0,i} + \lambda_1^t \cdot \hat{\beta}_{1,i} + \eta_i^t$

(2) $R_i^t = \lambda_{0,i} + \lambda_1^t \cdot \hat{\beta}_{1,i} + \lambda_2^t \cdot \hat{\beta}_{2,i} + \lambda_3^t \cdot \hat{\beta}_{3,i} + \eta_i^t$

(3) $R_i^t = \lambda_{0,i} + \lambda_1^t \cdot \hat{\beta}_{1,i} + \lambda_2^t \cdot \hat{\beta}_{2,i} + \lambda_3^t \cdot \hat{\beta}_{3,i} + \lambda_4^t \cdot \hat{\beta}_{4,i} + \lambda_5^t \cdot \hat{\beta}_{5,i} + \eta_i^t$

Coefficients are then weighted by the inverse of their standard error (Litzenberger and Ramaswamy correction), and time series averaged to obtain the risk premium of each factor.

Table 8: Pricing errors, 25 FF portfolios

Port.	CAPM	FF3F	FF5F
S1/V1	–.5130	–.3931	–.3092
S1/V2	–.2395	–.1533	–.1253
S1/V3	–.6691	–.5001	–.5338
S1/V4	.4443	.4442	.5085
S1/V5	1.0906	1.0268	1.1292
S2/V1	–.3120	–.0466	–.1135
S2/V2	–.1958	.0308	–.0269
S2/V3	.1107	.0651	.2192
S2/V4	–.1667	–.1890	–.0842
S2/V5	.7624	.5710	.6871
S3/V1	–.0018	.4109	.2756
S3/V2	.1495	.1979	.2857
S3/V3	.0852	.0239	.1308
S3/V4	.5252	.4525	.5521
S3/V5	.5427	.3998	.5222
S4/V1	–.3401	–.1184	–.1285
S4/V2	.1257	.1213	.2088
S4/V3	.5441	.4603	.5731
S4/V4	.8067	.6464	.7759
S4/V5	.6427	.4620	.6470
S5/V1	.0542	.1492	.1385
S5/V2	.4888	.4074	.5489
S5/V3	.7987	.6620	.7982
S5/V4	.3226	.0059	.1996
S5/V5	.1987	–.1578	.1284
Mean squared error	.3195	.2713	.2484
Chi-square	53.331		
p-value	.0008		
GRS	2.015	1.861	2.220
p-value	.0044	.0106	.0014

Average pricing errors (in percentage) from the Fama-MacBeth (1973) regressions reported in Table 7 for each of the 25 Fama-French portfolios. S1 denotes the portfolio with the smallest firms and S5 the largest. Similarly, V1 includes the firms with the lowest book-to-market portfolios and V5, firms with the highest. The optimal informed investors' portfolio is used as proxy for the market portfolio. The last five lines report the square root of average squared pricing error across all portfolios (average MSE[23]), the chi-square statistic for a test that the pricing errors are zero and the corresponding *p*-value, and finally the Gibbons-Ross-Shanken (1989) test statistic with the corresponding *p*-value. Data are monthly and the sample period is 8/1996 – 12/2014.

[23] $MSE = SCR/(T - K - 1)$ with SCR the sum of squared errors.