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On the effectiveness and usefulness of the portfolio insurance strategies for REITs

Charles-Olivier Amédée-Manesme · Philippe Bertrand · Jean-Luc Prigent

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Abstract Portfolio insurance refers to financial strategies whose goal is to protect portfolio against the downside market risk while allowing investor to benefit from potential market rises. So far the use of these specific strategies has not been the subject of research in the real estate context and there exists no consensus about their effectiveness for this asset class. This paper provides a performance evaluation of both the Constant Proportion Portfolio Insurance and Option-Based Portfolio Insurance techniques for the Real Estate Investment Trusts market. The effectiveness of the strategies is tested on historical return of NAREITs index for the period of Jan. 2000-Jun. 2016. We evaluate the performance of each strategy as a pure investment tool. Overall the results indicate that both PI strategies almost perfectly protect their insured amount and allow to catch-up part of the rise. Gap risk and cash-lock effects are discussed within the results. Robustness is confirmed with a block-bootstrap simulation.

Keywords: Portfolio Insurance, Real Estate, CPPI, OBPI, REITs.

JEL codes: D81, G11, G17, G32, R30, R33.

1 Introduction

Investors face investment risk among which idiosyncratic and systemic risks. It is straightforward to decrease idiosyncratic risk by the use of diversification: this way, investors can avoid the individual risk factor. However, it is difficult to get rid of systematic - or market - risk. It is exactly where the portfolio insurance takes its essence. The portfolio insurance concept is based on the premise that investors can be protected from loss when the market is falling but also can make a profit when market is rising.

C-O. Amédée-Manesme (contact author)
Université Laval, Department of Finance, Insurance and Real Estate, Canada
charles-olivier.amedee-manesme@fsa.ulaval.ca

P. Bertrand
CERGAM, IAE Aix-en-Provence and Kedge Business School, France
philippe.bertrand@univmed.fr

J-L. Prigent
THEMA, Université de Cergy-Pontoise, France
Jean-Luc.Prigent@u-cergy.fr

This paper is an empirical study examining the effectiveness of portfolio insurance strategies in the Real Estate Investment Trusts (REITs) market. Portfolio managers may adopt many strategies. The most simple is the 'buy & hold' strategy, where in reference to an investment horizon, a well-diversified portfolio is built-up, without readjustment till the end. On the opposite side, numerous active management techniques allow to shuffle assets and allocations in an attempt to increase overall returns (or to decrease the overall risk). Dynamic strategies that rebalance the portfolio according to market evolution can improve the performance. The general premise of dynamic asset allocation is to reduce the fluctuation risks and achieve returns that exceed the target benchmark. All these strategies might support large losses. In order to avoid large losses the manager may decide to "insure" a certain amount of the portfolio. Typically, portfolio insurance investment strategy can provide a capital protection while allowing to benefit from market rises.

Financial strategies which are designed to limit downside risk and at the same time to take advantage of rising markets belongs to the class of portfolio insurance strategies. Among others, Rubinstein & Leland (1981), Perold (1986), Grossman & Vila (1989), Basak (2002) and Bertrand & Prigent (2005) define portfolio insurance trading strategies as a strategy which guarantees a minimum level of wealth at a specified time horizon, but also participates in the potential gains of a reference portfolio. Portfolio insurances are thus investment strategies where various financial instruments - equities, debts and derivatives - are combined such that the portfolio value is protected. It implies buying and selling securities periodically in order to maintain limit of the portfolio value. The most prominent examples of such strategies are the Constant Proportional Portfolio Insurance (CPPI) - also known as Cushion method - and the Option Based Portfolio Insurance (OBPI). These strategies do address people's hopes and fears by being able to make money and by avoiding losing.

The market for investors to buy such a financial product lies in their risk aversion. Portfolio insurance strategy generally imply to give up some potential gains. The investors must thus have reasons to enter in this type of products. Indeed the optimality of an investment strategy depends on the risk profile of the investor. Portfolio insurances can be modelled by a classical utility maximization: for example, Black and Perold (1992) show that the CPPI strategy is optimal for a piecewise-HARA utility function with a minimum consumption constraint; if the guarantee constraint (i.e. the terminal portfolio value is above a specified wealth level) is exogeneous, the OBPI strategy can be optimal. Investors - institutional or individual - might have a strong aversion to the losses and may be tempted to pay a premium in exchange of some guarantee on their capital. Institutional investors may also have legal constraints imposed to them in a sense such that the value of the asset under management does not drop below a threshold. For example, Ahn et al. (1999) consider the problem of an institution optimally managing the market risk of a given exposure by minimizing its Value-at-Risk using options. More recently, Døskeland & Nordahl (2008) justify the existence of guarantees from the point of an investor through behavioural models.[1]

Equity REITs are companies that own, and in most cases actively manage, portfolios of properties. They are often traded on major exchanges like a stock exchange. REITs provide investors a liquid stake in real estate. They are considered as real estate firms as their assets and activities are restricted to real estate (to keep their REITs status). They invest in various types of real estate and earn money through rents, leases and capital gains. Mostly, REITs are focused on commercial real estate ownership such as apartment complexes, office buildings, timber land, warehouses, hotels and shopping malls. REITs are tax advantaged form of corporate organization. REITs do not pay tax on the distributable income they pay to investors, but pay tax on the income they keep. Usually, other types of income trusts and corporations are taxed on their income before distributions to investors, who are in turn taxed on the distributions they receive. Since REIT's distributions are

[1] In particular, they use cumulative prospect theory as an example where guarantees can be explained by a different treatment of gains and losses, i.e. losses are weighted more heavily than gains.

only taxed in the investors' hands, the marginal tax rate that is applied to investor distributions is less than the combined corporate and investor tax rates for corporations. In short, REITs are not subject to the double taxation applied to dividends. They typically offer high dividend yields as they are generally required by law to distribute a high percentage of their net income as common dividends. Individuals can invest in REITs either by purchasing their shares directly on an open exchange or by investing in a mutual fund that specializes in public real estate. Approximately 40% of total REIT equity is hold by dedicated REITs investors (i.e., asset managers exclusively focused on REITs).

While their is a high degree of confidence in the REITs sector, there are several key risks to bear in mind when investing in REITs. First, they heavily depend on capital markets: because REITs cannot retain earnings (dividend requirement), they are dependent on capital markets both to grow (through new equity issuance) and to refinance outstanding debt. Second, they are sensitive to interest rates as interest rates have an impact on both appraisal and funding cost. While REITs are not a fixed income investment, a rise in interest rates could decrease the value of its units, since investors can then choose investments with higher rates.[2] While distributions are usually stable, they are not guaranteed, as opposed to safer investments like guaranteed investment certificates. Third, they are obviously linked to real estate market. More precisely, traditional metrics such as earnings-per-share, growth or price-to-earnings multiple do not really apply. More common metrics are net asset value, funds from operations, leverage, average leases term, average cap rates and occupancy rates. Finally, REITs are not a great diversifier from stocks as evidenced by their roughly 78% correlation (see Management, 2016, Long-Term Capital Market Assumptions report) to stocks based on a comparison of the S&P 500 Index and the NAREIT Equity REIT Index over the ten years through the end of 2015. REITs, however, do provide good diversification in terms of low or negative correlations to core bonds, commodities and currencies.

The financial management industry extensively uses portfolio insurance methods to protect various risky portfolios: equities, bonds, structured credit products, hedge funds...In particular, since the 2008 sub-prime crisis which found its basis in real estate prices, number of structured protected funds have been launched. However, to the best of our knowledge, as of today, portfolio insurance strategies are not used in the REITs management industry and the effectiveness of portfolio insurance strategies has not led to any academic study in the case of REITs. This fact is questioning since investing in REITs is risky: the initial investment is not guaranteed.

The REITs industry has received many positive signs over the last years. The Global Industry Classification Standard recently granted REITs the status of being a separate asset class in 2016.[3] This soared their popularity. In addition, for the years 2000-2015, REITs over-perform S&P 500 with an average annual return of 12.9% for the NAREIT Equity REIT Index over that timeframe. However and even with all these positive signs about the REITs market, neither options market nor portfolio insurances products exist on the market. This paper intends to give some insights to better understand how, why and what can be done.

Recognizing the reality, this study begins with an inquiry: is it possible to built a product for REITs that protects from eventual losses and allow to participate in rise? To answer the inquiry, this study examines whether the two most famous portfolio insurance strategy CPPI and OBPI can be applied to REITs. Although these strategies have their applications to many of financial products,

[2] The spread between current cap rates (2016) and REITs funding costs is at historic wides. It would be able to sustain a reasonable amount of compression.

[3] Real Estate is being moved out from under the Financial Sector and being promoted to its own Sector (code 60). The Real Estate Investment Trusts Industry is being renamed to Equity Real Estate Investment Trusts (REITs), and excludes Mortgage REITs which remain in the Financial Sector.

they have not been used with REITs yet. Thus the main purpose of this paper is to implement CPPI and OBPI on REITs and to evaluate their effectiveness as an investment asset.

Our study complements and expands these earlier studies which evaluate and compare different portfolio insurance strategies for other asset classes. We compare two portfolio insurance strategies by presenting the outputs of the strategy and by using Kappa performance metrics (see 3.3) for the period 1999-2016 on the NAREITs index. Our results indicate that the two strategies offers effective protection and allow to participate to the rise, at a certain cost. Particular events - respectively called the "gap risk" and the "cash lock" - are highlighted and discussed.

While the utility of portfolio protection for direct real estate holdings is interesting in itself, its implementation is questioning given the liquidity concerns, the absence of real derivatives markets (or the impossibility to replicate) and the non divisibility of investments. In this paper, we focus on the REITs portfolio management industry and we concentrate on the effectiveness of the portfolio insurance strategies.

The outline of the paper is as follows. Section 2 provides a survey about the existing literature on portfolio insurance, real estate portfolio management and REITs. Then, in section 3, we recall portfolio insurance strategy theory and setup and review some of the fundamental properties of CPPI and OBPI strategies (introduced in continuous-time for simplicity).[4] Section 4 provides an empirical application, presents the results and discusses criteria which analyze the effectiveness of these strategies for REITs, in particular if the portfolio protection is valid. A data presentation and a robustness test complement this part. This part includes a sensitivity analysis of the capital protection from 100% to 90% with respect to model and strategy parameters. The gap risk and the cash lock effects are empirically discussed in this section.Finally, section 5 concludes the paper.

2 Literature review

Portfolio insurance arose as a portfolio management technique and became widely used following the work of Leland (1980) and Rubinstein & Leland (1981) for the OBPI strategy and Perold (1986) and Black & Perold (1992) for the CPPI strategy. Indeed, portfolio insurance strategies are the basis of the structured portfolio management.[5] Rubinstein & Leland (1981) demonstrate how it is possible to create the desired terminal pay-off through asset allocation between bonds, stocks and an European option. This approach is known as the OBPI method.[6] Later on, Perold (1986) and Perold & Sharpe (1988) propose the CPPI approach for fixed-income instruments. Then Black & Jones (1987) developed such CPPI strategies for equity instruments. This strategy uses a simplified method to allocate assets dynamically over time and has become very popular among practitioners. Both methods - OBPI and CPPI - guarantee that the portfolio current value dominates the discounted value of a pre-specified final floor. Most of the properties of portfolio insurance have been previously studied by Black & Perold (1992) when the risky asset follows a geometric Brownian

[4] In section 3, we consider a general framework of complete, arbitrage free and frictionless market evolving in continuous-time to recall some of the basic properties of portfolio insurance strategies. All this yields to closed-form solutions. We also analyze both the gap risk and the cash lock risk mainly in this framework. In the implementation part (section 4), time is discrete since it corresponds to discrete-time data.
We consider daily portfolio rebalancing.

[5] A former article published by Brennan & Schwartz (1976) already shows the intuition of portfolio insurance as an investment strategy with a minimum guaranteed value.

[6] Actually, Leland and Rubinstein did not use put options in order to provide portfolio insurance, as these did not exist at that time for entire portfolios. Instead, they replicated the put option according to the Black-Scholes formula and no-arbitrage arguments. This investment strategy is now known as the Synthetic Put Portfolio Insurance (SPPI) strategy

motion and by Bertrand & Prigent (2003) when the volatility is stochastic.[7]

Comparisons between standard portfolio insurance methods and empirical performance of portfolio insurance strategies have widely been the subject of studies in the literature. Bookstaber & Langsam (2000) compare OBPI and CPPI strategies and show that path independence and time invariance are desirable characteristics of insurance strategies. Cesari & Cremonini (2003) use simulation and empirical data to compare OBPI and CPPI. They compute various metrics to compare the methodologies and use different market situations and practice issues. They show that CPPI strategy generally dominates OBPI. A theoretical comparison of OBPI and CPPI is given in Bertrand & Prigent (2005). In particular, they compare the two methods with respect to various criteria, introducing systematically the probability distributions of the two portfolio values. They conclude that neither of the two strategies dominates the other one (first order stochastic dominance).[8] Balder et al. (2009) test for the effectiveness of the CPPI strategy with trading restrictions. For this purpose, they propose a discrete-time version of the CPPI strategies, this way they were able to account for transaction costs. They show that the CPPI strategy may break through the floor in incomplete market or when the portfolio may only be rebalanced at a finite number of trading dates. Bertrand & Prigent (2011) rely on downside risk measures, in particular the Omega performance measure, to compare portfolio insurance strategies. Mainly, they conclude that it is not so easy to rank the strategies and to assess the effectiveness. Finally, to sum up, the main conclusion is that it is not so easy to rank these strategies, except by their sensitivity Vega to the volatility of the risky asset (to the benefit of the OBPI strategy). However, the CPPI method is the best strategy when the market drops or increases by a significant amount.

To the best of our knowledge, real estate asset class as a whole has not been the subject of portfolio insurance strategies studies. In investment field, most of the researches have concentrated on the benefits of including real estate in mixed asset portfolios (both in terms of expected return increase and volatility reduction) and the puzzle between suggested versus actual (institutional) allocations in real estate. For instance this has been studied by Hudson-Wilson et al. (2003), Hoesli & Lekander (2005) and Lizieri (2013). Another strand of research has been the link between public and private real estate. In this line, Tuluca et al. (2000), Clayton & MacKinnon (2003), Giliberto (2009), Hoesli & Oikarinen (2012), or Ling & Naranjo (2015) can be consulted. Among this field, the hedgeability of REITs have led to some works such as Liang et al. (2009) or Lin Lee & Lee (2012).Finally other researches have sought to analyze the risk of real estate (see Amédée-Manesme et al., 2013) , the property derivatives market (see Fabozzi et al., 2010, 2012) or diversification issues (see Stevenson, 2009).

3 Portfolio Insurance

The portfolio manager is assumed to invest in two basic assets : a money market account, denoted by B, and a portfolio of traded assets such as a composite index, denoted by S. The period of time considered is $[0, T]$. The strategies are self-financing. The value of the riskless asset B evolves according to : $\mathrm{d}B_t = B_t r \mathrm{d}t$, where r is the constant interest rate.

[7] See e.g. Prigent (2007) for a detailed review of portfolio insurance.

[8] Note also that, using various stochastic dominance (SD) criteria up to third order and assuming that the risky underlying asset follows a GBM, Zagst & Kraus (2011) provide very specific parameter conditions implying the second-and third-order SD of the CPPI strategy (see also Maalej & Prigent, 2016).

3.1 The CPPI strategy

The CPPI method consists in managing a dynamic portfolio so that its value is above a floor F at any time t. The investor starts by setting a floor equal to the lowest acceptable value of the portfolio. It is assumed to evolve according to:[9]

$$\mathrm{d}F_t = F_t r \mathrm{d}t, \tag{1}$$

meaning that its rate is equal to the risk-free rate r. The initial floor F_0 is chosen such as to recover a guaranteed amount pV_0 at maturity T ($p \leq e^{rT}$). Thus, the initial floor F_0 is equal to pV_0e^{-rT} and is smaller than the initial portfolio value V_0^{CPPI}. The difference $V_0^{CPPI} - F_0$ is called the cushion, denoted by C_0. At any time t in $[0, T]$, its value C_t is given by:

$$C_t = V_t^{CPPI} - F_t. \tag{2}$$

Denote by e_t the exposure, which is the total amount invested in the risky asset. Both the floor and the multiple are functions of the investor's risk tolerance and are exogenous to the model. The standard CPPI method consists in letting $e_t = mC_t$ where m is a constant called the multiple ($m > 1$).[10] The multiple is the key control parameter of the CPPI. It governs the amount of exposure the portfolio has to the risky asset. [11]Typical multiplier values lie between 3 and 8. Both the floor and the multiple are functions of the investor's risk tolerance and are exogenous to the model. The interesting case is when $m > 1$, that is, when the pay-off function is convex allowing the portfolio to better benefit from market rises. The risky asset S is supposed to follow the dynamic of a jumps diffusion process:

$$\mathrm{d}S_t = S_t[\mu(t, S_t)\mathrm{d}t + \sigma(t, S_t)\mathrm{d}W_t + \delta(t, S_t)\mathrm{d}N_t], \tag{3}$$

where μ and σ are supposed to be adapted processes with standard integrability properties. $\left(\text{i.e. } \int_0^T r_t \,\mathrm{d}t < \infty, \int_0^T \sigma_t^2 \,\mathrm{d}t < \infty\right)$ and $(W_t)_t$ is a standard Brownian motion which is independent from the Poisson process with the measure of jumps N. [12] Then the cushion value C_t at time t is given by:

$$C_t = C_0 exp\left((1-m)rt + m\left[\int_o^t (\mu - 1/2m\sigma^2)(s, S_s)\mathrm{d}s + \int_o^t \sigma(s, S_s)\mathrm{d}W_s\right]\right) \tag{4}$$
$$\times \prod_{0 \leq T_n \leq t} (1 + m\delta(T_n, S_{T_n})),$$

and the portfolio value is given by:

$$V_t^{CPPI} = F_0.e^{rt} + C_t. \tag{5}$$

Relation (4) shows that the guarantee is satisfied as soon as the relative jumps satisfy $\delta(T_n, S_{T_n}) \geq -1/m$. Moreover, if the relative jumps satisfy the condition that $\delta(T_n, S_{T_n})$ are higher than a non-positive parameter d, then the condition $0 \leq m \leq -1/d$ implies the positivity of the cushion. For

[9] See e.g. Prigent (2007) for detailed presentation of portfolio insurance methods within various financial modellings.

[10] If $m = 1$, the strategy is the classical buy & hold one. If $F_0 = 0$, the strategy is the so-called constant-mix strategy that maintains an exposure to stocks that is a constant proportion of the porftolio over time).

[11] It can provide a proxy for the risk aversion of the investor since, under mild conditions, it is equal to the Merton's ratio.

[12] Recall that the sequence of times $(T_n)_n$ at which jumps occur has the following properties: the jump interarrival times $(T_{n+1} - T_n)$ are independent with the same exponential distribution associated with parameter λ. The relative jumps $\frac{\Delta S_{T_n}}{S_{T_n}}$ are equal to $\delta(T_n, S_{T_n})$, which are assumed to be strictly higher than (-1) (to guarantee the positivity of asset price S). The integral $\int_{-\infty}^{+\infty} \int_0^t S_{u-}.\delta(u, S_u)\mathrm{d}N$ is equal to the sum $\sum_{T_n \leq t} \Delta S_{T_n}$ of all jumps before time t.

example, if d is equal to -10%, then $m \leq 10$. If the maximum market drop is equal to -20%, then taking $m = 5$ allows to maintain the guarantee.

The multiple m is the parameter that determines the leverage of the portfolio since the total amount e_t invested in the risky asset is equal to mC_t with $C_t = (V_t - F_t)$. To better highlight the leverage effect, we note that the cushion value C_t satisfies:

$$\frac{\mathrm{d}C_t}{C_t} = m\frac{\mathrm{d}S_t}{S_t} + (1-m)r_t\mathrm{d}t. \tag{6}$$

Equation (6) shows how the leverage can be used through the multiplier m. More specifically, this means that CPPI strategies have dynamic leverage, compared with other strategies with principal protection such as the simple buy-and-hold strategy or the OBPI strategy. However, if the market suddenly drops, the portfolio return may dramatically decrease. Indeed, for a very short period (one day for example corresponding to time T_n), we have approximately:

$$\frac{\Delta V_{T_n}}{V_{T_{n-}}} = 1 + m\frac{\Delta S_{T_n}}{S_{T_{n-}}}.$$

For instance, with a multiple m equal to 5 (standard value) and a market drop $\frac{\Delta S_{T_n}}{S_{T_{n-}}}$ equal to -10%, the portfolio return $\frac{\Delta V_{T_n}}{V_{T_{n-}}}$ is equal to -50%. Thus we must take care of the cash-lock risk (the portfolio value provides a smaller return than a risk-free investment) and worst of the gap risk (i.e. the portfolio value is below the floor implying that the guarantee is no longer satisfied).

3.2 The OBPI strategy

As aforesaid, the OBPI was introduced by Rubinstein & Leland (1981). It has two objectives (like the CPPI): protecting the portfolio value at maturity and taking advantage of rises in the underlying strategic allocation. The objective is generally to recover a percentage p of the initial investment amount V_0 (with $p \leq e^{rT}$). The initial capital invested in the fund is then split into two parts: the first component corresponds to an amount invested on the risk-free asset; the second one is the optional component, namely the purchase of call options or equivalenty the purchase of the risky asset and that of the put option written on it (due to the put/call parity).

The OBPI method consists basically in purchasing an amount $q \times K$ invested on the money market account, and q shares of European call options written on asset S with maturity T and exercise price K.[13] The portfolio value V^{OBPI} is given at the terminal date by:

$$V_T^{OBPI} = q\left(S_T + (K - S_T)^+\right) = q\left(K + (S_T - K)^+\right) \tag{7}$$

given Put/Call parity. This relation shows that the insured amount at maturity is the exercise price times the number of shares, qK. The value V_t^{OBPI} of this portfolio at any time t in the period $[0, T]$ is:

$$V_t^{OBPI} = q\left(S_t + P(t, S_t, K)\right) = q\left(K.e^{-r(T-t)} + C(t, S_t, K)\right) \tag{8}$$

[13] Synthetic calls and/or puts here are understood in the sense of a trading strategy in basic (traded) assets which creates the put. The first difference between dynamic hedging and listed put options for portfolio insurance is that the delta value of a put option changes automatically while it must be adjusted continuously in a dynamic-hedging framework. Second difference is the insurance cost, for listed-put strategy is paid up front but for dynamic-hedging strategy is the forgone profits that result from shorting futures. Third difference is that listed-option strategy is confined to fixed interval exercise prices but dynamic-hedging strategy can be implemented around any exercise price.

where $P(t, S_t, K)$ and $C(t, S_t, K)$ are the no-arbitrage values calculated under a given risk-neutral probability Q (if coefficient functions μ, a and b are constant, $P(t, S_t, K)$ and $C(t, S_t, K)$ are the usual Black-Scholes values of the European Put and Call). Note that, for all dates t before T, the portfolio value is always above the deterministic level $qKe^{-r(T-t)}$.

Since $V_0 = q(K.e^{-rT} + C(0, S_0, K))$ and since the insured amount, pV_0, is equal to qK, K satisfies the relation:

$$pV_0 = pq(K.e^{-rT} + C(0, S_0, K)) = qK \tag{9}$$

By rewriting the first part of equation 9, we obtain the number of shares q:[14]

$$q = \frac{V_0}{S_0 + P(0, S_0, K(p))} \tag{10}$$

Thus, for any initial investment value V_0, the number of shares q is a decreasing function of the percentage p. Rewriting then the second part of equation 9, we obtain:

$$\frac{C(0, S_0, K)}{K} = \frac{1 - pe^{-rT}}{p} \tag{11}$$

Therefore, the strike K is a function $K(p)$ of the percentage p. It is straightforward to show that the function $K(p)$ is strictly increasing. This ensures only one exercise price K satisfying the equation 11. Note that the higher the guarantee p, the higher K (for fixed S_0 value).

3.3 Performance measurements analysis: Kappa Performance Measure

In the following, we use the Kappa measures introduced by Kaplan & Knowles (2004) to compare the performances of the CPPI and OBPI strategies with regards to REITs (considered as the risky asset here).[15] The Kappa measures involve downside risk measures, and are defined by:

$$\text{For } l = 1, 2, ..., \; Kappa_l(L) = \frac{\mathbb{E}_{\mathbb{P}}[R] - L}{\left(\mathbb{E}_{\mathbb{P}}\left[\left[(L - R)^+\right]^l\right]\right)^{\frac{1}{l}}}. \tag{12}$$

The index return is denoted by R and L is the return threshold chosen by the investor. Note that, when $l = 1$, the Kappa measure corresponds to the Sharpe Omega measure and when $l = 2$, to the Sortino ratio. Zakamouline (2014) also proves that performance measures based on partial moments such as Kappa measures correspond to measure based on piecewise linear plus power utility functions. When $l = 3$, the risk measure in the denominator of the Kappa appears like a semi-skewness and when $l = 4$, the risk measure in the denominator of the Kappa appears like a semi-kurtosis. The main advantage of the Kappa metric is that it takes the entire return distribution into account and thus consider the possible loss aversion of the investor.

As in Bertrand & Prigent (2011), we examine the choice of the threshold L involved in the Kappa ratios. The threshold L must be determined exogenously, for example with respect to risk aversion or aversion towards downside risk (values of outcomes lower than a given level). To avoid that the Kappa measures are increasing in the volatility, we must choose a "rational" threshold L that is lower than the expected performance of the CPPI and the OBPI portfolios. Here, we rely on a set of Kappa performance measures to assess the portfolio insurance strategies: the CPPI and the OBPI. More precisely, we compute the Kappa measures corresponding to powers from one to four. We choose to vary the threshold from 1% to 3%.

[14] Note that the portfolio value has the homogeneity property with respect to q. q may be normalized to 1 without loss of generality.

[15] Detailed explanations and motivations of these performance measures are refered to Bertrand & Prigent (2011).

3.4 The gap risk

CPPI and OBPI models are usually formulated in continuous-time, which assumes instantaneous trading and smooth price changes. However, in practice these assumptions may be violated. This introduces the notion of gap risk - the risk that the portfolio value will not meet the guaranteed amount at maturity.[16] Portfolio insurance strategies are dynamic by nature (with the exception of the direct purchase of a put option at the right strike and at the right maturity) and thus they should be adjusted continuously to get the desired result.

Discontinuities in the price of the risky asset, trading frictions, difficulties of making continuous rebalancing at appropriate timings and lack of liquidity all contribute to gap risk. All these frictions can be modelled in a setup where the price dynamic of the risky asset is described by a continuous-time stochastic process but trading is restricted to discrete time.On the one hand, one might think of an investor who accepts, because of market incompleteness, a strategy which gives the guaranteed amount with a certain success probability. On the other hand, one might think of retail products which are based on the CPPI or OBPI method. Normally, the buyer of such a product gets the guaranteed amount even in the case the strategy fails to provide it. Here, the issuer takes the gap risk and considers this in his product pricing. In both cases, the risk profile of the strategies regarding this point is of great interest.

It is necessary to compute risk measures which allow a characterization of the risk that such strategies do not fully protect their insured amount. In this context shortfall probability (VaR) and expected shortfall ($CVaR$) are often considered with threshold being the insured amount. Such risk metrics determine the effectiveness of the discrete-time strategies (by opposition to the continuous time one). The shortfall probability is the probability that the final value of the discrete-time strategies is less or equal to the guaranteed amount. Intuitively, one can also define a local shortfall probability (given that no prior shortfall happened before).[17] Table 1 in Section 4.3 illustrates empirically this gap risk. Note that the OBPI strategy generally avoids the gap risk since the option component is at worst null.[18]

3.5 Cash-lock risk

Recall that, if the gap risk takes place, the portfolio manager must immediately invest the whole portfolio value on the risk free asset for the remaining management period. Thus, the portfolio is monetized. Even if such event does not happen, the portfolio value may provide a smaller return than the risk-free investment at maturity, as mentioned previously. On the contrary to the OBPI, where the potential gain is known at the setting of the strategy, CPPI is path-dependent: i.e., the final pay-off depends on the path of underlying asset prices during the investment horizon. Once the allocation to REITs index falls to zero (or close to zero), CPPI has no chance to recover its market exposure while OBPI can restore it. This is why CPPI strategy performs poorly in a volatile market. This risk, usually called the cash-lock risk, can occur if for example a sudden market drop dramatically reduces the cushion. In such a case, the amount invested on the risky asset becomes very small, which does not allow to sufficiently benefit from future market rises. Therefore, the investor may be disappointed since his portfolio return is too weak with respect to the risk-free

[16] An analysis of gap risk is provided for example in Cont & Tankov (2009) or Balder et al. (2009).

[17] See e.g. Ben Ameur & Prigent (2014).

[18] If the portfolio manager must synthesize the option, he will be cautious by considering potential high volatility levels when determining the hedge of the call option.

investment. The cash-lock phenomenon can be observed in Figure 1c.

Note that specific CPPI strategies can be introduced to reduce the cash-lock risk, among them those based on margins (see e.g. Boulier & Kanniganti, 2005). For the standard CPPI strategy, the cash-lock risk can be controlled by choosing appropriate portfolio parameter values (floor and multiple). For the OBPI strategy, it corresponds mainly to the fact that the call option cannot be exercised (namely the risky asset value is smaller than the strike K at maturity) with a very significant probability.

4 Empirical application

4.1 Data and implementing assumptions

This paper simulates the performance of the CPPI and the OBPI strategies. We test the hypotheses by empirical simulation on market data on the FTSE NAREITs all equity US index. Our data consists of daily returns over the sample period from January 1999 to June 2016. As already documented, stock market returns deviate from the normal distribution as both series exhibit left-skewed returns and fat tails. The effect is much more pronounced for REITs (see Table 2). The risk free rate is the U.S. 3-month bond yield over the same period.

The strategy is set up each year of the dataset for one year (from the 1st of January to the end of December). In our setup, the simulations involve an investor with a one year horizon. We have considered floors of 100%, 95% and 90%. The model is implemented in a discrete time framework. We do not account for transaction costs.[19] The portfolio is self-financing, i.e., money is neither injected nor withdrawn during the investment horizon. To analyze more precisely the strategies without being influenced by calendar year, we use 100,000 circular block-bootstrap of 252 returns to avoid serial dependency and under-weighting issue (beginning and end of original time series). CPPI strategy is presented for three multiples: 1, 5 and 9. It is important to notice the strategy with $m = 1$ which may be compared to a buy & hold of the CPPI strategy where very limited part of the fund are invested in the risky asset. OBPI strategy is based on the premise that the option exists. Finally, four strategies are presented.

Important implementation points must be mentioned. First recognizing that in practice, the leverage is limited for the majority of investors, we limit the leverage to 100% of the investment. As a robustness check, we also implement the strategy without leverage restriction in order to allow the methodology to fully work. Interestingly, it does not change the results that much (see appendix A). Second, the option of the OBPI strategy is priced according to historical volatility and not rebalanced, supposing thus the existence of a market (in this case, the performance is either the guarantee or those of the underlying index). To be as close as possible to practice, the historical volatility of the previous year is used as a proxy for the volatility of REITs used for the initial pricing of the OBPI. In turbulent period, the initial estimated volatility may be too low and the options premium may increase during the year above the expectation. It may be possible to sell the option to make a profit. However, within the context of this work focusing on PI strategies, we will not consider this possibility.

[19] Transaction costs are generally not considered in the literature as they are difficult to model. In a discrete time framework, transaction costs can be reduced by considering trading bands, limited frequency (daily, weekly, monthly...) or threshold to avoid multiple rebalancing in a short period of time. Lower fees prolong or shorten the exposure to the risky asset by making the structure less prone to re/de-leveraging and allocating to riskfree asset.

The portfolio is rebalanced on a daily basis.[20] The required notional amount for rebalancing is assumed to be completely settled at the settlement price of the previous trading day in the next trading day. This assumption is generally taken for granted in the literature on the portfolio insurance.

4.2 Empirical results

Table 2 presents the summary statistics of the returns for all the strategies. This table raises many comments. There is no free lunch and no strategy is able to generate a higher return than the simple buy & hold strategy totally invested in the index. All the PI strategies call - on average - for a reduction in returns. This lower return illustrates the implicit cost inherent in PI investments. Overall the higher the requested protection (the floor), the higher the average returns reduction. In counterpart of this lower return, the volatility is itself reduced for all the PI strategies compared to the buy and hold strategy of the index. This leads to the analysis of the return-to-volatility ratio (Mean/Std. Dev.). The striking point is that the ratio is somehow the same along all the strategies (buy & hold totally invested on the REITs and PI). Exception to this point is the ratio for the CPPI without multiple ($m = 1$) where the ratio is close to 1. Indeed, in this case, a particularly low part of the total amount is invested in the risky asset and the risk (measured within this metric by the standard deviation) is very limited which boosts the ratio. In counterpart, it must be noted that the CPPI without leverage generates a particularly low return. Finally, this first analysis demonstrates that no strategy is clearly able to generate a higher risk-return performance and that no strategy clearly dominates the others.

INSERT TABLE 2 AROUND HERE

Turning to the rest of the distribution, the effectiveness of the PI strategies is confirmed. Skewness are higher for PI strategies than for the standard buy & hold, and kurtosis are lower. In particular, CPPIs exhibit globally lower volatility but higher skewness (positive) and kurtosis than the OBPI. All the strategies exhibit positive skewness, indicating that the risk of PI strategies is essentially a "good risk" as the fact that the mean is above median shows. Note that this is the contrary for the REITs index. Regarding the minimum return per year, while REITs show a minimum of -62%, standard OBPI and CPPI never fall below their floor since the maximum daily drawdown is -19%. The CPPI without leverage ($m = 1$) does even generate a higher minimum value than the predefined floor. PI techniques help actually to reduce the unexpected future loss. The lower decile confirms this result with a loss for the REITs of 12% and still a perfect hedge against loss from the PI strategies. It underlines the effectiveness of the PI strategies to avoid losses. On the other side of the distribution, quantile Q-75, Q-90 and Q-97.5 show clearly the implicit cost inherent in PI investments. In this sense, the CPPI strategy with leverage and a lower guaranty is less impacted. It may be attributed to the fact that the leverage gives more opportunity to catch up rising market.

In terms of protection, the OBPI strategy performs pretty well. The value of the floor is the present value of the specified number discounted using the riskless rate and the strategies involve the buying of a riskless asset and a call option. At maturity, the riskfree asset ensures the floor value and option allows to catch up part of the upside potential. The OBPI is really expensive when market is volatile: actually, the premium of the option is a direct increasing function of the volatility. Therefore, when the previous year (252 returns in our case) was very volatile, the option premium is

[20] Even if the transaction costs are not considered here, one point must be bear in mind. The frequency of rebalancing provides a trade-off between transactions cost and protection. Since the volatility of REITs is relatively high, the daily rebalancing may be acceptable to secure solid protection in spite of the increased transactions cost.

very expensive and therefore the remaining funds only allow to buy a small proportion of options. CPPI also shows good performance. CPPI strategy sells when the market is decreasing only and buys after the beginning of rise to see the market weaken again (CPPI strategy sells stocks when they fall and buys stocks when they rise in value).

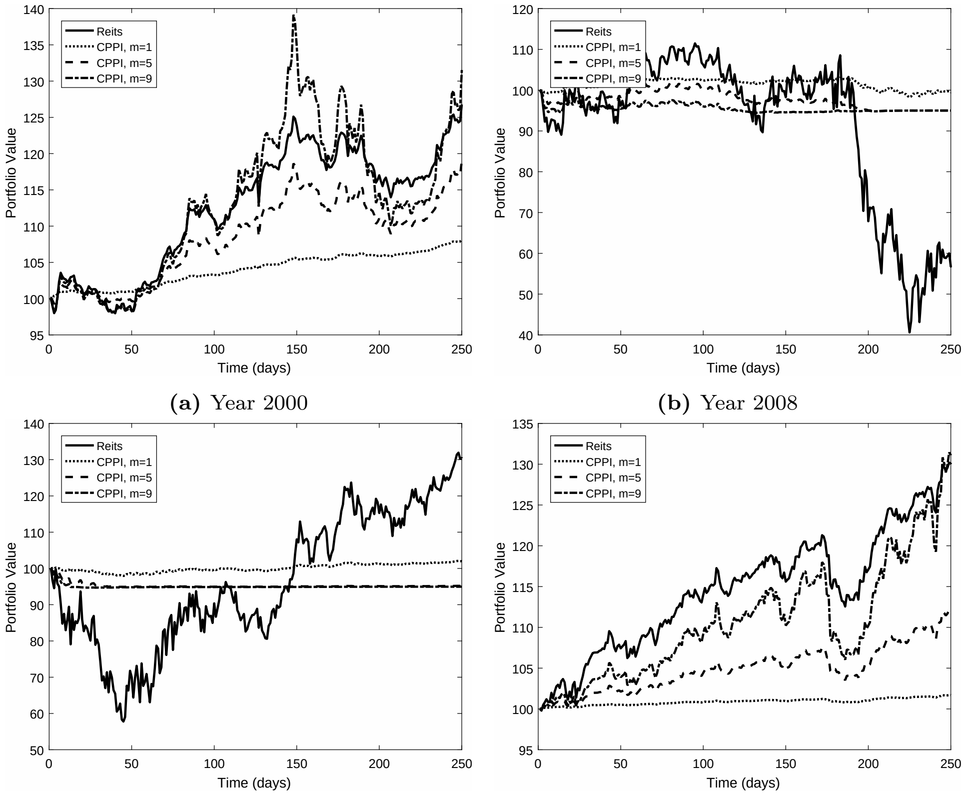
Figure 1 presents four paths of the REITs and for the four strategies (the OBPI strategy and the three CPPI strategies for multiple $m = 1$, 5 and 9) for four specific calendar years: 2000 (Figure 1a), 2008 (Figure 1b), 2009 (Figure 1c) and 2014 (Figure 1d). The results are mainly illustrative as the chosen dates - from the 1st of January to the end of December - give a result that may be specific to this time frame. The relative performances of the strategies depend thus on the performance of the market during the evaluation period. Nevertheless, the four chosen periods allow to exemplify what happens with the strategies according to various market conditions. Appendix C present the same results for 90% and 100% guaranty.

Globally OBPI is useful in all kinds of situation. The CPPI is useful for all kinds of situations as it reduces the risk by investing in the riskless asset. The conclusions can be summed up as follows. The floor is secured by CPPI strategy in all cases. In a declining market where performance may be negative, the CPPI strategy outperforms buy & hold (see Figure 1b for year 2008 where the loss is limited to 5%) as it allocates assets to bonds which ensure that the insured principal amount will be repaid to investors at maturity. In a rising market, it is difficult to conclude about the performances of CPPI. Index performance may be positive and additional exposure to the index may be gained through leverage. However, it crucially depends on the multiple and on the way the strategy is implemented as, as soon as the floor is reached, part of stocks must be sold. In 2000, after 150 days (see Figure 1a), the CPPI with a multiple of 9 outperforms the REITs index (possible by the use of leverage), but the contrary happens in year 2014 (Figure 1a). In a flat market, neither strategies have an obvious advantage (see 180 first days of 2008 Figure 1b).

Year 2009 is particularly interesting. It illustrates the cash-lock effect and show the conditions under which the return of the CPPI may become negative. Price reversals disadvantage CPPI investors as it can be seen in Figure 1c. Indeed, the CPPI strategy buys stock as it rises but buys stocks in a proportion allowed by the multiple and the cushion. When the market drops immediately after the setting of the strategy, the risky asset is liquidated and the cash-lock phenomenon become obvious.[21] This graph illustrates particularly how the cash-lock risk affects the CPPI strategy. CPPI strategy is path-dependent and may remain stuck on the floor (CPPI is sometimes said to be trend-following). Note that when the risky asset dynamics is described by a geometric Brownian motion, the probability to merely obtained the guaranteed amount at maturity for a CPPI portfolio is equal to zero. It is however possible to get a negative return from the PI strategy even if the REITs market exhibits positive returns. This interesting and troubling result has a theoretical foundation that lies in the geometric Brownian motion assumption. We prove this result in appendix B.

In order to more deeply analyze the performance of the strategies, table 3 presents a comparison between the REITs performance and the four portfolio insurance strategies OBPI and CPPI for multiple m =1, 5 and 9 based on the Kappa performance criteria (see part 3.3). Depending on the Kappa order, the rankings differ and lead to different analysis. The table must be analyzed with regard to four inputs: the power of the performance metric (K1, K2, K3 and K4), the choice of the strategy (buy & hold of index, OBPI, CPPI for m = 1, 5 and 9), the level of the protection (90,

[21] In the case of year 2000 with a multiple of 9 and a cushion equals to 5 (100-95), it is possible to invest initially 45 in stocks (9 x (100-95)). If the stocks fall of 3$ in price, the total asset value will reach 97 and therefore the new appropriate stock position will be 18 (=9(97-95)). This imply the sale of 22 of stocks and investment of the proceeds in riskfree asset. On the contrary, if stock prices rise in value, stocks should be bought.



(a) Year 2000 **(b)** Year 2008

(c) Year 2009 **(d)** Year 2014

Fig. 1: CPPI vs REITs performance with 95% guarantee

95, 100%) and the threshold ($L = 1$, 2 or 3%).

INSERT TABLE 3 AROUND HERE

The main results of the table 3 may be summed up as follows: first, under K1 criteria, the simple buy & hold of the REITs dominates almost all the PI strategies. The only exceptions to that are the three CPPI strategies when the threshold L equals 1% (whatever the level of protection) and the OBPI strategies when the protection is total (100%) for the 1% threshold. This dominance is straightforward to understand by the impact of loss aversion as a 100% protection and a multiple $m = 1$ provide a very high level of guaranty. Second, CPPI strategy generally dominates the OBPI strategy with the exception of the 100% protection when the threshold is above 1%. Third, under K2, K3 and K4 criteria, the results share common trends. PI strategies globally dominate the simple buy & hold of REITs index with CPPI results generally above those of OBPI even if this may be discussed for the CPPI without leverage. Fourth, for all the strategies, the higher the power of the Kappa, the more advantageous are the PI strategies. Moreover the higher the threshold the lower is the effectiveness of the strategies.

4.3 Gap risk and robustness tests

In the geometric Brownian motion (theoretical framework), the gap risk can never occur since the cushion is always positive. However, in continuous time, the gap risk can happen due to sudden market drops. In the discrete time setup, the gap risk can be due to the imperfect portfolio hedging. Thus, it is important to empirically examine the gap risk from observed data. Table 1 summarizes the robustness check results for the gap risk.

	CPPI		
	$m = 1$	$m = 5$	$m = 9$
90%	0%	0%	7,99%
95%	0%	0%	8,17%
100%	0%	0%	8,23%

Note: 100,000 simulations. OBPI is not subject to gap risk as the options premium cannot be negative.

Table 1: Gap Risk for CPPI

As it can be seen, the event happens only for $m = 9$. The percentage is very low. This shows how the method is effective to protect against losses.

5 Conclusion

The purpose of this paper is to propose a methodology allowing to invest in REITs while controlling down-side risk. In this work, we have empirically tested the effectiveness of the portfolio insurance strategies in the context of REITs investment. To the best of our knowledge, this work is the first to test these strategies for REITs. The implementation helps to understand the most common approaches of portfolio insurance strategies, i.e. OBPI and CPPI (for three different multiples). Each strategy is evaluated as a pure investment tool. In addition, to assess thoroughly the effect of the PI strategies, an analysis for the Kappa risk-adjusted performance is supplemented for comparison. Kappa incorporates both the return and risk properties of a given strategy and does not depend on any distributional assumption. The analysis is performed using circular block-bootstrap. We run historical return simulations over the period 2000-2016.

The main conclusions can be summed-up as follows. First the PI strategies - CPPI and OBPI - success to protect from losses and are effective to catch part of the REITs rise. Second, on the long term and on average, no PI strategy is able to generate a higher return than the simple buy & hold of the index. All the PI strategies call for a reduction in returns but for a lower volatility (skewness and kurtosis exhibit also positive movements). This results were expected as those strategies with a downside protection have a cost (implicit cost of security). Third, the PI strategies are effective to avoid huge sharp losses. Within this regards, the lowest return for PI equals the insured percentage compared to a realized -62% per year for the REITs index.

Finally, PI strategies reduce volatility, dampen losses and allow to participate in positive returns. These strategies are thus effective to avoid loss risk. PI strategies for REITs can therefore be used as an effective hedging tool against hight losses. After all, these strategies allow to include REITs in prudent and well-informed institutional investors portfolio seeking a precise control of their loss risk while benefiting from potential REITs rises.

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	Reits	OBPI			CPPI								
		90%	95%	100%	90%	95% $m = 1$	100%	90%	95% $m = 5$	100%	90%	95% $m = 9$	100%
Mean	14,6%	8,9%	7,1%	5,0%	3,4%	2,8%	2,2%	8,8%	5,7%	2,9%	11,2%	8,3%	3,9%
Std. Dev.	23,9%	14,3%	12,5%	8,0%	3,1%	2,4%	2,1%	13,4%	8,2%	3,7%	18,2%	13,6%	6,3%
Variance	0,06	0,02	0,02	0,01	0,00	0,00	0,00	0,02	0,01	0,00	0,03	0,02	0,00
Mean/Std. Dev.	0,61	0,62	0,57	0,63	1,09	1,16	1,01	0,66	0,70	0,78	0,62	0,61	0,61
Skewness	-0,21	0,32	0,71	1,36	-0,02	0,53	0,94	0,68	0,89	1,43	1,09	1,04	1,88
Kurtosis	6,08	2,22	2,40	3,58	3,26	2,59	2,70	3,63	3,57	4,14	6,42	4,02	5,74
Min	-62,4%	-10,0%	-5,0%	-1,0%	-5,7%	-2,6%	0,1%	-10,0%	-5,0%	0,0%	-10,0%	-5,0%	0,0%
Max	135,0%	50,7%	46,2%	33,6%	14,9%	8,8%	7,7%	102,8%	43,3%	18,5%	134,8%	112,9%	31,3%
Q-2.5	-47,5%	-10,0%	-5,0%	-0,8%	-3,6%	-1,2%	0,1%	-10,0%	-5,0%	0,0%	-10,0%	-5,0%	0,0%
Q-10	-12,4%	-10,0%	-5,0%	0,0%	0,3%	0,3%	0,2%	-8,6%	-4,1%	0,1%	-10,0%	-5,0%	0,0%
Q-25	4,9%	-3,1%	-5,0%	0,0%	1,3%	0,8%	0,3%	-1,6%	-0,5%	0,2%	-5,4%	-3,2%	0,1%
Median	16,7%	7,6%	3,2%	0,0%	3,1%	2,4%	1,4%	7,2%	4,0%	0,8%	10,5%	2,6%	0,5%
Q-75	27,1%	20,3%	17,7%	9,9%	5,6%	4,3%	3,8%	18,0%	10,9%	4,6%	23,9%	18,4%	4,8%
Q-90	36,5%	28,4%	25,3%	18,2%	7,8%	6,7%	5,6%	26,9%	16,9%	8,9%	33,1%	27,7%	13,7%
Q-97.5	58,1%	36,1%	32,6%	25,2%	9,1%	7,9%	7,2%	37,4%	24,9%	12,5%	48,1%	39,3%	22,8%

Note: All numbers are in percentage annual value. Mode is the most frequent value. 100,000 replications. Maximum leverage=100% of investment.

Table 2: Basic statistics of the REITs, the OBPI strategies and the three CPPI strategy for multiples 1, 5 and 9

	REITs				OBPI				CPPI, $m = 1$				CPPI, $m = 5$				CPPI, $m = 9$			
	K1	**K2**	**K3**	**K4**	**K1**	**K2**	**K3**	**K4**	**K1**	**K2**	**K3**	**K4**	**K1**	**K2**	**K3**	**K4**	**K1**	**K2**	**K3**	**K4**
$L = 1\%$																				
90%	3,36	1,08	0,70	0,56	2,96	1,53	1,21	1,07	7,00	2,08	1,30	1,01	3,80	1,86	1,41	1,21	3,42	1,92	1,55	1,38
95%	3,36	1,08	0,70	0,56	2,68	1,70	1,45	1,33	7,88	2,99	1,93	1,49	4,17	2,07	1,57	1,35	3,82	2,34	1,94	1,76
100%	3,36	1,08	0,70	0,56	5,95	4,68	4,24	3,97	4,05	2,53	2,14	1,95	5,18	3,52	3,04	2,80	6,30	4,58	4,06	3,81
$L = 2\%$																				
90%	2,98	0,98	0,64	0,51	2,28	1,21	0,96	0,85	2,29	0,93	0,62	0,49	2,83	1,45	1,11	0,96	2,74	1,56	1,27	1,14
95%	2,98	0,98	0,64	0,51	1,86	1,21	1,03	0,95	1,34	0,73	0,55	0,46	2,47	1,33	1,03	0,90	2,64	1,68	1,41	1,28
100%	2,98	0,98	0,64	0,51	2,27	1,82	1,68	1,61	0,20	0,14	0,12	0,12	0,96	0,70	0,63	0,59	1,75	1,36	1,24	1,18
$L = 3\%$																				
90%	2,62	0,88	0,57	0,46	1,73	0,95	0,75	0,67	0,39	0,20	0,15	0,12	2,07	1,11	0,86	0,75	2,19	1,27	1,04	0,93
95%	2,62	0,88	0,57	0,46	1,27	0,84	0,72	0,67	-0,20	-0,13	-0,11	-0,09	1,40	0,81	0,64	0,57	1,84	1,20	1,02	0,93
100%	2,62	0,88	0,57	0,46	1,01	0,81	0,75	0,72	-0,58	-0,45	-0,40	-0,38	-0,08	-0,06	-0,06	-0,05	0,51	0,41	0,38	0,36

Note: All numbers are in percentage annual value. 1,000,000 replications. Maximum leverage: 100%.

Table 3: Performance measures of the two strategies using Kappa measures with leverage restriction

	Reits	OBPI			CPPI								
		90%	95%	100%	90%	95% $m = 1$	100%	90%	95% $m = 5$	100%	90%	95% $m = 9$	100%
Mean	14,4%	8,8%	7,0%	5,0%	3,4%	2,8%	2,2%	8,3%	5,6%	2,9%	12,2%	8,0%	3,7%
Std. Dev.	23,7%	14,3%	12,5%	8,0%	3,1%	2,4%	2,1%	13,5%	8,1%	3,7%	33,8%	19,4%	6,1%
Variance	0,06	0,02	0,02	0,01	0,00	0,00	0,00	0,02	0,01	0,00	0,11	0,04	0,00
Mean/Std. Dev.	0,61												
Skewness	-0,28	0,34	0,72	1,37	-0,03	0,54	0,95	1,01	0,96	1,44	4,06	3,72	2,17
Kurtosis	5,93	2,24	2,43	3,61	3,22	2,61	2,72	4,73	4,02	4,15	25,85	22,46	7,71
Min	-62,4%	-10,0%	-5,0%	-1,0%	-5,7%	-2,6%	0,1%	-10,0%	-5,0%	0,0%	-10,0%	-5,0%	0,0%
Max	135,0%	50,7%	46,2%	33,6%	14,9%	8,8%	7,7%	69,9%	39,3%	18,5%	294,9%	163,4%	38,7%
Q-2.5	-47,6%	-10,0%	-5,0%	-0,8%	-3,7%	-1,2%	0,1%	-10,0%	-5,0%	0,0%	-10,0%	-5,0%	0,0%
Q-10	-12,4%	-10,0%	-5,0%	0,0%	0,4%	0,3%	0,2%	-8,6%	-4,1%	0,1%	-10,0%	-5,0%	0,0%
Q-25	4,8%	-3,1%	-5,0%	0,0%	1,3%	0,8%	0,3%	-1,7%	-0,5%	0,2%	-7,2%	-3,4%	0,1%
Median	16,5%	7,4%	3,0%	0,0%	3,1%	2,4%	1,4%	6,6%	4,0%	0,8%	1,1%	0,9%	0,5%
Q-75	27,0%	20,2%	17,6%	9,8%	5,5%	4,2%	3,8%	16,3%	10,6%	4,5%	20,4%	12,9%	4,4%
Q-90	36,4%	28,3%	25,3%	18,3%	7,8%	6,7%	5,6%	25,0%	16,4%	8,9%	41,5%	26,0%	12,3%
Q-97.5	57,6%	36,1%	32,6%	25,2%	9,1%	7,9%	7,2%	37,9%	24,2%	12,6%	88,7%	53,0%	22,8%

Note: All numbers are in percentage annual value. Mode is the most frequent value. 100,000 replications. No maximum leverage.

Table 4: Basic statistics of the REITs, the OBPI strategies and the three CPPI strategy for multiples 1, 5 and 9

	REITs				OBPI				CPPI, $m=1$				CPPI, $m=5$				CPPI, $m=9$			
	K1	**K2**	**K3**	**K4**	**K1**	**K2**	**K3**	**K4**	**K1**	**K2**	**K3**	**K4**	**K1**	**K2**	**K3**	**K4**	**K1**	**K2**	**K3**	**K4**
$L=1\%$																				
90%	3,32	1,07	0,69	0,55	2,93	1,52	1,20	1,06	6,91	2,05	1,28	1,00	3,51	1,74	1,32	1,13	2,96	1,91	1,61	1,47
95%	3,32	1,07	0,69	0,55	2,63	1,68	1,43	1,32	7,74	2,95	1,90	1,47	4,04	2,01	1,53	1,32	3,35	2,17	1,83	1,66
100%	3,32	1,07	0,69	0,55	5,88	4,63	4,20	3,93	3,99	2,50	2,12	1,94	5,13	3,50	3,02	2,78	5,79	4,22	3,75	3,52
$L=2\%$																				
90%	2,94	0,96	0,63	0,50	2,25	1,20	0,95	0,84	2,24	0,91	0,61	0,48	2,58	1,33	1,03	0,89	2,38	1,57	1,33	1,22
95%	2,94	0,96	0,63	0,50	1,82	1,19	1,02	0,94	1,29	0,71	0,54	0,45	2,37	1,28	1,00	0,87	2,29	1,54	1,32	1,21
100%	2,94	0,96	0,63	0,50	2,24	1,79	1,66	1,59	0,19	0,13	0,12	0,11	0,94	0,69	0,62	0,58	1,54	1,20	1,10	1,04
$L=3\%$																				
90%	2,58	0,86	0,57	0,45	1,70	0,93	0,75	0,66	0,36	0,19	0,14	0,11	1,86	1,01	0,79	0,68	1,91	1,28	1,10	1,01
95%	2,58	0,86	0,57	0,45	1,24	0,83	0,71	0,66	-0,21	-0,14	-0,11	-0,10	1,32	0,77	0,61	0,54	1,58	1,09	0,94	0,87
100%	2,58	0,86	0,57	0,45	0,99	0,80	0,74	0,71	-0,58	-0,45	-0,41	-0,38	-0,08	-0,07	-0,06	-0,06	0,38	0,31	0,29	0,27

Note: All numbers are in percentage annual value. Volatility estimated in log-returns. 1,000,000 replications. No maximum leverage.

Table 5: Performance measures of the two strategies using Kappa metrics without leverage restriction

A Appendix: Robustness check for the leverage in the CPPI

In this part, we illustrate the results of the CPPI without leverage restriction.

INSERT TABLE 4 AROUND HERE

INSERT TABLE 5 AROUND HERE

B Appendix: Proof of negative returns for CPPI when the risky asset exhibits large positive returns

The results show a troubling feature about CPPI and REITs returns. Indeed, it may be observed (for instance year 2009, 2011, 2013 and 2015) that the CPPI strategy exhibits negative returns even when the risky asset display pretty good returns. Even if this point may seem odd and counter-intuitive, it has a significant probability to occur. In this section, we demonstrate why this result can happen. Let's first note that this result holds even with a capital protection below 100% and remind our theoretical set-up is based on GBM.

We examine how the CPPI strategy value varies according to the risky asset fluctuations described by a GBM. We are especially interested to propose a theoretical explanation of the behaviour of the CPPI portfolio with an insured percentage less than one such that the 95% case during the years 2009, 2011, 2013 and 2015. Indeed, these CPPI portfolios return a negative annual performance while the REITs exhibits a positive one. First note that by construction, in case the whole initial portfolio value is insured ($p = 1$), the portfolio return cannot be negative. On the contrary, as soon as losses are allowed ($p < 1$), the situation can happen.

From equation 5 and 6, we deduce:

$$\ln\left(\frac{V_t}{V_0} - pe^{-r(T-t)}\right) = m\ln\left(\frac{S_t}{S_0}\right) + \left[\ln\left(\frac{C_0}{V_0}\right) + \left((1-m)\,rt + \frac{1}{2}\left(m - m^2\right)\sigma^2 t\right)\right] \tag{13}$$

At maturity, $t = T$, this relation becomes:

$$\ln\left(\frac{V_T}{V_0} - p\right) = m\ln\left(\frac{S_T}{S_0}\right) + \left[\ln\left(1 - pe^{-rT}\right) + \left((1-m)\,rT + \frac{1}{2}\left(m - m^2\right)\sigma^2 T\right)\right] \tag{14}$$

Therefore, in continuous-time, the CPPI portfolio excess logreturn[22] is equal to the risky (REITs) asset logreturn multiplied by the multiple m to which a negative correction term is added. Indeed, the CPPI portfolio return is an increasing function of the REITs index return but the correction term is negative as soon as the multiple m is higher than 1 since the logarithm term in the bracket is always negative knowing that $C_0 < V_0$. Thus, it is possible that the REITs index increases while at the same time the CPPI portfolio decreases. This is an event that is not always easy to understand for individual investors on such fund.

Proposition 1 *For all $t < T$, the probability P_t that the REITs index increases while the CPPI fund decreases is given by:*

$$P_t = \Phi\left(\frac{-\frac{a.t}{m} + \frac{1}{m}\ln(\alpha_t) - \left(\mu - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{-\left(\mu - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right). \tag{15}$$

Proof Denote $Y_t = \frac{V_t}{V_0}$, $X_t = \frac{S_t}{S_0}$, $a = (1-m)r + \frac{1}{2}\left(m - m^2\right)\sigma^2$ and $\alpha_t = \frac{1-pe^{-r(T-t)}}{1-pe^{-rT}} (\leq 1)$. Then, from equation 5, the CPPI strategy at time t fund value (Y_t) can be written as:

$$Y_t = pe^{-r(T-t)} + \left(1 - pe^{-rT}\right)X_t^m e^{a.t}. \tag{16}$$

Note that $a < 0$ as soon as $m > 1$. We are looking for the value $\mathbb{P}[(X_t > 1) \cap (Y_t < 1)]$. We observe that:

$$Y_t < 1 \Leftrightarrow pe^{-r(T-t)} + \left(1 - pe^{-rT}\right)X_t^m e^{a.t} < 1, \tag{17}$$

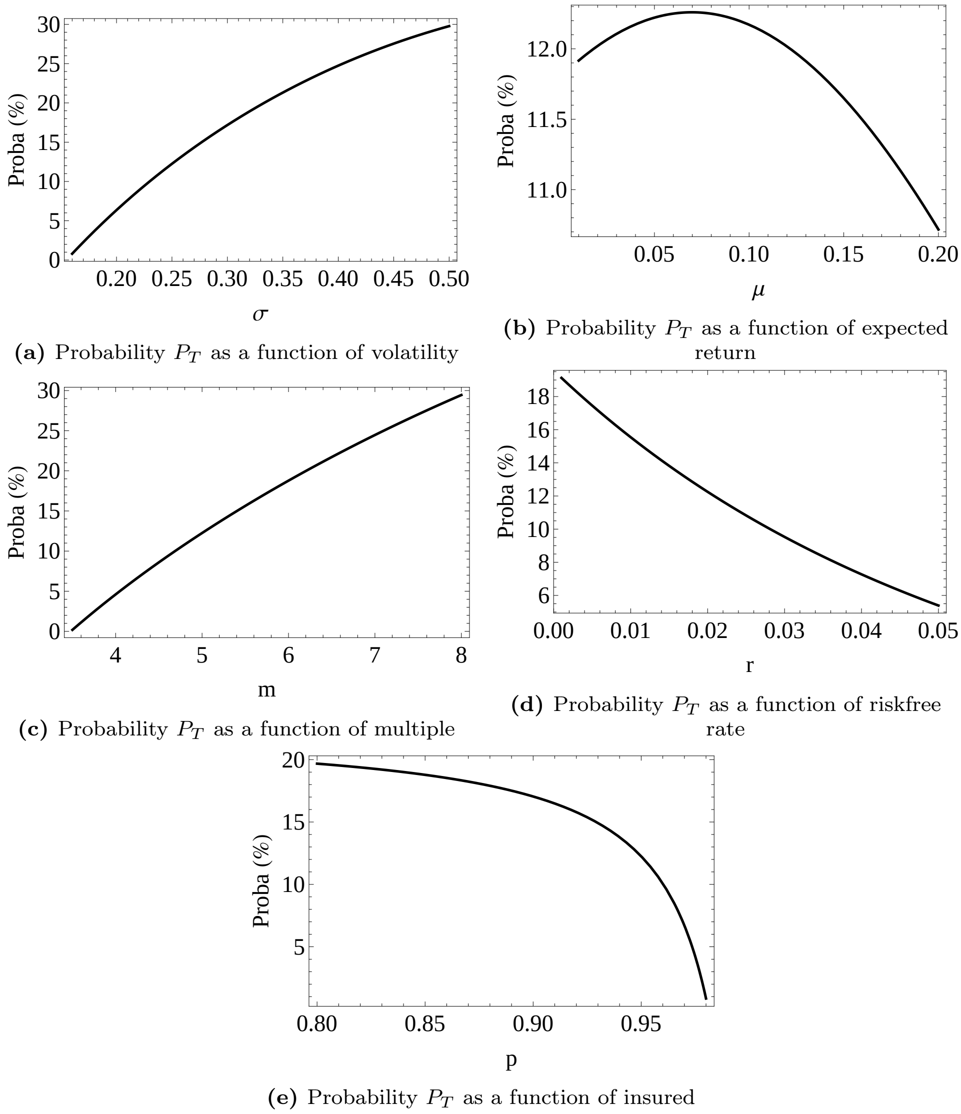
$$\Leftrightarrow X_t < \alpha_t^{\frac{1}{m}} e^{-\frac{a.t}{m}}. \tag{18}$$

For $\alpha_t^{\frac{1}{m}} e^{-\frac{a.t}{m}} > 1$, we must evaluate the probability that $1 < X_t < \alpha_t^{\frac{1}{m}} e^{-\frac{a.t}{m}}$, which is equivalent to: $0 < \ln(X_t) < -\frac{a.t}{m} + \frac{1}{m}\ln(\alpha_t)$. Using the equality $\ln(X_t) = \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}Z$ where Z has a standard Gaussian distribution, Relation (15) is proven.

[22] The CPPI portoflio return is in excess of the insured percentage. For instance, if $p = 0.95$ and the CPPI annual return is 3%, the excess return is equal to 8%.

Remark 1 For this probability to be defined, the condition $\alpha_t^{\frac{1}{m}} e^{-\frac{a.t}{m}} > 1$ must be fulfilled. For example, for the following parameter values, $r = 2\%$, $m = 5$, $p = 0.95$ and $t = T = 1$, the volatility must be higher than $15,47\%$. This threshold volatility value is increasing with the interest rate, r, the insured percentage, p, and the time, t. It is also decreasing with the multiple, m.

To illustrate the behaviour of this probability at maturity, P_T, we consider the following parameter values : $\mu = 6\%$, $\sigma = 25\%$, $r = 2\%$, $t = T = 1$, $m = 5$ and $p = 0.95$. We obtain a probability P_T equal to 12.25%. This value is not negligible and investors must be aware of this feature. Figure 2 displays the different sensitivities of the probability P_T starting from our aforementioned base case and letting one parameter value varying.



(a) Probability P_T as a function of volatility

(b) Probability P_T as a function of expected return

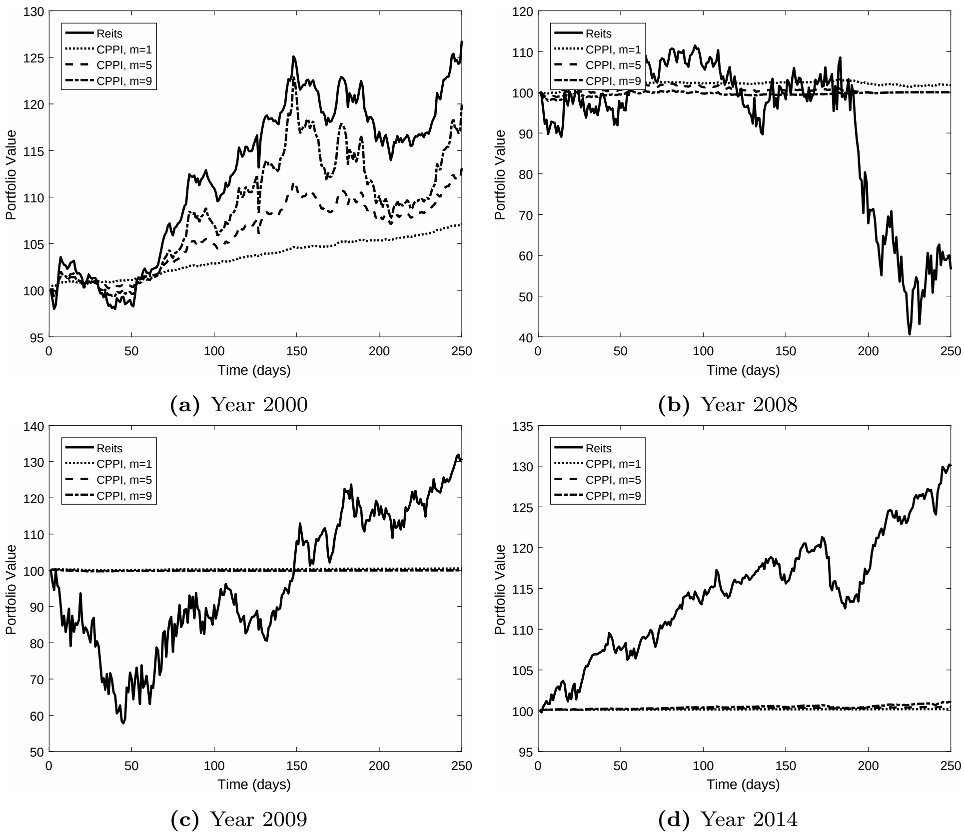
(c) Probability P_T as a function of multiple

(d) Probability P_T as a function of riskfree rate

(e) Probability P_T as a function of insured percentage

Fig. 2: Sensitivity of probability P_T to various parameters

C Appendix: Illustration of the strategies for a floor of 100% and 90%



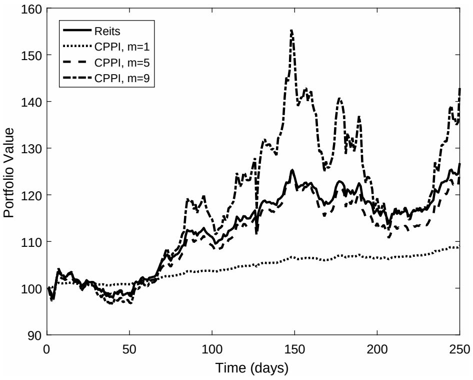
(a) Year 2000

(b) Year 2008

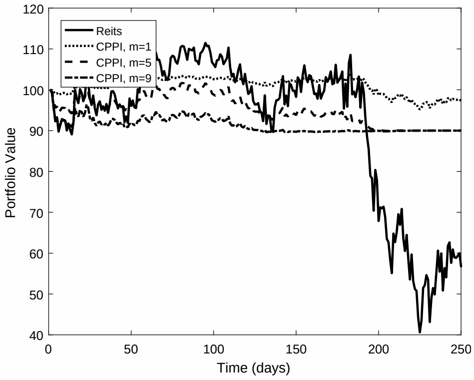
(c) Year 2009

(d) Year 2014

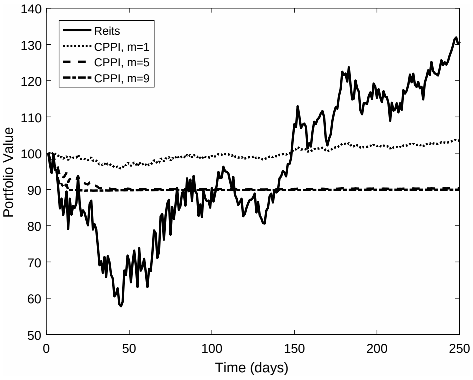
Fig. 3: CPPI vs REITs performance with 100% guarantee



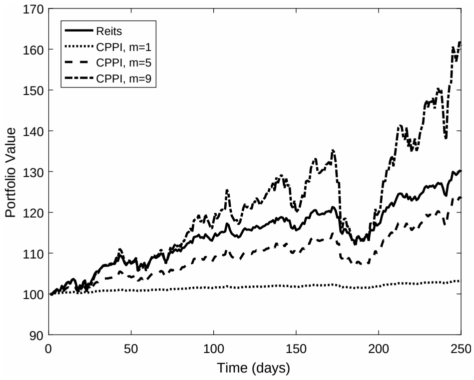
(a) Year 2000



(b) Year 2008



(c) Year 2009



(d) Year 2014

Fig. 4: CPPI vs REITs performance with 90% guarantee