Reaching medium risk exposure through non-fixed income ETFs[1]

Christophe Dispas[2], Amaury Goguel[3], Thomas Lanzi[4] and Hugo Pascail[5]

ABSTRACT: Fixed income investments represent a significant part of diversified portfolios from private to institutional investors. The current very low interest rate environment means high potential risk for these fixed income investment holders. In this paper, we propose an alternative investment solution for "medium risk" profile investors - defined as 50% equity and 50% fixed income portfolio - with comparable return and risk characteristics but without any fixed income products. The goal of this strategy is to protect medium risk portfolio against big losses in case of interest rate increase. We use Exchange Traded Funds (ETF) to compose our portfolios in order to provide an investable solution. To reach that goal, an originality of our research is to select ETFs by an algorithm, namely PcGets. Based on this selection, we minimize the tracking error between our ETFs selection and our medium risk benchmark. Within the block search extension, it also allows to deal with database where the number of exogenous variables is greater than the number of observations. From a methodological standpoint, we compare the optimized PcGets portfolio with another one composed from a standard quadratic optimization. To complete our analysis, we add some constraints on tracking-error volatility for each portfolios. We discuss the fact that adding constraints can substantially improve the performance of the portfolio selection. From an investment standpoint, a low volatility combination of non-fixed income ETFs seem to provide an interesting alternative to classical medium risk portfolio.

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[2] Université Côte d'Azur, SKEMA, France.
[3] Université Côte d'Azur, SKEMA, France.
[4] Université Côte d'Azur, SKEMA, France.
[5] Université Côte d'Azur, SKEMA, CNRS, GREDEG, France.

I. Introduction

After the major market turmoil of the financial crisis (2007-2008), interest rates have strongly decreased. Reduced risk appetite and very accommodative monetary policies around the world mainly explain it. In the current very low interest rate environment, a brutal increase of interest rates, or "interest rate risk", could translate in very negative returns for fixed income investors. Fixed income products are commonly used by investors as a diversification tool, allowing them to smooth the volatility of their portfolios. They are perceived as a safe way of investing. Unfortunately, after so many years of continuous positive performance of the bond market, interest rate risk is probably underestimated by many investors. In case of a strong rate increase, due to the negative relationship between fixed income products value and interest rates, the result could be very damaging for portfolio returns.

We propose an alternative investment solution for "medium risk" profile investors - defined in this paper as 50% equity and 50% fixed income portfolio - with comparable return and risk characteristics but without any fixed income products. The goal of this strategy is to protect medium risk portfolio against big losses in case of interest rate increase.

Medium risk portfolio investing finds its validation in the Capital Asset Pricing Model (CAPM) framework[6]. By adding the risk free rate to the Markowitz model (1952), the CAPM provides a new dominant portfolio solution by combining risky and risk free investments. In this paper, we have chosen a 50% equity and 50% fixed income benchmark as a proxy for a CAPM optimal portfolio of risky and risk free investments. The CAPM, developing the Markowitz model (1952), emphasizes the benefits of portfolio diversification to reduce risk.

To reach a medium risk portfolio without fixed income strategy, an originality of our analysis is the choice of a new flexible product who gains more and more attractiveness since few years, the exchange traded fund (ETF). An ETF is defined as a marketable security which tracks index, or baskets of assets, which is traded like common stocks, who experiences daily price changes with high liquidity and low fees. In other words, an ETF could be defined as a flexible investment fund.

Investment funds are widely used to invest in the financial market, from the smallest individual to the largest institutional. In the year 2000, mutual funds accounted for more than 95% of the US investment funds (representing around $7 trillion in assets under management) and were spread across more than 8,000 funds. At the same time, ETFs accounted only for 1% of the total

[6] See for example Sharpe, William F. (1964).

of $7 trillion AUM (which represented only 80 funds split in 66 billion assets under management). In the early 2015, ETFs investments represented 10.90% of the total with $1.98 trillion of AUM.

This success can be partially explained by specific characteristics. *"ETFs give to the investor a high diversification possibility*". As it is described by Abner (2015), there are more than 1500 diversified ETFs strategies. This diversity of strategies gives the ability to any investor to be invested on any asset class, sector, industry or geographical area. Moreover, *"ETFs experience high liquidity on the market"*. As it is a day to day traded product, ETFs experience daily price changes like common stocks. But one of the main advantage of the ETFs is their low structure of fees. Balchunas (2016) illustrates that the asset-weighted average annual fee is 0.30 %, which is less than half the asset-weighted average for active mutual fund (around 0.66%). In 2015, the 20 largest ETFs in the world, had an average annual fee of 0.19%. In comparison, alternative investments funds (like hedge funds), have a classical fees structure of 2% for the management fees and 20% for the performance fees (paid on quarterly or annual basis).

We have chosen ETFs because they represent an affordable and easy way for common investors to build a diversified portfolio. Unfortunately, ETFs selection can be quite difficult regarding the very large choice and the multiplicity of providers make it quite hard and opaque for non-informed investors.

The paper tries to provide answers to the need of finding a methodology to select the "best" ETFs through the total ETFs universe. Another originality of our research is to use a quantitative filter to select ETFs to invest in.

The quantitative filter (PcGets and Block search algorithm) is a methodology and an econometric algorithm that selects "best" ETFs through an iterative process. From ETFs selected, we performed a two steps analysis. We shall call this *Alternative tracking error volatility method*. Firstly, we did an initial quantitative selection by using the algorithm. Secondly, based on this initial selection, we defined the final weighting by quadratic optimization with a practical constraint of non-negativity. Indeed, an average investor will not have the opportunity to short ETFs and, even if possible, the cost of the short could be prohibitive. Indeed, Richard and Michaud (1989) have extensively discussed whether the optimized Markowitz portfolio is that optimal. It clearly appears that its practical value can be enhanced by imposing constraints based on fundamental investments considerations such as for instance, a long only constraint on the portfolio. Put it in another way, we try to provide a practical solution to replace "risky" fixed income assets. Shorting the market does not seem to provide a reasonable solution.

We compare the solution issue by the Alternative tracking error volatility method with the one issue from a standard quadratic optimization without any pre-selection of explicative variables. We shall call this method, the classic tracking error volatility method.

The two methodologies have in common the minimization of the tracking error between the monthly return earned with the replicated index of ETFs and the monthly return earned with the benchmark case constructed as a medium risk portfolio.

Our results conclude that the ATEV appears as an interesting "ETF's selector". We successfully replicate the benchmark characteristics with a low tracking error and acceptable risk and return for a medium risk investor. We find that these characteristics are close to those associated to an investment in "equity low volatility" strategies. Moreover, we find some similarity with a strategy composed by equally weighted equity and cash.

The paper is structured as follows. Section II discusses some related literature. Section III presents the methodology of replication and describes the process of PcGets and the block search method. Section IV presents the data sample and presents the robustness tests we use to validate our results. Section V contains the central empirical results of this study, analyzing the efficiency of the portfolios generated by our ETFs selecting methodology. Section VI discuss the results by adding constraints on optimized portfolio and Section VII concludes.

II. Related literature

A. Standard minimization of the tracking error

In this paper, we propose to construct portfolio by using the standard minimization of the tracking error. A portfolio's tracking error is basically the difference between the returns on: (i) the portfolio and (ii) the benchmark. In practice, portfolio managers commonly use objective functions that lead to the selection of portfolios with minimum tracking error variance for a given expected gain over the benchmark. However, in an influential paper, Roll (1992) shows that these portfolios are typically suboptimal from the perspective of investors. In particular, managers tend to select portfolios that are overly risky for investors (see Jorion, 2002, 2003). Roll (1992) proposes to add a constraint on the beta to improve the managed portfolio. Starting from Roll (1992), several papers propose new expectations for the tracking error methodology.

Rudolf and al. (1999) consider the first absolute moment and first absolute partial moments as relevant characteristics for the deviation between portfolio returns and benchmark returns leading to linear optimization problems. Jorion (2003) proposes adding a standard deviation constraint to Roll's model to move the optimal portfolio closer to the mean–variance efficient frontier. Alexander and Baptista (2006, 2008) propose to characterize optimal portfolio by adding a constraint in the minimization problem. In Alexander and Baptista (2006), they consider a drawdown constraint. In the context of Roll's model, a portfolio's maximum drawdown is the worst loss that the portfolio suffers relative to the benchmark over a given time period. Alexander and Baptista (2008) study a similar problem by adding a value at risk constraint. Baptista (2008) investigates conditions under which an investor can optimally delegate the management of his or her wealth to managers with mean tracking error variance objective functions. He provides an explanation for the use of these objective functions based on the effect of background risk on investors' optimal portfolios. Stoyan and al. (2008) provide a generalization of the currently existing approaches to benchmark tracking by using the methods of the theory of probability metrics.

Our contribution to this related literature is to add three constraints on the tracking error minimization: one on the beta, one on the information ratio and one on the value at risk.

B. PcGets methodology

The methodology used to select the ETFs is based on "Gets methodology". Gets is a process that automatically selects useful empirical models. The method is based on a general-to-specific modeling. This method consists to simplify an initially general model to a specific one in order to identify some empirical evidences within the theoretical framework. More precisely, the general model is reduced in complexity by eliminating statistically insignificant variables step by step, checking the validity of the reductions at each step to ensure congruence of the finally selected model. This method is also known as the LSE (or London School of Economics) approach and the acronym Gets is used to symbolize the LSE approach for general-to-specific modeling. The leading principle of the LSE approach is to apply consistently the theory of encompassing (Mizon 1984, Hendry 1988). Roughly, one specification encompasses another if it carries all of the information of the other specification in a more parsimonious form. The LSE approach also finds some sources in the theory of data reduction (Hendry 1995, chap. 9). The main question of these methods is always how the data can be characterized in a way that is partial or simpler than the true data-generating process without a loss of information relative to the questions of interest.

PcGets is a computer automation of such an econometric model-selection process. Automation of this method is due to Hoover and Perez's (1999) who have first developed and analyzed a computer algorithm for general-to-specific modeling. Based on the work of Hoover and Perez's (1999), Hendry and Krolzig (1999, 2005) and Krolzig and Hendry (2001), throw further light on several methodological issues related to the automated Gets.

The automation of this method follows the standard approach depicted by the general-to-specific modeling. Starting from a general dynamic statistical model, automatic statistical tests and procedures are used to reduce its complexity by eliminating statistically-insignificant variables, provided that the model passes chosen specification tests. This algorithm leads to a few models which can be selected by encompassing tests. An information criterion, such as Akaike (AIC), may be used to discriminate final models. Basically, such a criterion offers a relative estimate of the information lost when a given model is used to represent the process that generates the data.

Krolzig and Hendry (2001), showed through Monte-Carlo experiments that, starting from an information set including the "true model", the Gets algorithm leads to the right specification, if diagnostic tests and significance levels are correctly selected. Hendry and Krolzig (2005) give solutions to deal with intractable problems such as regressions with more variables than observations in regression analysis or perfectly collinear regressors.

Gets estimations are made under the liberal strategy and conservative strategy. The main difference lies in the p-value rejection of tests: the first approach (resp. the second) is based on a 1% (resp. 5%) threshold. See Hendry and Krolzig (2005) for further explanations. Specification tests are Doornick and Hansen normality test, Chow predictive failure test for a break at 50% and 90% of the sample, LM test of 1 to 4 order residual autocorrelation and quadratic in regressors heteroscedasticity test.

Readers may refer to Krolzig and Hendry (2001) for a full presentation and to Dubois and Michaux (2015) who provided a simplified but complete introduction to the algorithm.

In order to implement the Gets modelling, we use Grocer which is an econometric toolbox for Scilab. Dubois and Michaux (2015) proposed a whole presentation of this computer program.

The PcGets methodology and the block search variant is currently employed in the modelling of economic series applicate to macro-economy or GDP forecasting but less so in financial market and portfolio optimization. Only few research applied the Gets methodology in finance, see for instance Bauwens and Succarat (2010) or Escribano and Succarat (2012).

C. ETF portfolio management

Series of scientific researches were done around ETF portfolios. They can be divided in different categories: active and passive management, diversification, comovements and ETF hedging.

The first series of papers compared the difference between active and passive investment management. For example, Chen, Wong and Susai (2016) worked over the benefits of active management and its impact on ETF pricing. They found that active management is useful in ETF pricing efficiency, especially for fund managers, who wants to have transparency into fund prices. Pace, Hili and Grima (2016) worked on the benefit of choosing between active and passive investment structure, (776 equity funds, domiciled either in America or Europe). Their findings suggest that active management is equivalent to index replication in terms of risk-adjusted return but as active management have higher fees and transaction costs, low costs ETFs seem a better solution for a classical investor. Schizas (2014) also worked on risk, return and incentives of ETF investments versus mutual funds and hedge funds. His empirical results shows that active ETFs are not as active as they are viewed by the market but most of the time, ETF actively managed surpassed mutual funds in terms of returns.

Another series of authors worked on the benefit of diversification and the comovements between ETF and their benchmark. Lee, Hsu and Lee (2016) examined how trading location affects return comovements and the diversification benefits in Asian country ETFs and their MSCI indices. Their findings showed that series of factors impact return comovements such as investor sentiment, market conditions, and US economic fundamentals. They also demonstrated the evidence of a higher diversification benefit for the Asian MSCI indices than the Asian country ETFs. O'Hagan-Luff and Berrill (2015) worked on the investors' investment preference, home based investments or international diversification. The goal of their study was to mimic the benefits of international diversification via domestically traded products. They find that portfolios of US-traded products can replicate 36 of the 37 foreign countries indices so the classical US investors do not need to invest overseas to enjoy the benefits of international diversification.

Last scientific articles in portfolio management research are focused on the ETFs and hedging issues. Shank and Vianna (2016) examined the behavior of US currency hedged ETFs towards changes in the underlying benchmark and foreign exchange rates from July 2011 to November 2015. They found that investors are able to anticipate changes in future exchange rates and invest in currency hedged ETFs prior to changes. That suggests ETFs as a hedge against exchange rates volatility could be itself a source of volatility. Lots of work were done also on hedging comparing derivatives and ETFs. Leung and Lorig (2016) proposed a hedging framework involving ETFs, options, bonds. Carr and al (2016) worked on the usage of ETFs in hedging but focused on the insurance risks. They found good results, with low costs and stable hedge. Sukcharoen, Choi and Leatham (2015) worked on optimal hedge ratios for gasoline spot prices using gasoline ETFs and gasoline futures contracts. In their point of view, ETFs hedging seems more effective than futures during high-volatility periods, but they found that this is not always the case during the normal period.

D. Smart Beta

Smart beta indices refer to a set on investments based on alternative index construction, by contrast with traditional capitalization weighted indices. Those strategies, sometimes so called “strategic beta”, “fundamental indexing” or “factor investing” aim to provide a better risk-adjusted returns than traditional indices. The cons of traditional indices have been extensively documented in the academic literature with Tabnen (2007) referring to the excess of concentration with traditional index, or Goltz and Le Sourd (2011) highlighting market inefficiencies. To overcome main pitfalls of traditional indices, smart betas use strategies lying between equally weighting assets, equally weighted economic sectors, value stocks or low volatility approach.

Their management can be either active or passive even if Jacobs and Levy (2015) assert that contrary to a popular perception: “smart beta are never passive nor well diversified. Nor can be expected to perform consistently in all markets environment”. There are not passive since their purpose is to outperform the market. To beat the market, smart beta strategies replace capitalization weighting scheme with a scheme that emphasis certain security characteristic or factors. Contrary to passive strategy, smart beta increase the exposure to certain preselected factors (see for example Arnott, Hsu and Moore (2005), Jun and Malkiel (2007), Kaplan (2008), Blitz and Swinkels (2008)). The main question for smart beta strategy is how portfolio weights should be determined. Unlike passive portfolios, they require “periodic trading’s to rebalance the portfolio to its targeted weights as securities’ factor exposure change”. Smart beta has a mixed theoretical foundation, according to Jacobs and Levy (2014): “it is not clear whether excess returns are due to bearing systematic factor risk or stem from market inefficiencies”.

The frame in which stems our study is more related to the development of Smart Beta 2.0 strategies. Their purpose is to let investors controlling the risk of their investment in smart beta indices. Before smart beta 2.0, prepackage strategies were not transparent enough about asset selection and weighting process, leading to hamper rigorous analysis of the performances and risks factors. Performances comparisons used to be bias for most academic authors (Chow et al. (2011), Arnott (2011) and Amenc, Goltz and Martellini (2011). Rather than proposing only pre-packaged choices of alternative betas, the smart beta 2.0 approach allows investors to explore different smart beta index construction methods to construct a benchmark that corresponds to their own choice of risks. The investor may wish to reduce the exposure to a specific risk such as interest rate risks.

In that frame, new smart beta strategies have evolved with hybrid forms by adding new underlying assets, mainly ETF. Smart Beta based on ETF boils down to the following strategies: one is a dividend-oriented ETF that weights the allocation by screening for dividend levels or growth stocks, other aim to reduce risk with low-volatility and high-beta strategy ETFs. The current smart beta ETF assets are valued at $478 billion worldwide according to Morningstar (2016), and are estimated by BlackRock (2016) to reach $1 trillion and $2.4 trillion by 2020 and 2025 respectively.

Our approach is completely in line with the concept of smart beta 2.0. Two methodologies are tested to adjust optimized portfolios to “medium risk benchmark” with interest rate risk protection through a transparent ETFs selection process.

III. Methodology

We compare two methods to create a portfolio based on a combination of ETFs that replicates a medium risk strategy. The first method that we call the *Classic Tracking Error Volatility* (CTEV hereafter) is simply a standard quadratic allocation of optimal weights based on non-negativity constraint. The second method is an Alternative Tracking Error Volatility (ATEV hereafter) with an algorithmic selection of the assets and quadratic portfolio creation. In these two methods, the replication approach consists to create factor models by using a selection of ETFs. This involves expressing the medium risk strategy return for a particular period as a weighted sum of ETFs over the period. Thus, the medium risk strategy is a benchmark case that we want to replicate per a selection of ETFs.

A. Benchmark construction

The benchmark case is constructed as a combination of three indices: the MSCI World, the Bank of America (BOFA) Merrill Lynch US Corporate Index and the JP Morgan Global Aggregate Bond. Each of these indexes is used as a specific proxy. The MSCI World expresses the equity investment proxy whereas the BOFA Merrill Lynch US Corporate Index and the JP Morgan Global Aggregate Bond are respectively used to express the corporate and government bond proxy. According to this combination, we can construct a benchmark average return. Since we use monthly time series, we denote by $\overline{R_{\iota,BN}}$ the benchmark average monthly return such that

$$\overline{R_{\iota,BN}} = \alpha_{BN}^1 \bar{r}_i^1 + \alpha_{BN}^2 \bar{r}_i^2 + \alpha_{BN}^3 \bar{r}_i^3$$

In the previous expression, $\bar{r}_i^1, \bar{r}_i^2$ and $\bar{r}_i^3$ respectively denote the average monthly return for the month i of MSCI World, BOFA Merrill Lynch US Corporate Index and the JP Morgan Global Aggregate Bond. $\alpha_{BN}^1, \alpha_{BN}^2$ and α_{BN}^3 are respectively the weight allocated the three indexes. We denote by T the total data point expressed in month such that $i \in \{1, \dots, T\}$.

B. The replicated index construction

In this part, we detail the construction of the replicated index. We consider *ex ante* a N total number of ETFs. Each ETF j, with $j \in \{1, \dots, N\}$, is characterized by an average monthly return denoted by $\bar{r}_i^j$. Now we can give a general formulation of the replicated index average monthly return, denoted by $\bar{R}_\iota$:

$$\bar{R}_\iota = \alpha^1 \beta^1 \bar{r}_i^1 + \alpha^2 \beta^2 \bar{r}_i^2 + \cdots + \alpha^N \beta^N \bar{r}_i^N = \sum_{j=1}^{N} \alpha^j \beta^j \bar{r}_i^j$$

$\beta^j \in \{0,1\}$ denotes a binary variable allocated to each ETF such that if $\beta^j = 0$ the ETF j is excluded of the database and included otherwise. The value of β^j depends on the methodology we use to replicate the benchmark case. $\alpha^j \in [0,1]$ denotes the optimized weight according to the quadratic optimization of the minimization tracking error between the benchmark and the replicated index.

In general, the tracking error is a measure of how closely a portfolio follows the index to which it is benchmarked. The lower the tracking error, the more the replicated index resembles its benchmark's risk and return characteristics. According to this measure α^j is the solution of the following minimization program:

$$\min_{\alpha^j} \sqrt{\frac{1}{T-1}\sum_{i=1}^{T}\left(\overline{R_i} - \overline{R_{i,BN}}\right)^2}$$

Here, we measure an *ex-post* tracking error as the difference in the monthly return earned by the replicated index and the monthly return earned by the benchmark case. Mathematically, this defines a measure of the standard deviation.

C. The classic tracking error volatility (CTEV)

The CTEV consists to minimize the tracking error on the whole data set. Consequently, in this case we suppose that $\beta^j = 1$ and all the ETFs we select in the database are taking into account to create the optimal portfolio. Thus, in this method, the parameter is fixed exogenously. The method consists to endogenously determine the optimal value of $\alpha^j \in [0,1]$.

D. The alternative tracking error volatility (ATEV)

The ATEV is based on two steps. At the first step, we apply the automatic block search selection method (Hendry and Krolzig, 2005) across the N exogenous variables. We use the block search selection to overcome the identification problem reflecting the fact that we have more exogenous variables than data points. Based on an iteration process, this algorithm automatically selects the more explicative variables. Thus, β^j equals to 1 for the selected variables and 0 for the non-selected ones. At the second step, we apply a minimization of the tracking error on the selected variables in order to determinate the optimal value of $\alpha^j \in [0,1]$.

The automatic selection method was initially developed by Hoovers and Perez (1999) and extended by Hendry and Krolzig (1999, 2001). The method consists in considering a general model integrating all the explanatory variables and gradually reducing it by eliminating the non-significant variables, ensuring that the models obtained satisfy a number of specification tests.

We use the normality test of Doornick and Hansen (1994), a test of the autocorrelation Lagrange multiplier of order 4 of the residues (Godfrey, 1978), the quadratic heteroscedasticity test between regressors (Nicholls and Pagan, 1983) and the Chow predictive failure test on 50% and 90% of the estimation period. This approach leads us to retain a small number of models between which we will discriminate using embedding tests and, if not sufficient, information criteria.

The number of exogenous variables envisaged is quite important and exceeds the number of data points such that $N > T$. Consequently, the use of classical version of the GETS algorithm is not possible. The implementation of the algorithm begins with the estimation of the most general model, which requires a number of data points greater than the number of exogenous variables. In addition, some tests (the heteroscedasticity test and the Chow tests) cannot be applied if the number of data points is small compared to the number of exogenous variables. Therefore, we applied the block variant of the GETS method (Hendry and Krolzig, 2005).

In the classic GETS approach, as we said the full initial model is estimated, using the entire information set, where there is fewer variables k than observations T. In our case, we have an identification problem, with more variables than observations (137 exogenous variables for 124 data points). Hendry and Krolzig (2005) defined a solution to overcome the identification problem, splitting the variables in sub groups name blocks.

Thus, we apply the "block search variant" of PcGets. This variant allows to split the initial model in K randomly created set of variables, do model selection on each set, select surviving variables and repeat until k is sufficiently small.

To implement the automation of the variables selection, we use Grocer, a Scilab econometric toolbox from Dubois and Michaud (2005). This allows us to run the algorithm name "Automatic". The automatic function of scilab incorporate an implementation of the block search algorithm proposed by Doornik (2009) and Doornik and Hendry (2009). Grocer block search has been compared to Doornik Autometrics block search program (the algorithm commercial version). The results show that Grocer block search program in comparison with Autometrics provide almost equivalent result (Grocer, chap 15).

In details, in the variant we compose a K_1 group of exogenous variables randomly sort without replacement in the N available variables. We select K_1 as minimum number such as the number of variables of the block never surpass the maximal number of variables $\frac{N}{K_1} < T_{max}$ (so through the iterative process, the size of the block will diminish increasingly). For each group, we apply the Gets algorithm. To compensate the low adjustment quality and the autocorrelation of the residual which could result of the omission of the explicative variables (link to the separation in sub groups of the total variables), we use at that time, a high tests levels. The high levels used are those recommended by Hendry and Krolzig (2001) (10% for significant test and 5% for the specificity test). We save the selected exogenous variables in K_1 group. The composition of K_1 group could affect the variables choice, we reiterate this procedure L times. At the end of calculation, if the number of the selected variables at least one time M_1 ,overcomes again the maximal number of variables $M_1 > T_{max}$, we constitute another K_2 block of exogenous variables randomly sorted without replacement in the M_1 variables and we apply for each new group the Gets algorithm but using a lower tests level. In order to lower the tests level, we used the recommended parameters by Hendry and Krolzig (2001) of 5% for significant test and 1% for the specificity test. We repeat this L time randomly sorts till $M_t < T_{max}$. We finally apply the classic Gets methods to the M_t selected variables[7].

IV. Data description and robustness test

A. Data description

Our data are coming from Bloomberg, we worked with two types of data: the average monthly return (of the ETFs and the index) and the fundamental data of each ETF (asset class focus, strategies, assets under management…). We started from an initial database of 4909 active traded ETFs, which represented, the total actively traded ETFs universe (as June 2016). To stick with our problematic, we decided to narrow this universe with several filters.

The first filter we put in place is to select only ETFs which have an inception date before January 2006 in order to have at least 124 data points (124 monthly returns) of comparison for the analysis. January 2006 has been chosen as start date because ETF's asset class representation was really limited before it. This first filter decrease the global data set from 4909 to 393 ETFs. As our goal is to replicate medium risk portfolio without the usage of any fixed income investments, we put a filter who eliminated all ETFs with fixed income strategies. This create a new dataset of 360 ETFs. We restarted from the dataset of 360 ETFs representing all asset classes available excluding the fixed income. From this selection, we reduced again our dataset by concentrating on a selection of the 137 funds distributed by the 3 largest and most liquid providers: Blackrock Investment LLC, State Street Global Market LLC & Vanguard Marketing group. The last filter is put in place to allow to take only the largest and

[7] A document of 200 pages with all the details results of the algorithm which list steps, intermediate models, selected and omitted variables, and the different blocks compositions is available on demand to the authors.

the more traded ETFs in the market. Finally, the dataset is reduced 137 ETFs with 124 comparison data-points for each ETF.

In this paper, we use Exchange Traded Funds (ETF) in order to replicate a medium risk portfolio with a benchmark composed of 50% MSCI World - 25% JP Morgan Global Bond Index - 25% Merrill Lynch Corporate US. This benchmark is composed of some of the most commonly used indices in the world.

Table 1 present the breakdown of our entire dataset for each of the three largest providers. The tab allows to understand the repartition of all ETFs in term of market capitalization, strategies, asset classes and geographical breakdown.

Table 1: Total Dataset overview (per distributor)

	BlackRock Investments LLC/NY	**State Street Global Markets LLC**	**Vanguard Marketing Corp**	**#**
% ETF per distributor	68.6%	14.6%	16.8%	137
ETF market cap				
Broad Market	21%	25%	39%	34
Large-cap	56%	40%	43%	71
Mid-cap	10%	10%	4%	12
Small-cap	12%	20%	13%	18
Not define	1%	5%	0%	2
ETF strategy				
Blend	77%	65%	83%	104
Growth	11%	15%	9%	15
Precious Metals	1%	5%	0%	2
Value	12%	15%	9%	16
ETF asset classes				
Commodity	1%	5%	0%	2
Equity	99%	95%	100%	135
ETF geographical				
North America	61%	80%	87%	93
South America	3%	0%	0%	3
Europe	13%	10%	4%	15
Asia	10%	0%	4%	10
Australia	1%	0%	0%	1
Africa	1%	0%	0%	1
Global	7%	10%	0%	9
International	4%	0%	4%	5
Total ETF Number	94	20	23	137

Our overall entire dataset is finally composed with 137 ETFs from the three main distributors. ETF distributors are decomposed as follows: the main one is Blackrock Investment LLC/NY which account for around 69% of the overall ETFs. This is fully coherent with the market reality, in fact, Blackrock Investment LLC/NY since the development of the ETFs market provided and created the largest number of ETFs in the market. The rest is divided between Vanguard Marketing Group (17%) and State Street Global Markets LLC (15%).

Another point to highlight, our database is composed by 99% of ETF with equity focus. On the 137 ETFs, 135 are equity focused and only 2 are commodity focused. This is also quite normal, as ETFs in 2006 were relatively young types of products (for memory in 2000, there is only 80 ETFs in the worldwide market), the diversity in asset classes were not existent. The asset classes diversity has grown after the crisis. As expected, most the ETF have an US geographical focus for the same reason as asset classes diversity. In general, new financial products, start in the US and more new products are developed, more there are spread around the world.

B. Robustness test

We analyze the data with “in-sample analysis “and to double check our results, we perform also several robustness tests: such as “an out-sample analysis” and a Monte-Carlo simulation. Our in-sample analysis is done on the entire dataset (from January 2006 to April 2016), which allows us to work with 124 data points for comparison (124 Months). To test the robustness of our process, in “real conditions”, we perform an out-of-sample analysis. We use a monthly rebalancing, through a dynamic approach. The process is estimated each month, based on a 62-month rolling window. 62 months is just our sample divided in two parts, calibration and then rolling window estimation.

To go one step further, we perform a Monte Carlo simulation to simulate the expected return and volatility of each portfolios. The simulation is made with 10,000 trials, which allow to have statistical significance of the simulated expected return and volatility. The simulation was done through random correlated data using co-variance metrics and the famous Cholesky decomposition. This type of Monte-Carlo simulation approach is one of the most well know techniques applied in the financial market field[8]. For the benchmark case, we assume 0% expected return for corporate and government bonds meaning that we are quite cautious and not assuming a crash in the bond markets.

V. Results

In this part, we analyze and present results of the two replication methods. We firstly introduce some descriptive statistics of each portfolio composition (based on the two methods). This

[8] See for example J. E Gentle (1998), P.Glasserman (2004) and N.Soukher,B.Daafi,J.Bouyaghroumni and A.Namir (2014)

highlights fundamental data of each ETF selected. Especially their market cap focus, asset class focus, strategies types, geographical allocations, and distributors names.

After doing this preliminary step, we present the central results of our analysis which is focused on the comparison of the efficiency of replication of each method (through portfolios created). We present in-and-out sample results. We conclude the part by a presentation of expected returns and volatility for each portfolio using a Monte-Carlo simulation.

A. Portfolios selection descriptive statistics

Table 2 presents the portfolio selection descriptive statistics in comparison with the total dataset.

Table 2: Portfolio selection descriptive statistics

	Classic tracking error volatility	Δ*	Alternative tracking error volatility	Δ*	Total Dataset Breakdown
Initial ETF sample	137	-	137	-	137
Unconstrained selection	-	-	31	77%	
Long only selection	77	44%	9	93%	
ETF market cap focus					
Broad Market	22%	-3%	0%	-25%	25%
Large-cap	69%	17%	89%	37%	52%
Small-cap	1%	-7%	0%	-9%	9%
Mid-cap	5%	-8%	0%	-13%	13%
Not define	3%	1%	11%	10%	1%
ETF strategy					
Blend	77%	1%	78%	2%	76%
Precious Metals	3%	-8%	11%	0%	11%
Growth	12%	10%	0%	-1%	1%
Value	9%	-3%	11%	-1%	12%
ETF asset classes focus					
Commodity	86%	-13%	85%	-14%	99%
Equity	14%	13%	15%	14%	1%
ETF geographical focus					
North America	65%	-3%	56%	-12%	68%
South America	0%	-2%	22%	20%	2%
Europe	8%	-3%	11%	0%	11%
Asia	10%	3%	11%	4%	7%
Australia	0%	-1%	22%	21%	1%
Africa	1%	1%	0%	-1%	1%
Global	10%	4%	22%	16%	7%
International	5%	2%	0%	-4%	4%
ETF Distributors					
BlackRock Investments LLC/NY	10%	-58%	33%	-35%	69%
Vanguard Marketing Corp	71%	55%	67%	50%	17%
State Street Global Markets LLC	18%	4%	0%	-15%	15%

***Δ Delta is % of difference between the portfolio selection composition and the total dataset allocation**

The first point to highlight, is the number of ETFs selected in each portfolio. The CTEV selects a huge number of ETFs (77 of the 137 ETFs) which represents 66% of the global dataset. At contrary, the ATEV, pre-selects 31 ETFs using Pc-Gets algorithm. After the composition of a long only portfolio, this number decreases to 9 ETFs. If we talk just in term of the ETFs number selected, it is quite clear that the ATEV seems better for a medium risk investor as a portfolio with low number of assets is easier to manage.

In term of market capitalization focus, the classic tracking error volatility method portfolio seems to follow a similar breakdown than the global dataset, the main variation is between ETFs large cap focus (+17%) and ETFs small cap focus (-8%). For the ATEV, it is a totally different composition, 89% of the portfolio are ETFs with large cap focus, and the rest is not defined. The ETFs strategies allocated in the classic tracking error volatility portfolio are quite the same than the global breakdown of strategies in the dataset. One variation needs to be noted: the precious metal strategy decrease (-8%), and the growth strategy increase (+10%). Interestingly the ATEV portfolio follows the same breakdown as the global dataset strategy allocation. The ETFs asset class focus is quite the same for the two portfolio methods, 86% of equity ETFs for 14% of commodity ETFs for the classic tracking error volatility replication portfolio and 85% of equity ETFs for 15% of commodity ETFs for the alternative tracking error volatility portfolio. This result is not surprising, as 99% of the initial sample is composed by equity ETFs.

Last important focus who needs to be discuss, is the ETFs distributors. In fact, in the CTEV portfolio, Blackrock Investment LLC/NY is the less represented distributors (around 10%), but in the global dataset, Blackrock Investment LLC/NY represents 67% of the overall ETFs distributors. In the ATEV, Vanguard Marketing Corp is the main distributor of the ETFs with 67% of the overall portfolio. Interestingly here, the third provider disappears from the selection in the ATEV. There are only two providers in the allocation.

For more information and details of the composition of each portfolio, it is possible to refer to table 9 and 10 in the annex part. Table 9 is focused on the classic tracking error volatility methodology portfolio which includes ETFs names, weights allocations, descriptive statistics. Table 10 has the same data but focuses on the ATEV portfolio.

B. In-sample analysis

Our goal is to replicate the medium risk benchmark characteristics. The in-sample analysis compares the efficiency of the replication of optimized portfolios through the more common risk measures and significant statistics tests.

Table 3 presents the in-sample results.

Table 3: In-sample results (01/2006 – 04/2016)

In Sample	Benchmark	Classic tracking error volatility	Alternative tracking error volatility
Initial ETFs sample	3	137	137
Unconstrained selection	-	-	31
Long only selection	-	77	9
Annualized tracking error	-	7.27%	4.12%
Annualized return	4.60%	6.2%	6.0%
Annualized volatility	8.80%	14.8%	11.4%
Skewness	-1.03	-0.76	-1.03
Kurtosis	3.45	2.59	2.98
Sharpe ratio	0.46	0.38	0.48
Sortino ratio	1.99	1.74	2.13
Information ratio	0.52	0.42	0.53
Drawdown ratio	13.70%	22.3%	18.2%
Correlation	-	94.0%	95.0%
Beta	-	1.58	1.23
Jensen's Alpha* (annualized)	-	2.40%	1.72%
Jensen's Alpha* (t-stat)	-	1.4786	1.549
Jensen's Alpha* (p-value)	-	0.14	0.12
Mvar*	-11.67%	-20.5%	-15.6%
Mvar**	-26.1%	-42.3%	-32.6%

*confident level at 95%

**confident level at 99%

Let's start with the comparison between CTEV portfolio and the ATEV portfolio. In term of tracking error (TE), the first method achieves an annualized TE of 7.27% where the second method is at 4.12%. The ATEV portfolio seems to be more efficient. Interestingly, the annualized return of the first method is a little more important (6.20%) than the ATEV portfolio (6.00%) but for an important level of volatility (14.80% for the first portfolio, 11.80% for the second one). We clearly see, that the CTEV portfolio is way riskier. The high level of volatility (14.80%), the low Sharpe ratio (0.38%), the low Sortino ratio and the high level of the drawdown ratio (22.30%) not defined it as "medium risk portfolio". The two methods, seem to have approximately the same level of correlation with the benchmark. The underlying alpha is not significant for the two portfolios.

As we clearly see, the benchmark distribution and the two methods portfolios distribution are negatively skew and have excess kurtosis. As the distribution have fat tails and asymmetric return distribution, we cannot use classical value at risk. Because a classical value at risk assume that our portfolios are normally distributed. We decided instead to apply a Modified value at risk (Mvar hereafter), which consider the standardized third and fourth central moments of the return distribution (skewness and kurtosis) using the well know Cornish-Fisher expansion (Cornish and Fisher, 1937). As exhibit table 3, the Mvar for the CTEV is really important (-20.50% at 95% of confident interval and -42.3% at 99% of confident interval. The benchmark and the ATEV portfolio are way more close with a Mvar at 95% of confidence of -11.7% and -15.6% respectively and -26.1% versus -32.6% at 99% level of confidence.

In fact, the ATEV portfolio is slightly riskier (annualized volatility of 11.40% and drawdown ratio of 18.20%) than the benchmark (annualized volatility of 8.80% and drawdown ratio of 13.70%) but this is compensated by higher observed returns: 6.0% versus 4.60% for the benchmark.

From the in-sample analysis, the ATEV (which includes exogenous variables selected with PcGets) provides better characteristics in terms of replication compared of the CTEV.

Table 4 compares the benchmark with portfolios generated by the two methods.

Table 4: Portfolios characteristics versus benchmark – Asset class breakdown.

	Benchmark	Classic tracking error volatility	Alternative tracking error volatility
Equity	50%	85%	86%
Commodity	0%	15%	14%
Fixed income	50%	0%	0%

The equity part is much larger for our selection (CTEV and ATEV) with around 85% against 50% for the benchmark. Based on our 2006 sample of ETFs, this result is not surprising as 99% of the initial sample was ETFs equity strategy focused.

It means that even with such a low part of alternative assets classes, our quantitative selection succeeded to duplicate our benchmark with a tracking error of 4.12%. On the return side, the outperformance is easily explained by our overweight in equity in bull market conditions most of the sample. As a test of robustness, and to make our analysis in more realistic conditions, we perform the same analysis but out-of-sample.

C. Out-of-sample analysis

To test the robustness of our process in "real" conditions, we perform an out-of-sample analysis. We use a monthly rebalancing, with a dynamic approach. The process is estimated each month based on a 62-month rolling window. 62 months is just our sample divided in two parts, calibration and then rolling window estimation.

Table 5 presents the out-of-sample results.

Table 5: Out-of-sample results (01/2006 – 04/2016; 01/2006 – 02/2011: Calibration; 03/2011 – 04/2016: out-of-sample analysis)

	Benchmark	**Alternative tracking error volatility - out of sample**
Initial ETFs Sample	3	137
Unconstrained Selection	-	31
Long Only Selection	-	16
Annualized tracking error	-	1.92%
Annualized Return	4.9%	2.8%
Annualized Volatility	6.8%	6.8%
Skewness	-0.12	-0.07
Kurtosis	0.07	0.64
Sharpe Ratio	0.65	0.34
Sortino Ratio	2.94	1.48
Information Ratio	-	-1.09
Draw Down Ratio	9.3%	9.6%
Correlation	-	96.4%
Beta	-	0.97
Jensen's Alpha* (annualized)	-	-2.15%
Jensen's Alpha* (t-stat)	-	-2.70

Jensen's Alpha* (p-value)	-	0.01
Mvar*	-6.55%	-8.37%
Mvar**	-11.67%	-14.29%

*confident level at 95%

**confident level at 99%

Based on our 2006 ETF's sample, we select the ETFs to be included in our portfolio through the ATEV using a 62 months' calibration period. The final weightings are the results of a quadratic optimization process on this selection. We apply a dynamic approach with a monthly rebalancing based on a 62-month rolling window. 62 months correspond simply to the half of our sample (62 months calibration period, 62 months out-of-sample results).

The results seem quite disappointing on a return side with an annualized return of 2.83% vs 4.93% for the benchmark. But, fortunately, risks characteristics are perfectly in line with expectations with a 6.78% annualized volatility (slightly lower than the 6.85% of the benchmark) and a tracking error of 1.92%. Mvar results are interesting, with -6.55% for the benchmark when the ATEV portfolio is at -8.37% (at 95% of confidence). There is only ~2% of difference at 95%, and approximately ~3% of difference at 99% confidence level. Without fixed income products, these results are well in line with our expectations.

To be back on the return characteristics, the performance of the benchmark benefits from the exceptional performance of fixed income products during the period. This performance is nearly impossible to be reproduced in the future regarding the current very low interest rate level. It is exactly the reason we propose an alternative medium risk portfolio without fixed income products.

To demonstrate our point, we perform two type of test: we did a Monte-Carlo simulation to assess the expected return (one-year horizon) of our proposed portfolios and of the benchmark. As an alternative, we compare our portfolio performance (out-of-sample) with a benchmark closer to what could be expect in the future in terms of performance: 50% MSCI World and 50% T-notes 1 month. We could even argue that this simulation is quite cautious in the way we are not considering the scenario of a bond market crash in the year to come.

D. Expected returns & volatility: Monte-Carlo simulation

Table 6: Monte-Carlo simulated expected returns

	Benchmark	Classic tracking error volatility method	Alternative tracking error volatility method
Average	2.7%	9.0%	3.1%
Min	-1.9%	0.1%	-3.5%
Max	7.3%	17.6%	9.8%

The very interest of our strategy is to provide an alternative medium risk profile by excluding the "risky" fixed income products in a very low interest rate environment. Historical returns are highly misleading for investors in the way that fixed income products exhibited unusually high returns for a low risk profile. However, future returns are the one we want to assess.

We perform a Monte Carlo simulation to simulate the expected return and volatility of each portfolios. This simulation is performed with 10,000 trials, through random correlated data using co-variance metrics and Cholesky decomposition. For the benchmark, we assume 0% expected return for corporate and government bonds meaning that we are quite cautious and not assuming a crash in the bond markets.

Table 7: Monte-Carlo Expected Volatility

	Benchmark	Classic tracking error volatility method	Alternative tracking error volatility method
Average	7.9%	18.3%	7.0%
Min	1.3%	4.1%	1.2%
Max	19.3%	33.9%	19.3%

In this context, the ATEV provides return characteristics slightly higher than the benchmark with lower risk. CTEV provides interesting results from a return perspective but with a risk profile out-of-scope, providing new arguments in favor of the ATEV to replicate the benchmark case.

Of course, in case of a crash in the bond markets, the relative performance would be even more in favor of the portfolio generated by the ATEV.

Regarding these results, we decide to confirm it by comparing on an historical basis the portfolio generated by the ATEV with a benchmark composed of 50% MSCI World and 50% Cash (T-Notes 1 month). This benchmark is quite close from our Monte-Carlo simulation.

Table 8: Out-of-sample results (01/2006 – 04/2016; 01/2006 – 02/2011: Calibration; 03/2011 – 04/2016 : out-of-sample analysis)

	MSCI World (50%) T-Notes (50%)	**Alternative tracking error volatility method Out of sample**
Initial ETFs Sample	2	137
Unconstrained Selection	-	31
Long Only Selection	-	16
Annualized tracking error	-	1.12%
Annualized Return	2.5%	2.8%
Annualized Volatility	6.6%	6.8%
Skewness	-0.19	-0.07
Kurtosis	0.50	0.64
Sharpe Ratio	0.30	0.34
Sortino Ratio	1.27	1.48
Information Ratio	-	0.30
Draw Down Ratio	9.6%	9.6%
Correlation	-	0.99
Beta	-	0.96
Jensen's Alpha* (annualized)	-	0.39%
Jensen's Alpha* (t-stat)	-	0.78
Jensen's Alpha* (p-value)	-	0.44
Mvar*	-8.62%	-8.37%
Mvar**	-14.41%	-14.29%

*confident level at 95%
**confident level at 99%

The portfolio generated by the ATEV compared to the benchmark composed of 50% MSCI World and 50% Cash, has an annualized tracking error of 1.12%. The return and risk characteristics of each other are very of the close: the annualized return (6.6% versus 6.8%) and volatility (6.6% versus 6.8%) are almost the same. Moreover, the ATEV portfolio has the exact same drawdown ratio than the benchmark (9,6%). The Mvar is slightly better (almost the same) for the ATEV portfolio than the benchmark for the two level of confident (95% and 99%).

These results confirm the Monte-Carlo simulation. The return and risk characteristics are very close and the alpha of the regression is not significant. With a portfolio mainly composed of equity assets, we succeeded to replicate quite precisely the risk and return characteristics of this benchmark. To achieve this, our portfolio generated by the ATEV is composed of equity assets with low volatility characteristics.

VI. Discussion

According to the related literature, we propose to discuss the effect of imposing constraints on our optimized portfolios. From Roll (1992)[9] imposing constraint in a portfolio composition could improve the portfolio characteristics and its global efficiency notably in term of lowering the tracking error variance and the total portfolio volatility. Roll (1992) imposes two types of constrains: one on the beta and another one on the return. Contrary to him, we do not impose a constraint on the return[10] but just on the beta. We add two other constrains an information ratio constraint (Jorion, 2003), and a value at risk constraint (Alexander and Batista, 2008). Each new portfolio is compared with the unconstrained one. Each constrain is described below:

Beta: an interpretation of beta on portfolio management could be define as a financial elasticity. Meaning the sensitivity of the portfolio returns to the benchmark returns. When the optimized portfolio beta is greater than 1, it has tendency to exhibit greater moves in its returns (could be negative or positive) than those of the benchmark. When the optimized portfolio beta is equal to 1, the portfolio is the benchmark held long. This implies for a benchmark gains of 1%, the optimized portfolio will gain 1% also. When the optimized portfolio beta is from 0 to 1, the optimized portfolio returns are less than those of the benchmark. We propose to add a beta constraint define as beta equal to 1. Thus, we assume that the manager objective is to generate an optimized portfolio, who have same returns sensitivity than the benchmark.

Information ratio: it defines the ratio of the expected excess return to the tracking error minimum variance. It measures the optimized portfolio ability to generate excess returns relative to the benchmark. According to Grinold and Kahn (1995), information ratio is analogous to a normal bell-shaped curve with an information ratio equal to 0 as the mean of the distribution. The information ratio is commonly used to compare investment portfolios. We follow Grinold and Kahn (1995) to define our constraint. They asserted that an information ratio of 0.50 is “good”.

Value at risk: as we already said, using traditional value at risk assume that our portfolios distributions are normally distributed. As our distributions are negatively skew and have excess kurtosis, we decided to apply Mvar (which consider standardized third and fourth central moments of the return distribution. We define Mvar constraint for our portfolios comparison equal to -11.67% at 95% of confidence interval, which is exactly the Mvar of our benchmark.

[9] According to Roll (1992), several papers analyzed the effect of imposing constraints on optimized portfolio. See Jorion (2003) and Alexander and Batista (2008).
[10] Indeed, he specify an equal return between the optimized portfolio and the benchmark.

Table 9: Optimized portfolios constraints results

The table 9 provides details characteristics for the two methods portfolios adding several constraints. For each method, four types of optimized portfolios are presented: the first one unconstrained, the second one with a beta constraint equal to 1, the third one with an information ratio equal to 0.50 and a fourth one based on the Mvar equal to the one of the benchmark (-11.67%) at 95% level of confidence.

In Sample	**Benchmark**	**Classic tracking error volatility method**				**Alternative tracking error volatility method**			
		Unconstrained	**Beta = 1**	**IR = 0.5**	**Mvar*=-11.67%**	**Unrestricted**	**Beta = 1**	**IR = 0.5**	**Mvar*=-11.67%**
Initial ETFs sample	3	137	137	137	137	137	137	137	137
Unconstrained selection	-	-	-	-	-	31	31	31	31
Long only selection	-	77	12	18	31	9	6	11	6
Annualized tracking error	-	7.3%	4.5%	3.7%	4.1%	4.1%	5.5%	4.2%	4.9%
Annualized return	4.60%	6.2%	7.0%	6.5%	7.5%	6.0%	6.6%	6.7%	8.14%
Annualized volatility	8.80%	14.8%	9.9%	11.0%	10.5%	11.4%	10.4%	11.4%	10.9%
Skewness	-1.03	-0.76	-1.07	-0.94	-0.95	-1.03	-1.02	-0.99	-0.90
Kurtosis	3.45	2.59	3.55	2.77	3.16	2.98	3.18	2.95	3.09
Sharpe ratio	0.46	0.38	0.66	0.54	0.67	0.48	0.59	0.54	0.70
Sortino ratio	1.99	1.74	2.99	2.45	3.09	2.13	2.67	2.45	3.29
Information ratio	-	0.21	0.54	0.50	0.72	0.33	0.37	0.50	0.72
Drawdown ratio	13.70%	22.3%	17.7%	18.4%	18.2%	18.2%	17.6%	18.1%	17.6%
Correlation	-	93.6%	89.1%	95.4%	92.9%	95.0%	84.7%	94.7%	90.2%
Beta	-	1.58	1.00	1.19	1.10	1.23	1.00	1.22	1.11
Jensen's Alpha* (annualized)	-	2.40%	2.57%	2.14%	3.08%	1.72%	2.09%	2.47%	3.76%
Jensen's Alpha* (t-stat)	-	1.48	1.82	2.07	2.55	1.55	1.20	2.15	2.54
Jensen's Alpha* (p-value)	-	0.14	0.07	0.04	0.01	0.12	0.23	0.03	0.01
Mvar*	-11.67%	-20.5%	-11.3%	-13.7%	-11.7%	-15.6%	-12.6%	-14.3%	-11.7%
Mvar**	-26.1%	-42.3%	-27.6%	-30.1%	-28.3%	-32.6%	-29.0%	-31.7%	-28.9%

*confident level at 95%

**confident level at 99%

Results of the different imposed constraints on optimized portfolios of the two methods are detailed in table 9 above. We show that for the CTEV, resulting portfolio have a significant lower tracking than the unconstrained portfolio. More surprisingly, this not the case for ATEV, where tracking errors are greater. Interestingly, adding any constraints such as beta equal 1, information ratio equal 0.50 and Mvar equal -11.67% (at 95% of confidence) improve the replication efficiency of the CTEV portfolios. More precisely, table 9 exhibits that annualized tracking error decrease, as well as annualized return increase and annualized volatility decrease.

It seems that imposing the same constraints on ATEV portfolios generate more diverse results. Annualized returns and annualized volatility of each optimized portfolios tend move in the same way than the CTEV portfolios. We also note that, the maximal annualized return is reached with the Mvar constraint on the ATEV portfolio. However, by comparison with the unrestricted optimization portfolio, adding such constraints increase annualized tracking error. This result should be soften because the unconstrained ATEV portfolio annualized tracking error is one of the lowest (equal to 4.1%).

We can exhibit the fact, the better-optimized portfolio for each method are generated through Mvar constraint.

VII. Concluding remarks

In this article, we propose attractive alternatives solutions to classical medium risk portfolio with comparable return and risk characteristics but without any fixed income products. Indeed, we use non-fixed-income ETFs to compose our investment solutions to be protected against big losses in case of interest rate increase.

We generate these solutions using ETF optimized portfolios through two alternative methods. One is based on classical tracking error minimization (CTEV), the other one add to this classical method a pre-selection of explicative variables through PcGets approach and the block search algorithm (ATEV). General results are in favor of ATEV, notably when we do not impose any constraints on optimized portfolios. Imposing constraints significantly improve the result of CTEV portfolios whereas it is more soften for the ATEV ones.

From an investment standpoint, the ATEV appears as an interesting “ETF’s selector”, within the block search extension, allowing to deal with data where the number of exogenous variables

(ETFs) is greater than the number of observations (data points). Moreover, optimized portfolios based on that method, have always-lower number of ETFs selected. Thus, transaction costs for classical investors will be shrunken and these portfolios will be easier to manage. Another important feature of the ATEV method is that a selection of low volatility ETF's tend to succeeded to replicate medium risk characteristics. Last interesting finding is that this same selection of ETFs replicated even better a portfolio composed of 50% MSCI World and 50% Cash.

Regarding the risk of rate increase in the current low interest rate environment, we found that two strategies appear quite attractive as a substitute to classical medium risk portfolio (50% Equity – 50% Fixed Income). First solution, replacing fixed income by cash, second solution, investing in "low volatility" strategies. In case of stable or increasing rates, these strategies should beat the classical benchmark with better risk characteristics.

VIII. Annexes

Table 10: Classic tracking error volatility methodology portfolio composition

The table 10 provides a details composition of classic tracking error volatility portfolio. The three first columns contain the Bloomberg tickers, the ETF names, its weight allocation in the portfolio, the next 6 columns are for the descriptive statistics of each ETF such as inception date, strategy, geographical focus, market cap focus, asset class focus and distributor.

Portfolio Allocation		**ETF Fundamental Data**						
Ticker	**Weight**	**Name**	**Inception Date**	**Fund Strategy**	**Fund Geographical Focus**	**Fund Market Cap Focus**	**Fund Asset Class Focus**	**Distributor**
IBB US Equit	0.1%	ISHARES NASDAQ BIOTECHNOLOGY	2/9/2001	Blend	United States	Broad Market	Equity	BlackRock Investments LLC/NY
IAU US Equit	14.2%	ISHARES GOLD TRUST	1/28/2005	Precious Metals	Global	N.A.	Commodity	BlackRock Investments LLC/NY
GLD US Equi	0.4%	SPDR GOLD SHARES	11/18/2004	Precious Metals	Global	N.A.	Commodity	State Street Global Markets LLC
VPU US Equi	0.3%	VANGUARD UTILITIES ETF	1/30/2004	Blend	United States	Large-cap	Equity	Vanguard Marketing Corp
IDU US Equit	0.3%	ISHARES US UTILITIES ETF	6/20/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IGE US Equit	0.1%	ISHARES NORTH AMERICAN NATUR	10/26/2001	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
VOX US Equi	0.2%	VANGUARD TELECOM SERVICE ETF	9/29/2004	Blend	United States	Large-cap	Equity	Vanguard Marketing Corp
IXC US Equit	0.1%	ISHARES GLOBAL ENERGY ETF	11/16/2001	Blend	Global	Large-cap	Equity	BlackRock Investments LLC/NY
VDE US Equi	0.1%	VANGUARD ENERGY ETF	9/29/2004	Blend	United States	Broad Market	Equity	Vanguard Marketing Corp
EWC US Equ	0.0%	ISHARES MSCI CANADA ETF	3/18/1996	Blend	Canada	Large-cap	Equity	BlackRock Investments LLC/NY
IYZ US Equit	0.1%	ISHARES US TELECOMMUNICATION	5/26/2000	Blend	United States	Broad Market	Equity	BlackRock Investments LLC/NY
SDY US Equi	0.1%	SPDR S&P DIVIDEND ETF	11/15/2005	Blend	United States	Large-cap	Equity	State Street Global Markets LLC
IYE US Equit	0.1%	ISHARES U.S. ENERGY ETF	6/16/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
DVY US Equi	0.1%	ISHARES SELECT DIVIDEND ETF	11/7/2003	Value	United States	Broad Market	Equity	BlackRock Investments LLC/NY
EZA US Equi	0.0%	ISHARES MSCI SOUTH AFRICA ET	2/7/2003	Blend	South Africa	Broad Market	Equity	BlackRock Investments LLC/NY
IXP US Equit	0.2%	ISHARES GLOBAL TELECOM ETF	11/16/2001	Blend	Global	Large-cap	Equity	BlackRock Investments LLC/NY
EWT US Equ	0.0%	ISHARES MSCI TAIWAN ETF	6/23/2000	Blend	Taiwan	Broad Market	Equity	BlackRock Investments LLC/NY
VDC US Equi	0.2%	VANGUARD CONSUMER STAPLE ETF	1/30/2004	Blend	United States	Large-cap	Equity	Vanguard Marketing Corp
VWO US Equ	0.0%	VANGUARD FTSE EMERGING MARKE	3/10/2005	Blend	International	Broad Market	Equity	Vanguard Marketing Corp
EWM US Eq	0.2%	ISHARES MSCI MALAYSIA ETF	3/18/1996	Blend	Malaysia	Small-cap	Equity	BlackRock Investments LLC/NY
JKF US Equit	0.1%	ISHARES MORNINGSTAR LARGE-CA	7/2/2004	Value	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IYK US Equit	0.1%	ISHARES US CONSUMER GOODS ET	6/16/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IYJ US Equity	17.8%	ISHARES U.S. INDUSTRIALS ETF	7/14/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IUSV US Equi	0.0%	ISHARES CORE US VALUE ETF	8/4/2000	Value	United States	Broad Market	Equity	BlackRock Investments LLC/NY
IVE US Equit	0.0%	ISHARES S&P 500 VALUE ETF	5/26/2000	Value	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IWD US Equi	0.0%	ISHARES RUSSELL 1000 VALUE E	5/26/2000	Value	United States	Large-cap	Equity	BlackRock Investments LLC/NY
SPYV US Equ	0.0%	SPDR S&P 500 VALUE ETF	9/29/2000	Value	United States	Large-cap	Equity	State Street Global Markets LLC
EWS US Equi	0.0%	ISHARES MSCI SINGAPORE ETF	3/18/1996	Blend	Singapore	Large-cap	Equity	BlackRock Investments LLC/NY
VTV US Equi	0.0%	VANGUARD VALUE ETF	1/30/2004	Value	United States	Large-cap	Equity	Vanguard Marketing Corp
IJK US Equity	0.0%	ISHARES S&P MID-CAP 400 GROW	7/28/2000	Growth	United States	Mid-cap	Equity	BlackRock Investments LLC/NY
JKG US Equit	0.0%	ISHARES MORNINGSTAR MID-CAP	7/2/2004	Blend	United States	Mid-cap	Equity	BlackRock Investments LLC/NY
KLD US Equi	0.1%	ISHARES MSCI USA ESG SELECT	1/28/2005	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IVV US Equit	0.1%	ISHARES CORE S&P 500 ETF	5/19/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY

JKD US Equit	0.1%	ISHARES MORNINGSTAR LARGE-CA	7/2/2004	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IWB US Equi	0.1%	ISHARES RUSSELL 1000 ETF	5/19/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
VTI US Equit	0.0%	VANGUARD TOTAL STOCK MKT ETF	5/31/2001	Blend	United States	Broad Market	Equity	Vanguard Marketing Corp
VV US Equity	0.1%	VANGUARD LARGE-CAP ETF	1/30/2004	Blend	United States	Large-cap	Equity	Vanguard Marketing Corp
ITOT US Equ	0.0%	ISHARES CORE S&P TOTAL U.S.	1/23/2004	Blend	United States	Broad Market	Equity	BlackRock Investments LLC/NY
IYY US Equit	0.0%	ISHARES DOW JONES U.S. ETF	6/16/2000	Blend	United States	Broad Market	Equity	BlackRock Investments LLC/NY
IWV US Equi	0.0%	ISHARES RUSSELL 3000 ETF	5/26/2000	Blend	United States	Broad Market	Equity	BlackRock Investments LLC/NY
OEF US Equit	0.1%	ISHARES S&P 100 ETF	10/27/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
ONEK US Eq	23.8%	SPDR RUSSELL 1000 ETF	11/15/2005	Blend	United States	Large-cap	Equity	State Street Global Markets LLC
THRK US Eq	0.0%	SPDR RUSSELL 3000 ETF	10/10/2000	Blend	United States	Broad Market	Equity	State Street Global Markets LLC
IVW US Equi	0.1%	ISHARES S&P 500 GROWTH ETF	5/26/2000	Growth	United States	Large-cap	Equity	BlackRock Investments LLC/NY
VUG US Equi	0.1%	VANGUARD GROWTH ETF	1/30/2004	Growth	United States	Large-cap	Equity	Vanguard Marketing Corp
IWP US Equi	0.0%	ISHARES RUSSELL MID-CAP GROW	8/1/2001	Growth	United States	Mid-cap	Equity	BlackRock Investments LLC/NY
SPYG US Equ	0.1%	SPDR S&P 500 GROWTH ETF	9/29/2000	Growth	United States	Large-cap	Equity	State Street Global Markets LLC
IWF US Equit	13.4%	ISHARES RUSSELL 1000 GROWTH	5/26/2000	Growth	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IGV US Equit	0.0%	ISHARES NORTH AMERICAN TECH-	7/13/2001	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IUSG US Equi	0.1%	ISHARES CORE US GROWTH ETF	7/28/2000	Growth	United States	Broad Market	Equity	BlackRock Investments LLC/NY
JKH US Equit	0.0%	ISHARES MORNINGSTAR MID-CAP	7/2/2004	Growth	United States	Mid-cap	Equity	BlackRock Investments LLC/NY
VPL US Equit	0.1%	VANGUARD FTSE PACIFIC ETF	3/10/2005	Blend	Asian Pacific Region	Large-cap	Equity	Vanguard Marketing Corp
IXN US Equit	0.0%	ISHARES GLOBAL TECH ETF	11/16/2001	Blend	Global	Large-cap	Equity	BlackRock Investments LLC/NY
EWH US Equ	0.1%	ISHARES MSCI HONG KONG ETF	3/18/1996	Blend	Hong Kong	Large-cap	Equity	BlackRock Investments LLC/NY
IYC US Equit	0.0%	ISHARES U.S. CONSUMER SERVIC	6/28/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IGM US Equit	0.0%	ISHARES NORTH AMERICAN TECH	3/19/2001	Blend	United States	Broad Market	Equity	BlackRock Investments LLC/NY
DGT US Equi	0.1%	SPDR GLOBAL DOW ETF	9/29/2000	Blend	Global	Large-cap	Equity	State Street Global Markets LLC
IYH US Equit	0.2%	ISHARES U.S. HEALTHCARE ETF	6/16/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IOO US Equit	0.1%	ISHARES GLOBAL 100 ETF	12/8/2000	Blend	Global	Large-cap	Equity	BlackRock Investments LLC/NY
VGT US Equi	3.2%	VANGUARD INFO TECH ETF	1/30/2004	Blend	United States	Large-cap	Equity	Vanguard Marketing Corp
IXJ US Equity	0.2%	ISHARES GLOBAL HEALTHCARE ET	11/16/2001	Blend	Global	Large-cap	Equity	BlackRock Investments LLC/NY
EFG US Equit	0.1%	ISHARES MSCI EAFE GROWTH ETF	8/5/2005	Blend	International	Large-cap	Equity	BlackRock Investments LLC/NY
VHT US Equi	0.2%	VANGUARD HEALTH CARE ETF	1/30/2004	Blend	United States	Broad Market	Equity	Vanguard Marketing Corp
IYW US Equi	0.0%	ISHARES USTECHNOLOGY ETF	5/19/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
JPXN US Equ	0.1%	ISHARES JPX-NIKKEI 400 ETF	10/26/2001	Blend	Japan	Broad Market	Equity	BlackRock Investments LLC/NY
JKE US Equit	0.1%	ISHARES MORNINGSTAR LARGE-CA	7/2/2004	Growth	United States	Large-cap	Equity	BlackRock Investments LLC/NY
EWJ US Equi	0.2%	ISHARES MSCI JAPAN ETF	3/18/1996	Blend	Japan	Large-cap	Equity	BlackRock Investments LLC/NY
VFH US Equit	22.1%	VANGUARD FINANCIALS ETF	1/30/2004	Blend	United States	Large-cap	Equity	Vanguard Marketing Corp
EWL US Equ	0.1%	ISHARES MSCI SWITZERLAND CAP	3/18/1996	Blend	Switzerland	Large-cap	Equity	BlackRock Investments LLC/NY
FXI US Equit	0.1%	ISHARES CHINA LARGE-CAP ETF	10/8/2004	Blend	China	Large-cap	Equity	BlackRock Investments LLC/NY
EFA US Equit	0.0%	ISHARES MSCI EAFE ETF	8/17/2001	Blend	International	Large-cap	Equity	BlackRock Investments LLC/NY
EWU US Equ	0.1%	ISHARES MSCI UNITED KINGDOM	3/18/1996	Blend	United Kingdom	Large-cap	Equity	BlackRock Investments LLC/NY
FEU US Equit	0.0%	SPDR STOXX EUROPE 50 ETF	10/21/2002	Blend	European Region	Large-cap	Equity	State Street Global Markets LLC
IEV US Equit	0.0%	ISHARES EUROPE ETF	7/28/2000	Blend	European Region	Large-cap	Equity	BlackRock Investments LLC/NY
VGK US Equi	0.0%	VANGUARD FTSE EUROPE ETF	3/10/2005	Blend	European Region	Large-cap	Equity	Vanguard Marketing Corp
EFV US Equit	0.0%	ISHARES MSCI EAFE VALUE ETF	8/5/2005	Blend	International	Large-cap	Equity	BlackRock Investments LLC/NY
EWP US Equ	0.0%	ISHARES MSCI SPAIN CAPPED ET	3/18/1996	Blend	Spain	Large-cap	Equity	BlackRock Investments LLC/NY

Table 11: Alternative tracking error volatility methodology portfolio composition

The table 11 provides a details composition of alternative tracking error volatility portfolio. The three first columns contain the Bloomberg tickers, the ETF names, its weight allocation in the portfolio, the next 6 columns are for the descriptive statistics of each ETF such as inception date, strategy, geographical focus, market cap focus, asset class focus and the distributor.

Portfolio Allocation		**ETF Fundamental Data**						
Ticker	**Weights**	**Name**	**Inception Date**	**Fund Strategy**	**Geographical Focus**	**Market Cap Focus**	**Asset Class Focus**	**Distributor**
GLD US Equity	14.0%	SPDR GOLD SHARES	11/18/2004	Precious Metals	Global	N.A.	Commodity	State Street Global Markets LLC
IDU US Equity	22.1%	ISHARES US UTILITIES ETF	6/20/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
SPYV US Equity	12.6%	SPDR S&P 500 VALUE ETF	9/29/2000	Value	United States	Large-cap	Equity	State Street Global Markets LLC
IVV US Equity	7.3%	ISHARES CORE S&P 500 ETF	5/19/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IYC US Equity	10.3%	ISHARES U.S. CONSUMER SERVIC	6/28/2000	Blend	United States	Large-cap	Equity	BlackRock Investments LLC/NY
IXJ US Equity	13.0%	ISHARES GLOBAL HEALTHCARE ET	11/16/2001	Blend	Global	Large-cap	Equity	BlackRock Investments LLC/NY
EWJ US Equity	15.8%	ISHARES MSCI JAPAN ETF	3/18/1996	Blend	Japan	Large-cap	Equity	BlackRock Investments LLC/NY
MTK US Equity	4.5%	SPDR MORGAN STANLEY TECHNOLO	9/29/2000	Blend	United States	Large-cap	Equity	State Street Global Markets LLC
EWP US Equity	0.4%	ISHARES MSCI SPAIN CAPPED ET	3/18/1996	Blend	Spain	Large-cap	Equity	BlackRock Investments LLC/NY

Table 12: Monte-Carlo simulation expected returns outcome – 10,000 Simulation trials

The table 12 provides a details overview of the Monte-Carlo simulation outcomes for each of the replication methods in comparison to the expected return of the benchmark. The first column show an expected return scale classified from -3% to +21%. The next three column presented the outcome probability for each optimized portfolio.

#10,000 Simulation	**Expected returns breakdown probabilities**		
	Benchmark	**Classic tracking error volatility method**	**Alternative tracking error volatility method**
<-3%	0%	0%	0%
Between -3% and -1%	0%	0%	0%
Between -1% and 1%	5%	0%	7%
Between 1% and 3%	42%	0%	30%
Between 3% and 5%	46%	2%	42%
Between 5% and 7%	7%	12%	18%
Between 7% and 9%	0%	27%	3%
Between 9% and 11%	0%	33%	0%
Between 11% and 13%	0%	19%	0%
Between 13% and 15%	0%	6%	0%
Between 15% and 17%	0%	1%	0%
Between 17% and 19%	0%	0%	0%
>21%	0%	0%	0%

Table 13: Monte-Carlo simulation expected volatility outcome – 10,000 Simulation trials

The table 13 provides a details overview of the Monte-Carlo simulation outcomes for each of the replication methods in comparison to the expected volatility of the benchmark. The first column show an expected volatility scale classified from 1% to +25%. The next three column presented the outcome probability for each optimized portfolio.

#10,000 Simulation	**Expected volatility breakdown probabilities**		
	Benchmark	**Classic tracking error volatility method**	**Alternative tracking error volatility method**
<1%	0%	0%	0%
1% to 3%	2%	0%	8%
3% to 5%	12%	0%	21%
5% to 7%	26%	0%	26%
7% to 9%	30%	1%	23%
9% to 11%	20%	3%	14%
11% to 13%	7%	7%	6%
13% to 15%	2%	13%	2%
15% to 17%	0%	17%	1%
17% to 19%	0%	18%	0%
19% to 21%	0%	16%	0%
21% to 23%	0%	12%	0%
>25%	0%	6%	0%

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