Lead-lag relationship between spot and futures stock indexes: Intraday data and Regime Switching Models

1. Introduction

A key question in finance is the lead-lag relationship between spot and futures markets. Understanding information flow across markets, in addition to being of academic interest, is important for asset valuation, hedging, investment strategies and economic policy. A better comprehension of the information transmission provides investors with more efficient trading strategies (Kawaller et al., 1987). Although in past decades, a significant strand of both theoretical and empirical research has focused on the study of the dynamic relationship between futures and spot prices, the conclusions obtained remain ambiguous because the empirical evidence diverges across articles.

According to the efficient market theory, the price of an asset reflects all relevant information available about its intrinsic value. Numerous articles have focused on the study of deviations from the Cost of Carry model and have investigated the linkages between futures and spot prices (Kawaller et al., 1987; Ng, 1987; Stoll and Whaley, 1990 and Chan, 1992, among others). In an efficient market, there will be a simultaneously perfect relationship between spot index and index future contract price changes. Therefore, innovations would be synchronously reflected both in spot and futures prices, and there should be no lead-lag relationship between prices in the two markets. Notwithstanding, due to market imperfections, such as asymmetric information, transaction costs, liquidity and other market restrictions, one market may reflect information faster than the other one, and as a result of that, a lead-lag relationship exists. Hence, price discovery may be considered an indicator of the market efficiency (Tse, 1999).

The main goal and major contribution of this investigation includes analysing the lead-lag relationship between the futures and spot markets of the DAX30 considering three aspects, given that, as will be explained below, they are considered essential. These aspects include a) high frequency data (on a five-minute interval basis); b) regime switching, which will allow us to consider arbitrage opportunities changes, linking these opportunities of arbitrage to the magnitude of long run disequilibrium between spot and futures prices given by the error correction term (ECT); and c) regime dependent impulse response analysis, which has been implemented to deepen the understanding of how markets react to shocks in high ECT regimes (more arbitrage opportunities) and low ECT regimes (less arbitrage opportunities).

The connection between the ECT and arbitrage is not recent. In fact, arbitrage has been described as an attempt to benefit from the long run trading opportunities involved in the cointegration relationship (Bodarenko, 2003; Hogan et al., 2004)[1]. Moreover, as Kawaller et

[1] The ECT may be interpreted as an opportunity of arbitrage. When the futures price is higher than the spot price, arbitragers will buy in the spot market and sell in the futures market. If the futures price is lower, they will do the

al. (1987) highlight, *"the lead-lag relationship during periods when arbitrage activity is present might reasonably be expected to differ from the lead-lag relationships present when no arbitrage activity occurs*". Considering this approach, this study intends to determine whether there exists an asymmetric adjustment process between spot and futures prices depending on the magnitude of the deviation from the long run equilibrium and which market (spot or future) has more predictive capability. Previous research has documented that the presence of arbitrage opportunities has a noteworthy effect on the dynamics of the price discovery process and faster adjustment is expected when deviations are large enough to make arbitrage advantageous, that is to say, arbitrage is associated with more rapid convergence of the basis to the cost of carry (Dwyer et al., 1996 and Theissen, 2012). Consistent with the above outcome, this study finds that the nature of the price discovery process depends on the presence of regimes in the ECT and that in the German market; the leading role of the futures market in the price discovery process is noticeably pronounced when arbitrage opportunities are greater.

The traditional Vector Error Correction Model (VECM) implies that the speed of adjustment of prices towards long-run equilibrium does not depend on the size of the deviation. To overcome this restraint, a state-dependent error correction model is estimated. Not considering that disequilibrium of different magnitudes in the ECT and hence that arbitrage opportunities provokes asymmetric responses in these markets might lead us to misspecified models and misleading conclusions regarding the market leadership or even conclude that these markets are not cointegrated.

A relevant issue that should be considered when studying empirically the lead-lag relationship between two markets or assets, and thus, the presence of arbitrage opportunities, is the frequency of the analysed data. Numerous empirical studies suggest that the lead-lag relationship is an important stylized fact at high frequency data; nevertheless, it vanishes when the frequency of observations decreases (Huth and Abergel, 2014). Harris et al. (1995) show evidence that frequency is crucial to testing pricing dynamics between markets that are cointegrated due to the following two reasons: 1) if the time interval is too wide, it might provoke the error correction to occur inside an interval that had not been considered, so higher frequency trading strategies might not be detected considering daily prices; and 2) cointegration models allow establishing long-term relationships between temporal series that may diverge in very short periods but readjust to the long run equilibrium. Therefore, the use of data on a high frequency basis can disclose new facts that cannot be detected at lower frequencies; for this reason, it is considered to be the most convenient way to approach this investigation. In this paper, data on a five-minute interval basis of transaction prices from January 2, 2014 to September 30, 2015 for the DAX30 index for both the stock index and index futures are used.[2]

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reverse. This trading forces prices back towards equilibrium in such a way that at the time of futures contract expiration, the ECT becomes zero.

[2] As Andersen (2000) highlights, "*the 5-minute frequency is about the highest at which properties of the return series are not seriously distorted by irregular quoting, the discreteness of prices, and the tendency of foreign-exchange dealers to position their quotes with a view toward inventory control".*

Moreover, an increasing empirical strand of the literature suggests that the dynamic relationship between futures and spot prices may be characterized by a nonlinear specification, and failing to consider this might lead to biased results (Brooks, 1996; Hiesh, 1991). Regime Switching Models (RSM hereafter) have the ability to adequately characterize and capture unusual movements that appear in the relationship between spot and futures markets. These models can capture these changes of behaviour and the fact that the new dynamics of prices and fundamentals persist for several periods (Sarno and Valente, 2000[3]). Ang and Timmermann (2012) determined three reasons why RSM have become popular in financial modelling, as follows. 1) The idea of regime shifts is natural and intuitive. When this methodology is implemented in financial series, regimes determined by econometric methods are often identified with different periods in regulation, policy, and other secular changes (Hamilton, 1989). 2) RSM can capture the stylized behaviour of many financial return series. Finally, 3) they can exhibit nonlinear dynamics of asset returns in a framework based on linear specifications. Furthermore, to achieve more complete comprehension of differences between states, the regime dependent impulse response function is implemented in this study. The authors perform this analysis in periods of low and high ECT and examine whether a shock of the same magnitude has similar responses in states with different arbitrage opportunities.

Although high frequency data have been used in many previous studies, to the best of the authors´ knowledge, there is no study that has analysed the contribution of spot and futures markets to the price discovery process on a high frequency interval basis while using a) the nonlinear equilibrium correction model based on an extension of Markovian regime shifts in time series proposed by Hamilton (1988, 1989) to relax the restrictive linear assumption in the deviation from the long run equilibrium, and b) the regime dependent impulse response function to trace out how a shock affects prices when arbitrage opportunities differ.

The main results of our paper related to linear models (traditional VECM) might be summarized as follows: a) in the short run, the relationship is unidirectional from the futures to the spot; and b) the ECT is not significant, so consequently it may be concluded that both markets are not cointegrated and do not respond to deviations from the equilibrium relationship or the existence of arbitrage opportunities, and therefore, there would be no long run relationship binding the markets together. Likewise, nonlinear models (VECM-MS) indicate the following. a) The relationship is bidirectional in the short run, so that there is a two-way feedback relationship between both markets, although the futures market leads the spot market. b) The ECT becomes significant for both markets and regimes. c) In the long run, the futures market also leads the stock market, and moreover, the greater the ECT is, the faster the speed of adjustment is. Finally, d) the regime dependent impulse response function reveals the noticeable asymmetries across regimes when a shock hits the system, so the impact of unexpected shocks on prices is more pronounced when there are more arbitrage opportunities. In this regard, we can state that the results obtained and the conclusions that

[3]They find strong evidence against the hypothesis of linear dynamics and in favor of the capability of regime-switching-vector-equilibrium models (nonlinear models) to capture properly the time-series properties of the data.

can be drawn certainly underpin the importance of considering nonlinear models VECM-MS vs. the linear traditional VECM.

The remainder of this paper is organized as follows: section 2 contains a review of the literature; section 3 describes the data used, including some summary statistics; section 4 explains the methodology employed; section 5 presents the empirical results from the traditional VECM and VECM-MS; section 6 addresses the regime dependent impulse response function; and finally, section 7 summarizes the results and concludes.

2. Review of the literature

The lead-lag relationship between price movements of stock index futures and the underlying cash market describes how fast one market reflects innovation relative to the other one as well as the linkage between them. When one market responds faster to new information and the other market reacts later, a lead-lag relationship is observed (Chan, 1992). A significant body of the literature has attempted to determine whether price discovery occurs primarily in the spot or futures market. Some empirical studies support the leading role of the spot market. Frino, Walter and West (2000) note that traders with stock specific information will benefit more by trading in the spot market than in the futures market; therefore, the spot market might reflect innovation better. There are other articles that conclude that both markets contribute to price discovery, but the conclusion drawn from previous research has mostly been supportive of the price discovery role of the futures prices. The reason why the futures market is the main source of market-wide information is generally explained by its inherent low transaction costs, higher leverage effect and lack of short sales restrictions (Tse, 1999).

Kawaller et al. (1987) examine the intraday price relationship between the S&P 500 Index and S&P 500 Futures Index using minute-to-minute data for all trading days during 1984 and 1985 and conclude that the futures market leads the stock market by over twenty to forty-five minutes; however, movements in the spot market rarely affect futures by more than one minute.

Ng (1987) used the S&P 500 Index and S&P 500 Futures Index daily data for approximately 5 years and concludes that futures prices lead spot prices by one day, although the magnitude of the lead coefficients is rather weak. It has not been detected as the lead for spot prices.

Stoll and Whaley (1990) examine the time series properties of 5-minute intraday returns for approximately 5 years of stock index and stock index futures contracts and conclude that on the one hand, S&P500 Index and MM index futures returns tend to lead stock market returns over five minutes, on average, but sometimes 10 minutes or more. On the other hand, lagged stock index returns have a moderate predictive impact on futures returns, so the effect is bidirectional, but futures market has more predictive capability.

Chan, Chan and Karolyi (1991) studied, simultaneously, the intraday relationship between returns and returns volatility (utilizing the GARCH models) in the S&P 500 stock index and stock index futures market from 1984 to 1989. Each day, trading hours are partitioned into five-minute intervals. Their evidence is consistent with the hypothesis that both markets contribute to price discovery.

Chan (1992) studies the lead-lag relationship between intraday futures and cash index prices on a five-minute interval basis for two sample periods, August 1984–June 1985 and January 1987–September 1987. The article analyses data on the MMI and an index comprising 20 actively traded stocks. The author finds strong evidence that there is an asymmetric lead-lag relationship between the two markets with strong evidence that the futures index leads the cash index and weak evidence that the cash index leads the futures.

Engle and Granger (1987) demonstrate that cointegrated series have an error correction term[4] representation (also named speed of adjustment coefficient) that allows correcting in one period the disequilibrium detected in the previous one. Not only does the error correction term indicate the percentage of disequilibrium from one period that is corrected in the next period, but it also shows the relative magnitude of adjustments in both markets towards equilibrium. The rationale behind the concept of cointegration is that two variables may deviate in the short run from each other, but market forces will bring them back together, and therefore, there exists a long run equilibrium relationship between these two variables. If the error correction term is not considered, then the model could be misspecified. Roughly, spot and futures prices are cointegrated with an order of one, and the linear VECM has been traditionally used to investigate the error correction process between spot and futures prices.

Wahab and Lashgari (1993) extend the study of the lead-lag relationship by applying the cointegration approach to investigate the robustness of previous studies including an alternative model parameterization, the error correction model. They use daily closing spot and futures prices for both the S&P 500 Index and the Financial Times Index from January 4, 1988 to May 30, 1992. The authors find evidence that a two-way relationship exists between the cash and futures markets, which is consistent with the important price discovery role served by both the stock and index futures markets and confirm the hypothesis that both markets contribute to price discovery. Furthermore, they find that the price leadership of the spot market is stronger.

Tse (1999) examines the intraday price discovery process and volatility spillovers between the DJIA futures and index using minute-by-minute data for the six-month period of November 1997 to April 1998. The author uses the VECM to analyse the price discovery process and concludes that the informational contribution attributable to the futures market is 88,3% implying that DJIA futures dominate the cash market in price discovery. It is the spot

[4]One interpretation of the error correction term is that it reflects the effect of arbitrage. If the futures price is too low compared to the index value, arbitragers will sell the stocks underlying the index and buy the futures contract. On the contrary, if the futures price is too high, they will sell the futures contract and buy the stocks underlying the index.

price that makes the greater adjustment to re-establish the equilibrium, or, to put it another way, the futures price leads the cash price in price discovery.

Pardo and Climent (2000) and Blanco (2003) study the temporal relationship between the IBEX 35 Index and IBEX 35 Futures Contracts applying a cointegration parameterization and using minute-by-minute data for the entire year 1996 and five minute data from January 11, 1995 to October 27, 1995, respectively. They conclude that both markets contribute to price discovery, but there exists strong evidence that the predictive capability of the futures market is greater.

Several publications have appeared in the past decades documenting a new approach to answer the longstanding question in the research field regarding the lead-lag relationship between spot and futures prices. Plenty of articles, such as Sarno and Valente (2000), Li (2009) and Theissen (2012), among others suggest the presence of different regimes in financial markets and employ nonlinear models to account for the dynamic of the relationship between spot and futures prices.

Sarno and Valente (2000) examine the dynamic relationship between spot and futures prices in stock index futures markets using weekly data for the S&P 500 and the FTSE 100 indices from January 1, 1988 to December 26, 1997 and using nonlinear Markov-switching vector equilibrium correction models that allow for three regimes in the mean of the equilibrium correction model as well as in the variance-covariance matrix. They find strong evidence against the hypothesis of linear dynamics and in favour of the capability of regime-switching-vector-equilibrium models to capture the stylized behaviour of the financial series.

Li (2009) studied the dynamics of the relationship between spot and futures markets of three mature markets (S&P500, FTSE100, DAX) and two emerging markets (BOVESPA, BSI) from the period April 3, 1995 to December 12, 2005 using daily data. The author uses a traditional VECM and a Markov-switching vector error correction model (VECM-MS), in which the parameter of the deviation of spot-futures prices changes according to the stage of the volatility regime, and compares the results. When a conventional VECM is used to examine the spot-futures price discovery process, the conclusions among markets are inconsistent. However, when a VECM-MS is considered, the findings show the following. 1) During a high variance state, the spot-futures disequilibrium adjustment process depends mainly on the futures market and on the spot market in the low variance state; that is to say, the futures price leads the spot price in price discovery during stable periods; on the contrary, during volatile periods, the price discovery occurs in the spot market. This finding is robust for all markets. 2) The scale of price adjustment in the futures market during a high variance state is greater than that in the spot markets during a low variance state. 3) The correlation between the spot and futures markets for the high variance state is lower than that for the low variance state in all cases. The study also provides evidence that the price adjustment process between spot and futures markets occurs very quickly in mature markets; additionally, for emerging markets, the disequilibrium between spot and futures prices takes longer to diminish. This fact denotes why the deviation in the spot and futures prices in the two

emerging markets analysed is remarkably greater in absolute value than that in the three mature markets, particularly in the high variance state.

Theissen (2012) examines the intraday price discovery process of the following two data sets: 1) DAX index values from the spot equity market and DAX index futures data from the first quarter of 1999 at a frequency of 15 seconds and 2) DAX EX (the most liquid DAX ETF) and DAX index futures data from the last quarter of 2010 at a frequency of 1 minute. The datasets contain transaction prices, bid and ask quotes for 61 trading days. The author estimates a threshold error correction model to allow for arbitrage opportunities to have an impact on the return dynamics including two dummy variables to identify the arbitrage opportunities that require selling in both the spot and futures market. Bidirectional Granger causality is found, and the evidence shows that the spot market depends on the futures market much more than the converse. Consequently, the futures market leads in the process of price discovery, and furthermore, the presence of arbitrage opportunities has a strong impact on the dynamics of the price discovery process. Therefore, the results suggest that the leading role of the futures market in the price discovery process is noteworthy when arbitrage signals are present.

3. Data and preliminary analysis

This study uses high-frequency observations on a five-minute interval basis of transaction prices from DAX30 for both the stock index and index futures. The sample period extends from January 2, 2014 to September 30, 2015, and only data for the period of simultaneous operation of both markets are used in our analysis.

Additionally, it is frequent in the literature to exclude some observations at the beginning of each trading day. When the negotiation in the spot markets begins, volatility reaches its highest level; therefore, in this study, the first return of the trading day, 09:05 hour, which generally reflects the adjustment to information accumulated overnight and displays the highest average return variability, is deleted to avoid deleterious effects on the econometric analysis (see Andersen et al., 2000 and Lee and Mathur, 1999).

After cleaning the data[5], we obtain the continuously compounded returns at each 5-minute interval by taking the logarithms and subtracting the previous value. Returns at the n interval at day t, for $n = 1,2 \dots N$ and $t = 1,2, \dots T$ can then be calculated as follows:

$$R_{n.t} = 100\ x\ (log\ P_{n,t} \div log\ P_{n,t-1})$$

where, $P_{n,t}$ represents the spot ($S_{n,t}$) and futures ($F_{n,t}$) price level on interval n, at day t.

[5]This leave us with a sample of 438 days with 103 intraday five-minute returns per day.

Table 1 presents some statistical tests on a five-minute interval basis for the prices and the returns series used. Some results are remarkable. As can be appreciated in panel A, returns are clearly not normally distributed due to the asymmetric and leptokurtic patterns of the series, and mean returns are close to zero. Leptokurtosis may be considered to be a measure of the fatness of the tails of distribution with more extreme movements than would be predicted by a normal distribution. Panel B shows the Ljung-Box test statistic that detects autocorrelation for both prices and returns.

Panels C and D exhibit stationary and cointegration tests. Spot and futures prices are nonstationary; however, returns series are found to be stationary. Thus, the study reveals that both series contain a unit root and are integrated of order one. Additionally, when analysing cointegration, contradictory results are encountered. Residuals Z_t from the regression $S_t = \beta + \gamma F_t + Z_t$ are tested for the presence of unit root using Augmented Dickey Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPPS) tests. According to ADF test, the residuals from this regression (ECT) are stationary, that is to say, the ECT is I (0), and as a consequence of that, the spot and futures markets are cointegrated; nevertheless, the result of the KPPS test is just the opposite; it reveals that the ECT is not stationary, and therefore, these markets are not cointegrated.

The cointegration analysis computes the long run equilibrium relationship between two series; thus, a large dataset is needed to test for cointegration. Nonetheless, the main disadvantage is that the longer is the dataset analysed, the more likely the series include structural changes.

[INSERT TABLE 1]

Perron (1989, 1990) found that failure to allow for these structural changes in the data series leads to bias results in the ADF test. Additionally, Lee et al. (1997) showed that the stationarity test KPSS proposed by Kwiatkowski et al. (1992) is biased towards rejecting null hypothesis of stationarity repeatedly when the data generating process is stationary with a structural break. In other words, the power of unit root tests is sensitive to structural breaks in the data, and neglecting the presence of different regimes may lead to distorted values in the stationarity and cointegration tests (Bartley et al., 2001).

To gain more insight into this issue, the ECT for the German market is depicted in Plot 1.

[INSERT PLOT 1]

As can be appreciated in Plot 1, different regimes seem to be detected in the ECT. On the basis of this evidence, a state-dependent error correction model will be considered in our empirical study when comparing with the traditional VECM.

4. Methodology

This section explains the empirical models used in this study. First we describe the linear model, and then, we consider a nonlinear dynamic in the ECT. If two variables are cointegrated, they can be represented by a VECM that incorporates the last period error term as well as lagged returns of each variable. Thus, temporal causality may be evaluated by analysing the statistical significance and relative magnitude of lagged variables coefficients and ECT coefficients.

4.1 Traditional VECM

The first approach to analyse the price discovery process is the traditional Vector Error Correction Model (VECM), which assumes a permanent causal relationship between the spot and futures prices over the sample period.

$$\Delta S_t = a_s + \sum_{i=1}^{k} b_{si}\Delta S_{t-i} + \sum_{i=1}^{k} c_{si}\Delta F_{t-i} + \alpha_s Z_{t-1} + \varepsilon_{s,t} \qquad (1)$$

$$\Delta F_t = a_f + \sum_{i=1}^{k} b_{fi}\Delta S_{t-i} + \sum_{i=1}^{k} c_{fi}\Delta F_{t-i} + \alpha_f Z_{t-1} + \varepsilon_{f,t} \qquad (2)$$

In the study of the price discovery process, both the short run and the long run causality will be analysed. The short run relationship is captured by coefficients $b_{si}, c_{si}, b_{fi}, c_{fi}$. When these coefficients are significant, it implies that a lead-lag relationship exists, and therefore, lagged returns in one market might be used to predict futures returns in the other market. Additionally, the long run relationship is represented by the error correction term $Z_{t-1} = S_{t-1} - \beta - \gamma F_{t-1}$, where S_{t-i} and F_{t-i} are log lagged spot and futures prices, respectively.

The error correction coefficients α_s, α_f collect information regarding the direction of the casual relationship between two series and show the speed with which the departures from equilibrium are corrected in the short run. Cointegrating variables may deviate from their relationship in the short run, but in the long run, their association will return. If a departure from equilibrium arises, prices in one or both markets should adjust to correct the deviation; otherwise, the series would wander apart without bound.

Additionally, a_s and a_f represent the unconditional return, and $\varepsilon_{s,t}$ and $\varepsilon_{f,t}$ are the residuals in the spot and futures equations, respectively. The optimal lag length will be determined using the Akaike information criteria (AIC) and the Bayesian information criteria (BIC).

4.2 VECM-MS

Recent years have witnessed a remarkable increase in the popularity of nonlinear modelling (Sarno and Valente, 2000 and Ang and Timmermann 2012, among others). Most of the previous studies have usually neglected nonlinearities in their empirical models employing linear specifications. In this section, to account for the dynamic of the ECT, we use a Vector Error Correction Model Markov Switching (VECM-MS) in which the parameter of the long run deviation of spot-futures prices is dependent on 2 regimes[6].The nonlinear dynamic approach presented here implies that the degree and speed of adjustment towards long run equilibrium depends on the size of the deviation. Moreover, the main advantage of the Markov Switching methodology is that it endogenously determines the changes in the dynamic relationship without postulating exogenous structural changes. Thus, instead of conjecturing a known regime in a certain period, its probability in each point of time is estimated based on the information extracted from the sample.

The methodology is basically the result of extending the Markovian regime shifts to time series analysis originally proposed by Hamilton (1988,1989), considering changes in causality as random events governed by an exogenous Markov process. In the VECM-MS used in this study, we parameterize that the ECT comes from a particular causality regime with a certain probability, and it is specified as follows:

$$\Delta S_t = a_s + \sum_{i=1}^{k} b_{si}\Delta S_{t-i} + \sum_{i=1}^{k} c_{si}\Delta F_{t-i} + \alpha_{s,st} Z_{t-1} + \varepsilon_{s,t,st} \qquad (3)$$

$$\Delta F_t = a_f + \sum_{i=1}^{k} b_{fi}\Delta S_{t-i} + \sum_{i=1}^{k} c_{fi}\Delta F_{t-i} + \alpha_{f,st} Z_{t-1} + \varepsilon_{f,t,st} \qquad (4)$$

$$\begin{matrix}\varepsilon_{s,t,st} \\ \varepsilon_{f,t,st}\end{matrix} \mid \psi_{t-1} \sim BN\left(0, H_{t,st}\right) \qquad (5)$$

$$H_{t,st} = \begin{bmatrix} h_{s,s,st} & h_{s,f,st} \\ h_{f,s,st} & h_{f,f,st} \end{bmatrix} \qquad (6)$$

where ψ_{t-1} refers to the information available at time t-1, $H_{t,st}$ is the regime dependent variance-covariance matrix, and St is an unobservable state variable that can take the value 1

[6] To keep the number of parameters tractable, this investigation considers two regimes in the ECT.

and 2; the parameters $\alpha_{s,st}, \alpha_{f,st}$ accompanying the ECT depend on the regime $St = \{1(state\ 1), or\ 2\ (state2)\}$[7].

The VECM-MS is estimated using a two-step maximum likelihood procedure[8]. The error correction term Z_{t-1} is determined in the first step and is the same variable computed in the traditional VECM. The second step consists of the implementation of an expectation-maximization algorithm using maximum likelihood to estimate equations (3) to (6).[9]

5. Empirical findings

This section displays the main empirical results using the models described in the previous section. First, we will estimate the traditional VECM and then the VECM-MS, discussing the main differences afterwards.

5.1 Estimations for linear models

The use of the error correction models enables differentiation between short run and long run deviations from the equilibrium relationship. We initially investigate the short run and long run casual links between stock index futures returns and stock index returns using the model presented in section 4.1 (Traditional VECM). According to AIC/BIC criteria, the optimal lag length is set at 5.[10]

Table 2 represents the main results obtained for the traditional bivariate VECM.

[INSERT TABLE 2]

It is worth noting that short run causality is unidirectional from the futures to the spot market. Note in Table 2 that lagged spot return coefficients ($\boldsymbol{b_{fi}}$) are not significant in the futures equation. Therefore, the short run adjustments underscore the importance of futures prices in the price discovery process for the German market. Additionally, contemporaneous spot returns are affected negatively by lagged spot returns (b_{si}) and positively by lagged futures returns (c_{si}) (the same evidence is found by Theissen, 2012). In the futures market, this pattern is not observed.

[7] These states will be identified as 1=Low State and 2=High State.
[8] See Perlin, M. (2010) MS Regress - The MATLAB Package for Markov Regime Switching Models. Available at SSRN: http://ssrn.com/abstract=1714016.
[9] For more details about the algorithm used for drawing probabilistic inference about whether and when shifts might occur, see Hamilton, 1989.
[10] The same lag length is used in the VECM-MS

As far as the long run relationship is concerned, it is found to be not significant (see significance of alfa parameters $\alpha_{s,}, \alpha_f$ accompanying the ECT in Table 2). None of them is significant; hence, this means that both series are not cointegrated, and they deviate without a bound in the long run. Contradictory results in the unit root test ADF vs. KPPS are hereby confirmed. Because spot and futures prices concerning the same index react to the same information, short run deviations might be possible, but in the long run, spot and futures prices are expected to strike a balance, In this regard, and according to the VECM-MS results, which will be further explained, neglecting structural changes apparently present in the ECT (see Plot 1), may be leading us to inefficient estimations. This finding reinforces the idea that ECT regime switching should be considered; thus, in the next subsection, we postulate the existence of two regimes in the ECT (high and low regimes).

5.2 Estimations for nonlinear models (VECM-MS)

Table 3 presents the parameter estimates of VECM-MS[11]. As can be appreciated, in the spot equation, similar results to the linear estimation are obtained; lagged spot returns (b_{si}) have a negative impact while the impact of lagged futures returns is positive (c_{si}).

However, in the futures equation, the results differ from those in the linear estimation, considering that both lagged spot returns (b_{fi}) and lagged futures returns (c_{fi}) are significant. Thus, after considering regimes in the ECT, a two-way causality is detected in the German market, which means that price innovations in either the cash or futures markets might be able to predict the arrival of new information in the other market and both markets play important price discovery roles. However, note that there is an asymmetric lead-lag relationship between the two markets with strong evidence that the futures index leads the cash index and weak evidence in the opposite direction (see in Table 3 that the magnitude of c_{si} coefficients accompanying lagged futures returns in the spot equation is much greater, in absolute terms, than the lagged spot returns parameters b_{fi} in the futures equation, for instance, c_{s1} = 0,70512, whereas b_{f1} = 0,00213). Therefore, in the short run, the effect is bidirectional, but the futures market has more predictive capability. As one might expect, these findings are consistent with previous empirical studies that reinforce the idea of the leading role of the futures market (Kawaller et al., 1987; Ng, 1987; Stoll and Whaley, 1990; Chan, Chan and Karolyi, 1991; and Chan, 1992, among others).

The main difference between the estimations in the traditional VECM and the VECM-MS is encountered in the analysis of the long run relationship. Contrary to results found in the linear estimation, parameters accompanying the ECT ($\alpha_{s,st}$ and $\alpha_{f,st}$) have become statistically significant, suggesting that spot and futures prices may diverge temporarily but then readjust

[11] For the estimation of the VECM-MS, we use the specification-robust estimator of the variance-covariance matrix suggested by Bollerslev and Wooldridge (1992) to prevent the effect of heteroskedasticity and autocorrelation in the residuals.

to the cointegrated pattern, and any mispricing is driven back to the equilibrium by arbitrage forces; in other words, there exists a long run equilibrium relationship between spot and futures markets. The leading role of the futures market is also corroborated in the long run in both the high and low states. Note that the α parameters measure the speed of disequilibrium correction and the greater is the value of this coefficient (in absolute terms), the more informationally efficient the market is. Hence, it is inferred from Table 3 that the spot price makes the greater adjustment to re-establish the equilibrium, and moreover, this speed of convergence to equilibrium is faster in states with higher ECT than in states with lower ECT. That is to say, the futures market leads the cash market in price discovery, but the dynamic of the adjustment differs when there exist arbitrage opportunities, and arbitrage can be related to faster convergence of the basis to the cost of carry. Note that in Table 3, in the spot return equation parameter accompanying the ECT in the high state ($\alpha_{s,2}$ = 0,020933) is more than four times the magnitude of the parameter accompanying the ECT in the low state ($\alpha_{s,1}$ = 0,004472). These results are, therefore, consistent with those found by Theissen (2012)[12], who documents that the futures market leads in the process of price discovery and highlights that the leading role of the futures market in the price discovery process is especially pronounced when arbitrage opportunities arise.

As far as the expected duration of each regime is concerned, during states with lower ECT, the expected duration of the regime is approximately two hours, whereas it decreases to 25 minutes in states with more arbitrage opportunities (see in Table 3, state duration of 25,52 and 4,9 intervals of 5 minutes for the high and low states). This implies that regime with lower ECT is more persistent than the other one, and it takes a longer time to reach the new equilibrium.

To sum up, our findings reveal the importance of considering the magnitude of the deviation from the long run equilibrium; otherwise, estimations may be biased. Linear models might be misspecified if there are structural changes in the sample analysed and they are not considered.

[INSERT TABLE 3]

6. Regime dependent impulse response function

Since the valuable contribution of Sims (1980), the dynamic interaction between the variables and the disturbances in vector autoregressive models (VARs) has been widely explained by the impulse response or "error shock" methodology. Impulse response functions (IRFs) are

[12] This author analyses 2 datasets with frequencies of 15 seconds and 1 minute (each dataset comprises 61 trading days for the German stock market) and uses a threshold error correction model (TECM) to account for different magnitudes of the ECT. To do this, arbitrage signals are defined exogenously.

considered to be useful tools to study the effect of a shock on the variables in the model throughout time.

The analysis of IRFs in linear models has been extensively implemented; however, the study of nonlinear cases has been less covered. As Gallant et al. (1993) highlight, it is crucial to expand research of time series to nonlinear models. In this regard, these authors extend the analysis of IRFs to the nonlinear case and emphasize the importance of using these models to capture dynamics. Considering that characteristics of nonlinear models are different across regimes, the IRF will depend on the magnitude of the shock and the time that it occurs (Koop, 1996; Koop et al., 1996). Therefore, to gain more insight into the idiosyncrasy of each regime in the VECM-MS, we determine how the system will respond following a shock[13] when arbitrage opportunities differ.

To obtain the regime dependent impulse response function, we follow a two-stage procedure. First, we convert the VECM-MS back to a VAR[14] model, and then, the resulting VAR model is used to perform the regime dependent IRF (RDIRF)[15].

The RDIRF may be defined as follows:

$$\theta_{k,st,h} = \frac{\partial \mathrm{E}_t P_{t+h}}{\partial u_{k,t}} \mid S_t \qquad for\ h \geq 0 \ (7)$$

$$S_t = \{1 = low\ regime, 2 = high\ regime\}$$

where $u_{k,t}$ is the structural shock to the k-th variable, P_{t+h} are the spot (S_{t+h}) or futures (F_{t+h}) prices at time t+h[16] and $\theta_{k,st,h}$ is a k-dimensional response vector dependent on the regime S_t.

Given that two regimes are present in the ECT, our general model contains 4 regime dependent impulse response functions, which include spot and futures market IRFs in low and high regimes.

[INSERT PLOT 2]

Plot 2 illustrates the RDIRF for an unexpected shock in states with lower (subplot 2.1) and higher ECT (subplot 2.2). A noteworthy result is that the shock implies a temporary and permanent effect on prices, but note that the impact of a shock of one standard deviation magnitude has a greater reaction on regimes with higher ECT. As can be appreciated in subplot 2.1, after the shock hits the system in states with lower ECT, prices increase approximately 0,07%, whereas in regimes with more arbitrage opportunities (see subplot

[13] For each variable, a shock of one standard deviation magnitude is applied to the residuals.

[14] The VECM(5) is converted into a VAR(6); then, we can express the VAR(6) as an infinite moving average model.

[15] The Generalized impulse response function by Pesaran and Shin (1998) is used.

[16] The dynamic response for each variable is traced out over a period of 30 intervals of five minutes (two and one half hours).

2.2), prices increase between 0,15 and 0,16%. Moreover, the effect of the shock stabilizes after 40-45 minutes (8-9 periods), inducing a permanent increase in prices on both regimes, which rise to reach their new long run equilibrium level. Moreover, when a shock is applied to the futures prices, the market responses are more pronounced than when a shock is introduced into the spot market. The findings thus obtained in the impulse response analysis are coherent with the results of VECM-MS. As is illustrated in Plot 2, both futures and spot prices react to unexpected shocks in the spot and futures markets, which suggests that there exists bidirectional interaction between the DAX30 index and the DAX30 index futures; however, note that the response to a shock in futures prices is relatively larger than the response to a shock in spot prices (see that solid lines are above the dashed lines), indicating that the futures market leads the spot market and reinforcing the idea of the leading role of the futures markets in the price discovery process.

Therefore, summing up, the most interesting results of the RDIRF analysis are as follows: a) the response to a shock in futures prices is relatively larger than the response to a shock in spot prices; b) shocks induce a permanent increase in prices, which rise to reach their new long run equilibrium level; and c) the dynamic causal effect is remarkably different in low/high regimes, so as the arbitrage opportunities increase, the impact of unexpected shocks on prices increases.

These findings, therefore, support the importance of the RDIRFs to confirm causal relationships between spot and futures prices from VECM-MS and capture asymmetries in both regimes.

7. Conclusions

In this paper, we investigate the lead-lag relationship between the DAX30 stock index and DAX30 index futures. The development of high frequency databases has boosted interest in empirical market microstructure and provides great potential to improve our understanding of financial markets (Goodhart and O'Hara, 1997). In addition, increasingly growing empirical research has stressed the existence of relevant nonlinearities in both spot and futures returns (Sarno and Valente, 2000; Li, 2009 and Theissen, 2012 among others). This study contributes to the existing literature by using high frequency data and nonlinear models based on an extension of Markovian regime shifts to overcome the weakness of linear assumptions in the dynamic relationship between spot and futures prices widely used in previous studies. These nonlinear models allow us to consider the presence of different regimes in the deviation from the long run equilibrium.

From the outcome of our investigation, it is possible to conclude that linear models might be misspecified when structural changes are present in the data and they are neglected. Therefore, it is important to capture the different regimes detected in the ECT to provide a more suitable empirical model and a better understanding of the information transmission so that market participants can define more efficient trading strategies.

In the short run, the results support the hypothesis that innovations propagate in the futures market before that in the spot market; however, pronounced two-way causality is detected in the estimation of the VECM-MS suggesting bilateral interaction in the price discovery process. These results are in good agreement with previous empirical studies that reinforce the idea of the leading role of the futures market (Kawaller et al., 1987; Ng, 1987; Stoll and Whaley, 1990; Chan , Chan and Karolyi, 1991; and Chan, 1992, among others). However, when analysing the cointegration relationship, the results are completely different depending on the model used to make the estimations.

In the long run analysis, according to the estimations of the traditional VECM, spot and futures prices do not follow a common long term trend. Nevertheless, in the VECM-MS, parameters accompanying the time varying ECT suggest that spot and futures prices may diverge temporarily but both readjust to the cointegrated pattern, although it is the spot price that makes the greater adjustment to re-establish the equilibrium. Moreover, spot price adjustment accelerates in states with higher ECT, revealing faster convergence of the basis when arbitrage opportunities arise.

Finally, results from the impulse response analysis reinforce the idea that the dynamic causal effect is remarkably different in regimes with different arbitrage opportunities in such a way that as the arbitrage opportunities increase, the impact of unexpected shocks on prices increases.

Overall, these results underpin the importance of considering regimes present in the error correction term and the perils of strong linear assumptions when analysing the lead-lag relationship.

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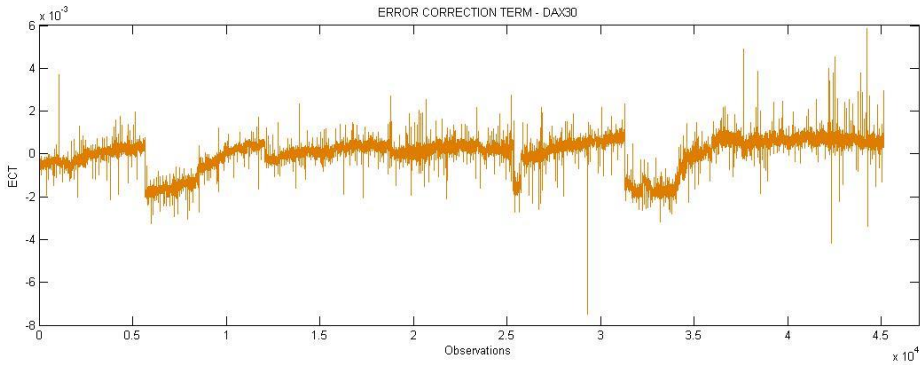
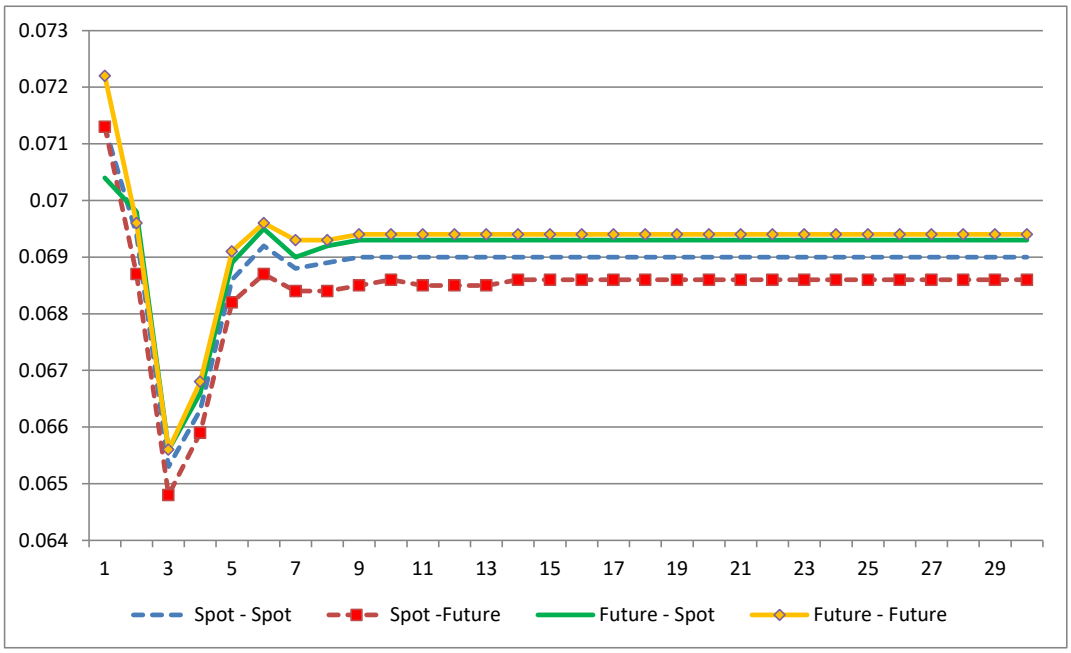
PLOT 1: Error correction term



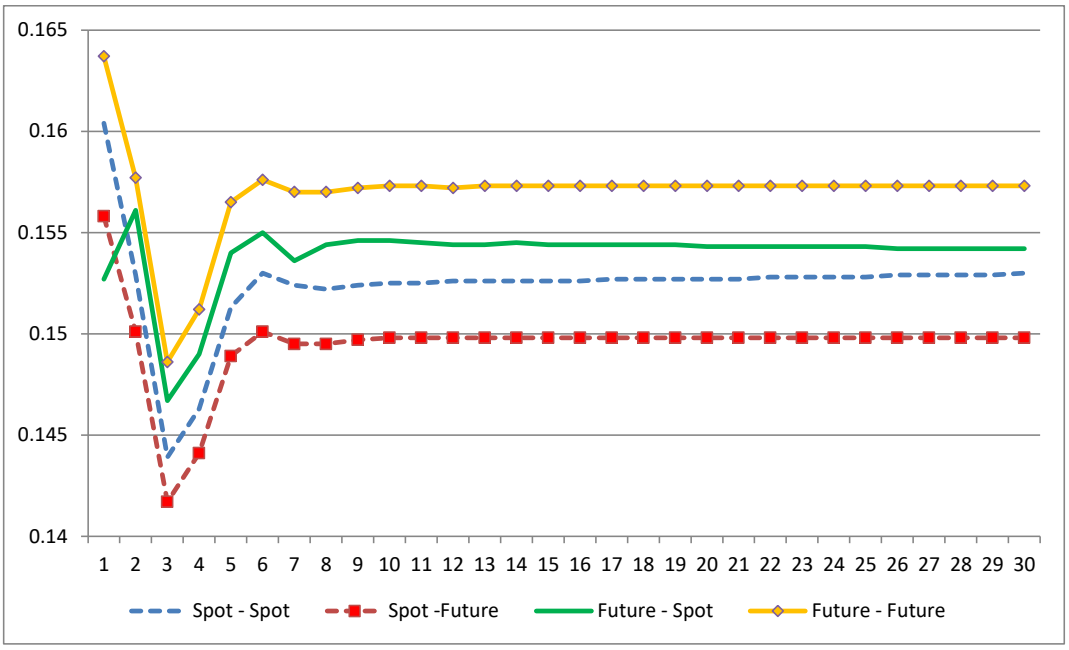
Plot 1 shows the error correction term for the DAX30 from January 2, 2014, to September 30, 2015. The horizontal axis represents the observations on a five interval basis, and the vertical axis measures the ECT.

PLOT 2: Regimen Dependent Impulse Response Function

Response in states with lower ECT (subplot 2.1)



Response in states with higher ECT (subplot 2.2)



Plot 2 gives the impulse responses to a one standard deviation shock in states with lower (subplot 2.1) and higher ECT (subplot 2.2). The horizontal axis represents the period in intervals of five minutes. The vertical axis represents the magnitude of the shock expressed as a percentage of the price increase. The solid line and the solid line with markers denote the impulse response of the spot and futures markets to a shock in the futures market, respectively. In addition, the dashed line and the dashed line with markers represent the impulse response of the spot and futures markets to a shock in the spot market, respectively. All impulses are based on the Generalized Impulse Response Function by Pesaran and Shin (1998).

TABLE 1: Statistics results

	Levels		Returns	
	Spot	**Futures**	**Spot**	**Futures**
Panel A: Summary Statistics				
Mean	10,195.49	10,201.05	-0.00034	-0.00031
Standard Deviation	927.45	930.61	0.099	0.100
Skewness	0.66	0.67	-0.389	-0.367
Kurtosis	2.12	2.15	16.78	17.021
Minimum	8,363.08	8,367.00	-2.100	-2.205
Maximum	12,385	12,422	1.245	1.292
Jarque Bera	4,741	4,746	358,054	370,532
Observations	45,115	45,115	45,114	45,114
Panel B: Autocorrelation test				
LB - $Q(20)$	900,772.62	900,775.64	98.41	114.15
LB - $Q^2(20)$	896,284.85	896,409.88	15,126.42	14,631.81
Panel C: Stationary test				
ADF(H0:Not stationary)	-0.0365	-0.0494	-215.4262*	-216.9964*
KPSS(H0:stationary)	347.51*	348.13*	0.1434	0.1336
Panel D: Cointegration test				
ADF(H0:Not stationary)	-38.537*			
KPSS (H0:stationary)	103.654*			

The ***** denotes the significance at 0.05.

Table 1 shows some statistical tests for DAX30 index prices and returns on a five-minute interval basis for both the spot and futures market from January 2, 2014, to September 30, 2015. Returns at the n interval at day t have been calculated as follows: $R_{n.t} = 100\ x\ (log\ P_{n,t} \div log\ P_{n,t-1})$, where $P_{n,t}$represents the price level on interval n, at day t. Panel A presents the main summary statistics and the Jarque-Bera test for normality. Panel B displays the results of the serial autocorrelation test Ljung-Box using 20 lags. Panel C performs the Augmented-Dickey Fuller and Kwiatkowski-Phillips-Schmidt-Shin stationary tests, and panel D shows the results of the cointegration tests (checking whether the error correction term is stationary).

TABLE 2: Parameter estimates of the linear VECM

		SPOT RETURN EQUATION	FUTURES RETURN EQUATION
Intercept $\boldsymbol{a_s}, \boldsymbol{a_f}$		-0.00037	-0.00033
ECT $\boldsymbol{\alpha_s}, \boldsymbol{\alpha_f}$		-0.011786	0.00222
Lagged Spot Return Coefficient	1	-0.551384*	0.00559
$\boldsymbol{b_{si}}, \boldsymbol{b_{fi}}$	2	-0.300798*	0.02479
	3	-0.182988*	0.00462
	4	-0.13084*	-0.03498
	5	-0.06758*	-0.02765
Lagged Futures Return Coefficient	1	0.54140*	-0.02712
$\boldsymbol{c_{si}}, \boldsymbol{c_{fi}}$	2	0.28443*	-0.04650
	3	0.20317*	0.01171
	4	0.14388*	0.04631
	5	0.07046*	0.02782

The ***** denotes significance at 0.05.

Table 2 presents the results of the linear VECM

$$\Delta S_t = a_s + \sum_{i=1}^{k} b_{si}\Delta S_{t-i} + \sum_{i=1}^{k} c_{si}\Delta F_{t-i} + \alpha_s Z_{t-1} + \varepsilon_{s,t} \qquad \text{Spot return equation}$$
$$\Delta F_t = a_f + \sum_{i=1}^{k} b_{fi}\Delta S_{t-i} + \sum_{i=1}^{k} c_{fi}\Delta F_{t-i} + \alpha_f Z_{t-1} + \varepsilon_{f,t} \qquad \text{Futures return equation}$$

where $\boldsymbol{S_{t-i}}$ and $\boldsymbol{F_{t-i}}$ are log lagged spot and futures prices, respectively; Δ is the first-difference lag operator, $\boldsymbol{a_s}$ and $\boldsymbol{a_f}$ represent the unconditional return; coefficients $b_{si}, c_{si}, b_{fi}, c_{fi}$ capture the short-run relationship; $\boldsymbol{Z_{t-1}}$, computed as $\boldsymbol{Z_{t-1} = S_{t-1} - \beta - \gamma F_{t-1}}$, is the error correction term (ECT); α_s, α_f are the error correction coefficients that collect information regarding the long-run relationship; and $\boldsymbol{\varepsilon_{s,t}}$ and $\boldsymbol{\varepsilon_{f,t}}$ are the residuals in the spot and futures returns equations, respectively. The optimal lag length has been determined using the Akaike information criteria (AIC) and the *Bayesian information* criteria (BIC) and has been set in five lags.

TABLE 3: Parameter estimates of the VECM-MS

Non-switching parameters		
	SPOT RETURN EQUATION	**FUTURES RETURN EQUATION**

Intercept a_s, a_f		-0.00004*	-0.00026*
Lagged Spot Return Coefficients b_{si}, b_{fi}	1	-0.73230*	0.00213
	2	-0.53412*	0.02000*
	3	-0.39150*	0.00601*
	4	-0.26318*	-0.00686*
	5	-0.12683*	0.00244*
Lagged Futures Return Coefficients c_{si}, c_{fi}	1	0.70512*	-0.03853*
	2	0.48213*	-0.07588*
	3	0.00500*	0.35822*
	4	0.29444*	0.03645*
	5	0.14066*	0.00860*
Switching parameters			
State with higher ECT $\alpha_{s,st}, \alpha_{f,st}$		0.020933*	-0.000317*
State with lower ECT $\alpha_{s,st}, \alpha_{f,st}$		0.004472*	0.000403*
Variance - covariance matrix $H_{t,st}$			
State with higher ECT		0.025739	0.024996
		0.024996	0.026798
State with lower ECT		0.005084	0.005085
		0.005085	0.005219
State duration			
State with higher ECT		4.90	
State with lower ECT		25.52	

The * denotes significance at 0.05.

Table 3 presents the results of the VECM - MS

$$\Delta S_t = a_s + \sum_{i=1}^{k} b_{si}\Delta S_{t-i} + \sum_{i=1}^{k} c_{si}\Delta F_{t-i} + \alpha_{s,st} Z_{t-1} + \varepsilon_{s,t,st}$$ Spot return equation

$$\Delta F_t = a_f + \sum_{i=1}^{k} b_{fi}\Delta S_{t-i} + \sum_{i=1}^{k} c_{fi}\Delta F_{t-i} + \alpha_{f,st} Z_{t-1} + \varepsilon_{f,t,st}$$ Futures return equation

$$\begin{matrix}\varepsilon_{s,t,st} \\ \varepsilon_{f,t,st}\end{matrix} | \psi_{t-1} \sim BN(0, H_{t,st})$$

$$H_{t,st} = \begin{bmatrix} h_{s,s,st} & h_{s,f,st} \\ h_{f,s,st} & h_{f,f,st} \end{bmatrix}$$

where $\boldsymbol{S_{t-i}}$ and $\boldsymbol{F_{t-i}}$ are log lagged spot and futures prices, respectively; Δ is the first-difference lag operator; $\boldsymbol{a_s}$ and $\boldsymbol{a_f}$ represent the unconditional return; coefficients $b_{si}, c_{si}, b_{fi}, c_{fi}$ capture the short-run relationship; $\boldsymbol{Z_{t-1}}$, computed as $\boldsymbol{Z_{t-1} = S_{t-1} - \beta - \gamma F_{t-1}}$, is the error correction term; $\alpha_{s,st}, \alpha_{f,st}$ are the parameters accompanying the ECT that depend on the regime $St = \{1 = Low\ State, 2 = High\ State\}$, which collect information regarding the long run relationship; $H_{t,st}$ is the variance-covariance matrix (s=spot market, f=futures market); and $\varepsilon_{s,t,st}$ and $\varepsilon_{f,t,st}$ are the residuals in the spot and futures returns equations, respectively. The same lag length set in the VECM is used in the VECM-MS (five lags).