Real options valuation and stress test analysis

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Abstract

In this paper we develop a novel valuation model and methodology to value a pharmaceutical R&D project based on real options approach. The real options approach enables the possibility of optimally abandon the project before completion whenever the investment cost turns out to be larger than the expected net cash flow stream. On the other hand, the proposed model accounts for two different sources of uncertainty, those are technical and economic risk. This model incorporates a novel economic state vector where each economic state captures the interaction among different market and economic forces using Fourier series as the particular basis for the economic function space. In this sense, Fourier series are considered as an aggregate of forces playing a relevant role in the process evolution determining the cash flow structure and also allowing us to properly define an economic scenario where the project will be developed.

Keywords: Real options, R&D, Economic risk, Fourier series, Pharmaceutical industry, Risk factor, Stress test

JEL classification: G12, G17, G32, C22, C63.

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1 Introduction

The pharmaceutical industry is one of the most dynamic and research-intensive industries in the world. One distinctive characteristic of this sector is the high level of investment in research and development, in fact the pharmaceutical industry has one of the highest R&D budget to sales revenue ratio across industries. The aim of this paper is to provide a comprehensive tool and methodology to value and stress test an R&D project considering that it is subjected to technical and economic uncertainty. Here we focus the attention on pharmaceutical R&D projects, however the model and methodology used can be easily extrapolated to any industry, say for instance mining projects.

Developing a new medicine is a challenging endeavour and the chances of success are extremely low, there are several complex forces, both economic and technical, governing the drug development process that are not entirely understood. The first obstacle arises during the early discovery stage when the company has to wisely assign the appropriate amount of both financial and scientific resources. Although the total cost to develop a new medicine varies from one to another it heavily depends on the kind of compound used, the drug under development, and the likelihood of failure. In terms of time to completion, a pharmaceutical R&D process can take, roughly speaking, between ten to fifteen years since the early-stage discovery of a new compound up to the marketing approval and market launch of the product, again it heavily depends on the drug or treatment. For some innovative drugs or treatments both cost and time to completion are a significant source of uncertainty and constitute the cost of innovation. On the other hand, many "new" medicines or treatments are just improvements on existing drugs, in this case the cost and time to completion are quite standardized and, although there is some uncertainty, the R&D financial and technological cost is considerable lower.

The pharmaceutical market is extremely complex and has divided the public opinion in several controversial topics such as animal testing, drug prices, lack of research interest for certain diseases, public funding, and so on. As any other private company, pharmaceutical companies are ultimately focused on increasing shareholder value. The public perception that privately research funding is solely motivated by profit has increased the friction between shareholders' return expectations and the public notion of fairness. On this regard, it is important to point out that no matter how big a pharmaceutical company is, it can only cover a small portion of breakthrough R&D projects. Therefore, most pharmaceutical companies have to choose with financial wisdom each project because simply they cannot afford to invest when the affected population is too poor to buy the drug or the market niche is just too small to achieve a reasonable return on the investment. Recently, the 2014 Ebola outbreak has revealed the lack of resources and effort assigned to fight this virus while it was limited or contained within the African border, and the increased interest when the virus crossed the European and American border and "opened a new market". In this

paper we are not going to discuss such controversial ethical issues, however our proposed model incorporates a novel economic state vector where each economic state captures the interaction among different market and economic forces using Fourier series as the particular basis for the economic function space. Hence, our model can be used to depict any extreme economic situation and properly value an R&D project targeting such market. Furthermore, since most drugs introduced by the pharmaceutical industry are developed with some contribution from the public sector, see for instance Cockburn and Henderson (2000), our model can be used to determine the appropriate amount of taxpayer's money to allocate in a specific project. On this regard, it is worth to mention that public opinion is a strong force which can heavily affect the project value, its effects can also be modelled with the appropriate terms in the Fourier expansion. Finally, considering the interaction between two main economic and financial variables such as the business cycle and market volatility, we perform a stress test analysis to determine the overall valuation impact and best timing to launch a project.

The objective of this paper is to provide a powerful and flexible valuation model and technology accounting for technical and economic risk and considering all those relevant forces playing a significant role in the project valuation and decision making process. The remainder of this paper is structured as follows: Section 2 is devoted to literature review. Section 3 presents the valuation model, technicalities, and implications. In Section 4 we perform a stress test analysis considering two main economic and financial forces. Finally, in Section 5 we make some concluding comments.

2 Literature review

There is a vast amount of literature based on real options and its application to R&D. Most of the academic literature based on real option valuation consider as exogenous variable the value of the project conditional to the successful completion of the research and development phase. For instance, Madj and Pinduck (1987) use a Geometric Brownian motion process to model the time evolution of the project's market value. The authors show that the arrival of new information might lead the firm to depart from the spending scenario originally planned, and conclude that traditional discounted cash flow criteria do not capture the managerial decision flexibility and for that reason are inadequate to properly value projects where the spending decisions and cash outlays occur sequentially over time, there is a maximum rate at which outlays and construction can proceed, and the project yields no cash return until it is actually completed. Furthermore, assuming that the gross project value follows a Geometric Brownian motion, Trigeorgis (1993) analyses the valuation of flexible capital budgeting projects with a collections of real options and examines the interactions among these options identifying situations where option interactions can be small or large, negative

or positive. Pennings and Sereno (2011) value a compound R&D option assuming a geometric Brownian motion process for the underlying value of the project and considering a Poisson random variable to depict the technical failure probability.

On this regard, our approach is closer to the work of Berk et al. (2003) and Schwartz (2004) where the cash flows from the R&D project are modelled. In more detail, Berk et al. (2003) develops and analyses a single R&D investment project modelling the cash flows from the project with two stochastic processes, one of them tracking any possible catastrophic event and the other process modelling the conditional cash flows the project would have produced if it were completed. The authors assume that the cash flows last forever allowing them to value the completed project using a continuously compounded version of the growing perpetuity formula. On the other hand, Schwartz (2004) implements a simulation approach to value patents and patent-protected R&D projects assuming two stochastic differential processes, one of them for the cost-of-completion and the other for the cash flows generated from the project, and introduces the probability of any catastrophic event with a Poisson probability. In this paper we consider the net cash flow as the underlying variable, however since this variable takes into consideration the production and marketing cost it could yield a negative cash flow stream. Therefore, we assume that the net cash flow of a successful project is given by an Arithmetic Brownian motion process plus a time dependent component depicts by the Fourier series. On this regard, Copeland and Antikarov (2001, Chapter 5) claim that cash flow streams, and thus present values, can be negative. Accordingly, Alexander et al. (2012) assume that the project's value does not necessarily remain positive during the whole project's life and model the intrinsic value of the project with an Arithmetic Brownian motion process which allows the underlying to become negative. Under this assumption, the authors find analytical formulas for European calls and puts on dividend-paying assets and provide a numerical algorithm for American-style options based on an Arithmetic Brownian motion process. It is worth point out that in this paper we model the stochastic process with an Arithmetic Brownian motion because we intentionally decided to model the net cash flow as the underlying variable, however the methodology applied here can be easily extrapolate to any other underlying variable following a Geometric Brownian motion or a mean reverting process.

An important feature of R&D projects is the uncertainty related with the cost to completion, for an in-depth look of this topic see, for instance, Hansen (1979), DiMasi et al. (1991), and particularly DiMasi et al. (2003) where the authors perform a thoroughly study of the research and development cost of 68 randomly selected new drugs of 10 different pharmaceutical companies and provide an estimate of the costs of pharmaceutical innovation. Also, Pindyck (1993) studies investment decisions when the project is subjected to two different sources of uncertainty, technical uncertainty and cost uncertainty. In this case, the author concludes that, although the sources and

amounts of cost uncertainty greatly varies across projects, cost uncertainty has a deeper impact than technical uncertainty in terms of its effect on the investment rule and the value of the investment opportunity.

Further relevant literature about real option valuation includes, Childs and Triantis (1999) who examine dynamic R&D investment policies and the valuation of R&D programs in a contingent claims framework. The authors study the interaction between multiple R&D projects cash flows and analyse how the firm may alter its funding policy over time. Smith and Nau (1995) compare the risk-adjusted discount-rate analysis, option pricing analysis, and decision analysis approaches for valuing risky projects. Posner and Zuckerman (1990) determine the optimal stopping time of an R&D project and characterize the expenditure strategy assuming a random R&D decision model without rivalry. McDonald and Siegel (1986) compare the optimal timing of investment for certain alternative combinations when the future net cash flow follows a Geometric Brownian motion process with and without jumps, and the cost of installation is fixed or stochastically modelled with also a GBM process. Gamba and Trigeorgis (2007) implement a multi-dimensional binomial algorithm for valuing options whose payoff depends on N-dimensional state variables following correlated Geometric Brownian processes.

3 R&D valuation model

Consider, for instance, a pharmaceutical R&D project for the development of a new drug. The very nature of such project and the potential impact on human health make the pharmaceutical industry quite unique and risky. There are several strict and well regulated stages since the early-stage drug discovery up to the marketing approval and market launch of the product. Figure 1 presents an illustrative schedule of a generic pharmaceutical R&D project.

[FIGURE 1 AROUND HERE]

The overall project's life can be divided into two mayor phases, firstly the research and development phase and secondly the market phase. During the early-stage of the research and development phase, a new compound which may potentially derive into a marketable drug is either discovered or designed. Once the compound is successfully identified as a potential drug and synthesized the project moves to the next stage. During the preclinical and clinical development the drug must successfully complete a number of well regulated stages. Firstly, the preclinical stage covers the laboratory and animal testing, and it is normally during this stage when the company applies for a patent. If and only if the drug successfully completes the preclinical stage, it accesses the clinical stage which can be divided into clinical phase I, II, and III. During the clinical phase I, the drug

or treatment is tested in a small group of healthy volunteers in order to determine the safe dosage, evaluate its safety, and to identify possible side effects and toxicity. During the clinical phase II, the drug or treatment is tested on a relative large group of subjects (100-300) with the condition that the drug is intended to treat in order to further evaluate its safety and efficacy. Finally, the clinical testing phase III consists of large scale trials, usually a few thousands, to confirm the safety and efficacy of the drug or treatment and to further monitor possible side effects. The final stage in the research and development phase is the marketing approval, once again if and only if the drug has successfully completed each and every preceding stage, the regulatory authority decides whether the drug is approved for patient use or not. If the marketing approval is granted the project moves to the market phase where the appropriate marketing strategy should be established and the product is market launched.

During the patent's life, the company is entitled to a set of exclusive rights protecting the project from market competitors for a limited period of time. However, market competition is not the only force that jeopardises the successful completion of the project. It is well established in the literature, see for instance Brealey and Myers (2000), that an R&D project faces two different sources of risk, those are the economic and technical risk. Technical or technological risk takes into account the inherent uncertainty about the successful completion of each stage during the drug development phase, for instance, an extreme side effect during the clinical testing would lead to a failure event. On the other hand, economic risk deals with both market uncertainty such as sales volume, pricing levels, market competitors, and other economic factors such as interest rates, inflation, growth rate. Indeed, in order to effectively value these sort of projects we have to be able to properly capture both sources of risk at the appropriate time.

3.1 Technical uncertainty

Technical or technological risk is the primary source of uncertainty during the drug development process, in fact, most drugs undergoing the preclinical and clinical stage do not obtain the regulatory authority's approval. Since each stage must be preceded by the successful completion of the previous one, the failure of one stage produces the overall project termination. On the other hand, we assume that once the drug successfully passes the preclinical and clinical test and finally achieves the regulatory authority's approval technical risk virtually vanishes. On this regard, it is widely spread the use of a Poisson process to model technical or technological risk (see for instance Pennings and Sereno (2011), Schwartz (2004), among others). The Poisson probability mass function (pmf) is given by

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{1}$$

where $\lambda > 0$ is the Poisson parameter, and $k = 0, 1, 2, ..., \infty$ defines the number of events. Generalizing $k = 1, 2, ..., \infty$ as any possible technical event and $k = 0$ as no technical event, we have that

$$\text{Probability of success} = e^{-\lambda} \tag{2}$$

$$\text{Probability of technical failure} = \sum_{k=1}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} = 1 - e^{-\lambda} \tag{3}$$

Hence, the expected project value *conditional to technical risk* is given as

$$E\left[V_t|\text{Technical Risk}\right] = V_t(k=0) \cdot e^{-\lambda} + V_t(k = 1, 2, ..., \infty) \cdot \left(1 - e^{-\lambda}\right) \tag{4}$$

where

- $V_t(k = 0)$ is the value of a successful project
- $V_t(k = 1, 2, ..., \infty)$ is the residual value of a failing project

Note that a failed project might increase the stock of knowledge of the company. However, it is common use to assume that the outcome of a failure is a worthless project. Under this assumption, technical risk can be consider as a premium over the risk free rate and during the development process the discount factor is given by $e^{-r_d t} = e^{-(r+\lambda)t}$, where λ represents the annual rate of failure and r the risk-free rate. Note that, as stated above, technical risk vanishes after the regulatory authority's approval, hence this premium is only valid during the drug development phase.

3.2 Economic and market uncertainty

So far things are fairly easy but we have only dealt with technical risk. Economic risk takes into account those factors affecting market conditions, not determining the successful completion in technological terms but defining the cash flow structure of a successful project which gives rise to the project's abandon option. In this fashion, economic risk not only comprehends macro and microeconomic figures but also certain project specific forces and circumstances driving the cash in and out flow, for instance an outbreak of influenza would drive an increase in market sales for those specific medicines or the 2014 Ebola outbreak that pushed the use of the experimental drug "ZMapp" in humans. Note that we have intentionally used the sentence "successful project", that is because we have divided the project into two mayor phases, the research and development and the market phase. As stated above, during the research and development phase technical risk is the dominant source of uncertainty and it vanishes as the drug successfully overcomes every single stage

in the development process and finally achieves the corresponding approval, those projects reaching the market phase are the "successful projects". Once the drug reaches the market phase, there are several forces playing a significant role and we have called this source of uncertainty economic risk, however it is important to remember that only a "successful project" will face economic risk.

We can easily realise that measuring economic risk is not a trivial endeavour, in fact, creating a framework where every force affecting the project is considered is literally impossible. On this regard, it is common use to model the evolution of the project or the evolution of the cash flow as a stochastic differential equation

$$dC_t = \mu(C,t)dt + \sigma(C,t)dW \tag{5}$$

where the process can take the form of a Geometric Brownian motion, Arithmetic Brownian motion, or an Ornstein-Uhlenbeck process. We can also find a more realistic and sophisticated framework, as the one proposed by Schwartz (2004), where the author models both the cash flow and the cost of completion with a stochastic differential equation.

As long as we stick with one stochastic factor, all these models share that the only source of uncertainty comes from a random walk weighted by $\sigma(C,t)$, that is the diffusion term.[1] It seems fairly obvious that a simple diffusion model cannot account for a realistic variety of forces playing a key role during the project's market phase. In particular, neither of these models can properly account for any seasonal component which, for instance, plays a primary role in the outbreaks of the flu, plus neither consider the effect of the business cycle nor any other relevant force. At this point it is worth to wonder whether such models are an oversimplification and which forces do really make an impact in terms of project valuation. Of course there is not one right answer, each project must be analysed in excruciating detail to determine the appropriate set of relevant forces, but it seems fair to conclude that a simple diffusion model is just a naive simplification of the market structure.

In what follows we consider that the net cash flow stream, C_t, of a successful project is given by a latent variable, Y_t, depicted by an Arithmetic Brownian motion process plus a time dependent component described by the Fourier series, that is

$$C_t = f(t) + Y_t \tag{6}$$

$$dY_t = \mu dt + \sigma dW_t \tag{7}$$

$$f(t) = \text{Fourier Series} \tag{8}$$

[1]The option pricing literature is very fructiferous in terms of models with two, or even three stochastic factors, see for instance Chen 1996

where $\{(\mu,\ \sigma) \in \mathbb{R}\}$. Note that, applying Ito's lemma to equation [6] the net cash flow dynamic is depicted by

$$dC_t = \left(\mu + \frac{df}{dt}(t)\right) dt + \sigma dW_t \tag{9}$$

The net cash flow stream takes into consideration the production and marketing cost, in consequence it could yield a negative rate, thus an Arithmetic Brownian motion process is a suitable representation of the underlying process. Under this framework, the solution of the underlying process and the net cash flow at any given time t, under the risk neutral probability $\mathbb{P}^Q$, is represented by

$$Y_t = Y_0 e^{rt} + \sigma \int_0^t e^{r(t-s)} dW_s^Q \tag{10}$$

$$C_t = \Big(C_0 - f(0)\Big) e^{rt} + f(t) + \sigma \int_0^t e^{r(t-s)} dW_s^Q \tag{11}$$

where W_t^Q is a standard Wiener process under the risk-neutral measure $\mathbb{P}^Q$.

In the same fashion as in Schwartz (2004), the cash flow stream starts when the R&D project is market launched, before this stage the process describes the net cash flow that the project would have produced if it were successfully completed. Once the medicine or treatment is market launched the value of the project depends exclusively on the net cash flow generated. Hence, using the Merton (1973) no-arbitrage technique the project value, $V(C_t, t)$, must satisfy the following partial differential equation

$$\frac{\partial V}{\partial t} + rC\frac{\partial V}{\partial C} + \frac{\sigma^2}{2}\frac{\partial^2 V}{\partial C^2} - rV = 0 \tag{12}$$

subject to the appropriate terminal condition $V(C, T)$, where T represents the patent expiration.

The novel component in this model is the *ad hoc* incorporation of the Fourier series, $f(t)$, accounting for any economic, market, and specific force affecting the project and not captured by the underlying stochastic differential equation. In this sense Fourier series should be considered as an aggregate of forces playing a relevant role in the process evolution and determining the cash flow structure. Note that Fourier series provides a great deal of flexibility as, by Carleson's theorem, it converges almost everywhere for a L^2 function. Therefore, $f(t)$ allows us to properly define a scenario where the project will be developed, such scenario is tailor made based on the characteristics of each project, the influence and exposure to certain forces, and so on. On this regard, we might not have a precise *ex-ante* projection of such scenario, for instance, we might know that the business cycle represents a risk factor but we might not know how deeply it affects the cash flow stream. Hence, let us represent the economic uncertainty by the state vector

$$\Phi^{(j)} \text{ with } j \in \mathbb{N} \tag{13}$$

where each state defines a case scenario depicted by a concrete selection of terms in the Fourier expansion and represents the aggregate of forces. It is important to stress out that a state scenario does not attempt to replicate a precise future outcome but rather establishes an alternative future development. Each state determines the cash flow structure of a successful project and consequently the managerial decision of ceasing or continuing the project. Thus, the expected patent value conditional to a certain economic state is given as

$$V\left(t, C_t, I_t; \Phi^{(j)}\right) = E\left[V \mid \Phi^{(j)}\right] \text{ with } j \in \mathbb{N} \tag{14}$$

where C_t and I_t represent the net cash flow structure once the drug obtains the marketing approval and the investment structure during the research and development phase, respectively.

Note that the conditional patent value is constrained to the future development of certain state, which of course is uncertain. Therefore, since Φ is defined as a discrete state vector, an essential piece of the puzzle is the appropriate definition of its mass probability function. On this regard, Huchzermeier and Loch (2001) define a one-dimensional parameter i to model the product performance. The authors claim that this performance may unexpectedly improve with probability p, or it may deteriorate with probability $(1-p)$ and they generalize the binomial distribution by allowing the performance "improvement" and "deterioration" over N performance states. We can easily accommodate a similar probability mass function defining two states in the economic state vector, that is, $j = 1,\ 2$. However, as stated above, each state represents the aggregate of forces acting over the project, and therefore is very project specific, so we will implement a rather Bayesian approach and assign a prior probability to each scenario. Note that each state can be defined in several ways, we can tailor made it based on our own expectations, we can define it based on analyst expectations, and so on. Hence, let us define the state vector probability mass function in general terms as

$$g\left(\Phi^{(j)}\right) = Pr\left(\Phi = \Phi^{(j)}\right) = p_j \tag{15}$$

where p_j represents the probability that the state $\Phi^{(j)}$ turns out real. Hence, under this framework the patent value is determined by

$$\text{Patent Value} = \sum_j V\left(t, C_t, I_t; \Phi^{(j)}\right) \cdot g\left(\Phi^{(j)}\right) \tag{16}$$

4 Stress Test

Economic forces not only affect the number of investment opportunities available in the Pharmaceutical industry but also play a key role in the cash-flow determination of a successful R&D project. Consequently, economic uncertainty represents an essential risk factor affecting Pharmaceutical stock returns. This section is devoted to stress test the impact of some economic forces to the overall project value and also to determine the best timing to launch a project based on the interaction among these forces.

4.1 Economic forces

There is a vast number of economic variables that can potentially affect an R&D project. For the sake of simplicity, in this paper we analyse two major forces, namely, i) the business cycle, defined as the cyclical movement of the GDP around its long-term trend, and ii) the VIX index which is considered as the barometer of investor sentiment and market volatility.

i) *Business cycle*

The first variable under consideration is the interaction with the business cycle, as already stated in the previous section, the business cycle is defined as the cyclical movement of the GDP around its long-term trend, so the first step is to disentangle the cyclical behaviour from the long-term trend. For this endeavour we use a standard Hodrick-Prescott (1997) filter being the most commonly used tool to do so, and we perform an spectral analysis to the cyclical component of the GDP using nonparametric estimates of the population spectrum as in Hamilton (1994). The data set includes 278 quarterly GDP observations ranging from January 1947 to April 2016, obtained from the Federal Reserve Bank of St. Louis web page. Figure 2 presents the cyclical component time series and the corresponding spectra.

[FIGURE 2 AROUND HERE]

The spectral analysis reveal a peak at a frequency of 0.1871Hz, representing a cyclical period of 5.35 years, which is very much line with similar studies (see for instance, Groth et. al).

ii) *Market volatility*

The second variable under consideration is the market volatility, for this matter we use 318 monthly observations of the VIX index from January 1990 to June 2016 downloaded from the CBOE web page. We apply the same procedure as for the GDP time series, that is, a Hodrick-Prescott filter to disentangle the long-term from the cyclical component, and then we perform a

spectral analysis using nonparametric estimates of the population spectrum. Figure 3 presents the cyclical component time series and the corresponding spectra.

[FIGURE 3 AROUND HERE]

This spectra reveal two dominating peaks with period of 1.4 and 3.8 years. Interestingly, in contrast to GDP time series, the VIX long-term component also presents a cyclical behaviour but with a much longer period. Figure 4 presents the long-term fluctuation and the corresponding spectra.

[FIGURE 4 AROUND HERE]

Indeed, we can observe a peak at a rather short frequency (0.0755Hz) representing a cycle of 13.25 years.

Note that the parameters included in the each term of the Fourier expansion and defining the behaviour of the economic force, and hence of each factor, are the frequency (f) and the phase (ϕ) parameter. On the other hand, the amplitude parameter defines the intensity of such force or cycle over the net cash flow stream, indeed a project dependent parameter.

4.2 Stress test Analysis

Launching a project involves a great deal of decision making, first we have to evaluate whether the project represents a valid investment opportunity, for that matter we have to consider the embedded abandon options and technical failure. Secondly and equally important, we have to identify the optimal timing to launch the project. Launching the project under certain economic conditions might impact the project value dramatically, it might lead us to reject an attractive project just for choosing a poorly timing. Note that timing the launching rises an option to delay. The objective of the stress test analysis is to determine the optimal launching time and the impact of the option to delay over the project valuation. Hence, we will shift the phase parameter to determine the proportional change if the project is launched at different phases of each cycle under consideration, for instance, we will examine the impact of launching the project at the peak or the bottom of the business cycle. Note that we do not intent to price an specific project but rather to identify the optimal launching time, hence we will use average market parameters for a generic R&D project as described below.

Let's assume that the research team has already identified a compound which may potentially be used to engineer a new medication. At this stage the Board has to face the first abandon option, that

is, they have to decide whether this project constitutes a valid investment opportunity and apply for a patent protection or drop it before going any farther into the development phase. But also they have to decide whether to immediately launch the project or postpone it, for this purpose we are going to stress test the launching date and stablish the optimal timing considering the different stages in the business cycle and the market volatility. For the sake of simplicity, let us assume that there is no uncertainty about the time and cost to completion if the project successfully overcomes every stage in the development process. Note that most of the investment cost is spent to develop the drug and it can also be modelled stochastically, see for instance Schwartz (2004). However, we prefer to keep the numerical example as simple as possible and focus the attention on timing the project kick-off rather than development issues, although a stochastic process for the cost and time to completion could be easily implemented. According to the "Tufts Center for the Study of Drug Development" (see DiMasi et al. 2014), the total out-of-pocket cost per approved new compound is about 1.400 Millions (in 2013 $). Based on this information, Table 1 summarises the representative out-of-pocket investment cost and schedule by year.

[TABLE 1 AROUND HERE]

4.2.1 Technical uncertainty

In the previous section we have established that during the development phase the project can either fail or be abandoned. Technical risk accounts for the probability of a failure event due to a technical or technological reason within the development phase, and we have generalized the Poisson distribution allowing for the probability of success and technical failure. According to the "2015 biopharmaceutical research industry profile" report, provided by PhRMA, the average time to develop a drug is about 10 years and the percentage of drugs entering clinical trials resulting in an approved medicine is less than 12 %. Hence, assuming that only 12 % of such projects successfully overcome every stage in the development phase and a development period of 10 years, the annual rate of failure is given as

$$e^{-10\cdot\lambda} = 0.12 \tag{17}$$

$$\lambda = 0.2120 \tag{18}$$

We have also assumed that the outcome of a failure is a worthless project, hence, during the development process the discount factor is given by $e^{-r_d t} = e^{-(r+0.2120)t}$, where r represents the risk-free rate.

4.2.2 Economic and market uncertainty

We have determined that the net cash flow stream from sales revenues, marketing and production cost starts when the medication gets the marketing approval and it is launched, which is expected to occur on period 10. Let us assume that the patent will be granted in 4 years right after the application and for a limited period of 20 years. When the patent expires market competition forces sales to virtually zero, meaning that based on the schedule the company can only benefit from this project for 14 years starting at market launch. This assumption generates the boundary condition, $V(T) = 0$, on equation [12], where T represents the patent expiration. In addition, we consider that the initial cash flow parameter, C_0, in equation [11] is 100 millions, while the process volatility, σ, is fixed at 20 millions.

Regarding the economic variables, in previous section we have identified the characteristic of each economic variable, hence fixing the amplitude parameter at 10, we have the following process in the Fourier component

$$f(t) = 10\left\{\cos\left(1.1753 \cdot t + \phi_1\right) + \cos\left(1.6597 \cdot t + \phi_2\right) + \cos\left(4.5049 \cdot t + \phi_3\right) + \cos\left(0.4742 \cdot t + \phi_4\right)\right\} \tag{19}$$

where each ϕ_i; $i = 1, 2, 3, 4$ defines the phase factor. The usual benchmark for the risk-free rate is the treasury constant maturity provided by the Federal Reserve, however since the beginning of the financial crisis the US treasury yield is close to zero. Hence, we will use a risk-free rate of 1.5% although the current value is much lower.

4.3 Implementation

Having defined and calibrated all the input parameters, we are ready to compute the value of this project. By assumption, we consider that the underlying process, C_t, defines the monthly net cash flow stream. Then, we simulate 100.000 paths considering a time increment of $\Delta t = 1/12$, that is monthly increment. The discrete cash flow at any time t is given by equation [11]. Once the marketing approval is granted, the marketing and production cost are accounted into the net cash flow process. Therefore, discounting all the discrete cash flows up to market launch and summing them up could yield an aggregated negative value, for that reason it is considered an abandon option at market launch although there is no further investment in developing the drug. Note that the probability of an aggregated negative cash flow at market launch is the consequence of considering an Arithmetic Brownian Motion process plus the impact of the Fourier component over such process, therefore such probability tends to decrease as the economic state improves. Accordingly, at market launch the abandon option is given by

$$V\left(t_{ML}, C_t, I_t; \Phi^{(j)}\right) = \text{Max}\left\{\sum_{t=t_{ML}}^{T} C_t \cdot e^{-r(t-t_{ML})}\ , 0\right\} \tag{20}$$

where t_{ML} and T represent the market launch and patent expiration time, respectively.

The exercise time for the subsequent abandon options is defined on yearly basis and the option is evaluated conditional on not having been abandoned before, therefore the time increment during the development phase is give by $\Delta t^* = 1$. The backward procedure consists on discounting[2] the project value to the exercise time and evaluating the optimal abandon option, that is

$$V\left(t, C_t, I_t; \Phi^{(j)}\right) = \text{Max}\left\{V\left(t + \Delta t^*, C_{t+\Delta t^*}, I_{t+\Delta t^*}; \Phi^{(j)}\right) \cdot e^{-(r+\lambda)\Delta t^*} - I_t\ , 0\right\} \tag{21}$$

The procedure continues rolling back up to the present time for those paths that are not optimally abandoned on previous interactions.

4.4 Business Cycle stress test

In this section we analyse the impact of launching the project on different phases of the business cycle. In more detail, we study the project evolution when launching i) at the peak of the cycle and entering into recession, that is $\phi_1 = 0$, ii) at the trough of the cycle and entering into the recovery phase, that is $\phi_1 = \pi$ and iii) when launching at an intermediate phase, $\phi_1 = \pi/2$

Considering 100.000 path simulations and following the above mentioned procedure, the expected patent value conditional to each phase in the business cycle is given as in Table 2

[TABLE 2 AROUND HERE]

We can clearly see that timing the project launch has a dramatic impact over the patent value. Launching the project at the peak of the business cycle and entering into a recession phase yields a much lower expected value, roughly speaking 22% lower than launching at the trough phase and 11% than launching at an intermediate phase. But timing the project kick-off not only affects the overall project value, it also affects the value of the embedded abandon option. Table 2 panel B shows the project value when the abandon option is not considered, indeed, we can observe that proportionally the abandon option has a higher value when the project is launched at the peak of the phase. Table 3 disaggregates by state and period the number of paths optimally abandoned, that is the number of abandon options exercised. We have already stated that the first exercise date is at market launch. Since the net cash flow stream takes into consideration not only the sales revenues but also the production and marketing cost this variable can, and indeed does, become negative for some paths.

[2]Note that during the development phase the discount factor is given by $e^{-r_d t} = e^{-(r+\lambda)t}$, where λ represents the annual rate of failure and can be considered as a technical or technological risk premium.

Hence, it may be optimally exercised the abandon option although there is no further investment in developing the drug at market launch. As expected the number of optimally abandoned paths is significantly higher when the phase factor is $\phi_1 = 0$, making the abandon option considerable more valuable when the project is launched entering into a recession state.

[TABLE 3 AROUND HERE]

4.5 Market volatility stress test

In this section we study the effect of market volatility over an R&D project. In the previous section we proxied market volatility using the VIX index and found that there two short- to medium-term cycles of 1.4 and 3.8 years and a long-term cycle of 13.25 years. In order to simplified the analysis we concentrate the study on the effect of the long-term cycle.

Similarly as with the Business cycle, we study the project evolution when launching i) at the peak of the volatility cycle, that is $\phi_1 = 0$, ii) at the trough of the cycle, that is $\phi_1 = \pi$ and iii) when launching at an intermediate phase, $\phi_1 = \pi/2$.

Considering 100.000 path simulations the expected patent value conditional to each phase in the volatility cycle is given as in Table 4

[TABLE 4 AROUND HERE]

We observe a similar behaviour as for the business cycle analysis, launching the project at the peak of the volatility cycle yields a lower patent value, roughly speaking 20% lower than launching at the trough phase and 11% than launching at an intermediate phase. Table 5 disaggregates by state and period the number of paths optimally abandoned conditional to each phase value. Indeed, we observe a higher abandon rate when the project is launched at the peak of the cycle.

[TABLE 5 AROUND HERE]

The stress test is quite revealing, we observe that for both cycles (business cycle of 5.35 years and long-term volatility cycle of 13.25 years) the optimal strategy is launching the project at the trough of each cycle. By launching at this point we achieve the higher expected patent value and the lowest abandon rate. However, synchronizing both cycles might not be possible and we still have to decide the optimal launching time conditional on certain economic conditions. Figure 5 presents all possible different combinations of the each phase and the corresponding patent value. As already stated, the best possible combination is launching the project when both the business and the volatility cycle phase parameter is equal to π, that is at the trough of both cycles. However, synchronizing both cycles might not be a possibility and we have to decide the best available combination.

[FIGURE 5 AROUND HERE]

5 Concluding remarks

In this paper we have developed a novel valuation model and methodology to value a pharmaceutical R&D project based on real options approach. The posited model takes into account the interaction of market and economic forces, and the effect of these risk factors in terms of asset pricing. In order to incorporate these risk factors and account for economic risk, we have incorporated a novel economic state vector where each economic state captures the interaction among different market and economic forces using Fourier series as the particular basis for the economic function space. In this sense, Fourier series allows us to properly define an economic scenario where the project will be developed and it is considered as an aggregate of forces playing a relevant role in the process evolution and determining the cash flow structure. On this regards, Fourier series is a powerful mathematical instrument which allows us to define the economic state scenario as much sophisticated as we want increasing the number of forces affecting the evolution of the project. In fact, Fourier series provides a great deal of flexibility as, by Carleson's theorem, it converges almost everywhere for a L^2 function.

In Section 4 we first have analysed two main economic forces, namely the GDP and the VIX index. Using a Hodrick-Prescott filter we have disentangled the long-term from the cyclical component of each variable, and then we have performed a spectral analysis using nonparametric estimates of the population spectrum. Our findings reveal a cyclical component of 5.35 years for the GDP variable, which is very much line with similar studies and represents the business cycle effect. On the other hand, the VIX index reveals two dominating cyclical periods of 1.4 and 3.8 years, respectively, in addition with a rather medium-term period of 13.25 years. Finally, we have performed a stress test analysis to determine whether the project represents a valid investment opportunity to identify the optimal launching time under certain economic conditions, indeed, poorly timing might lead us to reject an attractive project.

The model and methodology presented in this paper constitute a powerful and yet simple valuation instrument with strong practical applications. As stated above, the pharmaceutical industry is extremely complex and competitive and most companies have to choose with financial wisdom each project. There are several forces, both economic and technical, driving the drug development process that are not fully understood. On this regards, our proposed model tackles all those forces playing a significant role in the project valuation process in a very simple manner and provides a comprehensive tool for the decision making process. The model and methodology here proposed can be easily extrapolated to any other industry or corporate project.

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Appendix of tables

Development phase schedule

Development stage	Preclinical testing		Clinical Phase I		Clinical Phase II		Clinical Phase III			Regulatory review
Period	0	1	2	3	4	5	6	7	8	9
Investment	60	60	82	82	196	196	174	174	174	11

Table 1: This table presents the work schedule and budget for the whole development process including the regulatory approval.

Conditional Expected Patent Value. Business cycle

Business Cycle Phase	Panel	
	A	B
$V(t, C_t, I_t; \phi_1 = 0)$	1027.9 (3.3)	785.5 (4.4)
$V(t, C_t, I_t; \phi_1 = \pi)$	1324.2 (3.4)	1169.9 (4.4)
$V(t, C_t, I_t; \phi_1 = \pi/2)$	1157.0 (3.5)	959.05 (4.4)

Table 2: This table presents the patent value conditional to the phase parameter in the Business Cycle.
Panel A: With abandon option
Panel B: Without abandon option

Abandon rate. Business cycle

Development stage	Preclinical testing		Clinical Phase I		Clinical Phase II		Clinical Phase III			Regulatory review	Market launch
Period	0	1	2	3	4	5	6	7	8	9	10
Investment	60	60	82	82	196	196	174	174	174	11	0
$\phi_1 = 0$	1473	1193	1214	953	1697	1374	895	703	537	30	18645
	28714	27241	26048	24834	23881	22184	20810	19915	19212	18675	18645
$\phi_1 = \pi$	1171	958	992	711	1324	1014	724	533	412	22	12099
	19960	18789	17831	16839	16128	14804	13790	13066	12533	12121	12099
$\phi_1 = \pi/2$	1345	1089	1072	839	1592	1202	776	641	442	22	15394
	24414	23069	21980	20908	20069	18477	17275	16499	15858	15416	15394

Table 3: This table presents the number of optimally abandoned projects out of 100.000 path simulations. In light gray the disaggregated by state and period number of paths optimally abandoned. In gray the aggregated by period number of paths optimally abandoned.

Conditional Expected Patent Value. Volatility cycle

Volatility Cycle Phase	Panel	
	A	B
$V(t, C_t, I_t; \phi_1 = 0)$	1327.6 (3.6)	1173.0 (4.4)
$V(t, C_t, I_t; \phi_1 = \pi)$	1651.5 (3.9)	1568.2 (4.4)
$V(t, C_t, I_t; \phi_1 = \pi/2)$	1495.4 (3.8)	1369.0 (4.4)

Table 4: This table presents the patent value conditional to the phase parameter in the Volatility Cycle.
Panel A: With abandon option
Panel B: Without abandon option

Abandon rate. Volatility cycle

Development stage	Preclinical testing		Clinical Phase I		Clinical Phase II		Clinical Phase III			Regulatory review	Market launch
Period	0	1	2	3	4	5	6	7	8	9	10
Investment	60	60	82	82	196	196	174	174	174	11	0
$\phi_1 = 0$	1174	872	933	665	1336	1006	665	536	360	22	12100
	19669	18495	17623	16690	16025	14689	13683	13018	12482	12122	12100
$\phi_1 = \pi$	877	681	719	551	1002	735	482	374	297	9	7403
	13130	12253	11572	10853	10302	9300	8565	8083	7709	7412	7403
$\phi_1 = \pi/2$	1038	768	839	597	1151	859	579	480	313	25	9523
	16172	15134	14366	13527	12930	11779	10920	10341	9861	9548	9523

Table 5: This table presents the number of optimally abandoned projects out of 100.000 path simulations. In light gray the disaggregated by state and period number of paths optimally abandoned. In gray the aggregated by period number of paths optimally abandoned.

Appendix of figures

Figure 1: This figure presents a general pharmaceutical process for the development of a new drug

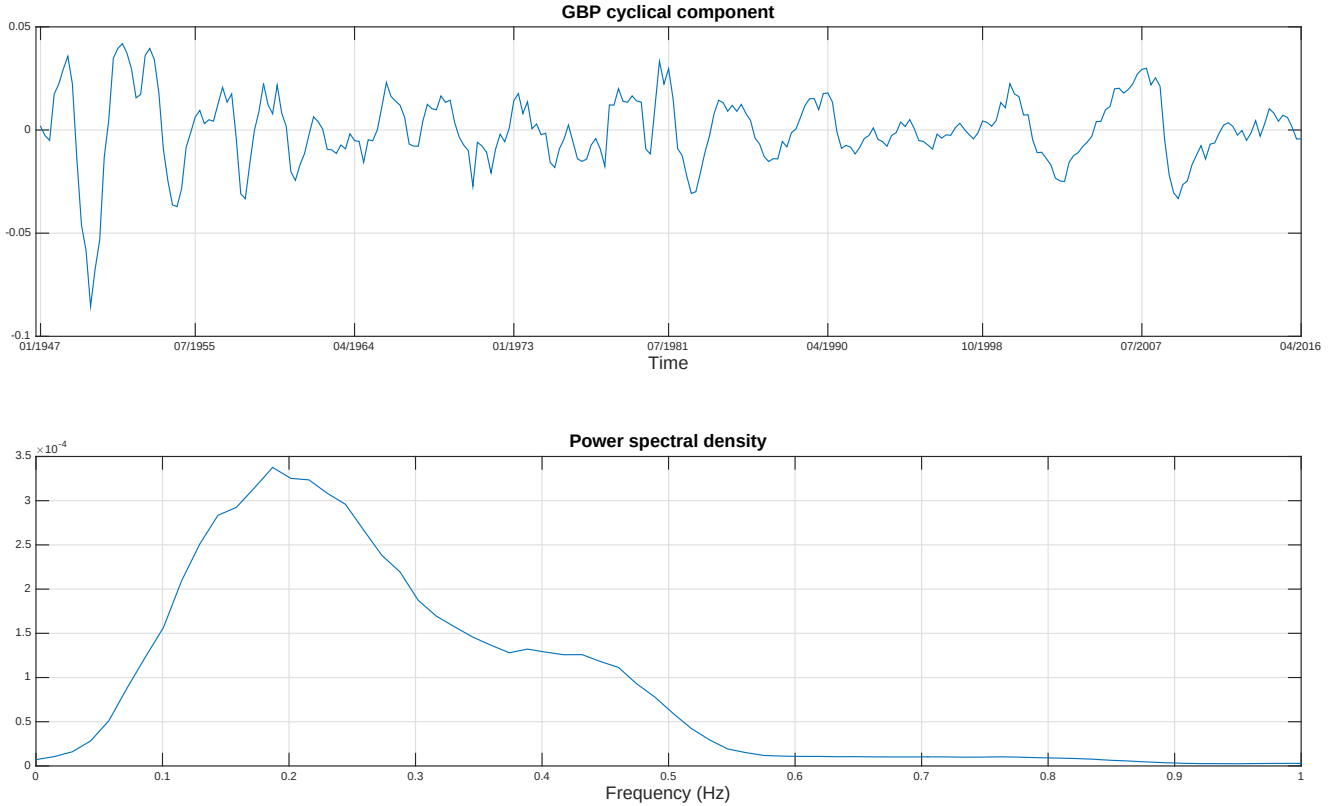

Figure 2: This figure presents the cyclical component of the GDP and its power spectral density.

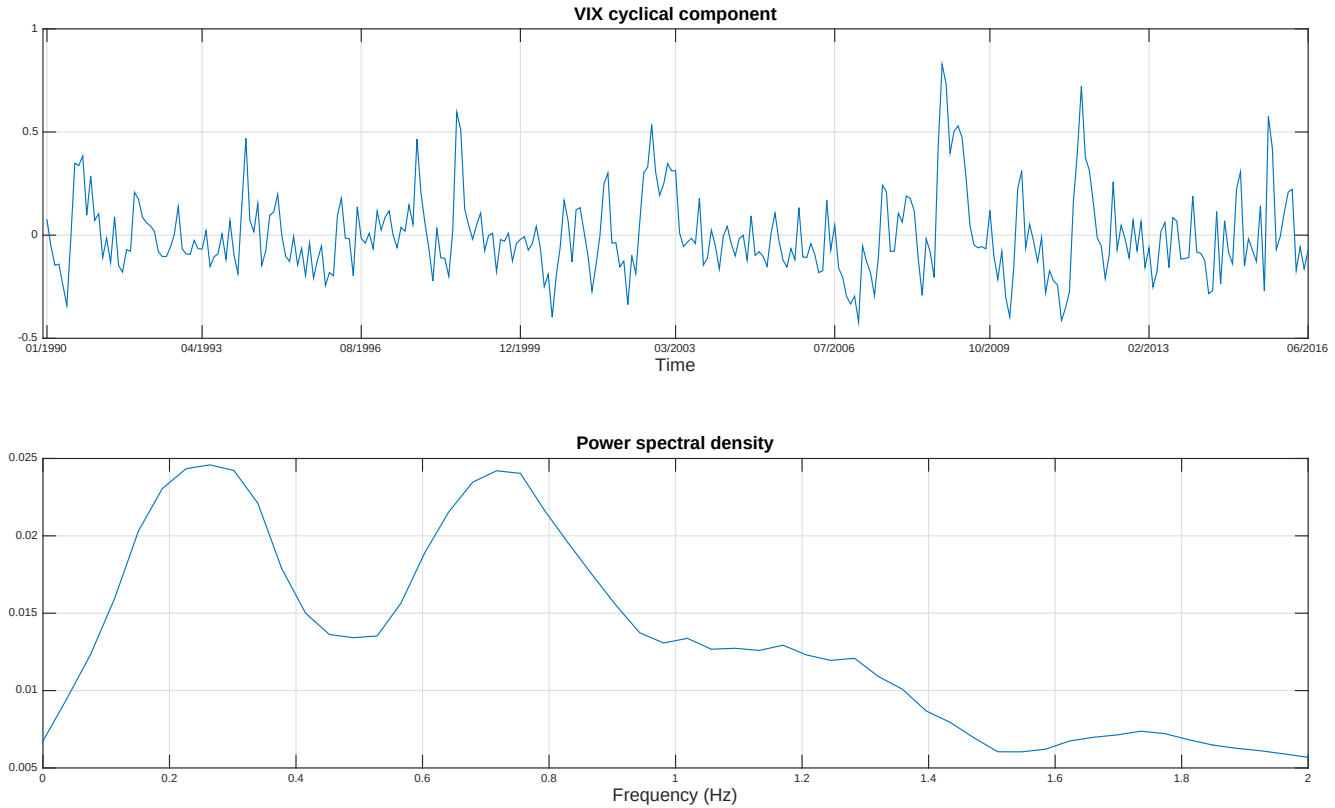

Figure 3: This figure presents the cyclical component of the VIX index and its power spectral density.

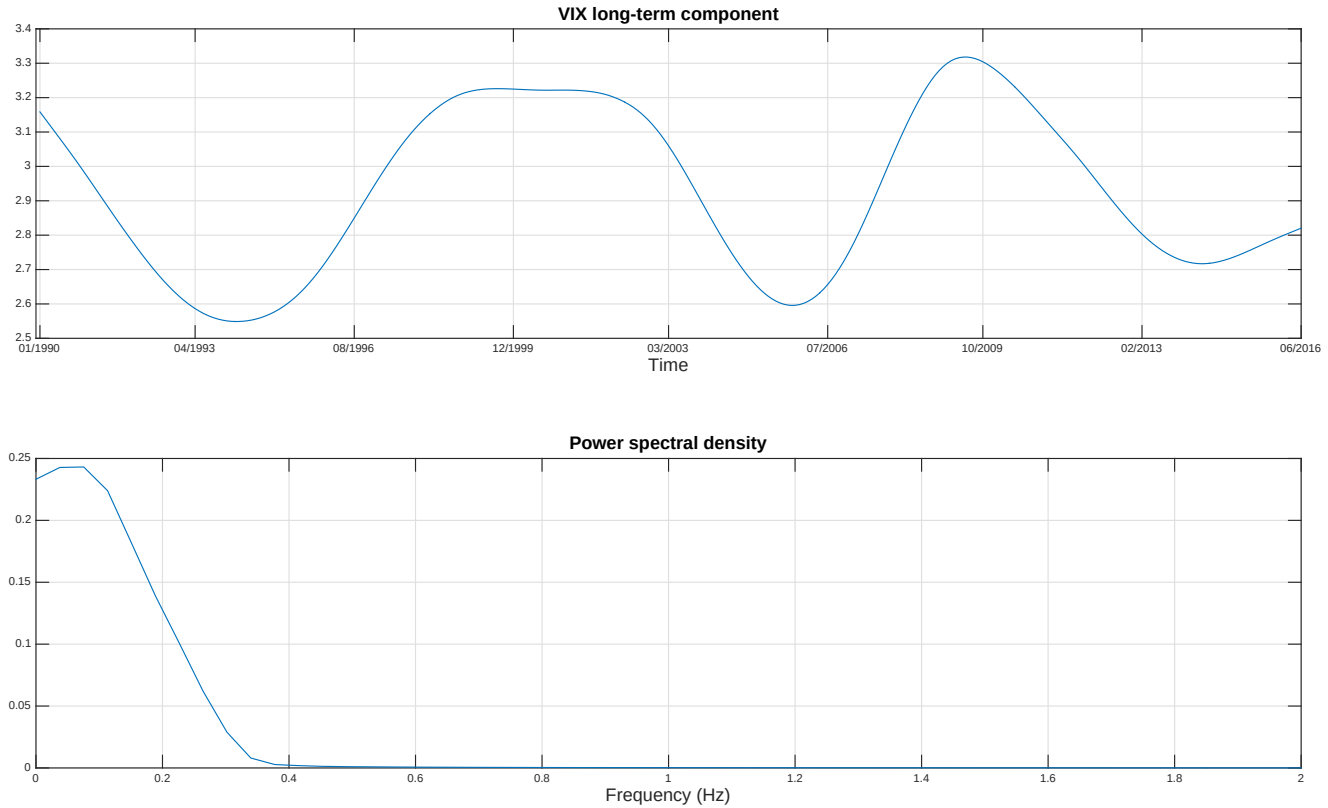

Figure 4: This figure presents the long-term fluctuation of the VIX index time series and its power spectral density.

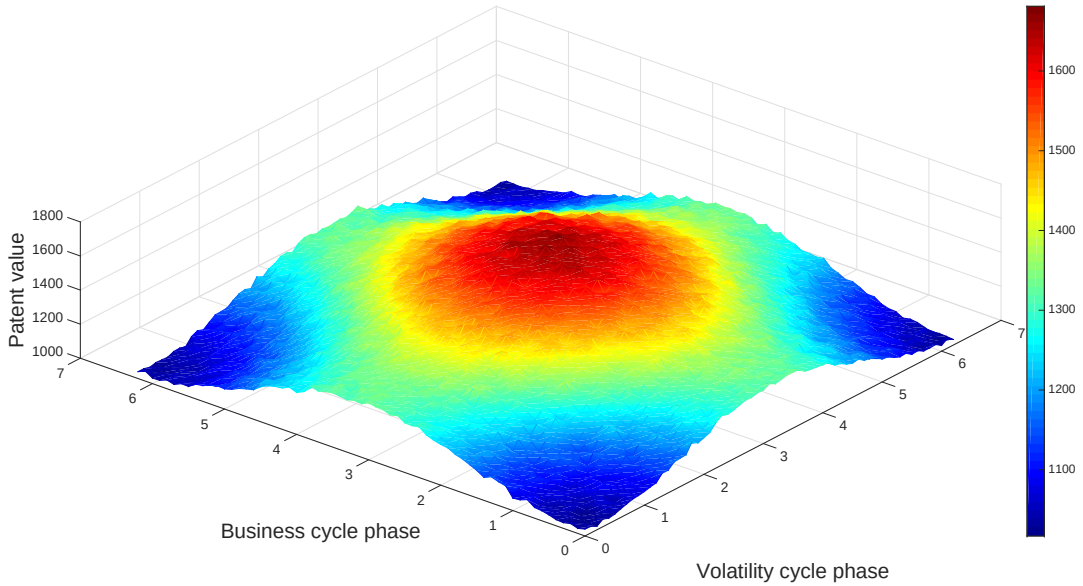

Figure 5: This figure presents the conditional patent value sensitivity considering different combination in the business and volatility cycle parameter