Asset Pricing with Housing Booms and Disasters*

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Comments/Suggestions are welcome

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Abstract

We introduce a new housing consumption-based asset pricing model (Housing CCAPM) that incorporates rare disaster events into the dynamics of non-housing consumption and rare boom/disaster events into housing consumption. Our analytical framework hinges on two key ideas. First, we extend existing Cumulant Generating Function-based pricing formulas to a two-good economy. Second, we develop an exponential affine approximation for the price-dividend ratio. These techniques enable us to derive intuitive closed form solutions for the risk-free rate, risk premia, the volatility of excess returns and the term structure of interest rates. Using monthly U.S. aggregate consumption data and a maximum likelihood estimation approach, we show that, while rare disaster events in non-housing consumption contribute only marginally to explaining the low risk-free rate, the high equity premium and high equity volatility observed in the data, rare boom/disaster events in housing expenditures solve these puzzles for moderate levels of risk aversion and intratemporal elasticity of substitution. Additionally, we show that the Housing CCAPM framework with CRRA utility will consistently produce theoretical upward-sloping real yield curves, and interpret the standard CCAPM risk-free rate as the rate that would prevail during extreme recessions.

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1 Introduction

The standard consumption-based capital asset pricing model (CCAPM), although a major branch of the asset pricing literature, has a notoriously poor empirical record as it leads to well established counter-factual predictions: a too high risk-free rate, too low and constant risk premia, a too small volatility of asset prices and a flat real yield curve. Many extensions to the model have been provided in the last three decades to help solving the puzzles, with various degrees of success. Among them, two strands of research are of particular interest for this paper, the introduction of rare disasters in the consumption process pioneered by Rietz (1988) and Barro (2006), and the Housing CCAPM proposed by Piazzesi et al. (2007).

In his influential contribution, Barro (2006) revived Rietz (1988)'s seminal work by vindicating the potential positive impact of consumption crashes on the equity risk premium when the probability of a disaster is estimated from cross-country observations. As the distribution of consumption growth is no longer Gaussian but exhibits excess kurtosis and negative skewness, the investors' demand for precautionary savings increases - disasters are rare but extremely harmful - and that for risky assets decreases, which lowers the risk-free rate and raises risk premia. This renewed interest in crashes occurred only a few months before the financial crisis of 2007 – 2008, which in turn triggered the Great Recession. Until then, the mainstream consumption-based asset pricing literature used, as its key macroeconomic variable, aggregate consumption (mainly non-durables and services consumption) in endowment or production economies. Since it is generally agreed that the recent financial and subsequent economic crisis was caused by the real-estate market, it seems natural to investigate the role of housing expenditures, separately from the other components of aggregate consumption, especially during periods of severe economic downturns.

Real estate is a particular asset as it both provides crucial housing services and can serve as collateral for borrowing against future income. There are several channels through which the bursting of the real-estate bubble propagated to the rest of the economy. One is leverage. For instance, Mian et al. (2013) shows that home equity-based borrowing is responsible for the sharp rise of U.S. household leverage during the period 2002 – 2006 which was used for financing non-housing consumption and home improvement. Another is a wealth effect. Mian and Sufi (2014) reports that, due to the sharp decline in real-estate prices in 2006, households drastically reduced their spending either because of a direct net worth effect or through tightened borrowing constraints due to a fall in collateral value. This in turn caused a sharp drop in non-tradable (i.e. local) employment. Also, an increase in the volatility of real estate impacts that of aggregate consumption and consequently the market risk premium. For example, Iacoviello and Neri (2010), analyzing spillovers of the housing market onto the rest of the economy, find that while fluctuations in the housing market represented a mere 4% of consumption growth variance before the financial liberalization of the mortgage market in the 1980s, they accounted for 12% afterwards. Another channel is bank securitization. As documented in Acharya and Richardson (2009), big financial institutions securitized wide pools of mortgages to alleviate capital adequacy requirements. However, instead of selling out these securities, they kept large portions of them on their balance sheets. When the housing bubble popped, their value fell sharply, rendering some of these institutions insolvent. As argued in Hall (2012), this triggered a massive increase in financial frictions and a severe reduction in business and residential investment expenditures.

Therefore, it appears that the real estate sector plays a crucial role in the fluctuations of the rest of the economy and that the dynamics of housing expenditures is driven by mechanisms that markedly differ from non-housing consumption.[1] Yet, the standard CCAPM does not segregate these two goods but instead bundles them up in a single basket, aggregate consumption, whose growth rate is assumed to be i.i.d. and Gaussian. This suggests that a more realistic CCAPM would postulate distinct dynamics for housing and non-housing consumption and take into account the economic agents' relative preferences over these two types of expenditures. Such a model, dubbed Housing CCAPM, has been developed by Piazzesi et al. (2007) in an endowment economy. In that framework, agents aggregate non-housing consumption and housing expenditures through a constant intratemporal elasticity of substitution. In the presence of two goods, they are concerned not only about consumption risk but also about composition risk. The latter is associated with the evolution of

[1]Pakos (2004) and Yogo (2006) also distinguish durable and non-durable consumption goods but their definition of durables is larger than real estate, defined here as the goods that produce housing services as dividends. As noted, housing has played a central role in the building up of the 2007-2008 financial and subsequent economic crises, which motivated us to focus our analysis on it.

the relative share of non-housing consumption over housing expenditures (referred to below as the expenditure ratio). If the intratemporal elasticity of substitution is sufficiently high (making the two goods imperfect substitutes), agents do not only dislike recessions, when non-housing consumption is weak, but they loathe severe recessions when housing expenditures are even lower. This will affect both the risk-free rate and the equity risk premium.

The Housing CCAPM has been derived under Gaussian assumptions. However, as documented in Constantinides and Ghosh (2014), household consumption growth is negatively skewed. Therefore, inspired by Rietz (1988) and Barro (2006), we extend the model by allowing the dynamics of both housing and non-housing consumption to be affected by rare disaster events. Our goal thus is to assess the overall impact of these events, and the relative influence of housing busts and booms and non-housing crashes, on asset prices and volatilities and on the level and slope of the yield curve.

We contribute to the literature in several ways. First, our housing consumption-based asset pricing model allows fat-tailed crashes to affect the dynamics of non-housing expenditures (using the power law) and busts and booms to affect the dynamics of housing services (using the Laplace distribution), thereby nesting both the Housing CCAPM of Piazzesi et al. (2007) where the two types of consumption are Gaussian and Barro (2006)'s rare disaster event framework where the only good is aggregate consumption. The achieved level of generalization is an important step towards understanding the dynamics of asset prices and bond rates.

Second, our analytical framework is partly based on the use of cumulant generating functions (CGF) and extends the work of Martin (2013) to a two-good economy. CGFs constitute a powerful tool for deriving asset pricing results in a non-Gaussian environment in which accounting for distribution moments of high order (possibly far beyond the fourth) is relevant to the representative investor's optimization program. Combined with the assumption that the price-dividend ratio is an exponential affine function of the log expenditure ratio, CGFs enable us to derive compact and intuitive closed-form solutions for the risk-free rate, the risk premium and its volatility, and the term structure of interest rates. These in turn allow us to clearly identify the contribution of rare disaster events to these variables.

Third, while estimating our model, we circumvent the implausible assumption that consumption disasters are similar across markedly different economies. This assumption is pervasive in the extant literature because crashes in aggregate consumption so seldom happen that, when yearly data are used, a pool of international data is needed to enlarge the information set. Instead, we use only U.S. monthly non-housing and housing consumption data over the period January 1959 to November 2015, which yields a workable sample of 682 observations. Furthermore, our sample contains several periods of large economic changes, most notably the dot com and housing booms and busts. Our sample is thus granular enough to contain sufficiently reliable information about the distribution of crashes/booms in both types of consumption. Besides, we derive closed-form solutions for the p.d.f. of non-housing consumption and the expenditure ratio that permit us to perform maximum likelihood estimations (MLE) of these quantities, unlike for instance Barro and Jin (2011) who perform MLE of the (exogenously defined) *crashes* in aggregate consumption.

Fourth, our model predicts levels of the risk-free rate and the equity premia that are uniformly more realistic than those offered in the literature, for a given level of risk aversion. An issue here is that empirical tests crucially rely on the measurement of the intratemporal elasticity of substitution between non-housing and housing consumption. This measurement is made difficult by a substantial misspecification in the definition of housing services, as pointed out by Prescott et al. (1997). Nevertheless, studies by Ogaki and Reinhart (1998) on aggregate durable consumption, by Bajari et al. (2013) on housing expenditures and by Benhabib et al. (1991) and McGrattan et al. (1997) on home production give confidence that the intratemporal elasticity is clearly above 1. This finding is vindicated by our own empirical tests. For such an intratemporal elasticity of substitution, we show that the fear of housing rare disasters amplifies the intertemporal substitution mechanism, thereby producing a lower interest rate and a higher and more volatile risk premium.

However, disaster events impact asset prices variously depending on whether they occur in housing services or in other expenditures. More precisely, we document that the severity of crashes in the distribution of U.S. non-housing consumption is neither as pronounced nor as frequent as international data on aggregate consumption could suggest. Consequently, non-housing disaster events help solving the asset pricing puzzles only marginally. By contrast, housing rare disaster events or, equivalently, crashes/booms in the expenditure ratio, strongly affect asset prices and produce in particular lower interest rates and higher and more volatile risk premia. For example, for a coefficient of relative risk aversion of 3 and an IES of 1.03, housing disaster events imply an annual real risk-free rate of 0.87%, very close to its average level found in the data (0.94%). For the same levels of risk aversion and IES, the model produces an annualized equity risk premium of 5.42%, close to its historical average of 6%. With the same parameters, however, the model predicts an annualized volatility of the risk premium equal to 24.2%, which overshoots its historical mean value of 15.3%. We also check the impact of rare disasters across a variety of risky assets and obtain consistent results: Most of the increase in risk premia is due to crashes in the housing sector. This finding vindicates the relevance of splitting aggregate consumption in its housing and non-housing components.

Finally, we justify on theoretical grounds that the housing consumption framework with CRRA utility tends to imply upward-sloping real yield curves, irrespective of whether rare disaster events are included or not in the dynamics of the two types of consumption. The presence of housing rare disasters (busts and booms) induces steeper yield curves. We also find that when the expenditure ratio decreases, which for instance happens during periods of strong expansion, the term spreads tend to increase because short term bonds are in stronger demand as they provide a better hedge against future severe recessions than long term bonds do. Additionally, we propose a new economic interpretation for the standard CCAPM risk-free rate as the - very high - interest rate that prevails *during* extreme recessions: in these times when marginal utility is large, the risk-free asset is relatively unattractive and the demand for it consequently low. As the economy drifts away from these dire straits, the demand for riskless bonds rallies and the risk-free rate keeps falling.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes our economic framework and the dynamics of consumption in a rare event environment. In Section 4 we derive asset pricing formulas for the risk-free rate and risky assets associated with a general dividend structure. Section 5 presents our data and our estimation procedure and shows numerically how rare events impact the dynamics of consumption and asset prices. In Section 6, we obtain a closed-form solution for the term structure of interest rates and discuss the impact of rare events on its level and slope. The last section concludes. Most proofs are relegated to a mathematical appendix.

2 Related work

Our consumption dynamics are subject to infrequent, sudden and large downward moves. This type of processes, referred to as rare disaster events, has been extensively covered in the important, and more general, strand of research that has attempted to show that departures from normality have a significant impact on asset returns. Influential examples include the seminal work of Rietz (1988) and Barro (2006) and, more recently, Barro (2009), Branger et al. (2016), Gabaix (2012), Martin (2013), Wachter (2013) and Lewis and Liu (2016), among many others.

Since rare disasters seldom happen by definition, they are notoriously difficult to estimate. That is why most of the literature on this topic relies on a multi-country consumption data set methodology designed originally by Barro and Ursua (2008). Modeling *aggregate* consumption growth as a single exponential crash model, Barro and Jin (2011) for instance use consumption from 24 countries and a multi-year peak-to-trough methodology to estimate the probability of crash (3.8%) and the tail exponent of the crash distribution (6.27). Their approach requires a subjective choice of a minimum crash size (10%). With these parameters, a risk aversion coefficient of 4.5 and a subjective discount factor of 0.94, the model is able to generate a premium for the consumption claim of 6% and a risk-free rate of 0.94%. However, their methodology poses several theoretical problems. First, the assumption that consumption distribution parameters are similar across a wide variety of countries with strongly heterogeneous economic developments is hard to justify. Second, as commented by Constantinides on a previous paper by Barro and Ursua (2008), the technique of using multi-year peak-to-trough variations and deducing from them a yearly crash size tends to exaggerate the effect of disasters on the equity premium. Third, the authors run a maximum likelihood estimation on the crash component of the consumption innovation. This implies that they are able to disentangle the Gaussian component from the crash element, which is virtually impossible. By contrast, our estimation approach is based on monthly

U.S. consumption data only, which implies a more realistic crash distribution, and apply the maximum likelihood estimation to the whole distribution, building on the closed-form expression for the consumption growth p.d.f. we derive.

Recently, several consumption-based models have been proposed which nest that of Barro (2009). On the one hand, Gourio (2012), Wachter (2013) and Lewis and Liu (2016) allow for time-varying disaster probabilities. The last two papers emphasize the importance of disentangling disasters common to all countries included in their international sample from country-specific disasters. Gabaix (2012) also introduces time-variation by assuming that the severity of disasters is stochastic. On the other hand, Branger et al. (2012) and Barro and Jin (2016) build general models which include long-run risk à la Bansal and Yaron (2004) and rare disaster events and assess the contribution of each component to solving asset pricing puzzles. Both papers show that rare disaster events mostly explain the equity premium puzzle while long-run risk contributes only moderately. This motivated our choice not to include the latter in the model. Our results rather stress the crucial importance of rare disaster events in the dynamics of *housing* consumption. All these models help solving the pricing puzzles with various degrees of success.

Julliard and Ghosh (2012), using a non-parametric maximum likelihood method applied to a finite sample of potential U.S. consumption crashes, is a singular instance where the rare disaster event hypothesis is empirically rejected. Yet, their approach only considers moderately fat-tailed rare event distributions since they implicitly assign a zero probability to crashes that are lower than the worst event present in their data. Thus, rare disaster events may still play a important role in explaining the equity premium puzzle because they are akin to a Peso problem, as explained in Danthine and Donaldson (1999). In this respect, the single power law distribution for crashes used in Barro and Jin (2011) illustrates that some distributions with truly pronounced fat tails are able to generate a substantial equity premium, even with moderate risk aversion. This induced us to adopt the power law distribution to model our rare disaster events for non-housing consumption.[2]

As noted by Piazzesi and Schneider (2016), since the early 2000s a rapidly growing body

[2]For a detailed account on the use of this distribution in financial economics, see Gabaix (2009).

of new research assigns a specific and important role for housing in economics.[3] In particular, one strand of literature provides an in-depth analysis of why and how housing, as both a consumption good an a collateralizable asset, influences savings and portfolio decisions and thereby the stochastic discount factor and asset prices. The Housing CCAPM has been derived by Piazzesi et al. (2007) with the insight that introducing housing generates, in addition to the standard consumption risk, a composition risk that justifies a risk premium of its own. We extend their model by allowing crashes to affect non-housing expenditures and busts and booms (to conform to actual observations) to affect housing services. This in particular allows us to generate volatile enough risky asset prices with reasonable risk aversion levels, unlike Gaussian rational expectations models with housing. The way our model induces time-variation in the pricing kernel resembles the approach taken by Campbell and Cochrane (1999)'s external habit formation model. The latter specifies the dynamics of a persistent consumption surplus that enters the utility function and implies a low interest rate and a high counter-cyclical risk premium.

Other types of housing equilibrium models have been derived. For instance, Lustig and Van Nieuwerburgh (2005) builds a general equilibrium model with housing collateral constraints. The collateral mechanism helps explain the time-variation in asset prices as well as the cross-sectional dispersion of stock returns. Favilukis et al. (2017) constructs a general equilibrium model with housing and non-housing sectors where households' bequests are heterogeneous. Households aggregate housing and non-housing consumption intratemporally through a Cobb-Douglas utility function. This implies a constant non-housing to housing expenditures ratio, which is a special case of Piazzesi et al. (2007) and ours. Their model is nonetheless able to generate sizable equity and housing risk premia and a low risk-free rate.

Another strand of research focuses on portfolio choice with housing under various assumptions. In Flavin and Yamashita (2002), investors are myopic and housing is an illiquid asset. Cocco et al. (2005), Yao and Zhang (2005), Cocco (2005) and Flavin and Nakagawa (2008) study the consumption and/or portfolio decisions of owners or tenants over the life

[3] Piazzesi and Schneider (2016) provides an extensive survey of the links between housing and macroeconomics. In particular, they present a general framework for housing consumption-based pricing models with heterogeneous households and market frictions that nests most of the extent housing literature without explicit crash modeling.

cycle with or without borrowing constraints. Here, we restrict our analysis to the impact of housing and rare disaster events on asset prices and the term structure of interest rates.

The issue of the *nominal* term structure of interest rates has been addressed under many different economic frameworks, including ones that allow for the presence of consumption disaster events and possibly of positive jumps in the expected inflation rate [see for instance the recent paper by Tsai (2015)]. Much less common are models of the *real* yield curve. In our model, time variation in bond risk premia is generated by the expenditure ratio, an observable datum, and the level and slope of the real term structure are determined only by this ratio. This may be viewed as consistent with Cochrane and Piazzesi (2005) who establish that a single counter-cyclical factor is able to predict movements in the (nominal) yield curve better than the usual level, slope and curvature representation of the term structure. Consumption-based studies usually report that the real yield curves are fairly flat, as in Ang et al. (2008), or downward sloping as in Piazzesi and Schneider (2007). By contrast, our model implies upward sloping curves under normal economic conditions, although other shapes are possible under unusual circumstances.

Our analytical framework relies heavily on cumulant generating functions as it involves non-Gaussian processes. CGFs have recently received a noticeable attention in the finance literature, in face of the strong evidence that macroeconomic data and asset returns depart from normality. For instance, Constantinides and Ghosh (2011) use CGFs to perform a non-parametric estimation of the stochastic discount factor in general consumption asset pricing models. Shaliastovich and Tauchen (2011) use CGFs to derive closed-form solutions for asset prices, assuming business time is driven by a Levy process. Martin (2013) generalizes these findings to arbitrary i.i.d. processes in a one-good economy. We, in turn, generalize his results to a two-good economy and to some non i.i.d. processes. Backus et al. (2014) uses CGFs to characterize the entropy of general asset pricing models as a measure of the pricing kernel dispersion. Ghosh et al. (2016) uses the entropy CGF representation to estimate non-parametrically the stochastic discount factor and, applying it in particular to Piazzesi and Schneider (2007)'s Housing CCAPM, shows that, contrary to ours, that model requires very high levels of risk aversion to explain the equity premium.

3 Economic setting

3.1 Preferences

We consider a frictionless endowment economy with two consumption goods,[4] in which a representative agent optimizes her lifetime utility defined on aggregate consumption:[5]

$$J(C_t) = E_t\left[\sum_{i=0}^{\infty}\beta^i U(C_{t+i})\right]$$
$$U(C_t) = \frac{C_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}$$

where C_t represents the consumption bundle, ψ is the intertemporal elasticity of substitution and β is the subjective discount factor. The lower the value of ψ, the more the agent is unwilling to substitute her consumption bundle over time, i.e. the more she smooths consumption over time. As demonstrated in Stokey (2009), in this environment void of frictions, the coefficient of relative risk aversion is constant and equal to $1/\psi$. The bundle C_t aggregates two distinct goods, namely non-housing consumption, c_t, which comprises all non-durables and non-housing services, and housing expenditures or shelter, s_t, through a Constant Elasticity of Substitution (CES) intratemporal utility function:

$$C_t = \left(c_t^{\frac{\varepsilon-1}{\varepsilon}} + \omega s_t^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}$$

where ω represents a preference parameter that controls the share of housing consumption present in the bundle, relative to non-housing services. The parameter ε represents the intratemporal elasticity of substitution between c_t, which from now on will be referred to as non-housing consumption, and housing services s_t. When $\varepsilon > 1$, the two goods are imperfect substitutes and when $\varepsilon < 1$ they are imperfect complements. When $\varepsilon = 1$, we recover the Cobb-Douglas utility function. As in Piazzesi et al. (2007) we assume that the two goods

[4]We ignore frictions for tractability. Two arguments can be invoked to minimize the role of frictions. First, home improvement by owner-occupiers, which is an alternative to new home construction and is essentially frictionless, represents a large share of housing activity and up to 2% of U.S. GDP according to Choi et al. (2014). Second, according to Iacoviello and Pavan (2013), about 35% of households are tenants who, unlike owners, move frequently without suffering material frictions.

[5]See subsections 3.1 and 3.2 of Piazzesi et al. (2007) for details.

can be exchanged in frictionless markets.

In equilibrium, consumption and output coincide such that:

$$(c_t, s_t) = (\overline{c}_t, \overline{s}_t)$$

where $(\overline{c}_t, \overline{s}_t)$ represents the endowment.

Combining the definition of the utility function and the intratemporal aggregator, the agent's preferences over the two goods are summarized by:

$$U(C_t) = \frac{\left(c_t^{\frac{\varepsilon-1}{\varepsilon}} + \omega s_t^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1-\frac{1}{\psi}}{1-\frac{1}{\varepsilon}}}}{1-\frac{1}{\psi}} \tag{2}$$

We note that if $\psi = \varepsilon$, i.e. the intratemporal and intertemporal elasticities coincide, utility is separable. This limit case, however, makes the two-goods assumption much less relevant.

3.2 Intratemporal first order condition

The quantities and prices of non-housing consumption and housing services are constrained by the representative individual's intratemporal first order condition. Let us denote the prices of non-housing and housing consumption by $p_{c,t}$ and $p_{s,t}$, respectively. $p_{s,t}$ corresponds to the price of renting one unit of real estate. The (static) intratemporal first order condition (FOC) establishes the following relationship between relative prices and relative quantities

$$\frac{p_{c,t}}{p_{s,t}} = \frac{1}{\omega}\left(\frac{c_t}{s_t}\right)^{-\frac{1}{\varepsilon}} \tag{3}$$

which reflects the standard equalization of the Marginal Rate of Substitution (MRS) between the two consumption goods with their price ratio.

Let z_t denote the expenditure ratio of non-housing consumption to housing services defined by

$$z_t = \frac{p_{c,t}c_t}{p_{s,t}s_t} \tag{4}$$

The intratemporal FOC (3) allows us to express the expenditure ratio as a function of consumed quantities only:

$$z_t = \frac{1}{\omega} \left(\frac{c_t}{s_t} \right)^{1-\frac{1}{\varepsilon}} \tag{5}$$

Equation (5) shows that, depending on whether the intratemporal elasticity ε is smaller or larger than 1, the expenditure ratio and housing consumption will move in the same or opposite directions, for a given level of non-housing consumption. In accordance with previous results by Benhabib et al. (1991), McGrattan et al. (1997), Ogaki and Reinhart (1998), Piazzesi et al. (2007) and Bajari et al. (2013), and as confirmed in the empirical section 6 below, the estimate of ε is larger than 1, implying that non-housing consumption and housing services are substitutes. We will therefore adopt this parametrization ($\varepsilon > 1$) throughout the paper[6].

Rewriting the expression for the consumption bundle as

$$C_t = \left(c_t^{\frac{\varepsilon-1}{\varepsilon}} + \omega s_t^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} = c_t \left(1 + \frac{1}{z_t} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

or in log form

$$\log C_t = \log c_t + \frac{\varepsilon}{\varepsilon - 1} \log \left(1 + \frac{1}{z_t} \right)$$

The evolution of the economy over the period $[t, t+1]$ is characterized by:

$$\log C_{t+1} - \log C_t = (\log c_{t+1} - \log c_t) + \frac{\varepsilon}{\varepsilon - 1} \left(\log \left(1 + \frac{1}{z_{t+1}} \right) - \log \left(1 + \frac{1}{z_t} \right) \right) \tag{6}$$

A recession is defined as a period where non-housing consumption decreases substantially. If in addition the expenditure ratio increases, the recession becomes severe (since $\varepsilon > 1$). Symmetrically, an economic expansion is characterized by an increase in non-housing consumption, and an economic boom by a simultaneous decrease in the expenditure ratio.

Incidentally, we note that in the Cobb-Douglas specification, as the one used by Berger

[6] The inequality must be strict since we are using the expenditure ratio rather than housing consumption. For the ratio to be well defined, $\varepsilon > 1$ must be satisfied. We can though compute the limit of our results when converges toward 1^+ making them comparable to Favilukis et al. (2017) who assume $\varepsilon = 1$

et al. (2015) and Favilukis et al. (2017) amongst others, ($\varepsilon \to 1^+$), expression (5) for the expenditure ratio degenerates to:

$$z_t = \frac{p_{c,t} c_t}{p_{s,t} s_t} = \frac{1}{\omega} \tag{7}$$

Consequently, the decision variables (c_t, s_t) cannot be equivalently represented by (c_t, z_t). In such a setting, one would need to resort to modeling directly the dynamics of non-housing consumption and housing expenditures. Alternatively, since the expenditure ratio is not constant in the data, result (7) leads us to assume that the representative agent is not endowed with a Cobb-Douglas utility function.

3.3 Stochastic discount factor

In the presence of two (or more) goods and the absence of money, we need to choose a numéraire. In view of the agent's Euler equation, further results are simpler if we adopt the price of non-housing consumption, $p_{c,t}$, as our numéraire. Henceforth, all asset prices will be deflated accordingly. Under the preferences specified by (2), the stochastic discount factor (SDF), is fully determined by the growth of non-housing consumption and the expenditure ratio. We first establish that:

$$M_{t+1} = \beta \frac{\frac{\partial U}{\partial c}(c_{t+1}, s_{t+1})}{\frac{\partial U}{\partial c}(c_t, s_t)} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{1}{\psi}} \left(\frac{1 + \omega \left(\frac{s_{t+1}}{c_{t+1}} \right)^{\frac{\varepsilon - 1}{\varepsilon}}}{1 + \omega \left(\frac{s_t}{c_t} \right)^{\frac{\varepsilon - 1}{\varepsilon}}} \right)^{\frac{\psi - \varepsilon}{\psi(\varepsilon - 1)}} \tag{8}$$

and then, using the intratemporal constraint (5), we rewrite the SDF as:

$$M_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{1}{\psi}} \left(\frac{1 + \frac{1}{z_{t+1}}}{1 + \frac{1}{z_t}} \right)^{\frac{\psi - \varepsilon}{\psi(\varepsilon - 1)}} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{1}{\psi}} \left(\frac{\frac{z_{t+1}}{1+z_{t+1}}}{\frac{z_t}{1+z_t}} \right)^{\frac{\varepsilon - \psi}{\psi(\varepsilon - 1)}} \tag{9}$$

The quantity $\frac{z_t}{1+z_t} = \frac{p_{c,t} c_t}{p_{c,t} c_t + p_{s,t} s_t}$ represents the share of non-housing expenditures relative to total expenditures. It is an increasing function of the expenditure ratio and is therefore positively correlated with the latter. Its presence in the SDF reflects the agent's aversion to composition risk, in addition to the non-housing consumption risk captured by the first multiplicative term in (9). Note that if utility were separable ($\varepsilon = \psi$), assuming that log consumption growth is i.i.d. and Gaussian would lead to the standard CCAPM applied to non-housing consumption.

Consider now an arbitrary risky asset whose one-period gross return is denoted by R_{t+1}. In equilibrium, the Euler equation prevails:

$$E_t \left[M_{t+1} R_{t+1} \right] = 1 \tag{10}$$

To ease the interpretation of asset pricing formulas, let us log-linearize this equation:

$$0 = \log E_t \left[M_{t+1} R_{t+1} \right] \simeq \log E_t \left[M_{t+1} \right] + \log E_t \left[R_{t+1} \right] + cov_t \left(M_{t+1}, R_{t+1} \right) \tag{11}$$

The term $EP_{t+1} = \log E_t \left[R_{t+1} \right] + \log E_t \left[M_{t+1} \right] = \log E_t \left[R_{t+1} \right] - r_{f,t+1}$ represents the risk premium on the risky asset where $r_{f,t+1}$ denotes the logarithmic risk-free rate. Equation (11) then can be written as:

$$\begin{aligned} EP_{t+1} &\simeq -cov_t \left(M_{t+1}, R_{t+1} \right) \\ &\simeq -cov_t \left(\log M_{t+1}, \log R_{t+1} \right) \\ &\simeq \frac{1}{\psi} cov_t \left(\Delta \log c_{t+1}, \log R_{t+1} \right) - \frac{1}{\psi} \frac{\varepsilon - \psi}{\varepsilon - 1} cov_t \left(\Delta \log \frac{z_{t+1}}{1 + z_{t+1}}, \log R_{t+1} \right) \end{aligned} \tag{12}$$

Agents like assets whose payoffs are high in bad times. Hence, as in the CCAPM, they value assets whose return co-varies *negatively* with non-housing consumption growth. In the presence of composition risk ($\varepsilon \neq \psi$), they may also value assets whose return co-varies *positively* with changes in the expenditure ratio, or, equivalently, in the non-housing expenditures share. For the latter property to hold, equation (12) imposes an additional constraint on their preference parameters:

$$\frac{1}{\psi} \frac{\varepsilon - \psi}{\varepsilon - 1} > 0 \tag{13}$$

Given that $\varepsilon > 1$, constraint (13) translates into:

$$\varepsilon > \psi \tag{14}$$

This condition means that intertemporal smoothing dominates intratemporal smoothing: agents are less willing to substitute total consumption across time than they are to substitute non-housing expenditures for housing consumption at a given time. Since ψ is the inverse of the coefficient of relative risk aversion, the condition is extremely likely to be met under the adopted parametrization $\varepsilon > 1$.

The risk premium on an arbitrary asset is determined by how its return co-moves with non-housing consumption growth and relative changes in the expenditure ratio. To assess the respective contribution of these variates to asset prices, we present in the next subsection the dynamics assumed for the data-generating process of (c_t, z_t).

3.4 Consumption and expenditure ratio dynamics

First, we assume that non-housing consumption is i.i.d. and obeys the following dynamics:

$$\begin{aligned}
\Delta \log c_{t+1} &= \mu_c + u_{c,t+1} + v_{c,t+1} \\
u_{c,t+1} &\sim N\left(0, \sigma_c^2\right) \\
v_{c,t+1} &= \begin{cases} 0 & \text{with probability } 1 - p_c \\ -v_{c,t+1}^d & \text{with probability } p_c \end{cases}
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
v_{c,t+1}^d &= \widehat{v}_{c,t+1}^d + v_{c,\min}^d \\
v_{c,\min}^d &\geq 0 \\
\widehat{v}_{c,t+1}^d &\sim \exp\left(\alpha_c\right)
\end{aligned} \tag{16}$$

This law of motion is the same as in Chibane et al. (2016) and is mathematically equivalent to Barro and Jin (2011)'s single power law rare disaster event dynamics. Non-housing consumption growth is driven most of the time by Gaussian innovations but is sometimes subject to crashes with a small constant probability p_c. The crash process is represented by a minimum level, $v_{c,\min}^d$, augmented by an independent, exponentially distributed variable $v_{c,t+1}$. The tail exponent of this distribution, α_c, determines how severe the crash may be.

The lower is α_c, the fatter is the left tail of the non-housing consumption growth distribution.

The dynamics of the expenditure ratio are given by:

$$\begin{aligned} \Delta \log z_{t+1} &= (\rho - 1)(\log z_t - \mu_z) + u_{z,t+1} + v_{z,t+1} \\ u_{z,t+1} &\sim N\left(0, \sigma_z^2\right) \\ v_{z,t+1} &= \begin{cases} 0 & \text{with probability } 1 - p_z \\ v_{z,t+1}^L & \text{with probability } p_z \end{cases} \end{aligned} \tag{17}$$

where $v_{z,t+1}^L$ obeys a standard Laplace distribution with diversity parameter $\frac{1}{\alpha_z}$.The distribution of $v_{z,t+1}^L$ is given by:

$$f_L(x) = P\left(v_{z,t+1}^L \in [x, x + dx]\right)/dx = \frac{1}{2}\alpha_z \exp\left(-\alpha_z |x|\right) \tag{18}$$

The distribution of the non-Gaussian component $v_{z,t+1}^L$ is symmetric around 0. Parameter α_z controls the fatness of the left and right tails: the lower α_z, the fatter the tails. This parameter is the counterpart of the exponential distribution tail exponent. We assume that processes $u_{z,t+1}, v_{z,t+1}$ are mutually independent and independent of all other processes.

Consequently, process $\log z_t$ is essentially a non-Gaussian $AR(1)$, with the restriction $0 < \rho < 1$. Its innovation has a similar structure as that of non-housing consumption: the log expenditure ratio is also conditionally Gaussian almost always but is occasionally subject to a sudden upward or downward jump with constant probability p_z. The expenditure ratio thus is not the slow-varying smooth process assumed by Piazzesi et al. (2007). The presence of rare disaster events in the dynamics of non-housing consumption and of booms and disasters in the dynamics of the expenditure ratio generates skewness and excess kurtosis in the distribution of excess returns that the Gaussian components cannot produce on their own. In fact, in the presence of rare disaster events, higher order cumulants may matter even beyond the fourth order. For instance, Chibane et al. (2016) shows that when consumption is truly subject to exponential crashes, cumulants up to order 12 do contribute significantly to excess returns and to the risk-free rate. Hence, in allowing for housing exponential busts and booms, we implicitly take into account the contribution of higher order cumulants to asset risk premia.

The intratemporal first order condition (5) rewrites:

$$\begin{aligned}\Delta \log s_{t+1} &= \Delta \log c_{t+1} - \frac{\varepsilon}{\varepsilon - 1} \Delta \log z_{t+1} \\ &= \mu_c - \frac{\varepsilon}{\varepsilon - 1} (\rho - 1) (\log z_t - \mu_z) + u_{c,t+1} \\ &\quad - \frac{\varepsilon}{\varepsilon - 1} u_{z,t+1} + v_{c,t+1} - \frac{\varepsilon}{\varepsilon - 1} v_{z,t+1}\end{aligned} \tag{19}$$

The dynamics of housing consumption is driven by two Gaussian innovations and by non-Gaussian innovations stemming from two sources which add up. The first source originates in fact in the non-housing sector of the economy and is represented by the term $v_{c,t+1}$. The second source is related to composition risk and reflected in the term $-\frac{\varepsilon}{\varepsilon-1} v_{z,t+1}$. Since we assume $\varepsilon > 1$, a boom (a crash) in the expenditure ratio translates into a crash (a boom) in housing consumption

Our model is quite general and nests several well-known approaches. For example, when $p_c = p_z = 0$, the joint evolution of non-housing consumption and the expenditure ratio coincides with the homoscedastic version of the Housing CCAPM introduced in Piazzesi et al. (2007). When utility is separable ($\psi = \varepsilon$), our model degenerates into the rare disaster event model of Barro and Jin (2011) applied to non-housing consumption. This justifies the name of "Housing Rare Event CCAPM" we give it.[7]

4 Asset pricing

Equipped with the preferences defined by (2) and the consumption and expenditure ratio dynamics (15) and (17), we can derive closed-form expressions for the risk-free rate, the price-dividend ratio and the risk premium on any asset whose dividend growth is determined by the evolution of $\Delta \log c_{t+1}$ and $\Delta \log z_t$. We use these expressions to analyze the relative impact of non-housing crashes and expenditure ratio booms/busts on equilibrium asset prices. We recall that the analysis is performed under the parametrization $\varepsilon > 1$ and $\varepsilon > \psi$.

[7] In fact, we can derive two of our results, namely the risk-free rate and the risk premium on a dividend paying asset, in an even more general framework. In order not to disrupt our emphasis on rare events, however, we relegate these two more general results to footnotes. See section 4 below.

4.1 The risk-free rate

Proposition 1. *In our Housing Rare Event CCAPM, and under conditions:*

$$\frac{1}{\psi} < \alpha_c \tag{20}$$

$$\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)} < \alpha_z \tag{21}$$

the risk free rate is:

$$\begin{aligned} r_{f,t+1} \simeq\; & -\log\beta+\frac{1}{\psi}\mu_c-\frac{1}{2}\left(\frac{1}{\psi}\right)^2\sigma_c^2 \\ & -\log\left(1-p_c+p_c\frac{\alpha_c}{\alpha_c-\frac{1}{\psi}}e^{\frac{1}{\psi}v_{c,\min}^d}\right) \\ & -\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}(\rho-1)(\log z_t-\mu_z) \\ & -\frac{1}{2}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2\sigma_z^2 \\ & -\log\left(1-p_z+p_z\frac{1}{1-\left(\frac{\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}}{\alpha_z}\right)^2}\right) \end{aligned} \tag{22}$$

Proof. See Appendix B. □

Expression (22) generalizes the risk-free rate formula obtained in Piazzesi et al. (2007) where all processes are conditionally Gaussian.[8] It exhibits terms that only depend on non-housing consumption risk (first two rows of (22)) and terms that only depend on composition risk (last three rows). This is due to the assumed independence between $\Delta\log c_{t+1}$ and

[8]For an arbitrary dynamics of (independent) non-housing consumption growth and expenditure ratio, the risk-free rate is given by the following approximate analytical expression:

$$r_{f,t+1} \simeq -\log\beta - K_c\left(-\frac{1}{\psi}\right) - K_{z,t}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right) \tag{23}$$

where K_c and $K_{z,t}$ refer respectively to the CGFs of non-housing consumption growth and expenditure ratio growth:

$$\begin{aligned} K_c(\xi) &= \log E\left[e^{\xi\Delta\log c_{t+1}}\right] \\ K_{z,t}(\xi) &= \log E_t\left[e^{\xi\Delta\log z_{t+1}}\right] \end{aligned}$$

See Appendix B for the proof. This expression extends the CGF framework established by Martin (2013) to the case of a two-good economy. Appendix B also shows that Proposition 1 is a special case of this expression.

$\Delta \log z_{t+1}$.

The first line of (22) is identical to the CCAPM expression for the risk-free rate and its interpretation straightforward. The first component, $-\log \beta$, represents the rate of impatience, the second, $\frac{1}{\psi}\mu_c$, reflects the demand for immediate consumption and the third, $-\frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_c^2$, accounts for the demand for precautionary savings against the variability of the consumption stream. The last two effects are stronger for a higher risk aversion ($\frac{1}{\psi}$).

Lines three and four of (22), also present in Piazzesi et al. (2007), are due to the presence of composition risk. These elements make the risk-free rate time-varying as a (non-linear) function of the expenditure ratio. The term in the third line has the same sign as $(\log z_t - \mu_z)$ and reflects an intertemporal substitution effect. For instance, in a severe recession ($\Delta \log c_{t+1} < 0$ and $\log z_t > \mu_z$) the agent anticipates that the economic situation will improve as $\log z_t$ reverts to its long term mean μ_z, and consequently saves less (or borrows more), which increases the risk-free rate. During expansions, the opposite is true. However, this term is nil on average and does not contribute to the interest rate level. The fourth line reflects additional precautionary savings due to uncertainty about the composition of the optimal consumption bundle. The representative agent, being averse to this uncertainty, saves more to account for it, which lowers the equilibrium interest rate. Therefore, the introduction of composition risk is a step towards solving the risk-free rate puzzle.

When there are no crashes in non-housing consumption and no booms/busts in housing expenditures, (22) thus degenerates into the approximate expression obtained in the homoscedastic Housing CCAPM of Piazzesi et al. (2007). In the presence of rare events, however, the risk-free rate expression accounts for new, additional effects. We note that the two logarithms in the second and last lines of (22) are strictly positive by construction, so that these two terms are negative, which further lowers the interest rate. The second line in (22) can be understood as a loss-aversion precautionary savings. Indeed, in the presence of potential crashes in non-housing consumption, economic agents not only fear uncertainty in consumption growth but also loathe the possibility of sudden sizable crashes and consequently increase their demand for savings, thereby reducing the risk-free rate. The fifth line of (22) is an additional term due to potential positive or negative jumps in the expenditure ratio. Agents not only fear sudden drops in non-housing consumption but they also hedge against booms/crashes in housing relative to non-housing consumption by saving more.

The restrictions (20) and (21) deserve some remarks. Knowing that $\frac{1}{\psi}$ is the coefficient of relative risk aversion, condition (20) reflects the fact that if the representative agent is too risk averse, the risk-free rate becomes $-\infty$. Since $z_t > 0$, a sufficient condition for (21) to hold uniformly is:

$$\frac{\varepsilon - \psi}{\psi\,(\varepsilon - 1)} < \alpha_z \tag{24}$$

Condition (24) expresses a complex interplay between intratemporal and intertemporal elasticities of substitution. To gain intuition, consider the case where $\varepsilon \to \infty$, i.e. non-housing and housing consumptions become perfect substitutes. Then (24) degenerates to:

$$\frac{1}{\psi} < \alpha_z \tag{25}$$

which imposes an additional bound, similar to (20), on the coefficient of relative risk aversion. Hence, a sufficient condition for the risk-free rate to exist under the perfect substitution case is:

$$\frac{1}{\psi} < \min\left(\alpha_c, \alpha_z\right)$$

Our general result given by equation (22) nests several interesting cases in addition to the homoscedastic Housing CAPM of Piazzesi et al. (2007). First, when utility is separable, i.e. $\varepsilon = \psi$, expression (22) is exact and degenerates to:

$$\begin{aligned} r_{f,t+1} \quad = \quad & -\log\beta + \frac{1}{\psi}\mu_c - \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_c^2 \\ & -\log\left(1 - p_c + p_c \frac{\alpha_c}{\alpha_c - \frac{1}{\psi}} e^{\frac{1}{\psi} v_{c,\min}^d}\right) \end{aligned} \tag{26}$$

Second, for a small non-housing consumption disaster probability and a weak crash severity $(p_c, v_{c,\min}^d)$, expression (26) recovers the approximation obtained in Barro and Jin (2011).

Finally, in absence of crashes in non-housing consumption, ($p_c = 0$), an extreme recession ($z_t \to \infty$) is characterized by:

$$\lim_{z_t \to \infty} r_{f,t+1} = -\log \beta + \frac{1}{\psi}\mu_c - \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_c^2$$

The standard CCAPM risk-free rate thus can be interpreted as the rate that prevails during extreme recessions. The intuition behind this result is that, under these exceptional circumstances, economic agents rationally foresee that very good times are ahead and then reduce their current savings, which increases the risk-free rate.

4.2 Risk premia

We now turn to analyzing the excess returns on risky assets. We consider securities whose dividend process is exogenous and defined by:

$$\begin{aligned} \Delta \log d_{t+1} &= k_c \Delta \log c_{t+1} + k_z \Delta \log z_{t+1} + u_{d,t+1} \\ u_{d,t+1} &\sim N\left(0, \sigma_d^2\right) \end{aligned} \tag{27}$$

where k_c and k_z are constant and u_d is a Gaussian process independent of all other innovations.

Specification (27) is rather general and represents a wide range of risky assets. For instance, if $k_c = 1, k_z = 0, u_{d,t+1} \equiv 0$, then the asset is the one that pays non-housing consumption as a dividend.

A second example is the real estate asset, which pays housing services as dividends:

$$d_t^{re} = \frac{p_{s,t}}{p_{c,t}} s_t = \frac{p_{s,t} s_t}{p_{c,t} c_t} c_t = \frac{c_t}{z_t}$$

It thus corresponds to $k_c = 1, k_z = -1, u_{d,t+1} \equiv 0$, and equation (27) is equivalent to:

$$\Delta \log d_{t+1} = \Delta \log re_{t+1} = \Delta \log c_{t+1} - \Delta \log z_{t+1}$$

A third example is the asset that pays the whole consumption bundle as a dividend. As proved in Appendix C, the dynamics of the consumption bundle can be approximated by:

$$\begin{aligned}\Delta \log C_{t+1} &\simeq k_c \log \Delta c_{t+1} + k_z \Delta \log z_{t+1} \\ k_c &= 1 \\ k_z &= -\frac{\varepsilon}{\varepsilon - 1}\frac{e^{-\mu_z}}{1+e^{-\mu_z}}\end{aligned} \tag{28}$$

Equation (28) can be rewritten as:

$$\begin{aligned}\Delta \log C_{t+1} &\simeq \mu_c + x_t + \eta_{C,t+1} \\ x_t &= k_z (\rho - 1) (\log z_t - \mu_z) \\ \eta_{C,t+1} &= u_{C,t+1} + v_{C,t+1} \\ u_{C,t+1} &= u_{c,t+1} + k_z u_{z,t+1} \\ v_{C,t+1} &= v_{c,t+1} + k_z v_{z,t+1}\end{aligned} \tag{29}$$

where the process x_t obeys the following dynamics:

$$\begin{aligned}x_{t+1} &= \rho x_t + \eta_{x,t+1} \\ \eta_{x,t+1} &= k_z (\rho - 1) (u_{z,t+1} + v_{z,t+1})\end{aligned}$$

Expression (29) is reminiscent of Bansal and Yaron (2004)'s dynamics where x_t is the persistent long run risk affecting the economy. Therefore (29) suggests that the expenditure ratio may identify this risk in a context where consumption is aggregated according to a CES intratemporal utility. Since, as noted by Piazzesi et al. (2007), z_t is an observable macroeconomic variable and not a financial variable computed from asset price data, contrary to many predictors in the finance literature, this interpretation is of particular interest.

4.2.1 The price-dividend ratio

To derive the expected excess return on a risky asset, we must obtain first an analytical expression for the associated price-dividend ratio. Consider the dividend process (d_t) given by equation (27). For the asset price process (p_t), the Euler equation writes:

$$p_t = E_t \left[M_{t+1} \left(p_{t+1} + d_{t+1} \right) \right]$$

Rescaling by the current dividend yields:

$$\frac{p_t}{d_t} = E_t \left[M_{t+1} \frac{d_{t+1}}{d_t} \left(1 + \frac{p_{t+1}}{d_{t+1}} \right) \right] \tag{30}$$

Since non-housing consumption is i.i.d., the time-varying price-dividend ratio must be a function of the log expenditure ratio $\log z_t$. Postulating for the price/dividend ratio an exponential affine form leads to the following Proposition.

Proposition 2. *In the Housing Rare Event CCAPM, the price-dividend ratio associated with dynamics* (27) *is given by:*

$$\frac{p_t}{d_t} = a e^{b(\log z_t - \mu_z)} \tag{31}$$

where coefficients a and b solve the following fixed point problem:

$$a = \beta \left(1 + a \right) e^{K_c\left(-\frac{1}{\psi}+k_c\right)+K_{z,t}\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}+k_z+\frac{ba}{1+a}\right)+\frac{1}{2}\sigma_d^2} \tag{32}$$

and

$$\begin{aligned} b =& \frac{ba}{1+a} + (\rho - 1) \left(\frac{1}{1+e^{\mu_z}} \frac{\varepsilon - \psi}{\psi \left(\varepsilon - 1 \right)} + k_z + \frac{ba}{1+a} \right) \\ & - \frac{\varepsilon - \psi}{\psi \left(\varepsilon - 1 \right)} \left(\frac{e^{\mu_z}}{1+e^{\mu_z}} \right)^2 \\ & \times \left(\left(\frac{1}{1+e^{\mu_z}} \frac{\varepsilon - \psi}{\psi \left(\varepsilon - 1 \right)} + k_z + \frac{ba}{1+a} \right) \sigma_z^2 + \frac{\partial K_{v_z}}{\partial \xi} \left(\xi \right) |_{\xi = \frac{1}{1+e^{\mu_z}} \frac{\varepsilon - \psi}{\psi(\varepsilon - 1)} + k_z + \frac{ba}{1+a}} \right) \end{aligned} \tag{33}$$

with:

$$\begin{aligned} K_c\left(\xi\right) &= \xi \mu_c + \frac{1}{2} \xi^2 \sigma_c^2 + K_{v_c}\left(\xi\right) \\ K_{z,t}\left(\xi\right) &= \xi \left(\rho - 1\right) \left(\log z_t - \mu_z\right) + \frac{1}{2} \xi^2 \sigma_z^2 + K_{v_z}\left(\xi\right) \\ K_{v_c}\left(\xi\right) &= \log \left(1 - p_c + p_c e^{\xi v_{c,\min}^d} \frac{\alpha_c}{\alpha_c + \xi} \right) \end{aligned}$$

$$K_{v_z}(\xi) = \log\left(1 - p_z + p_z \times \frac{1}{1-\left(\frac{\xi}{\alpha_z}\right)^2}\right)$$

Proof. See Appendix D. □

In general, we would expect the price-dividend ratio to decrease during severe recessions, i.e. when $\log z_t$ increases along with a sharp decrease in $\log c_t$. We can see that, as the intratemporal elasticity of substitution ε approaches 1^+, and in the presence of booms/crashes in the log expenditure ratio, the sign of b is guaranteed to be *negative*. Under this setting, agents facing a severe recession sell their assets to increase their current consumption, thereby depressing market prices. As dividends are likely in general to depend only peripherally on the expenditure ratio, the price-dividend ratio decreases.

When utility is separable ($\varepsilon = \psi$), equations (32) and (33) degenerate to:

$$\begin{aligned} a &= \beta(1+a)\, e^{K_c\left(\frac{-1}{\psi}+k_c\right)} \\ b &= 0 \end{aligned}$$

and the price-dividend ratio is constant and equal to:

$$\frac{p_t}{d_t} = a = \frac{\beta e^{K_c\left(\frac{-1}{\psi}+k_c\right)}}{1-\beta e^{K_c\left(\frac{-1}{\psi}+k_c\right)}} \tag{34}$$

The closed-form expression (34), which our equations (32) and (33) generalize, was established by Martin (2013) in the special case of a one-good economy with i.i.d. consumption growth. It nevertheless encompasses the standard CCAPM price-dividend ratio as well as the price-dividend ratio obtained in the rare disaster event framework of Barro (2009).

4.2.2 Risk premium on the dividend paying asset

We are now able to find an analytical expression for the risk premium.

Proposition 3. *In the Housing Rare Event CCAPM, the risk premium on the risky asset* *yielding the dividend process* (27) *is given by the following analytical expression:*

$$\begin{aligned} EP_{t+1} = & \frac{k_c}{\psi}\sigma_c^2 \\ & + K_{v_c}\left(k_c\right) + K_{v_c}\left(-\frac{1}{\psi}\right) - K_{v_c}\left(-\frac{1}{\psi} + k_c\right) \\ & - \frac{1}{1+z_t}\frac{\varepsilon - \psi}{\psi\left(\varepsilon - 1\right)}\left(k_z + \frac{ab}{1+a}\right)\sigma_z^2 \\ & + K_{v_z}\left(k_z + \frac{ab}{1+a}\right) + K_{v_z}\left(\frac{1}{1+z_t}\frac{\varepsilon - \psi}{\psi\left(\varepsilon - 1\right)}\right) \\ & - K_{v_z}\left(\frac{1}{1+z_t}\frac{\varepsilon - \psi}{\psi\left(\varepsilon - 1\right)} + k_z + \frac{ab}{1+a}\right) \end{aligned} \tag{35}$$

Proof. See Appendix E.[9] □

Expression (35) contains rewards for several types of risk. The first compensation is identical to that of the CCAPM and reflects the agent's aversion to consumption growth risk in normal times. The second line of rewards reflects the agent's aversion to crashes in non-housing consumption. The sum of three terms is guaranteed to be positive if:

$$k_c > 0 \tag{37}$$

Since dividends are generally positively correlated with non-housing consumption, condition (37) is verified. The agent fears sudden, sharp negative changes in consumption and demands an extra premium for bearing this risk. All the above components of the risk premium are constant. However, as functions of the expenditure ratio, the other components make the risk premium vary over time. The term in the third line reflects the agent's

[9] As in the case of the risk-free rate, a more general result obtains for arbitrary processes for non-housing consumption growth and expenditure ratio. The risk premium can be expressed in the following closed-form:

$$\begin{aligned} EP_{t+1} = & \log E_t\left[R_{t+1}\right] - r_{f,t+1} \\ \simeq & K_c\left(k_c\right) + K_c\left(-\frac{1}{\psi}\right) - K_c\left(-\frac{1}{\psi} + k_c\right) \\ & + K_{z,t}\left(k_z + \frac{ab}{1+a}\right) + K_{z,t}\left(\frac{1}{1+z_t}\frac{\varepsilon - \psi}{\psi\left(\varepsilon - 1\right)}\right) \\ & - K_{z,t}\left(\frac{1}{1+z_t}\frac{\varepsilon - \psi}{\psi\left(\varepsilon - 1\right)} + k_z + \frac{ab}{1+a}\right) \end{aligned} \tag{36}$$

This expression generalizes the expression of Martin (2013) to the case of i.i.d. non-housing consumption and time-varying expenditure ratio. Appendix E also shows that Proposition 3 is a special case of this expression.

fear of changes in his consumption composition during normal times. It is positive provided that:

$$k_z + \frac{ab}{1+a} < 0 \tag{38}$$

Since k_z is negative (see (28)) and a positive, for b nil (separable utility), negative (ε close to 1^+) or small enough, the risk premium will increase with the undesirable variability of consumption composition.

The three terms in the fourth and fifth lines of (35) express the fact that the agent not only dislikes sudden negative jumps in her non-housing consumption but also abhors sudden positive or negative moves in housing consumption. So she requires an even higher risk premium. Indeed, condition (38) also guarantees that the sum of these terms is positive. To see this, let us first recall the definition of the *crash/boom* process's CGF:

$$\begin{aligned} K_{v_z}(\xi) &= \log E\left[e^{\xi v_z}\right] \\ v_z &> 0 \end{aligned}$$

Since the term $\xi_1 := k_z + \frac{ab}{1+a}$ is negative while the term $\xi_2(z_t) := \frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}$ is positive (because of the restrictions $\varepsilon > 1, \varepsilon > \psi$), the random variables $e^{\xi_1 v_z}$ and $e^{\xi_2(z_t)v_z}$ must be negatively correlated. Consequently:

$$cov\left(e^{\xi_1 v_z}, e^{\xi_2(z_t)v_z}\right) = E\left[e^{\xi_1 v_z}e^{\xi_2(z_t)v_z}\right] - E\left[e^{\xi_1 v_z}\right]E\left[e^{\xi_2(z_t)v_z}\right] < 0$$

or equivalently:

$$E\left[e^{(\xi_1+\xi_2(z_t))v_z}\right] < E\left[e^{\xi_1 v_z}\right]E\left[e^{\xi_2(z_t)v_z}\right]$$

Taking logs on both sides of the equation, we get:

$$\log E\left[e^{(\xi_1+\xi_2(z_t))v_z}\right] < \log E\left[e^{\xi_1 v_z}\right] + \log E\left[e^{\xi_2(z_t)v_z}\right]$$

which is equivalent to stating that:

$$K_{v_z}\left(k_z+\frac{ab}{1+a}\right)+K_{v_z}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi\left(\varepsilon-1\right)}\right)-K_{v_z}\left(k_z+\frac{ab}{1+a}+\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi\left(\varepsilon-1\right)}\right)>0$$

Consequently, as intuition suggests, the presence of a boom/crash in the dynamics of z_t increases the risk premium required in equilibrium. We note at the same time that conditions (20) and (24) which ensure that the risk-free rate is not minus infinity also guarantee a finite risk premium.

Given condition (38), detailed scrutiny of equation (35) shows that, in our framework, the risk premium decreases with the *level* of the expenditure ratio. In other words the risk premium is pro-cyclical w.r.t. to a severe recession (since z_t then is large) or pronounced expansion. This is because both the volatility of the Gaussian component and the probability of disasters are constant in the expenditure ratio dynamics.[10] A richer and more realistic model equipped with time-varying volatility and disaster probability would help induce a counter-cyclicality into the risk premium.

We also find that, even when future dividends do not depend on the current expenditure ratio ($k_z = 0$), composition risk still contributes to the risk premium through the last three lines of (35). Indeed, the agent is still exposed to composition risk through its influence on the stochastic discount factor, hence on asset prices and expected returns.

4.3 Variance of excess returns

Our framework readily applies to the analysis of the variance of excess returns. We define the excess return on a risky asset as:

$$ER_{t+1}=\log R_{t+1}-r_{f,t+1}$$

where:

$$R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t}$$

[10] Recall that the risk premium is constant in the standard CCAPM.

$$
\begin{aligned}
\frac{D_{t+1}}{D_t} &= e^{k_c \Delta \log c_{t+1} + k_z \Delta \log c_{t+1} + u_{d,t+1}} \\
u_{d,t+1} &\sim N\left(0, \sigma_d^2\right)
\end{aligned}
$$

The dividend process is identical to (27) since rates are left unaffected by a change of numéraire.

Proposition 4. *In the Housing Rare Event CCAPM, the variance of the excess return on a risky asset is approximately given by:*

$$
\begin{aligned}
var\left(ER_{t+1}\right) \simeq\; & k_c^2\left(\sigma_u^2 + \frac{1}{\alpha_c^2}\right) + \sigma_d^2 \\
& + \left(\left(k_z + \frac{ab}{1+a}\right)(\rho - 1) - \left(\frac{b}{1+a} + \frac{\partial r_{f,t+1}}{\partial \log z_t}|_{\log z_t = \mu_z}\right)\right)^2 \frac{1}{1-\rho^2}\left(\sigma_z^2 + \frac{1}{\alpha_z^2}\right) \\
& + \left(k_z + \frac{ab}{1+a}\right)\left(\sigma_z^2 + \frac{1}{\alpha_z^2}\right)
\end{aligned}
\tag{39}
$$

where

$$
\begin{aligned}
\frac{\partial r_{f,t+1}}{\partial \log z_t}|_{\log z_t = \mu_z} =\; & -\frac{1}{1+e^{\mu_z}} \frac{\varepsilon - \psi}{\psi(\varepsilon - 1)} \\
& + \frac{e^{\mu_z}}{(1+e^{\mu_z})^3}\left(\frac{\varepsilon - \psi}{\psi(\varepsilon - 1)}\right)^2 \sigma_z^2 \\
& + 2p_z\alpha_z^2 \left(\begin{array}{c}
\left(\frac{1}{\alpha_z^2 - \left(\frac{1}{1+e^{\mu_z}} \frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)} \frac{e^{\mu_z}}{(1+e^{\mu_z})^2} \frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right) \\
\times \left(\frac{\frac{1}{\alpha_z^2 - \left(\frac{1}{1+e^{\mu_z}} \frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2}}{1 - p_z + p_z \frac{\alpha_z^2}{\alpha_z^2 - \left(\frac{1}{1+e^{\mu_z}} \frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2}}\right)
\end{array}\right)
\end{aligned}
$$

Proof. See Appendix F. □

The first term of (39) corresponds to the variance of excess returns under the standard i.i.d. model with one consumption good. The last two terms reflect the contribution of composition risk. The second term is always strictly positive. The sign of the third term is indeterminate as k_z is positive and b negative. However, as simple inspection suggests, this term should be small relative to the second one, so that the introduction of a second good in the economy and thereby composition risk should increase the variance of excess returns. This is what our empirical analysis shows in subsection 5.6. In addition, it is worth mentioning that, even though future dividends would not depend on the expenditure ratio ($k_z = 0$), composition risk would still affect expression (39) through its influence on the pricing kernel.

5 Empirical setting

5.1 Data

Our data set covers the period January 1959 to November 2015. We use U.S. monthly consumption data from the BEA's National Income and Product Accounts (NIPA)'s Table 2.4.5U. The latter provides nominal personal consumption expenditures and price indexes for a very granular variety of consumption goods and services, including housing services, non-durables, and services (as a whole). Based on these pieces of data, we can convert expenditures into quantities. Yet, one of our two measures of consumption merges non-durables and non-housing services. We do not have a specific price index for this bundle but, since it represents 70% of total expenditures, we use the CPI as the relevant index.

Housing expenditure data correspond to rental expenses for tenants and to the owner-equivalent rent for owners/occupiers. The latter reflects the amount for which the owner would rent out his dwelling. These data are obtained from the Census Bureau annual surveys[11]. The housing price index is computed from the rent component of the CPI. Data on housing suffer from a variety of measurement errors. First, drastic technological advances in the quality of housing services such as provision of electricity, water, and so on are not taken into account. Moreover, besides the number of rooms and living surface, there are other components to housing services, such as the quality of surrounding amenities, which depend on location. Last, Gordon and Van Goethem (2005) argues that the CPI rent index is downward biased, which adds more uncertainty to the measurement of housing services consumption. All these arguments cast doubt on the approach of directly using housing services consumption as a variable of our model. Fortunately, as noted in Piazzesi et al. (2007) and Fillat (2008), the expenditure ratio does not suffer from these shortcomings

[11] Complete details can be found in November 2014 version of a document published as Updated summary NIPA methodologies by the U.S Bureau of Economic Analysis in `http://www.bea.gov/scb/pdf/2014/11\%20November/1114_summary_of_nipa_methodologies.pdf`

because it does not need information about price indexes to be calculated. This justifies why we use non-housing consumption and the expenditure ratio as the two processes driving our economy.

In Figure 1, we plot the evolution of the log expenditure ratio over the period January 1959 to November 2015.

[Figure 1 about here.]

This process appears to be quite stationary and it thus seems appropriate to introduce some persistence into its dynamics as we did in (17). Furthermore, we can observe two sub-periods, $2001-2003$ and $2008-2010$, where sharp declines are followed by strong rises. These correspond to the bursting of the dot com and the housing bubbles, respectively. This justifies why we have included a non-Gaussian process in the dynamics of the expenditure ratio that can account for booms and crashes in housing consumption. Financial data were downloaded from Kenneth French's website. We use the return on the one-month Treasury Bill as the risk-free rate. The market equity index is the value-weighted return extracted from the CRSP database. The stream of dividends is also extracted from this database. Nominal return rates are deflated by the CPI growth rate.

To clearly identify the effect of the $2007-2008$ financial crisis, we create a pre-crisis sample that spans January 1959 to December 2006. Table 1 displays the descriptive monthly statistics on non-housing consumption, standard non-durables and services consumption as reported by the BEA and on the expenditure ratio, the risk-free rate and the market index excess return for both the full and pre-crisis samples.

[Table 1 about here.]

We first notice that over the full sample, non-housing consumption exhibits an equivalent annual volatility of 1.64% versus 1.25% for non-durables and services. All quantities but the risk-free rate display significant excess kurtosis, which indicates departures from normal distributions. Additionally, the skewness of non-housing consumption decreases significantly if the post-crisis period is included, which suggests that it has been subject to more frequent negative shocks after 2006. Last, we note that, consistent with standard empirical results, the average historical real risk-free rate is about 0.94% annually while the market equity premium is about 6% annually. The volatility of the risk-free rate is equivalent to 0.74% annually while the volatility of the equity premium is approximately 15.3%. The market excess return exhibits a significant skewness of -0.52, which by itself suggests that incorporating non-Gaussian negatively skewed innovations in the dynamics of consumption is justified. We now turn to estimating our model parameters.

5.2 Estimation of intratemporal preference parameters

We recall, in log form, the intratemporal first order condition (5), which relates the expenditure ratio with non-housing and housing consumption:

$$\log z_t = -\log \omega + \frac{\varepsilon - 1}{\varepsilon} \log \frac{c_t}{s_t} \tag{40}$$

We basically use equation (40) to estimate intratemporal preference parameters by maximum likelihood. Since, as we argued in subsection 5.1, the measurement of consumption of housing services is plagued by a substantial inaccuracy, we assume the measurement error on the expenditure ratio are accounted for by a Gaussian noise, which yields:

$$\begin{aligned} \log z_t &= -\log \omega + \frac{\varepsilon - 1}{\varepsilon} \log \frac{c_t}{s_t} + \xi_t \\ \xi_t &\sim N\left(0, \eta^2\right) \end{aligned}$$

Then the probability density function of $\log z_t$, conditional on non-housing consumption and housing expenditures, is given by:

$$\begin{aligned} g_{z,\varepsilon,\omega,\eta}\left(x, (c_t, s_t)\right) &= P\left(\log z_t \in [x, x + dx]\right)/dx \\ &= \frac{1}{\eta} n \left(\frac{x + \log \omega - \frac{\varepsilon - 1}{\varepsilon} \log \frac{c_t}{s_t}}{\eta} \right) \end{aligned} \tag{41}$$

Given our set of data $(z_t, c_t, s_t)_{t=1,\ldots,T}$ on expenditure ratio and both types of consumption for the period January 1959 to November 2015, we run the following maximum likelihood estimation program:

$$\begin{aligned}
\left(\widehat{\varepsilon},\widehat{\omega},\widehat{\eta}\right) &= \underset{\varepsilon,\omega,\eta}{\arg\max} L_z\left(\varepsilon,\omega,\eta,\left(z_t,c_t,s_t\right)_{t=1,\ldots,T}\right) \\
L_z\left(\varepsilon,\omega,\eta,\left(z_t,c_t,s_t\right)_{t=1,\ldots,T}\right) &= \prod_{t=1}^{T} g_{z,\varepsilon,\omega,\eta}\left(\log z_t,\left(c_t,s_t\right)\right)
\end{aligned}$$

The estimation results are displayed in Table 2 for both the full and pre-crisis samples.

[Table 2 about here.]

We find that the elasticity of intratemporal substitution ε is significantly above one at around 1.4, which implies that non-housing and housing consumptions are imperfect substitutes. This result is consistent with Piazzesi et al. (2007) who base their estimation on the co-integrating relationship implied by the intratemporal FOC (3) and report an estimate of $\varepsilon = 1.27$ with a standard error of 0.16. It is also in line with Bajari et al. (2013) who provides an estimate of $\varepsilon = 1.32$, and with Fillat (2008) who is led to consider two scenarios: $\varepsilon = 1.4$ and $\varepsilon = 1.8$. ω is consistently estimated at 0.254, hinting at a share of non-housing consumption in the representative agent preferences close to 20%. These estimates are similar to those used in the calibration of Berger et al. (2015) (1 - 0.853 = 0.147) or Favilukis et al. (2017) (1 - 0.70 = 0.30).

5.3 Estimation of consumption and expenditure ratio dynamics

Since non-housing consumption and the expenditure ratio are independent, they can be estimated separately. As we have obtained the probability densities of log consumption growth and expenditure ratio analytically, we can choose the convenient maximum likelihood method to estimate our model.

Proposition 5. *Under the consumption dynamics* (15)*, the p.d.f. of non-housing consumption growth is equal to:*

$$f_c\left(x\right) = \frac{1}{\sigma_c} n\left(\frac{x-\mu}{\sigma_c}\right)\left(\left(1-p_c\right) + p_c\alpha_c\sigma_c e^{\alpha_c v^d_{c,\min}} \frac{N\left(-\frac{v^d_{c,\min}+x-\mu_c+\alpha_c\sigma_c^2}{\sigma_c}\right)}{n\left(\frac{x-\mu_c+\alpha_c\sigma_c^2}{\sigma_c}\right)}\right) \tag{42}$$

Proof. The derivation of the closed-form equation (42) can be found in Chibane et al. (2016). □

Proposition 6. *Under the expenditure ratio dynamics* (17)*, the p.d.f. of the expenditure ratio growth is equal to:*

$$f_z(x, z_t) = \frac{1}{\sigma_z}\left((1-p_z)\, n\left(\frac{\widetilde{x}}{\sigma_z}\right) + \frac{1}{2} p_z \sigma_z \alpha_z \left(\begin{array}{c} e^{\frac{1}{2}\left(\alpha_z^2\sigma_z^2 + 2\alpha_z \widetilde{x}\right)} N\left(-\frac{\widetilde{x}+\alpha_z\sigma_z^2}{\sigma_z}\right) \\ +e^{\frac{1}{2}\left(\alpha_z^2\sigma_z^2 - 2\alpha_z \widetilde{x}\right)} N\left(\frac{\widetilde{x}-\alpha_z\sigma_z^2}{\sigma_z}\right) \end{array}\right)\right) \tag{43}$$

Proof. See Appendix A. □

Given our consumption data set, denoted by $(\Delta \log c_t, \Delta \log z_t)_{t=1,..,T}$ where T represents the sample size, we can perform the following two maximum likelihood estimations:

$$\left(\widehat{\mu}_c, \widehat{\sigma}_c, \widehat{p}_c, \widehat{v}^d_{c,\min}, \widehat{\alpha}_c\right) = \underset{\left(\mu_c,\sigma_c,p_c,v^d_{c,\min},\alpha_c\right)}{\arg\min} L_c\left((\Delta \log c_t)_{t=1,...T}, \left(\mu_c, \sigma_c, p_c, v^d_{c,\min}, \alpha_c\right)\right) \tag{44}$$

$$L_c\left((x_t)_{t=1,...T}, \left(\mu_c, \sigma_c, p_c, v^d_{c,\min}, \alpha_c\right)\right) = \prod_t^T f_c(x_t)$$

and

$$\left(\widehat{\mu}_z, \widehat{\sigma}_z, \widehat{\rho}_z, \widehat{p}_z, \widehat{\alpha}_z\right) = \underset{(\mu_z,\sigma_z,\rho_z,p_z,\alpha_z)}{\arg\min} L_z\left((\Delta \log z_t)_{t=2,...T}, (\mu_z, \sigma_z, \rho_z, p_z, \alpha_z)\right)$$

$$L_z\left((x_t)_{t=1,...T}, (\mu_z, \sigma_z, \rho_z, p_z, \alpha_z)\right) = \prod_{t=2}^T f_z(x_t, z_{t-1})$$

The results of our estimations are reported in Tables 3, 4 and 5 for non-housing consumption, the expenditure ratio, and non-durables and services (NDS) respectively, for the full and the pre-crisis samples. The reason why we have added the estimates for NDS is that this aggregate is the one commonly used in tests of the standard CCAPM to which in particular our more general model will be compared in later subsections. Also for the sake of comparison, these Tables display the estimation results of the corresponding Gaussian models.

[Table 3 about here.]

[Table 4 about here.]

[Table 5 about here.]

In the full sample, we can identify the presence of a crash in non-housing consumption with a probability of occurrence p_c around 1.4%, a tail exponent of 252 and a minimum crash size $v^d_{c,\min}$ of 1%. These estimates are significant at the 10% confidence interval. Our results from the pre-crisis sub-sample for non-housing consumption show that the crash probability and crash minimum size are less significant. This vindicates the impression that the post-crisis data contain more crashes, which improves the overall quality of our estimation.

Our findings are also in line with the conventional understanding of crashes. In effect, the probability of a crash in non-housing consumption is low enough to correspond to a rare event. Also, its p-value is acceptable (9.5%). Moreover, the severity of the crash is about twice the standard deviation of the Gaussian component, so that it can be considered as an extreme event, relative to a purely Gaussian consumption growth rate. Interestingly, the tail exponent (252) is large, which means that the left tail of non-housing consumption growth is not as pronounced as found by Barro and Jin (2011) in their cross-country setting.

The analysis of the expenditure ratio parameters reveals a similar pattern. The probability of a boom/crash is rather low at 2.4% in the pre-crisis sample and 1.2% in the full sample. The tail exponent α_z is moderate in the pre-crisis sample but quite low in the full sample confirming the fact that more drastic variations have taken place after 2006. We stress that p-value for the boom/crash probability is under 5% for both samples and that all parameters p-values are under the 1% cutoff.

Inspection of Table 5 relative to non-durables and services leads to a markedly different conclusion. Although the tail exponent of the crash process differs now more substantially between the pre-crisis and the full periods, decreasing from 270 to 120, the probability of a crash is divided by two, from 1.16% to 0.61%. Moreover, the p-value of the crash probability is 15% in the sub-sample and increases to 18% in the full sample. The degradation of the p-value for the crash severity is even worse, increasing from 9% in the sub-sample to 26.4% in the full sample. Therefore, unlike the calibration used in part of the literature, e.g. Barro (2006) and Barro and Jin (2011), the presence of crashes in NDS appears neither pronounced nor significant in the U.S. data we use.

To test how our rare disaster event models for the dynamics of non-housing consumption and the expenditure ratio perform relative to the Gaussian Housing CCAPM, we computed the p-values associated with the likelihood ratio test for nested models. Our results are reported in Table 6.

[Table 6 about here.]

For the pre-crisis period, we cannot reject the hypothesis that non-housing consumption and the expenditure ratio are jointly Gaussian at the 10% confidence level. For the full sample, however, the null hypothesis is rejected at the 1% level. The result obtained on the full sample thus tends to vindicate the relevance of our alternative hypothesis. We now use our framework to appreciate how exactly our housing rare event model performs empirically relative to the standard CCAPM model, the Housing CCAPM of Piazzesi et al. (2007) and the rare disaster event model of Barro and Jin (2011).

5.4 Rare events and the risk-free rate

First, for various levels of risk-aversion, we compare the Gaussian version of the housing model (Housing CCAPM), where we have shut down the rare disaster events in both non-housing consumption and the expenditure ratio, to the standard CCAPM based on non-durables and services (NDS). Second, we compare our housing rare event model, where both non-housing consumption and the expenditure ratio are subject to rare disaster events, with the one-good model of Barro and Jin (2011) based on NDS. For both exercises, we use $\varepsilon = 1.4$ in accordance with the estimation results of subsection 5.2. Our results are reported in Figure 2.

[Figure 2 about here.]

We can see that, for reasonable levels of risk aversion, the Housing CCAPM implies a risk-free rate that is substantially and consistently lower than the standard CCAPM in both the full and pre-crisis samples. For example, for $\varepsilon = 1.4, \gamma = 5$, in the full sample estimation the Housing CCAPM predicts an annualized risk-free rate of 8.6% while the standard CCAPM implies one of 11%. These rates are very far away from the 0.94% rate that we observe in the data. The lowest risk-free rate we can obtain for the Housing CCAPM is 5.3%, for $\gamma = 3$, which is still much too high. The results are almost identical in the pre-crisis estimation. We would expect an improvement from introducing rare events in the models but, for this level of intratemporal elasticity of substitution, we can hardly distinguish the Housing rare event from the Housing CCAPM, as shown in Figure 3. Similarly, the risk-free rate implied by the one-good model of Barro and Jin (2011) is almost identical to that obtained with the standard CCAPM, confirming that crashes are not so present in the dynamics of U.S. non-durables and services.

[Figure 3 about here.]

We now compare the interest rate predictions obtained from the four possible versions of our model. These versions are generated by changing the distribution type of either non-housing consumption or the expenditure ratio. We refer to these models as "GG", "DG", "GD" and "DD", where the first letter refers to the distribution of non-housing consumption and the second letter to the distribution of the expenditure ratio. "G" denotes a Gaussian distribution and "D" a rare Disaster event distribution. For example, "GG" stands for the homoscedastic version of the Housing CCAPM whereas "GD" denotes the model where non-housing consumption is Gaussian and the expenditure ratio distribution includes a boom/crash component. "DD" thus is our more general model. Figure 3 displays the annualized risk-free rate implied by these four models for various levels of relative risk aversion. We still use the value of 1.4 for the intratemporal elasticity of substitution, and the parameter estimates for the consumption dynamics obtained over the full and pre-crisis samples. Not surprisingly, the worst performing model is the Housing CCAPM (GG) and the best performing is the full-fledged Housing Rare Event model (DD). However, differences are disappointingly tiny.

A possible explanation for all versions of housing consumption models to imply too high a risk-free rate is that we use the relatively large value of 1.4 for ε, which is based on inaccurate housing data and thus is subject to a substantial measurement error. Consequently, following Piazzesi et al. (2007), we use two other values for ε: 1.05 and 1.03. In Figures 4 and 5 we plot the predicted risk-free rate as a function of relative risk aversion for, respectively, $\varepsilon = 1.05$ and $\varepsilon = 1.03$.

[Figure 4 about here.]

[Figure 5 about here.]

We can see that lowering ε improves the ability of all housing consumption models to lower the risk-free rate. We also note that, with these preferences, it is the boom/crash on the ratio rather than the crash on the non-housing consumption that prominently reduces the risk-free rate, relative to the Housing CCAPM: the DG model is rather close to GG, while DD is significantly better. For example, for $\varepsilon = 1.05$ and $\gamma = 5$, in the full sample estimation the Housing CCAPM (GG) predicts an annualized risk-free rate of 5.9% while introducing rare disaster events in both types of consumption (DD) leads to 2.1%. The intuition behind this result is that the demand for precautionary savings is exacerbated by the perspective of being exposed to an even bigger crash on housing (which is both a shelter for the family and a collateral asset for borrowing) than on non-housing when the latter is already severe enough.

We also note that using $\varepsilon = 1.05$ leads to negatively sloped curves and thus negative interest rates for risk aversion in the full sample. As ε decreases to 1, the two types of consumption become increasingly more imperfect substitutes, which boosts further the demand for precautionary savings.

In Tables 7 and 8 we show numerical examples of the implied annualized risk-free rate and its standard deviation for our three values of ε (1.03, 1.05 and 1.4) and for $\gamma = 3$ and 5.

[Table 7 about here.]

[Table 8 about here.]

For the preference parameters $\varepsilon = 1.03, \gamma = 3$ we obtain with our preferred, more general model (DD) an annualized risk-free rate of 0.87%, which is very close to its historical average of 0.94% (see Table 7). However, the risk-free rate standard deviation implied by this set of parameters, as reported in Table 8, exceeds 1.3% for all models, which is a little far from the historical standard deviation of 0.74%. Recall that the one-good models (CCAPM^{NDS} and BJ^{NDS}) lead to a constant risk-free rate. For the higher value of $\varepsilon = 1.4$, the risk-free rate annualized standard-deviation is 0.21% for the DD model, which is more in line with observations. Consequently, there is a tension between the mean and the standard deviation of the risk-free rate in the housing models. To be small, the former requires a low level of *intra*-temporal elasticity of substitution (ε) while the latter needs a relatively high one. An explanation for this is that the agent's preferences are represented by a power utility which constrains the *inter*temporal elasticity of substitution (ψ) to be the reciprocal of the coefficient of relative risk-aversion. This forces ψ to be too low, implying a very strong consumption smoothing motive and consequently a high risk-free rate. In our framework, however, the agent can compensate intertemporal smoothing with higher intratemporal smoothing (a low ε) so that she has to borrow less against future consumption, which reduces the risk-free rate. Thus, for a low enough ε, the interest rate is small. This, however, comes at the cost of increasing its volatility. This is due to the fact that as ε decreases, the intratemporal smoothing motive becomes stronger: hence any shock to the housing component will force the agent to adjust his non-housing consumption more and borrow/lend accordingly against future consumption. This increasingly changing levels of demand for borrowing/savings will consequently impact the volatility of interest rates. Introducing recursive preferences as in Epstein and Zin (1989) could alleviate this problem by disentangling risk aversion and intertemporal substitution.

5.5 Rare events and the price-dividend ratio

We have used specification (27) to approximate the dividend process associated with the equity index and estimated the parameters k_c, k_z and σ_d by OLS from the CRSP database. To mitigate the errors introduced by seasonality, we first summed monthly dividend growth data over one year and then corrected standard errors by using Newey-West robust error adjustments (with 12 lags) to account for overlapping data. Our results are reported in Table 9.

[Table 9 about here.]

Our estimates are statistically significant and careful analysis of coefficients k_c and k_z indicates that dividends depend more on non-housing consumption than on the expenditure ratio. In Figures 6, 7, 8 and 9, for all four versions of our housing consumption models, we plot the price-dividend ratio as a function of the log expenditure ratio for claims on the consumption bundle, non-housing consumption, the equity index and real estate, respectively.

[Figure 6 about here.]

[Figure 7 about here.]

[Figure 8 about here.]

[Figure 9 about here.]

We have assumed $\varepsilon = 1.03$, $\gamma = 3$ and $\beta = 0.999$. The Figures show that, as expected from the theoretical discussion in subsection 4.2.1, for all assets and in all models, the price-dividend ratio is a decreasing function of the expenditure ratio, as the sign of b in equation (57) is negative.

Also, the price-dividend ratio is markedly higher if the expenditure ratio is subject to potential booms/crashes (GD and DD) than if it is Gaussian (GG and DG). This is because, in the presence of such booms/crashes, even if future dividends decrease, the possibility of a strong recovery due to housing is still possible (through a crash in the expenditure ratio). Hence asset prices do not fall as much as in the Gaussian expenditure ratio framework.

Finally, the price-dividend ratio is consistently higher when non-housing consumption is affected by disasters than when it is Gaussian, keeping the distribution of the expenditure ratio unchanged. This rather counter-intuitive result can be explained as follows. In the presence of crashes in non-housing consumption, the stochastic discount factor M_{t+1} increases (towards 1) relatively to the Gaussian case because the term c_{t+1} present in the expression of the pricing kernel (see equation (9)) is expected to be relatively smaller. This causes the price-dividend ratio to be higher. It is noteworthy that this undesirable characteristic of M_{t+1} is closely associated with power utility. Adopting more complex preferences that disentangle intertemporal elasticity of substitution from risk aversion, such as Epstein-Zin's for example, would lead to a different result.

5.6 Rare events and the risk premium

In this section we study the impact of rare disaster events on the risk premium for our four models (GG, DG, GD and DD) and the four risky assets defined in the preceding subsection.

We analyze the annualized expected excess return on the underlying asset as a function of risk aversion. For consistency, we have used throughout $\varepsilon = 1.03$. The results are presented in Figures 10, 11, 12 and 13. Note that results obtained from the full sample are very close to those obtained from the pre-crisis sample, except for the non-housing consumption risk premium which is uniformly larger for the full sample which includes the financial and economic crises.

[Figure 10 about here.]

[Figure 11 about here.]

[Figure 12 about here.]

[Figure 13 about here.]

These Figures consistently show that the model that produces the highest risk premium for all assets is the housing consumption model (DD) which allows for disasters on both non-housing consumption and housing expenditures. In particular, we observe again that it is the presence of booms/crashes in the expenditure ratio that essentially increases the implied premium. The contribution of disasters in non-housing consumption is only marginal.

The CRSP equity index deserves a special attention as it justifies a comparison between our models and alternative one-good models. We computed the risk premium for our three values of ε (1.03, 1.05 and 1.4) and for $\gamma = 3$ and 5 for the four housing consumption models as well as for the standard CCAPM and the one-good single power law model of Barro and Jin (2011) (BJ). The consumption used in the last two models is the standard non-durables and services aggregate. Our results appear in Table 10.

[Table 10 about here.]

Neither the CCAPM nor the BJ model are able to produce an annualized risk premium above 0.1% across all sets of parameters. By contrast, for $\varepsilon = 1.05$ and $\gamma = 5$, both GD and DD models deliver a risk premium of 7.4%, which is 1.4% above historical observations while the Housing CCAPM GG imply 3.6%, which is 2.4% below the target. For parameters $\varepsilon = 1.03$ and $\gamma = 3$, models GD and DD imply an expected excess return of 5.4%, which is much in line with historical observations while the Housing CCAPM GG is quite far away at 2.8%. Therefore, with the set of preference parameters $\varepsilon = 1.03, \gamma = 3$ and $\varepsilon = 1.05, \gamma = 5$ our housing consumption DD model goes a long way in solving simultaneously the risk-free rate puzzle and the equity premium puzzles.

In Table 11 we report the standard deviation of annualized excess returns implied by all six models. We find that all housing consumption models imply standard deviations that are higher than in the CCAPM and the BJ model, in accordance with our theoretical model (see equation (39) of Proposition 4). Also, not surprisingly, both non-housing and housing rare disaster events contribute to increasing the volatility of excess returns. We find that for the GD and DD models, the combination $\varepsilon = 1.03, \gamma = 3$ produces a standard deviation of 24.2%, which overshoots the historical level (15.3%). For the value of ε estimated from the data (1.4), all standard deviations are about 14.3%, close to the average level.

[Table 11 about here.]

6 Term structure of interest rates

Our analytical framework can also be used to derive a novel closed-form expression for the term structure of interest rates. This being done, we can assess precisely to what extent the presence of rare disaster events impacts the level and shape of the yield curve.

6.1 Rare disaster events and the yield curve

The n-period yield is defined as:

$$y_{t,t+n} = -\frac{1}{n} \log E_t \left[\prod_{i=0}^{n-1} M_{t+i+1} \right] \tag{45}$$

The quantity $y_{t,t+n}$ corresponds to the yield associated with the discount bond paying 1 unit of non-housing consumption at time $t+n$. For $n = 1$, the n-period yield is the risk-free rate:

$$y_{t,t+1} = r_{f,t+1} \tag{46}$$

For longer periods we derive the following closed-form expression.

Proposition 7. *In the Housing Rare Event CCAPM, the n-period yield is approximated by:*

$$
\begin{aligned}
y_{t,t+n} = & -\log\beta + \frac{1}{\psi}\mu_c - \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_c^2 \\
& -K_{v_c}\left(-\frac{1}{\psi}\right) \\
& -\frac{1}{n}\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\left(\rho^n-1\right)\left(\log z_t - \mu_z\right) \\
& -\frac{1}{2}\frac{1}{n}\frac{1-\rho^n}{1-\rho}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2 \sigma_z^2 \\
& -\frac{1}{n}\sum_{i=0}^{n-1} K_{v_z}\left(\rho^i \frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)
\end{aligned}
\tag{47}
$$

and

$$
\begin{aligned}
K_{v_c}(\xi) &= \log\left(1-p_c+p_c e^{\xi v^d_{c,\min}}\frac{\alpha_c}{\alpha_c+\xi}\right) \\
K_{v_z}(\xi) &= \log\left(1-p_z+p_z\frac{\alpha_z^2}{\alpha_z^2-\xi^2}\right)
\end{aligned}
$$

Proof. See Appendix G. □

Expression (47) carries several intuitions. First, the slope of the term structure of interest rates is time-varying as it is a function of the expenditure ratio. Second, since non-housing and housing expenditures are, as already noted, observable macroeconomic variables, we can relate the yield curve to the information relative to consumption composition. More precisely, we can establish the following pattern:

Proposition 8. *Under the housing rare disaster event assumption, during normal economic conditions, i.e. when* $\log z_t = \mu_z$*, the yield curve is increasing.*

Proof. See Appendix H. □

However, during severe recessions, that is when the expenditure ratio is high, the third line of (47) tends to mitigate this effect and push short term rates higher than during normal economic conditions. Long term rates will be less affected because, since the expenditure ratio is persistent, good times will return in the long run and the effect of precautionary savings due to the presence of housing will decrease with the bond maturity.

The fifth line of (47) corresponds to the precautionary savings due to booms/crashes in the expenditure ratio. This effect is significant in the short term but vanishes gradually at longer maturities. Therefore, our model predicts that in the presence of housing rare events, the yield curve is steeper than in the Housing CCAPM.

Although steep in its short term end, the yield curve flattens in its long term end since:

$$\lim_{n\to\infty} y_{t,t+n} = -\log\beta + \frac{1}{\psi}\mu_c - \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_c^2 - K_{v_c}\left(-\frac{1}{\psi}\right) \tag{48}$$

as proved in Appendix H.

In other words, long term yields converge to the risk-free rate implied by Barro and Jin (2011)'s i.i.d. single power law rare disaster event model. In particular, in the absence of disasters, equation (48) reduces to:

$$\lim_{n\to\infty} y_{t,t+n} = -\log\beta + \frac{1}{\psi}\mu_c - \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_c^2 \tag{49}$$

which corresponds to the (constant) risk-free rate implied by the standard CCAPM. Equation (49) provides an intuitive explanation why the standard model implies such a high risk-free rate: it corresponds to the long term equilibrium rate implied by the Housing CCAPM model.

6.2 Empirical Findings

Figure 14 displays the yield curves during normal economic conditions, i.e. when $\log z_t = \mu_z$, for all four housing consumption models (GG, DG, GD and DD). We have used $\varepsilon = 1.03$ and $\gamma = 3$, as this couple of parameters is our choice set for the anchor of the term structure, the risk-free rate. Note that only results from the full sample are reported, for the same reason as above.

[Figure 14 about here.]

As expected, all curves are upward-sloping and flattening as maturity increases. In particular, in the full sample the spread between the ten-year and the one-month yields is 1.1% for the Housing CCAPM (GG) and 2.7% for the Housing Rare Event CCAPM (DD) model, figures which are rather realistic. These results are at odds with those of Piazzesi and Schneider (2007), Ang et al. (2008) and Gillman et al. (2015), who predict either flat or downward-sloping yield curves. A plausible explanation for our housing framework to imply upward-sloping real curves during normal economic conditions is the assumed absence of persistence in non-housing consumption. Indeed, if the latter were persistent, future non-housing consumption growth would be correlated with current growth and this correlation would decrease with the horizon. During relatively good times, for instance, agents who foresee that less favorable times are looming would hedge against a fall in non-housing consumption by buying assets whose return are less correlated with current consumption growth. They would consequently increase their demand for long term bonds, thereby exerting a downward pressure on long term yields. Therefore, adding persistence into the dynamics of non-housing consumption would reduce the positive slope of the yield curve. Another, non mutually exclusive route would consist in relaxing the independence assumption between the growth rates of non-housing consumption and the expenditure ratio. Let us assume they are positively correlated. Taking the same example, during relatively good times non-housing consumption would then increase more than housing expenditures do. Agents would again hedge against a relative decrease in non-housing consumption by buying assets whose return are less correlated with current consumption growth, with the same effect as above on long term yields.

A related question is how current economic conditions affect the term spreads in our Housing Rare Event (DD) model. For instance, during pronounced expansions where non-housing consumption is high and $\log z_t$ is smaller than μ_z, agents fear future bad times and expect their expenditure ratio to increase in the short term. Although bond coupons do not depend on this variable, the value of short term bonds will increase as the discount rate decreases (the pricing kernel M_{t+1} increases). Long term bonds will be less affected because of the long horizon de-correlation effect. We therefore expect term spreads to decrease when the expenditure ratio is high and to increase when it is low. This is what we do observe in Figure 15 where the yield curve obtained from the Housing Rare Event model (DD) is plotted for three different economic conditions characterized by the level of $\log z_t$: The ten-year/one-month spread is 2.7% during normal economic conditions, 2.85% when the expenditure ratio is low and 2.53% when it is high.

[Figure 15 about here.]

7 Conclusion

We build a new housing consumption-based asset pricing model that allows for fat-tailed crashes to affect the dynamics of both non-housing and housing expenditures, and thereby nests both the housing consumption CCAPM where the two types of consumption are Gaussian and the one-good rare disaster event framework. Extending the cumulant based analytical approach to a two-good economy enables us to derive closed-form solutions for the risk-free rate, the risk premium, the volatility of excess returns and the yield curve. Estimating the model using U.S. consumption data over the period January 1959 to November 2015 leads to levels of the risk-free rate, the equity risk premium and its volatility that are more realistic than those implied by nested models. In particular, we find that, although introducing non-housing rare disaster events solves the standard asset pricing puzzles only marginally, modeling housing rare disaster events is genuinely effective in producing lower interest rates and higher and more volatile risk premia than what is currently offered in the literature. In addition, we justify on theoretical grounds that the housing consumption framework with CRRA utility tends to produce upward-sloping yield curves, and interpret the standard CCAPM risk-free rate as the rate that prevails during extreme recessions.

While estimating our rare disaster events model, we dispense of the implausible assumption that disasters are similar across countries. By contrast, using monthly, not annual, observations allows us to consider U.S. consumption data exclusively and to focus on the distinction between non-housing expenditures and housing services.

Our versatile analytical framework, based on cumulant generating functions and exponential affine approximations, is relevant to other contexts and issues. In effect, whenever the CGFs of the data-generating processes are known analytically and the state variables are persistent and mildly volatile, our techniques can readily be applied. In particular, we could analyze asset pricing issues within a monetary economy in which a special role is devoted to the Central Bank. Policy recommendations as to the specificity of the real estate part of the economy would be vindicated, as in particular this sector provides a powerful hedge against inflation.

Nevertheless, in the present housing consumption framework, CRRA preferences impose strong constraints on the agents' intertemporal elasticity of substitution and structurally limit the ability of even our more general (DD) model to solve simultaneously the risk-free rate, risk premium and volatility puzzles. A natural and desirable extension of our housing RDE model would allow for recursive preferences as in Epstein and Zin (1989). This research is under way.

Other avenues are promising as well. For instance, it would be interesting to investigate how introducing (relatively large) frictions in the housing market impacts the magnitude and significance of our results. Also, an elegant way to generalize the consumption model would consist in allowing for time-varying probability of disaster and volatility in the dynamics of the expenditure ratio.

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Appendix

A Derivation of the expenditure ratio's conditional probability density function

In this section we derive the analytical expression for the expenditure ratio growth probability density function. We start by deriving the exact expression for the cumulative distribution function F_z.

Using the fact that all processes are mutually independent, we can write, using Bayes'rule:

$$P\left(\Delta \log z_{t+1} < x | z_t\right) = \int_{-\infty}^{+\infty} P\left(\Delta \log z_{t+1} < x | v_{z,t+1} = v, z_t\right) f_{v_z}(v)\, dv$$

$$f_{v_z}(v) = (1 - p_z)\,\delta(v) + p_z f_L(v)$$

Consequently:

$$\begin{aligned} P\left(\Delta \log z_{t+1} < x | z_t\right) &= \int_{-\infty}^{+\infty} N\left(\frac{x - ((\rho - 1)(\log z_t - \mu_z) + v)}{\sigma_z}\right) f_{v_z}(v)\, dv \\ &= (1 - p_z)\, N\left(\frac{x - (\rho - 1)(\log z_t - \mu_z)}{\sigma_z}\right) \\ &+ p_z \int_{-\infty}^{+\infty} N\left(\frac{x - ((\rho - 1)(\log z_t - \mu_z) + v)}{\sigma_z}\right) f_L(v)\, dv \end{aligned} \tag{50}$$

We deduce the following semi-analytical expression for the expenditure ration p.d.f.:

$$f_z(x, z_t) = \frac{\partial P}{\partial x}\left(\Delta \log z_{t+1} < x | z_t\right) \tag{51}$$

$$= \frac{1}{\sigma_z}\begin{pmatrix} (1 - p_z)\, n\left(\frac{x - (\rho - 1)(\log z_t - \mu_z)}{\sigma_z}\right) \\ + p_z \int_{-\infty}^{+\infty} n\left(\frac{x - ((\rho - 1)(\log z_t - \mu_z) + v)}{\sigma_z}\right) f_L(v)\, dv \end{pmatrix} \tag{52}$$

The integral term on the r.h.s. of (52) can be simplified as follows:

$$\int_{-\infty}^{+\infty} n\left(\frac{x - ((\rho - 1)(\log z_t - \mu_z) + v)}{\sigma_z}\right) f_L(v)\, dv$$

$$
\begin{aligned}
&= \frac{1}{2}\frac{\alpha_z}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-\frac{1}{2}\left(\frac{x-((\rho-1)(\log z_t-\mu_z)+v)}{\sigma_z}\right)^2-\alpha_z|v|}dv \\
&= \frac{1}{2}\frac{\alpha_z}{\sqrt{2\pi}}\left(\int_{-\infty}^{0} e^{-\frac{1}{2}\left(\frac{x-((\rho-1)(\log z_t-\mu_z)+v)}{\sigma_z}\right)^2+\alpha_z v}dv+\int_{0}^{+\infty} e^{-\frac{1}{2}\left(\frac{x-((\rho-1)(\log z_t-\mu_z)+v)}{\sigma_z}\right)^2-\alpha_z v}dv\right) \\
&= \frac{1}{2}\frac{\alpha_z}{\sqrt{2\pi}}\begin{pmatrix} e^{\frac{1}{2}\left(\alpha_z^2\sigma_z^2+2\alpha_z\widetilde{x}\right)}\int_{-\infty}^{0} e^{-\frac{1}{2\sigma_z^2}\left(v-\left(\widetilde{x}+\alpha_z\sigma_z^2\right)\right)^2}dv \\ +e^{\frac{1}{2}\left(\alpha_z^2\sigma_z^2-2\alpha_z\widetilde{x}\right)}\int_{0}^{+\infty} e^{-\frac{1}{2\sigma_z^2}\left(v-\left(\widetilde{x}-\alpha_z\sigma_z^2\right)\right)^2}dv \end{pmatrix} \\
\widetilde{x} &= x-(\rho-1)\left(\log z_t-\mu_z\right)
\end{aligned}
$$

By changing variables we can write:

$$
\begin{aligned}
\int_{-\infty}^{0} e^{-\frac{1}{2\sigma_z^2}\left(v-\left(\widetilde{x}+\alpha_z\sigma_z^2\right)\right)^2}dv &= \sigma_z\int_{-\infty}^{-\frac{\widetilde{x}+\alpha_z\sigma_z^2}{\sigma_z}} e^{-\frac{u^2}{2}}du \\
\int_{0}^{+\infty} e^{-\frac{1}{2\sigma_z^2}\left(v-\left(\widetilde{x}-\alpha_z\sigma_z^2\right)\right)^2}dv &= \sigma_z\int_{-\frac{\widetilde{x}-\alpha_z\sigma_z^2}{\sigma_z}}^{+\infty} e^{-\frac{u^2}{2}}dv = \sigma_z\int_{-\infty}^{\frac{\widetilde{x}-\alpha_z\sigma_z^2}{\sigma_z}} e^{-\frac{u^2}{2}}dv
\end{aligned}
$$

Hence:

$$
\int_{-\infty}^{+\infty} n\left(\frac{x-((\rho-1)(\log z_t-\mu_z)+v)}{\sigma_z}\right)f_L(v)\,dv = \frac{1}{2}\sigma_z\alpha_z\begin{pmatrix} e^{\frac{1}{2}\left(\alpha_z^2\sigma_z^2+2\alpha_z\widetilde{x}\right)}N\left(-\frac{\widetilde{x}+\alpha_z\sigma_z^2}{\sigma_z}\right) \\ +e^{\frac{1}{2}\left(\alpha_z^2\sigma_z^2-2\alpha_z\widetilde{x}\right)}N\left(\frac{\widetilde{x}-\alpha_z\sigma_z^2}{\sigma_z}\right)\end{pmatrix}
$$

and finally get the expression for the expenditure ratio p.d.f:

$$
f_z(x,z_t) = \frac{1}{\sigma_z}\left((1-p_z)\,n\left(\frac{\widetilde{x}}{\sigma_z}\right)+\frac{1}{2}p_z\sigma_z\alpha_z\begin{pmatrix} e^{\frac{1}{2}\left(\alpha_z^2\sigma_z^2+2\alpha_z\widetilde{x}\right)}N\left(-\frac{\widetilde{x}+\alpha_z\sigma_z^2}{\sigma_z}\right) \\ +e^{\frac{1}{2}\left(\alpha_z^2\sigma_z^2-2\alpha_z\widetilde{x}\right)}N\left(\frac{\widetilde{x}-\alpha_z\sigma_z^2}{\sigma_z}\right)\end{pmatrix}\right)
$$

B Derivation of the risk-free rate

We start from the dynamics of non-housing consumption, equations (15) and (16), and of the expenditure ratio, equations (17) and (18). We want to derive the risk-free rate defined by:

$$r_{f,t+1} = -\log E_t \left[M_{t+1}\right]$$

We assume that the following conditions on the tail exponents hold:

$$\begin{aligned}
&\frac{1}{\psi} < \alpha_c \\
&\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi\left(\varepsilon-1\right)} < \alpha_z
\end{aligned}$$

We recall the exact expression for the SDF:

$$\begin{aligned}
M_{t+1} &= \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\psi}}\left(\frac{\frac{z_{t+1}}{1+z_{t+1}}}{\frac{z_t}{1+z_t}}\right)^{\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}} \\
&= \beta e^{-\frac{1}{\psi}\Delta\log(c_{t+1})+\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\log\left(\frac{\frac{z_{t+1}}{1+z_{t+1}}}{\frac{z_t}{1+z_t}}\right)}
\end{aligned}$$

By expanding the log expenditure share, $\log\frac{z_{t+1}}{1+z_{t+1}}$, to the first order around the log expenditure ratio $\log z_t$, we can write:

$$\log\left(\frac{\frac{z_{t+1}}{1+z_{t+1}}}{\frac{z_t}{1+z_t}}\right) \simeq \frac{1}{1+z_t}\Delta\log z_{t+1}$$

and find a more convenient expression for the SDF:

$$M_{t+1} \simeq \beta e^{-\frac{1}{\psi}\Delta\log(c_{t+1})+\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\Delta\log z_{t+1}}$$

We deduce:

$$\begin{aligned}
E_t\left[M_{t+1}\right] &= \beta E\left[e^{-\frac{1}{\psi}\Delta\log(c_{t+1})}\right]E_t\left[e^{\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\Delta\log z_{t+1}}\right] \\
&= \beta e^{K_c\left(-\frac{1}{\psi}\right)}e^{K_{z,t}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)} \\
&\Leftrightarrow \\
\log E_t\left[M_{t+1}\right] &= \log\beta + K_c\left(-\frac{1}{\psi}\right)+K_{z,t}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi\left(\varepsilon-1\right)}\right)
\end{aligned}$$

where:

$$\begin{aligned} K_c(\xi) &= \log E\left[e^{\xi\Delta\log c_{t+1}}\right] \\ K_{z,t}(\xi) &= \log E_t\left[e^{\xi\Delta\log z_{t+1}}\right] \end{aligned}$$

K_c and $K_{z,t}$ denote respectively the CGF of the consumption and the expenditure ratio's growth rates. Under our rare disaster event assumption they can be obtained in closed form:

$$\begin{aligned} K_c\left(-\frac{1}{\psi}\right) &= \log E\left[e^{-\frac{1}{\psi}(\mu_c+u_{c,t+1}+v_{c,t+1})}\right] \\ &= \log\left(e^{-\frac{1}{\psi}\mu_c+\frac{1}{2}\left(\frac{1}{\psi}\right)^2\sigma_c^2}\right)+K_{v_c}\left(-\frac{1}{\psi}\right) \end{aligned}$$

We note that rare disaster event CGF K_{v_c} is defined on $]-\infty,\alpha_c[$. Therefore we require that $\frac{1}{\psi}<\alpha_c$, for the risk-free rate to be finite. Under this setting we have:

$$K_{v_c}\left(-\frac{1}{\psi}\right) = \log\left(1-p_c+p_c e^{-\frac{1}{\psi}v^d_{c,\min}}\frac{\alpha_c}{\alpha_c-\frac{1}{\psi}}\right) \tag{53}$$

$$K_c\left(-\frac{1}{\psi}\right) = -\frac{1}{\psi}\mu_c+\frac{1}{2}\left(\frac{1}{\psi}\right)^2\sigma_c^2+\log\left(1-p_c+p_c e^{\frac{1}{\psi}v^d_{c,\min}}\frac{\alpha_c}{\alpha_c-\frac{1}{\psi}}\right) \tag{54}$$

Similarly we derive the expenditure ratio's CGF in Appendix B:

$$\xi\in\,]-\alpha_z,\alpha_z[\,:K_{z,t}(\xi)=\xi(\rho-1)(\log z_t-\mu_z)+\frac{1}{2}\xi^2\sigma_z^2+\log\left(1-p_z+p_z\frac{1}{1-\left(\frac{\xi}{\alpha}\right)^2}\right)$$

Therefore:

$$\begin{aligned} K_{z,t}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right) &= \frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}(\rho-1)(\log z_t-\mu_z) \\ &\quad+\frac{1}{2}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2\sigma_z^2 \\ &\quad+\log\left(1-p_z+p_z\frac{1}{1-\left(\frac{\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}}{\alpha}\right)^2}\right) \end{aligned}$$

We deduce the expression for the risk-free rate:

$$
\begin{aligned}
r_{f,t+1} &= -\left(\log\beta + K_c\left(-\frac{1}{\psi}\right) + K_{z,t}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)\right) \\
&= -\log\beta + \frac{1}{\psi}\mu_c - \frac{1}{2}\left(\frac{1}{\psi}\right)^2\sigma_c^2 - \log\left(1-p_c+p_c e^{\frac{1}{\psi}v^d_{c,\min}}\frac{\alpha_c}{\alpha_c-\frac{1}{\psi}}\right) \\
&\quad -\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\left((\rho-1)(\log z_t-\mu_z)\right) \\
&\quad -\frac{1}{2}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2\sigma_z^2 \\
&\quad -\log\left(1-p_z+p_z\frac{\alpha_z^2}{\alpha_z^2-\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2}\right)
\end{aligned}
$$

Derivation of the expenditure ratio growth rate's CGF

The conditional CGF of the expenditure ratio growth rate is defined by:

$$
\begin{aligned}
K_{z,t}(\xi) &= \log E_t\left[e^{\xi\Delta\log z_{t+1}}\right] \\
&= \log E_t\left[e^{\xi((\rho-1)(\log z_t-\mu_z)+u_{z,t+1}+v_{z,t+1})}\right] \\
&= \xi(\rho-1)(\log z_t-\mu_z) + \frac{1}{2}\xi^2\sigma_z^2 + K_{v_z}(\xi)
\end{aligned}
$$

where:

$$K_{v_z}(\xi) = \log E\left[e^{\xi v_{z,t+1}}\right]$$

We recall the definition of the non-gaussian component:

$$
\begin{aligned}
v_{z,t+1} &= \begin{cases} 0 & \text{with probability } 1-p_z \\ v^L_{z,t+1} & \text{with probability } p_z \end{cases} \\
f_L(x) &= P\left(v^L_{z,t+1}\in[x,x+dx]\right)/dx = \frac{1}{2}\alpha_z\exp(-\alpha_z|x|)
\end{aligned}
$$

The expectation $E\left[e^{\xi v_{z,t+1}}\right]$ is easy to compute since:

$$E\left[e^{\xi v_{z,t+1}}\right]=(1-p_z)\times e^0+p_z\times E\left[e^{\xi v^L_{z,t+1}}\right]$$

the expectation $E\left[e^{\xi v^L_{z,t+1}}\right]=\int_{-\infty}^{+\infty}\frac{1}{2}\alpha_z e^{-\alpha_z|x|+\xi x}dx$ is well defined only for $\xi\in\left]-\alpha_z,\alpha_z\right[$ and its exact expression is:

$$E\left[e^{\xi v^L_{z,t+1}}\right]=\frac{1}{1-\left(\frac{\xi}{\alpha}\right)^2}$$

We conclude:

$$\xi\in\left]-\alpha_z,\alpha_z\right[:K_{z,t}(\xi)=\xi(\rho-1)(\log z_t-\mu_z)+\frac{1}{2}\xi^2\sigma_z^2+\log\left(1-p_z+p_z\frac{1}{1-\left(\frac{\xi}{\alpha}\right)^2}\right)$$

C Derivation of the approximate bundle dynamics

From the definition of the consumption bundle

$$C_t=\left(c_t^{\frac{\varepsilon-1}{\varepsilon}}+\omega s_t^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

and using expression (5) for the expenditure ratio, the bundle can be rewritten as:

$$C_t=c_t\left(1+z_t^{-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Taking the logs yields:

$$\begin{aligned}\log C_t &= \log c_t+\frac{\varepsilon}{\varepsilon-1}\log\left(1+z_t^{-1}\right) \\ &= \log c_t+\frac{\varepsilon}{\varepsilon-1}\log\left(1+e^{-\log z_t}\right)\end{aligned} \tag{55}$$

Since the expenditure ratio is very persistent as demonstrated in Section 5 it is reasonable to expand $\log\left(1+e^{-\log z_t}\right)$ to the first order around μ_z so that we can write:

$$\log\left(1+e^{-\log z_t}\right)\simeq\log\left(1+e^{-\mu_z}\right)-\frac{e^{-\mu_z}}{1+e^{-\mu_z}}(\log z_t-\mu_z) \tag{56}$$

Combining equations (55) and (56), we can write the bundle log growth as:

$$\Delta \log C_{t+1} = \log C_{t+1} - \log C_t = \Delta \log c_t - \frac{\varepsilon}{\varepsilon - 1} \frac{e^{-\mu_z}}{1 + e^{-\mu_z}} \Delta \log z_{t+1}$$

which proves Equation (28).

D Derivation of the price-dividend ratio

We derive the price-dividend ratio for a risky asset whose dividend process (d_t) is exogenously defined by equation (27). The price-dividend ratio, henceforth denoted by X_t, is assumed to have the following exponential affine form (see equation (31)):

$$X_t = a e^{b(\log z_t - \mu_z)} \tag{57}$$

We note that, $\log z_t$ being highly persistent, it is reasonable to also approximate $1 + X_t$ as exponential affine since:

$$\begin{aligned}
1 + X_t &= 1 + a e^{b(\log z_t - \mu_z)} \\
&\simeq 1 + a \left(1 + b \left(\log z_t - \mu_z\right)\right) \\
&\simeq (1 + a) \left(1 + \frac{ab}{1 + a} \left(\log z_t - \mu_z\right)\right) \\
&\simeq (1 + a)\, e^{\frac{ab}{1+a} (\log z_t - \mu_z)}
\end{aligned}$$

Rewriting the fundamental equation (30) yields:

$$\begin{aligned}
a e^{b(\log z_t - \mu_z)} &= E_t \left[M_{t+1} \frac{d_{t+1}}{d_t} (1 + a)\, e^{\frac{ba}{1+a} (\log z_{t+1} - \mu_z)} \right] \\
&= E_t \left[\beta (1 + a)\, e^{\left(-\frac{1}{\psi} + k_c\right) \Delta \log c_{t+1} + \left(\frac{1}{1+z_t} \frac{\varepsilon - \psi}{\psi(\varepsilon - 1)} + k_z + \frac{ba}{1+a}\right) \Delta \log z_{t+1} + u_{d,t+1} + \frac{ba}{1+a} (\log z_t - \mu_z)} \right]
\end{aligned}$$

which boils down to

$$\begin{aligned}
a e^{b(\log z_t - \mu_z)} &= \beta (1 + a)\, e^{\frac{ba}{1+a} ((\log z_t - \mu_z)) + K_c\left(-\frac{1}{\psi} + k_c\right) + K_{z,t}\left(\frac{1}{1+z_t} \frac{\varepsilon - \psi}{\psi(\varepsilon - 1)} + k_z + \frac{ba}{1+a}\right) + \frac{1}{2}\sigma_d^2} \qquad (58) \\
K_c(\xi) &= \log E\left[e^{\xi \Delta \log c_{t+1}}\right] = \mu_c \xi + \frac{1}{2} \xi^2 \sigma_c^2 + K_{v_c}(\xi)
\end{aligned}$$

$$K_{z,t}(\xi) = \log E_t\left[e^{\xi\Delta\log z_{t+1}}\right] = \xi(\rho-1)(\log z_t - \mu_z) + \frac{1}{2}\xi^2\sigma_z^2 + K_{v_z}(\xi)$$

Since the log expenditure ratio is quite persistent, we can approximate (58) by expanding it to the first order around μ_z so that we get:

$$\begin{aligned} a + ab(\log z_t - \mu_z) =& \beta(1+a)\left(e^{K_c\left(-\frac{1}{\psi}+k_c\right)+K_{z,t}\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}+k_z+\frac{ba}{1+a}\right)+\frac{1}{2}\sigma_d^2}\right) \qquad (59)\\ &\times\left(\begin{array}{c} 1 \\ +\left(\begin{array}{c} \frac{ba}{1+a} + \frac{\partial K_{z,t}}{\partial\log z}\big|_{\log z=\mu_z,\xi=\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}+k_z+\frac{ba}{1+a}} \\ -\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\frac{e^{\mu_z}}{(1+e^{\mu_z})^2}\frac{\partial K_{z,t}}{\partial\xi}\big|_{\log z=\mu_z,\xi=\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}+k_z+\frac{ba}{1+a}} \end{array}\right)(\log z_t - \mu_z) \end{array}\right) \\ \frac{\partial K_{z,t}}{\partial\log z}(\xi) =& \xi(\rho-1) \\ \frac{\partial K_{z,t}}{\partial\xi}(\xi) =& (\rho-1)(\log z_t-\mu_z) + \xi\sigma_z^2 + \frac{\partial K_{v_z}}{\partial\xi}(\xi) \end{aligned}$$

Equation (59) is equivalent to:

$$a = \beta(1+a)\,e^{K_c\left(-\frac{1}{\psi}+k_c\right)+K_{z,t}\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}+k_z+\frac{ba}{1+a}\right)+\frac{1}{2}\sigma_d^2} \qquad (60)$$

and

$$\begin{aligned} b =& \frac{ba}{1+a} + (\rho-1)\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)} + k_z + \frac{ba}{1+a}\right) \qquad (61)\\ &-\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\frac{e^{\mu_z}}{(1+e^{\mu_z})^2}\times\left(\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}+k_z+\frac{ba}{1+a}\right)\sigma_z^2 + \frac{\partial K_{v_z}}{\partial\xi}(\xi)\big|_{\xi=\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}+k_z+\frac{ba}{1+a}}\right) \end{aligned}$$

E Derivation of the risk premium

We consider the risky asset associated to the dividend process defined by equation (27) and want to compute the associated risk premium:

$$EP_{t+1} = \log E_t[R_{t+1}] - r_{f,t+1}$$

First we rewrite the gross return R_{t+1} as:

$$R_{t+1} = \frac{d_{t+1}}{d_t} \frac{1 + \frac{p_{t+1}}{d_{t+1}}}{\frac{p_t}{d_t}}$$

Using the exponential affine functional form for the price-dividend ratio we get:

$$R_{t+1} = \frac{1+a}{a} e^{k_c \Delta \log c_{t+1} + \left(k_z + \frac{ab}{1+a}\right) \Delta \log z_{t+1} + \left(\frac{ab}{1+a} - b\right)(\log z_t - \mu_z) + u_{d,t+1}}$$

and deduce:

$$E_t \left[R_{t+1}\right] = \frac{1+a}{a} e^{K_c(k_c) + K_{z,t}\left(k_z + \frac{ab}{1+a}\right) + \frac{1}{2}\sigma_d^2 + \left(\frac{ab}{1+a} - b\right)(\log z_t - \mu_z)} \tag{62}$$

We recall that the price dividend ratio verifies the fundamental equation:

$$X_t = E_t \left[M_{t+1} \frac{d_{t+1}}{d_t} \left(1 + X_{t+1}\right)\right]$$

which translates into:

$$a e^{b(\log z_t - \mu_z)} = \beta \left(1 + a\right) e^{K_c\left(-\frac{1}{\psi} + k_c\right) + K_{z,t}\left(\frac{1}{1+z_t} \frac{\varepsilon - \psi}{\psi(\varepsilon - 1)} + \frac{ab}{1+a} + k_z\right) + \frac{ab}{1+a}(\log z_t - \mu_z) + \frac{1}{2}\sigma_d^2} \tag{63}$$

Combining (63) and (62) yields:

$$E_t \left[R_{t+1}\right] = \frac{1}{\beta} e^{K_c(k_c) - K_c\left(-\frac{1}{\psi} + k_c\right) + K_{z,t}\left(k_z + \frac{ab}{1+a}\right) - K_{z,t}\left(\frac{1}{1+z_t} \frac{\varepsilon - \psi}{\psi(\varepsilon - 1)} + k_z + \frac{ab}{1+a}\right)} \tag{64}$$

Using (64) and the general expression (23) for the risk-free rate, we get the following closed-form expression for the risk premium:

$$\begin{aligned} EP_{t+1} &= K_c\left(k_c\right) + K_c\left(-\frac{1}{\psi}\right) - K_c\left(-\frac{1}{\psi} + k_c\right) \\ &\quad + K_{z,t}\left(k_z + \frac{ab}{1+a}\right) + K_{z,t}\left(\frac{1}{1+z_t} \frac{\varepsilon - \psi}{\psi\left(\varepsilon - 1\right)}\right) - K_{z,t}\left(\frac{1}{1+z_t} \frac{\varepsilon - \psi}{\psi\left(\varepsilon - 1\right)} + k_z + \frac{ab}{1+a}\right) \end{aligned}$$

In the presence of rare disaster events the risk premium can be expressed analytically as:

$$EP_{t+1} = \frac{k_c}{\psi} \sigma_c^2$$

$$
\begin{aligned}
&+K_{v_c}\left(k_c\right)+K_{v_c}\left(-\frac{1}{\psi}\right)-K_{v_c}\left(-\frac{1}{\psi}+k_c\right) \\
&-\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi\left(\varepsilon-1\right)}\left(k_z+\frac{ab}{1+a}\right)\sigma_z^2 \\
&+K_{v_z}\left(k_z+\frac{ab}{1+a}\right)+K_{v_z}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi\left(\varepsilon-1\right)}\right)-K_{v_z}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi\left(\varepsilon-1\right)}+k_z+\frac{ab}{1+a}\right)
\end{aligned}
$$

where:

$$
\begin{aligned}
K_{v_c}\left(\xi\right) &= \log\left(1-p_c+p_c e^{\xi v_{c,\min}^d}\frac{\alpha_c}{\alpha_c+\xi}\right) \\
K_{v_z}\left(\xi\right) &= \log\left(1-p_z+p_z\frac{\alpha_z^2}{\alpha_z^2-\xi^2}\right)
\end{aligned}
$$

F Derivation of the excess return variance

We define the excess return associated with the risky asset by:

$$
ER_{t+1}=\log R_{t+1}-r_{f,t+1}
$$

where:

$$
\begin{aligned}
R_{t+1} &= \frac{P_{t+1}+D_{t+1}}{P_t} \\
\frac{D_{t+1}}{D_t} &= e^{k_c\Delta\log c_{t+1}+k_z\Delta\log c_{t+1}+u_{d,t+1}} \\
u_{d,t+1} &\sim N\left(0,\sigma_d^2\right)
\end{aligned}
$$

Using the exponential affine form defined in (57) we can rewrite the gross return as:

$$
\begin{aligned}
R_{t+1} &= \frac{P_{t+1}+D_{t+1}}{P_t} \\
&= \frac{D_{t+1}}{D_t}\frac{1+\frac{P_{t+1}}{D_{t+1}}}{\frac{P_t}{D_t}} \\
&= \frac{1+a}{a}e^{k_c\Delta\log c_{t+1}+\left(k_z+\frac{ab}{1+a}\right)\Delta\log z_{t+1}-\frac{b}{1+a}\left(\log z_t-\mu_z\right)+u_{d,t+1}}
\end{aligned}
$$

or equivalently in log form:

$$
\begin{aligned}
\log R_{t+1} = & \log\frac{1+a}{a} + k_c \Delta \log c_{t+1} + \left(k_z + \frac{ab}{1+a}\right)\Delta \log z_{t+1} \\
& - \frac{b}{1+a}\left(\log z_t - \mu_z\right) + u_{d,t+1}
\end{aligned}
$$

As expression (22) is rather complex, we adopt the following first order approximation

$$
r_{f,t+1} = r_{f,t+1}\left(\log z_t\right) \simeq r_{f,t+1}\left(\mu_z\right) + \frac{\partial r_{f,t+1}}{\partial \log z_t}|_{\log z_t = \mu_z}\left(\log z_t - \mu_z\right)
$$

and deduce the following approximate expression for the excess return:

$$
\begin{aligned}
ER_{t+1} &= \log\frac{1+a}{a} - r_{f,t+1}\left(\mu_z\right) + k_c \Delta \log c_{t+1} \\
&\quad + \left(k_z + \frac{ab}{1+a}\right)\Delta \log z_{t+1} - \left(\frac{b}{1+a} + \frac{\partial r_{f,t+1}}{\partial \log z_t}|_{\log z_t = \mu_z}\left(z_t\right)\right)\left(\log z_t - \mu_z\right) \\
&\quad + u_{d,t+1} \\
&= \log\frac{1+a}{a} - r_{f,t+1}\left(\mu_z\right) + k_c \mu_c \\
&\quad + k_c\left(u_{c,t+1} + v_{c,t+1}\right) \\
&\quad + \left(\left(k_z + \frac{ab}{1+a}\right)(\rho - 1) - \left(\frac{b}{1+a} + \frac{\partial r_{f,t+1}}{\partial \log z_t}|_{\log z_t = \mu_z}\left(z_t\right)\right)\right)\left(\log z_t - \mu_z\right) \\
&\quad + \left(k_z + \frac{ab}{1+a}\right)\left(u_{z,t+1} + v_{z,t+1}\right) + u_{d,t+1}
\end{aligned}
$$

We deduce the approximate variance of the excess return:

$$
\begin{aligned}
var\left(ER_{t+1}\right) &\simeq k_c^2\left(\sigma_u^2 + var\left(v_{c,t+1}\right)\right) + \sigma_d^2 \\
&\quad + \left(\left(k_z + \frac{ab}{1+a}\right)(\rho - 1) - \left(\frac{b}{1+a} + \frac{\partial r_{f,t+1}}{\partial \log z_t}|_{\log z_t = \mu_z}\right)\right)^2 var\left(\log z_t\right) \\
&\quad + \left(k_z + \frac{ab}{1+a}\right)\left(\sigma_z^2 + var\left(v_{z,t+1}\right)\right)
\end{aligned}
$$

Since $v_{c,t+1}$ and $v_{z,t+1}$ are respectively exponentically and Laplace distributed, we have:

$$
\begin{aligned}
var\left(v_{c,t+1}\right) &= \frac{1}{\alpha_c^2} \\
var\left(v_{z,t+1}\right) &= \frac{2}{\alpha_z^2}
\end{aligned}
$$

Furthermore, recalling the dynamics of the expenditure ratio:

$$\log z_{t+1} = \rho \log z_t + (1-\rho)\,\mu_z + u_{z,t+1} + v_{z,t+1}$$

it is easy to show that the unconditional variance of $\log z_t$ is:

$$var\left(\log z_t\right) = \frac{var\left(u_{z,t+1}+v_{z,t+1}\right)}{1-\rho^2} = \frac{1}{1-\rho^2}\left(\sigma_z^2 + \frac{2}{\alpha_z^2}\right)$$

We also have:

$$\begin{aligned}
\frac{\partial r_{f,t+1}}{\partial \log z_t}\Big|_{\log z_t=\mu_z} &= -\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)} \\
&\quad + \frac{e^{\mu_z}}{(1+e^{\mu_z})^3}\left(\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2\sigma_z^2 \\
&\quad + 2p_z\alpha_z^2\left(\begin{array}{c}\left(\frac{1}{\alpha_z^2-\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)}\frac{e^{\mu_z}}{(1+e^{\mu_z})^2}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right) \\ \times\left(\frac{\frac{1}{\alpha_z^2-\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2}}{1-p_z+p_z\frac{\alpha_z^2}{\alpha_z^2-\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2}}\right)\end{array}\right)
\end{aligned}$$

We can therefore conclude:

$$\begin{aligned}
var\left(ER_{t+1}\right) &\simeq k_c^2\left(\sigma_u^2+\frac{1}{\alpha_c^2}\right)+\sigma_d^2 \\
&\quad + \left(\left(k_z+\frac{ab}{1+a}\right)(\rho-1)-\left(\frac{b}{1+a}+\frac{\partial r_{f,t+1}}{\partial \log z_t}\Big|_{\log z_t=\mu_z}\right)\right)^2\frac{1}{1-\rho^2}\left(\sigma_z^2+\frac{2}{\alpha_z^2}\right) \\
&\quad + \left(k_z+\frac{ab}{1+a}\right)\left(\sigma_z^2+\frac{2}{\alpha_z^2}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial r_{f,t+1}}{\partial \log z_t}\Big|_{\log z_t=\mu_z} &= -\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)} \\
&\quad + \frac{e^{\mu_z}}{(1+e^{\mu_z})^3}\left(\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2\sigma_z^2 \\
&\quad + 2p_z\alpha_z^2\left(\begin{array}{c}\left(\frac{1}{\alpha_z^2-\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)}\frac{e^{\mu_z}}{(1+e^{\mu_z})^2}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right) \\ \times\left(\frac{\frac{1}{\alpha_z^2-\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2}}{1-p_z+p_z\frac{\alpha_z^2}{\alpha_z^2-\left(\frac{1}{1+e^{\mu_z}}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2}}\right)\end{array}\right)
\end{aligned}$$

G Derivation of the yield curve

We define the yield for the period $[t, t+n]$ by:

$$y_{t,t+n} = -\frac{1}{n} \log E_t \left[\prod_{i=0}^{n-1} M_{t+i+1} \right] \tag{65}$$

We first consider:

$$\begin{aligned}
E_t \left[\prod_{i=0}^{n-1} M_{t+i+1} \right] &= E_t \left[\prod_{i=0}^{n-1} \left(\beta \left(\frac{c_{t+i+1}}{c_{t+i}} \right)^{-\frac{1}{\psi}} \left(\frac{\frac{z_{t+i+1}}{1+z_{t+i+1}}}{\frac{z_{t+i}}{1+z_{t+i}}} \right)^{\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}} \right) \right] \\
&= \beta^n E_t \left[\left(\frac{c_{t+n}}{c_t} \right)^{-\frac{1}{\psi}} \left(\frac{\frac{z_{t+n}}{1+z_{t+n}}}{\frac{z_t}{1+z_t}} \right)^{\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}} \right] \\
&= \beta^n E_t \left[e^{-\frac{1}{\psi} \Delta \log c_{t+n} + \frac{\varepsilon-\psi}{\psi(\varepsilon-1)} \Delta \log \left(\frac{z_{t+n}}{1+z_{t+n}} \right)} \right] \\
\Delta \log c_{t+n} &= \log c_{t+n} - \log c_t \\
\Delta \log \left(\frac{z_{t+n}}{1+z_{t+n}} \right) &= \log \left(\frac{z_{t+n}}{1+z_{t+n}} \right) - \log \left(\frac{z_t}{1+z_t} \right)
\end{aligned}$$

Since non-housing consumption growth and the expenditure ratio are independent, we can separate expectations to get:

$$\begin{aligned}
E_t \left[\prod_{i=0}^{n-1} M_{t+i+1} \right] &= \beta^n E \left[e^{-\frac{1}{\psi} \Delta \log c_{t+n}} \right] E_t \left[e^{\frac{\varepsilon-\psi}{\psi(\varepsilon-1)} \Delta \log \left(\frac{z_{t+n}}{1+z_{t+n}} \right)} \right] \\
&= \beta^n E \left[e^{-\frac{1}{\psi} \Delta \log c_{t+1}} \right]^n E_t \left[e^{\frac{\varepsilon-\psi}{\psi(\varepsilon-1)} \Delta \log \left(\frac{z_{t+n}}{1+z_{t+n}} \right)} \right]
\end{aligned}$$

Expanding $\log \left(\frac{z_{t+n}}{1+z_{t+n}} \right)$ to the first order around $\log(z_t)$, we can write:

$$\Delta \log \left(\frac{z_{t+n}}{1+z_{t+n}} \right) = \frac{1}{1+z_t} \Delta \log z_{t+n}$$

Furthermore, simple recursion yields:

$$\Delta \log z_{t+n} = (\rho^n - 1)(\log z_t - \mu_z) + \sum_{i=0}^{n-1} \rho^{n-i} (u_{z,t+i} + v_{z,t+i})$$

We deduce:

$$E_t\left[e^{\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\Delta\log\left(\frac{z_{t+n}}{1+z_{t+n}}\right)}\right]=e^{\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}(\rho^n-1)(\log z_t-\mu_z)+\frac{1}{2}\frac{1-\rho^n}{1-\rho}\sigma_z^2\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)^2+\sum_{i=0}^{n-1}K_{v_z}\left(\rho^i\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)}$$

The term structure of yields therefore is given by:

$$\begin{aligned}
y_{t,t+n} &= -\log\beta+\frac{1}{\psi}\mu_c-\frac{1}{2}\left(\frac{1}{\psi}\right)^2\sigma_c^2-K_{v_c}\left(-\frac{1}{\psi}\right) \\
&\quad -\frac{1}{n}\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi\left(\varepsilon-1\right)}\left(\rho^n-1\right)\left(\log z_t-\mu_z\right) \\
&\quad -\frac{1}{2}\frac{1}{n}\frac{1-\rho^n}{1-\rho}\left(\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi\left(\varepsilon-1\right)}\right)^2\sigma_z^2 \\
&\quad -\frac{1}{n}\sum_{i=0}^{n-1}K_{v_z}\left(\rho^i\frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi\left(\varepsilon-1\right)}\right)
\end{aligned} \tag{66}$$

H Proof of proposition 8

We prove here that during normal economic conditions the yield curve is increasing. The expression for the n-period yield is given by equation (47). We note that for any strictly decreasing sequence $(x_i)_{1\leq i\leq\infty}$ we have:

$$\forall n\geq 1:\frac{1}{n}\sum_{i=1}^{n}x_i\leq\frac{1}{n-1}\sum_{i=1}^{n-1}x_i$$

Indeed:

$$\begin{aligned}
\frac{1}{n}\sum_{i=1}^{n}x_i &= \left(\frac{1}{n}\sum_{i=1}^{n-1}x_i\right)+\frac{x_n}{n} \\
&= \frac{n-1}{n}\left(\frac{1}{n-1}\sum_{i=1}^{n-1}x_i\right)+\frac{x_n}{n} \\
&= \left(\frac{1}{n-1}\sum_{i=1}^{n-1}x_i\right)+\frac{1}{n}\left(x_n-\frac{1}{n-1}\sum_{i=1}^{n-1}x_i\right)
\end{aligned}$$

Since the sequence is decreasing, we must have:

$$\left(x_n - \frac{1}{n-1}\sum_{i=1}^{n-1} x_i\right) < 0$$

and consequently:

$$\frac{1}{n}\sum_{i=1}^{n} x_i \leq \frac{1}{n-1}\sum_{i=1}^{n-1} x_i$$

Also we note that since $v_z > 0$, K_{v_z} must be strictly increasing. Hence, applying the above results to the fourth and fifth lines of (47) we deduce:

$$\begin{aligned}
&\frac{1}{n}\frac{1-\rho^n}{1-\rho} = \frac{1}{n}\sum_{i=0}^{n-1}\rho^i < \frac{1}{n-1}\sum_{i=1}^{n-2}\rho^i = \frac{1}{n-1}\frac{1-\rho^{n-1}}{1-\rho} \\
&\frac{1}{n}\sum_{i=0}^{n-1} K_{v_z}\left(\rho^i \frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right) < \frac{1}{n-1}\sum_{i=0}^{n-2} K_{v_z}\left(\rho^i \frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)
\end{aligned}$$

which guarantees that during normal economic conditions, i.e. $\log z_t = \mu_z$, the yield curve must be upward sloping.

We note that since both sequences $\left(\frac{1}{n}\frac{1-\rho^n}{1-\rho}\right)_{1\leq n}$ and $\left(\frac{1}{n}\sum_{i=0}^{n-1} K_{v_z}\left(\rho^i \frac{1}{1+z_t}\frac{\varepsilon-\psi}{\psi(\varepsilon-1)}\right)\right)_{1\leq n}$ are bounded and strictly decreasing, the yield curve flattens in the long term and its limit is given by:

$$\lim_{n\to\infty} y_{t,t+n} = -\log\beta - \frac{1}{\psi}\mu_c - \frac{1}{2}\left(\frac{1}{\psi}\right)^2\sigma_c^2 - K_{v_c}\left(-\frac{1}{\psi}\right)$$

Table 1: **Descriptive statistics for monthly consumption and financial data**

Panel A: Sub-sample statistics

	Mean (%)	**Standard Deviation (%)**	**Skewness**	**Excess Kurtosis**
$\Delta \log C_{t+1}^{NDS}$	2.504	1.317	0.015	1.467
$\Delta \log C_{t+1}^{NDNHS}$	2.005	1.685	0.127	1.215
$\Delta \log z_{t+1}$	-0.203	1.657	-0.027	1.074
$r_{f,t+1}$	1.307	0.741	0.131	1.031
$r_{m,t+1} - r_{f,t+1}$	5.845	15.092	-0.487	2.012

Panel B: Full sample statistics

	Mean (%)	**Standard Deviation (%)**	**Skewness**	**Excess Kurtosis**
$\Delta \log C_{t+1}^{NDS}$	2.236	1.253	0.127	1.704
$\Delta \log C_{t+1}^{NDNHS}$	1.782	1.639	0.007	1.767
$\Delta \log z_{t+1}$	-0.172	1.615	-0.229	1.669
$r_{f,t+1}$	0.941	0.745	0.311	0.703
$r_{m,t+1} - r_{f,t+1}$	6.082	15.259	-0.519	1.869

This table describes the summary statistics of consumption and financial data for the sub-sample (Panel A) and full sample (Panel B) periods. Superscript ^{NDS} refers to non-durables and services while ^{NDNHS} refers to non-housing consumption. Sample mean and standard deviation have been annualized.

Table 2: **Estimated intratemporal preference parameters**

	ε	ω	η (%)
Sub-sample estimate	1.427	0.254	3.142
	(0.000)	(0.000)	(0.000)
Full sample estimate	1.408	0.254	3.347
	(0.000)	(0.000)	(0.000)

This Table reports our intratemporal preference parameter estimates and their associated one-sided p-values. Results were obtained by performing a maximum likelihood estimation using the conditional probability density function for the expenditure ratio detailed in (41). η is the standard deviation of the expenditure ratio measurement error introduced in subsection 5.2. Numbers between parentheses are p-values.

Table 3: **Maximum likelihood estimates of non-housing consumption**

Panel A: maximum likelihood estimates of non-housing consumption in a model with crash

	μ_c (%)	σ_c (%)	p_c (%)	α_c	$v^d_{c,\min}$ (%)
Sub-sample estimate	0.179	0.472	1.017	222.000	0.761
	(0.000)	(0.000)	(0.228)	(0.000)	(0.150)
Full sample estimate	0.166	0.447	1.368	252.000	0.953
	(0.000)	(0.000)	(0.095)	(0.000)	(0.011)

Panel B: maximum likelihood estimates of non-housing consumption in a Gaussian model

	μ_c (%)	σ_c (%)
Sub-sample estimate	0.167	0.486
	(0.000)	(0.000)
Full sample estimate	0.148	0.473
	(0.000)	(0.000)

Panel A displays the estimates and associated one-sided p-values of the parameters of non-housing consumption dynamics in a single power law crash model as in Barro and Jin (2011). The estimation is performed by maximum likelihood based on the probability density function expressed in (42). Panel B reports the estimates and associated one-sided p-values of the parameters of non-housing consumption dynamics in the corresponding Gaussian model. The estimation is performed by maximum likelihood based on the Gaussian probability density function. Numbers between parentheses are p-values.

Table 4: **Maximum likelihood estimates of the expenditure ratio**

Panel A: maximum likelihood estimates of the expenditure ratio in a model with booms and crashes

	μ_z (%)	σ_z (%)	ρ	α_z	p_z (%)
Sub-sample estimate	158.102	0.450	0.990	80.000	2.440
	0.000	0.000	0.000	0.000	0.047
Full sample estimate	158.106	0.439	0.989	36.000	1.248
	0.000	0.000	0.000	0.000	0.034

Panel B: maximum likelihood estimates of the expenditure ratio in a Gaussian model

	μ_z (%)	σ_z (%)	ρ
Sub-sample estimate	157.933	0.477	0.991
	(0.000)	(0.000)	(0.000)
Full sample estimate	157.509	0.465	0.991
	(0.000)	(0.000)	(0.000)

Panel A reports the estimates and associated one-sided p-values of the parameters of the expenditure ratio dynamics in a Laplace distributed boom/crash model as described in dynamics (17). The estimation is performed by maximum likelihood based on the probability density function expressed in (43). Panel B displays the estimates and associated one-sided p-values of the parameters of the expenditure ratio dynamics in a homoscedastic $AR(1)$ Gaussian model as described in Piazzesi et al. (2007). The estimation is performed by maximum likelihood based on the Gaussian conditional probability density function. Numbers between parentheses are p-values.

Table 5: **Maximum likelihood estimates of non-durables and services consumption**

Panel A: maximum likelihood estimates of non-durables and services consumption in a model with crashes

	μ_c (%)	σ_c (%)	p_c (%)	α_c	$v^d_{c,\min}$ (%)
Sub-sample estimate	0.220	0.365	1.159	269.999	0.647
	(0.000)	(0.000)	(0.150)	(0.000)	(0.091)
Full sample estimate	0.193	0.351	0.611	119.999	0.551
	(0.000)	(0.000)	(0.182)	(0.000)	(0.264)

Panel B: maximum likelihood estimates of non-durables and services consumption in a Gaussian model

	μ_c (%)	σ_c (%)
Sub-sample estimate	0.209	0.380
	(0.000)	(0.000)
Full sample estimate	0.186	0.361
	(0.000)	(0.000)

Panel A displays the estimates and associated p-values of the parameters of non-durables and services consumption dynamics in a single power law crash model as in Barro and Jin (2011). The estimation is performed by maximum likelihood based on the probability density function expressed in (42). Panel B reports the estimates and associated p-values of the parameters of non-durables and services consumption dynamics in the Gaussian model. The estimation is performed by maximum likelihood based on the Gaussian probability density function. Numbers between parentheses are p-values.

Table 6: ***p*-values associated with the likelihood ratio test for nested housing consumption models**

	Sub-sample	**Full sample**
***p*-value** (%)	46.481	0.104

This table summarizes the results obtained by performing a likelihood ratio test for nested housing consumption models. The null hypothesis corresponds to the pure Gaussian model for both non-housing consumption and the expenditure ratio and the alternative is a model where the dynamics of these variates exhibit a rare disaster event component according to model (15) and (17). We perform the test on the pre-crisis sub-sample and on the full sample. p-values are one-sided.

Table 7: **Risk-free rate for various preference parameters**

ε	γ	β	$CCAPM^{NDS}$(%)	BJ^{NDS}(%)	**GG (%)**	**DG (%)**	**GD (%)**	**DD (%)**
1.030	3.000	1.000	6.740	6.670	3.470	3.450	0.890	0.870
1.050	5.000	1.000	11.080	10.960	5.920	5.870	2.150	2.100
1.408	3.000	1.000	6.740	6.670	5.300	5.280	5.180	5.150
1.408	5.000	1.000	11.080	10.960	8.590	8.550	8.330	8.280

This table displays the model-implied annualized risk-free rate in % for all our housing consumption models as well as for the standard CCAPM (CCAPM^{NDS}) and Barro and Jin (2011)'s one-good single power law model (BJ^{NDS}) for various intratemporal elasticities of substitution and relative risk aversion coefficients. Both one-good models characterize the dynamics of non-durables and services consumption. We use the parameters of non-housing consumption, expenditure ratio and non-durables and services dynamics estimated over the full sample, and an annualized subjective discount factor $\beta = 0.99$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Table 8: **Risk-free rate standard deviation for various preference parameters**

ε	γ	$CCAPM^{NDS}$(%)	BJ^{NDS}(%)	**GG (%)**	**DG (%)**	**GD (%)**	**DD (%)**
1.030	3.000	0.000	0.000	1.330	1.330	1.960	1.960
1.050	5.000	0.000	0.000	1.630	1.630	2.420	2.420
1.408	3.000	0.000	0.000	0.150	0.150	0.210	0.210
1.408	5.000	0.000	0.000	0.280	0.280	0.400	0.400

This table displays the model-implied standard deviation of the annualized risk-free rate in % for all our housing consumption models as well as for the standard CCAPM (CCAPM^{NDS}) and Barro and Jin (2011)'s one-good single power law model (BJ^{NDS}) for various intratemporal elasticities of substitution and relative risk aversion coefficients. Both one-good models characterize the dynamics of non-durables and services consumption. We use the parameters of non-housing consumption, expenditure ratio and non-durables and services dynamics estimated over the full sample, and an annualized subjective discount factor $\beta = 0.99$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Table 9: **Dividend Growth**

Panel A: CRSP dividend regression on non-housing consumption and expenditure ratio

	k_c	k_z	σ_d (%)
Sub-sample	1.055	-0.057	3.972
	(0.000)	(0.561)	(0.000)
Full sample	1.229	0.533	4.042
	(0.000)	(0.102)	(0.000)

Panel B: CRSP dividend regression on non-durables and services

	k_c	σ_d (%)
Sub-sample	0.880	3.995
	(0.001)	(0.000)

This table displays the coefficients resulting from regressing the CRSP index log dividend growth on non-housing growth and expenditure ratio (Panel A) and on non-durables and services growth (Panel B). Standard errors are corrected for overlapping data using Newey-West robustness adjustments with 12 lags. Numbers between parentheses are one-sided p-values.

Table 10: **Equity index risk premium for various preference parameters**

ε	γ	$CCAPM^{NDS}$(%)	BJ^{NDS}(%)	**GG (%)**	**DG (%)**	**GD (%)**	**DD (%)**
1.030	3.000	0.054	0.058	2.778	2.781	5.419	5.423
1.050	5.000	0.090	0.096	3.651	3.659	7.438	7.451
1.408	3.000	0.054	0.058	0.128	0.130	0.155	0.157
1.408	5.000	0.090	0.096	0.256	0.259	0.343	0.346

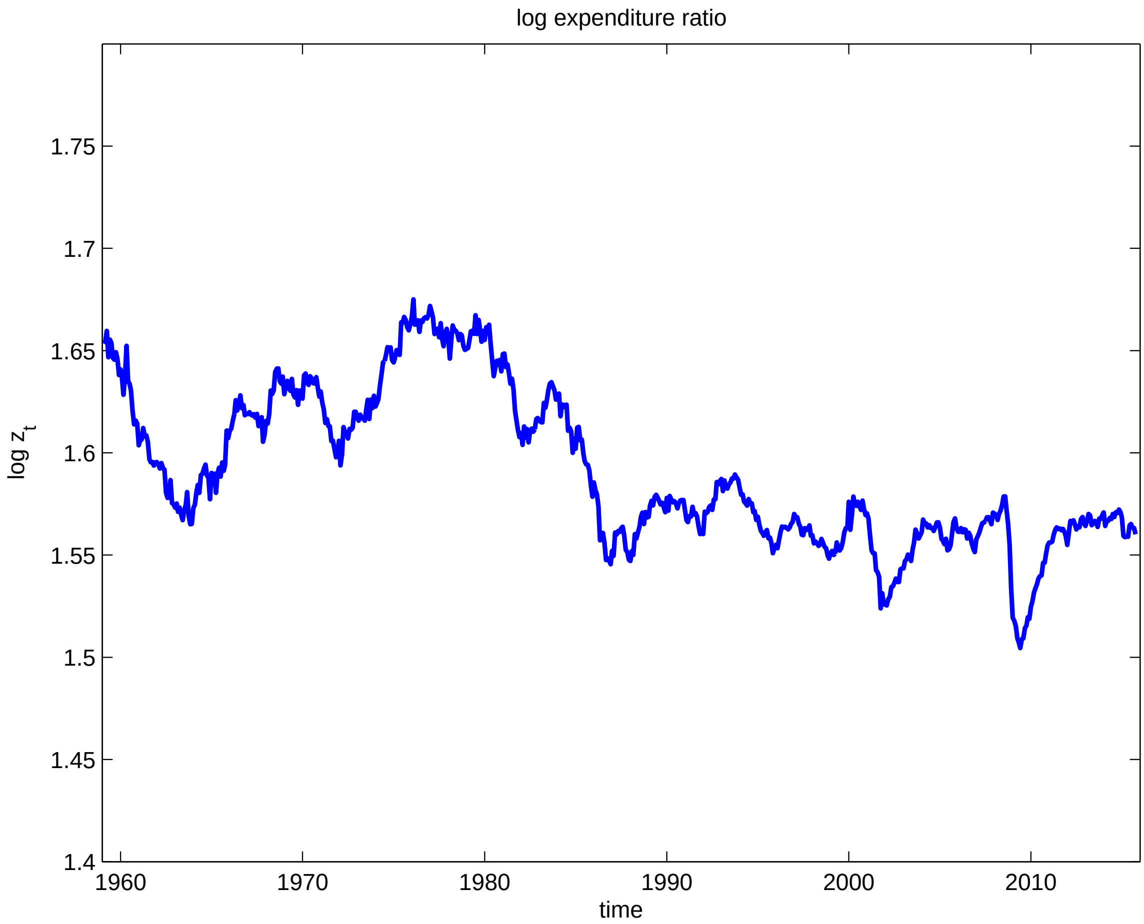
This table displays the model-implied annualized risk premium in % for the CRSP equity index for all our housing consumption models as well as for the standard CCAPM (CCAPM^{NDS}) and Barro and Jin (2011)'s one-good single power law model (BJ^{NDS}) for various intratemporal elasticities of substitution and relative risk aversion coefficients. In housing consumption models, the dividend structure assumes the functional form described in (27) whereas for CCAPM^{NDS} and BJ^{NDS}, we calibrate the dividend structure to the constrained version of (27), where $k_z = 0$ and consumption is the standard non-durables and services aggregate. We use the parameters for the non-housing consumption, expenditure ratio and non-durables and services dynamics estimated over the full sample. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Table 11: **Excess return standard deviation for various preference parameters**

ε	γ	$CCAPM^{NDS}$(%)	BJ^{NDS}(%)	**GG (%)**	**DG (%)**	**GD (%)**	**DD (%)**
1.030	3.000	14.270	14.280	19.850	19.850	24.220	24.230
1.050	5.000	14.270	14.280	20.510	20.520	25.640	25.670
1.408	3.000	14.270	14.280	14.210	14.210	14.280	14.280
1.408	5.000	14.270	14.280	14.320	14.320	14.510	14.520

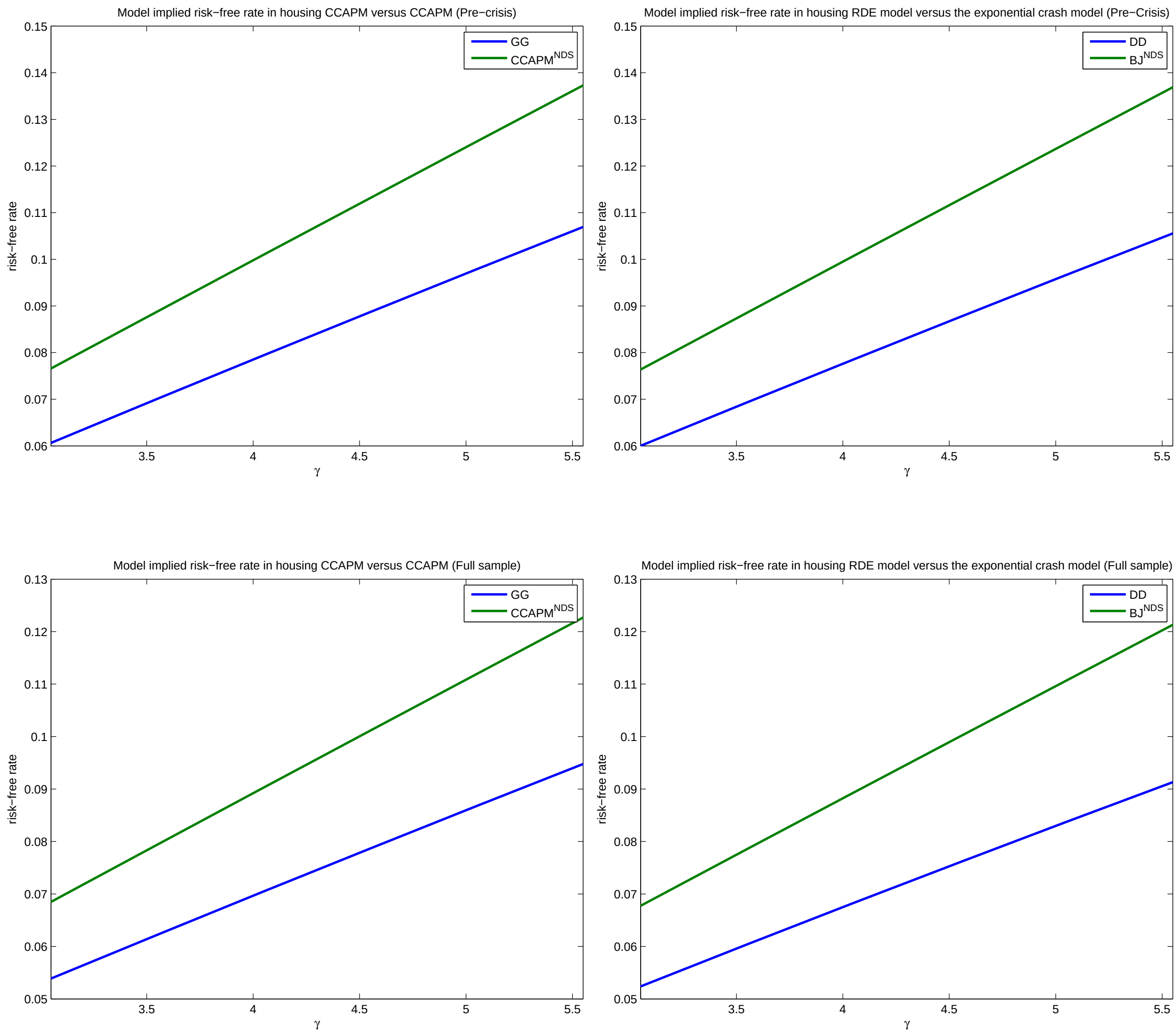
This table displays the model-implied volatility in % of the annualized excess return for the CRSP equity index for all our housing consumption models as well as for the standard CCAPM (CCAPM^{NDS}) and Barro and Jin (2011)'s one-good single power law model (BJ^{NDS}) for various intratemporal elasticities of substitution and relative risk aversion coefficients. In housing consumption models, the dividend structure assumes the functional form described in (27) whereas for CCAPM^{NDS} and BJ^{NDS}, we calibrate the dividend structure to the constrained version of (27), where $k_z = 0$ and consumption is the standard non-durables and services aggregate. We use the parameters for the non-housing consumption, expenditure ratio and non-durables and services dynamics estimated over the full sample. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Figure 1: **Log Expenditure Ratio**



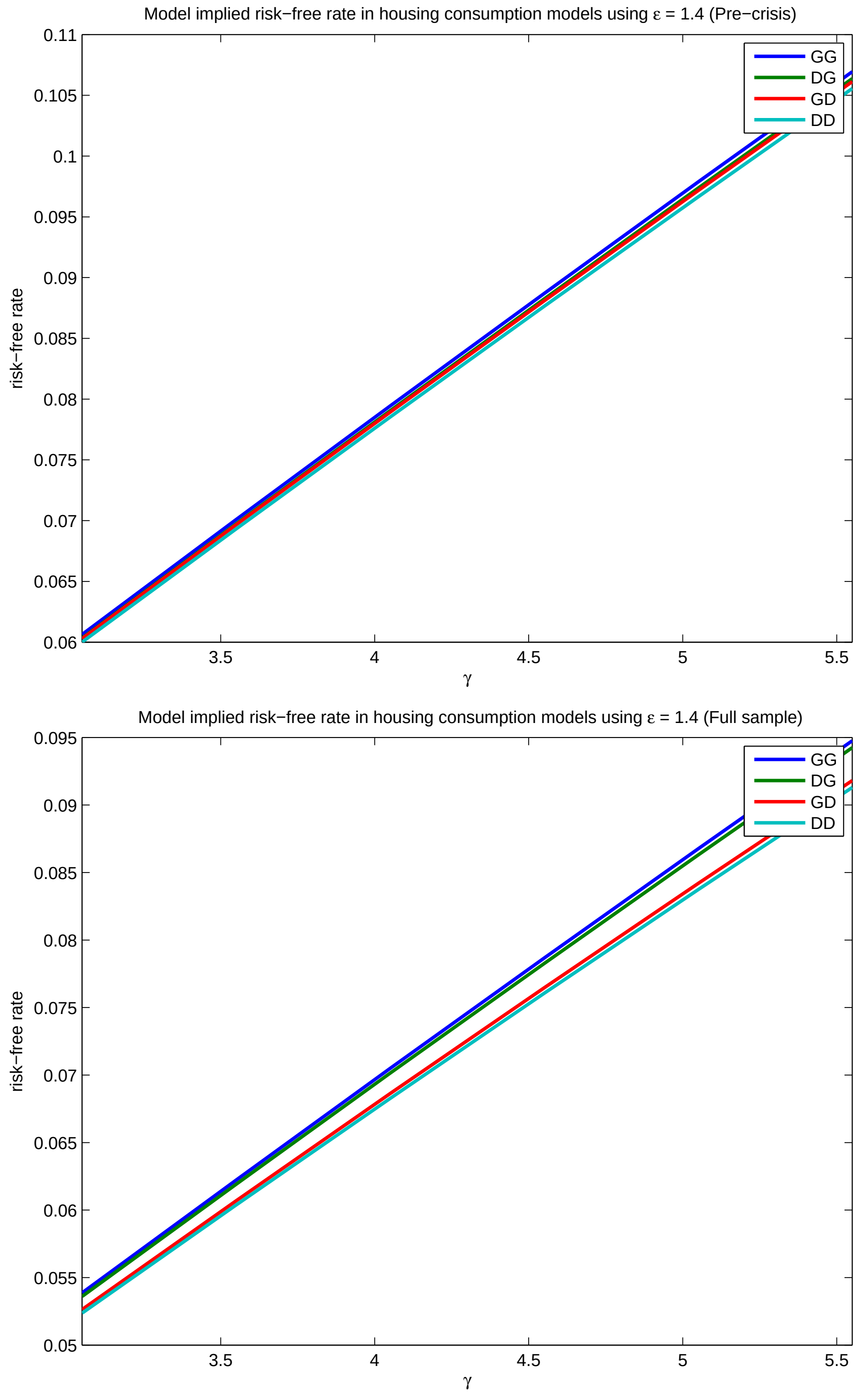
This figure displays the log expenditure ratio over the period [01-1959,11-2015]. Data are monthly.

Figure 2: **Risk-free rate comparison between one and two goods CCAPM models**



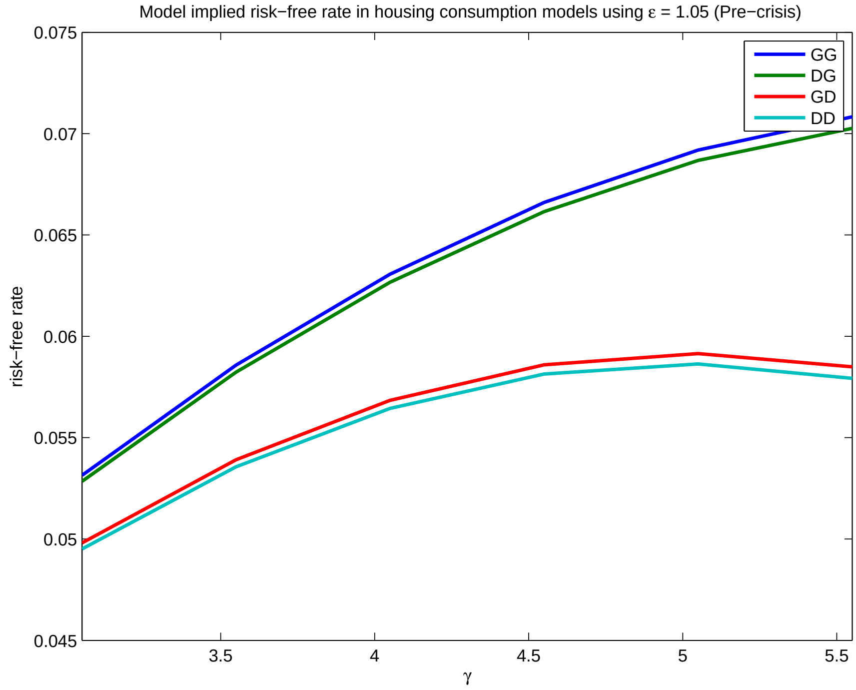
This figure compares the model-implied annualized risk-free rate between housing consumption models and one-good models for various levels of risk-aversion. Estimation is performed over the sub-sample for the top graphs and over the full sample for the bottom ones. The left-hand graph compares the Housing CCAPM (GG) versus the standard CCAPM (CCAPM^{NDS} based on non-durables and services). The right-hand graph compares the Housing Rare Event Model (DD) against the single exponential crash model of Barro and Jin (2011)'s (BJ^{NDS}, also based on non-durables and services). Acronyms GG and DD refer to the assumed probability distributions, where the first letter describes the distribution of non-housing consumption and the second letter describes that of the expenditure ratio. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio. We have used $\varepsilon = 1.4$. The annualized subjective discount factor is $\beta = 0.999$.

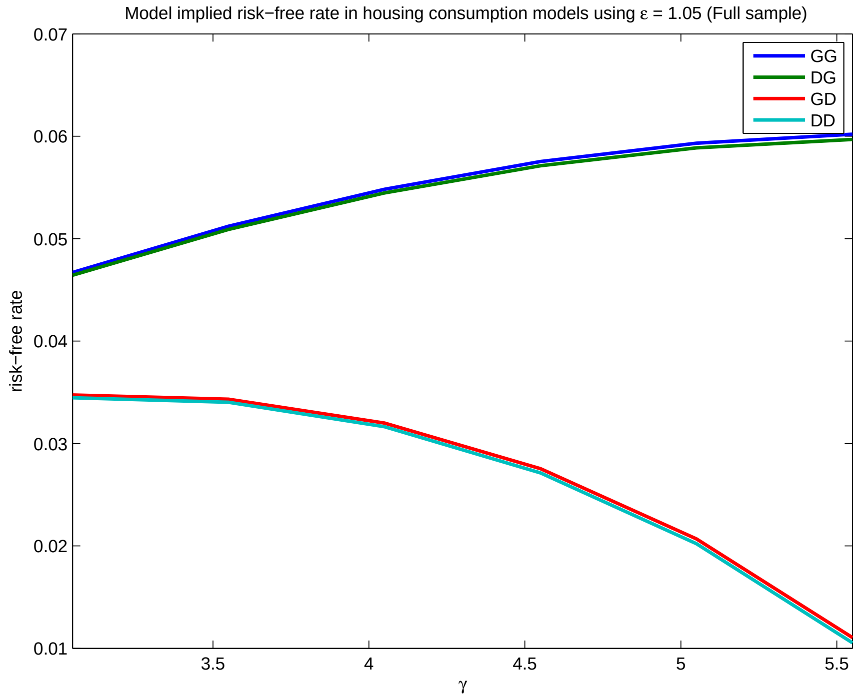
Figure 3: **Risk-free rate comparison between different housing consumption models for $\varepsilon = 1.4$**



This figure displays the model-implied annualized risk-free rate for all our housing consumption models. We have used $\varepsilon = 1.4$. The annualized subjective discount factor is $\beta = 0.999$. The top graph uses non-housing consumption and expenditure ratio parameters estimated over the sub-sample and the bottom graph uses the full sample. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

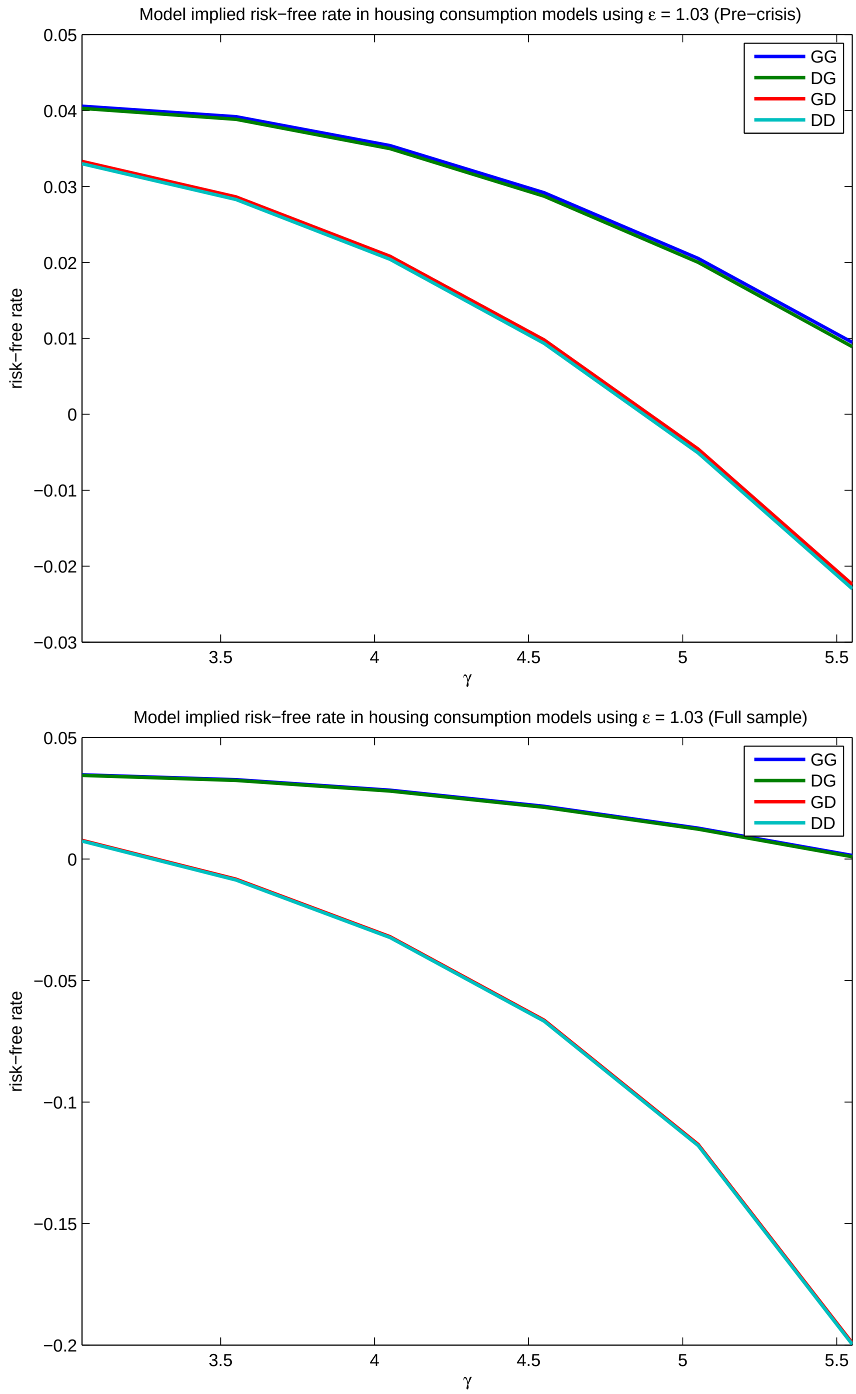
Figure 4: **Risk-free rate comparison between different housing consumption models for** $\varepsilon = 1.05$





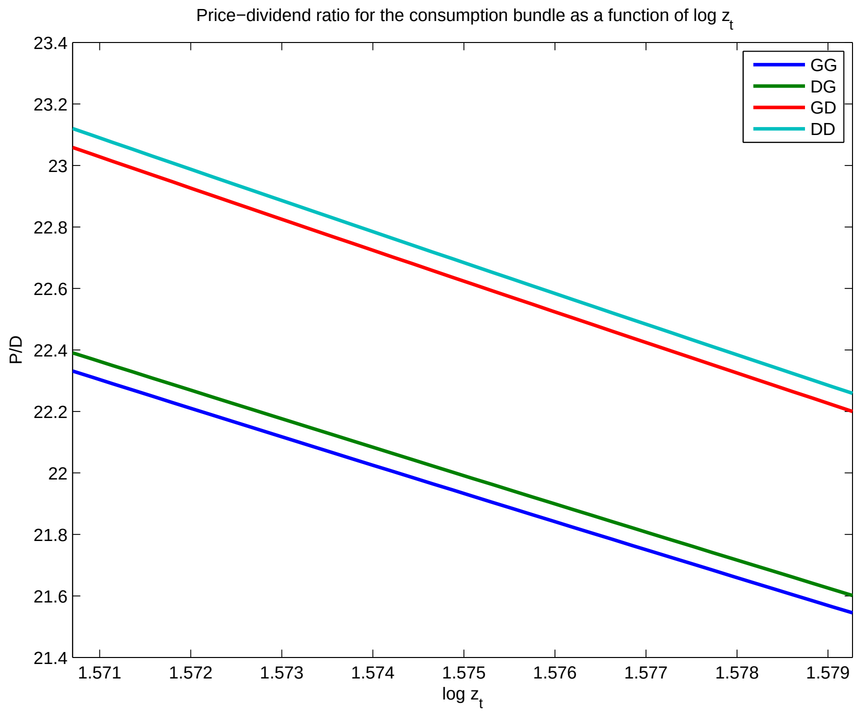
This figure displays the model-implied annualized risk-free rate for all our housing consumption models. ε is equal to 1.05. The annualized subjective discount factor is $\beta = 0.999$. The top graph uses non-housing consumption and expenditure ratio parameters estimated over the sub-sample and the bottom graph uses the full sample. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Figure 5: **Risk-free rate comparison between different housing consumption models for $\varepsilon = 1.03$**



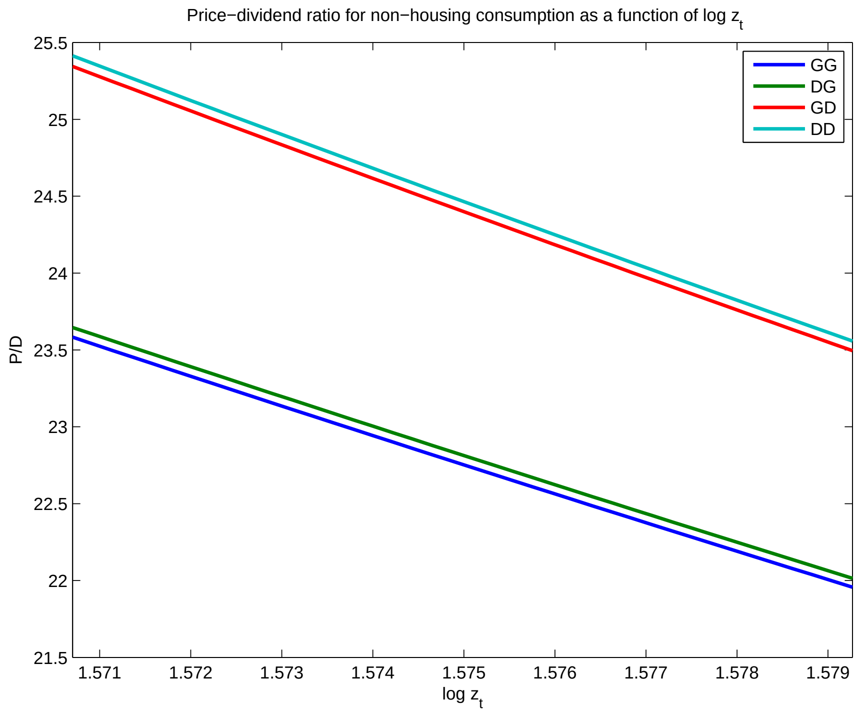
This figure displays the model-implied annualized risk-free rate for all our housing consumption models. We have used $\varepsilon = 1.03$. The annualized subjective discount factor is $\beta = 0.999$. The top graph uses non-housing consumption and expenditure ratio parameters estimated over the subsample and the bottom graph uses the full sample. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Figure 6: **Price-dividend ratio associated with the consumption bundle as a function of the expenditure ratio**



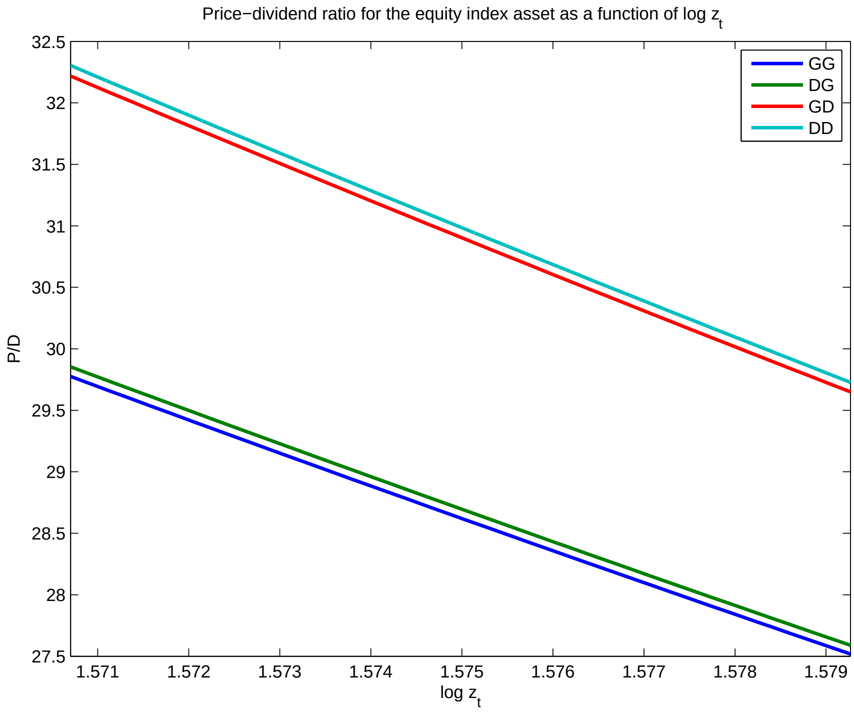
This figure displays the model-implied price-dividend ratio associated with the consumption bundle dividend as a function of $\log z_t$. We have used non-housing consumption and expenditure ratio parameters estimated over the full sample. We have used $\varepsilon = 1.03, \gamma = 3$ and an annualized subjective discount factor $\beta = 0.999$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Figure 7: **Price-dividend ratio associated with non-housing as a function of the expenditure ratio**



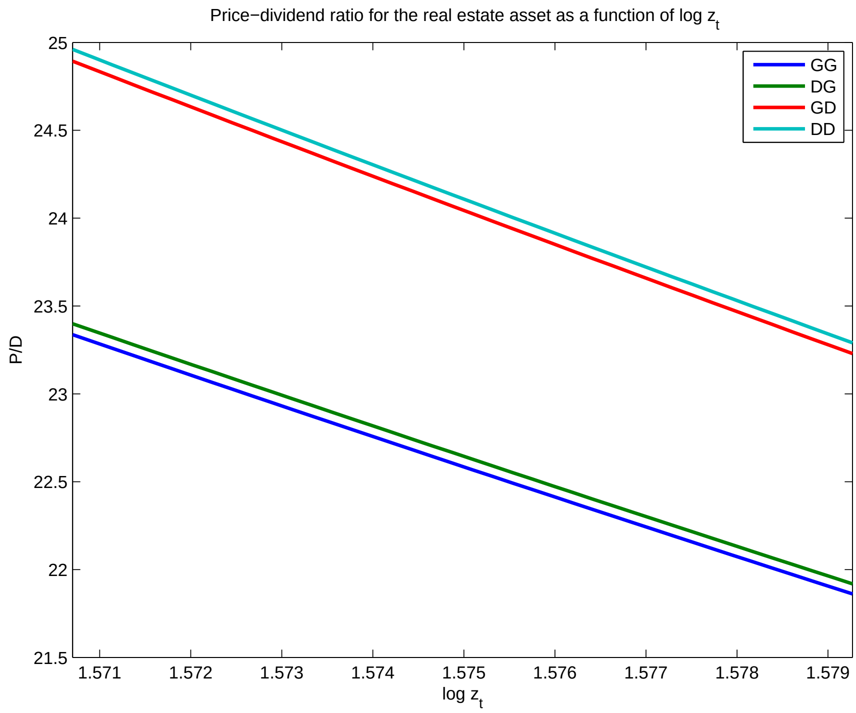
This figure displays the model-implied price-dividend ratio associated with the non-housing consumption dividend as a function of $\log z_t$. We have used non-housing consumption and expenditure ratio parameters estimated over the full sample. We have used $\varepsilon = 1.03, \gamma = 3$ and an annualized subjective discount factor $\beta = 0.999$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Figure 8: **Price-dividend ratio associated with the equity index as a function of the expenditure ratio**



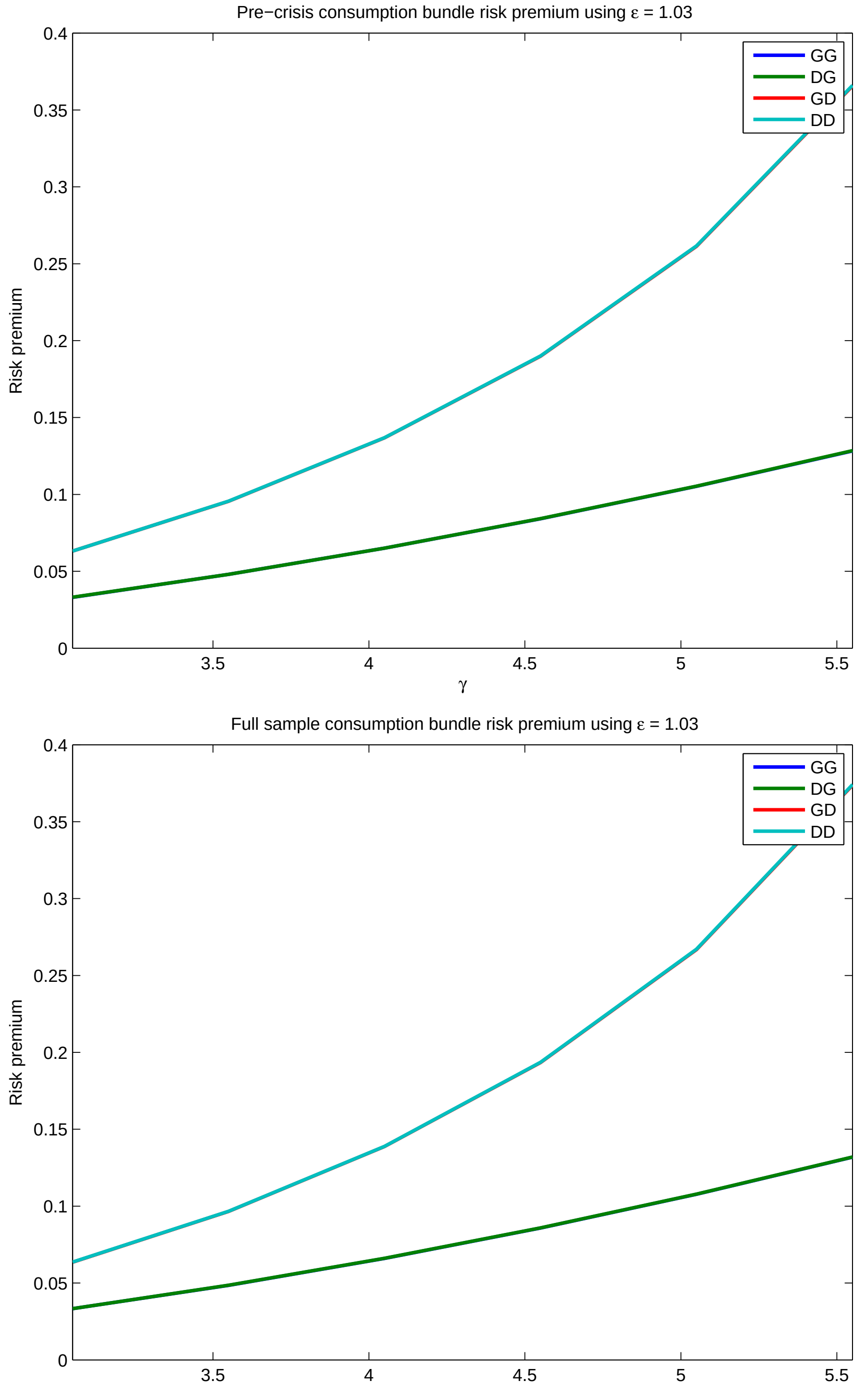
This figure displays the model-implied price-dividend ratio associated with equity index as a function of $\log z_t$. The dividend structure used for this asset is the affine form introduced in (27), which is estimated based on the CRSP value-weighted index. The resulting OLS coefficients are reported in Table 9. We have used non-housing consumption and expenditure ratio parameters estimated over the full sample. We have used $\varepsilon = 1.03, \gamma = 3$ and an annualized subjective discount factor $\beta = 0.999$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Figure 9: **Price-dividend ratio associated with real estate as a function of the expenditure ratio**



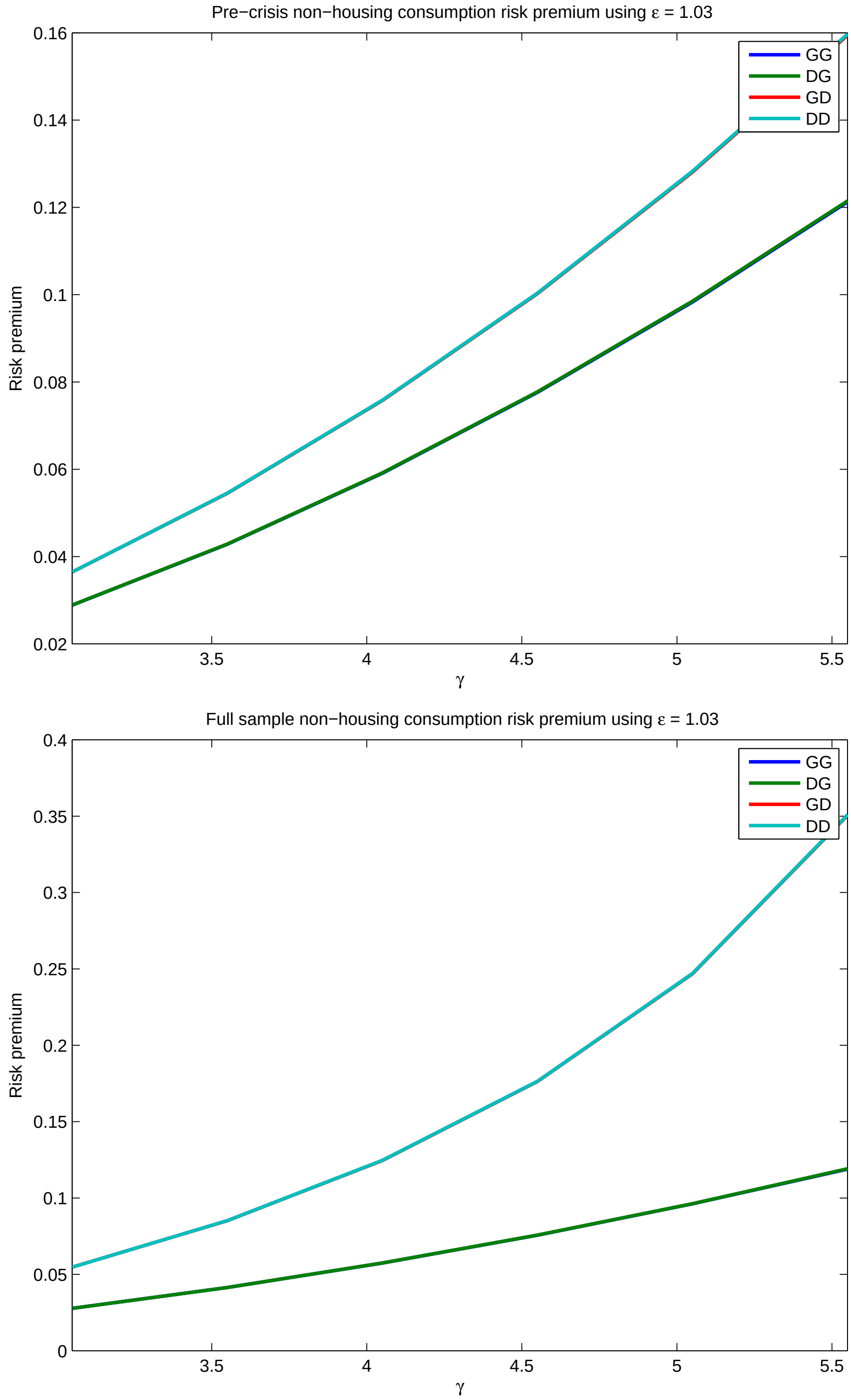
This figure displays the model-implied price-dividend ratio associated with real estate as a function of $\log z_t$. We have used non-housing consumption and expenditure ratio parameters estimated over the full sample. We have used $\varepsilon = 1.03, \gamma = 3$ and an annualized subjective discount factor $\beta = 0.999$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Figure 10: **Consumption bundle risk premium**



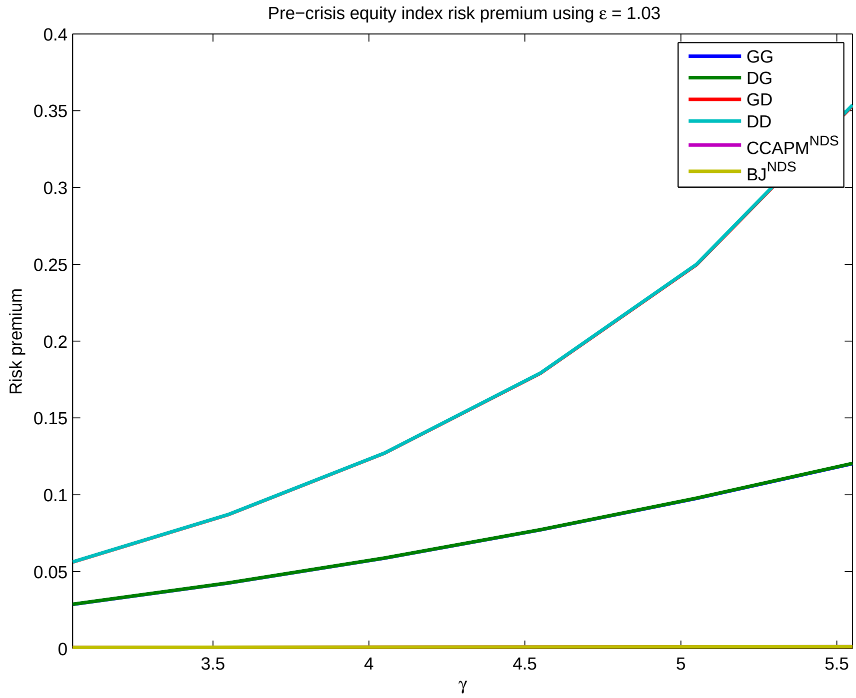
This figure displays the model-implied annualized risk premium associated with the consumption bundle asset as a function of γ. The graph uses the parameters of non-housing consumption and expenditure ratio dynamics estimated over the full sample. We have used $\varepsilon = 1.03$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

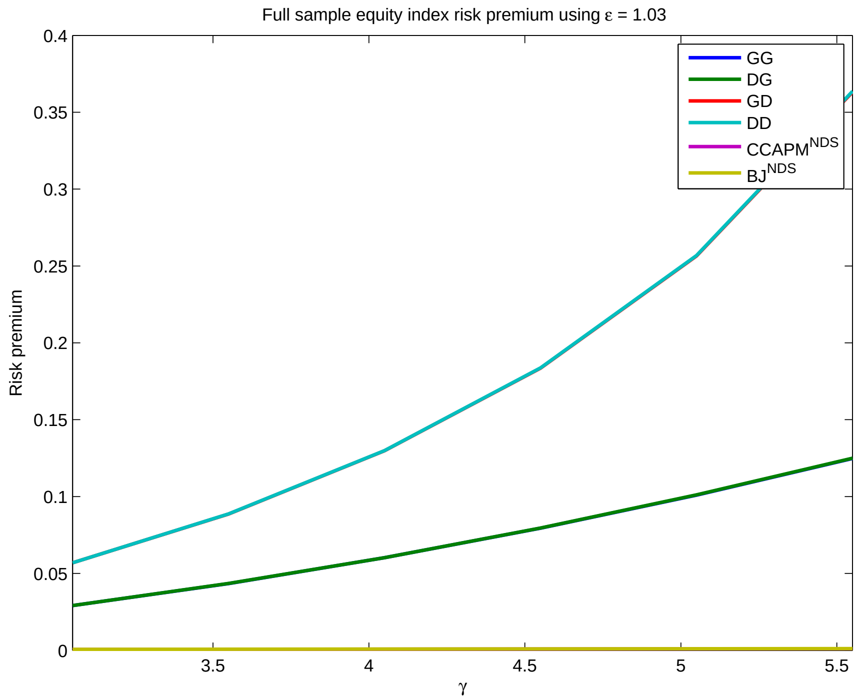
Figure 11: **Non-housing consumption risk premium**



This figure displays the model-implied annualized risk premium associated with the non-housing consumption asset as a function of γ. The graph uses the parameters of non-housing consumption and expenditure ratio dynamics estimated over the full sample. We have used $\varepsilon = 1.03$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

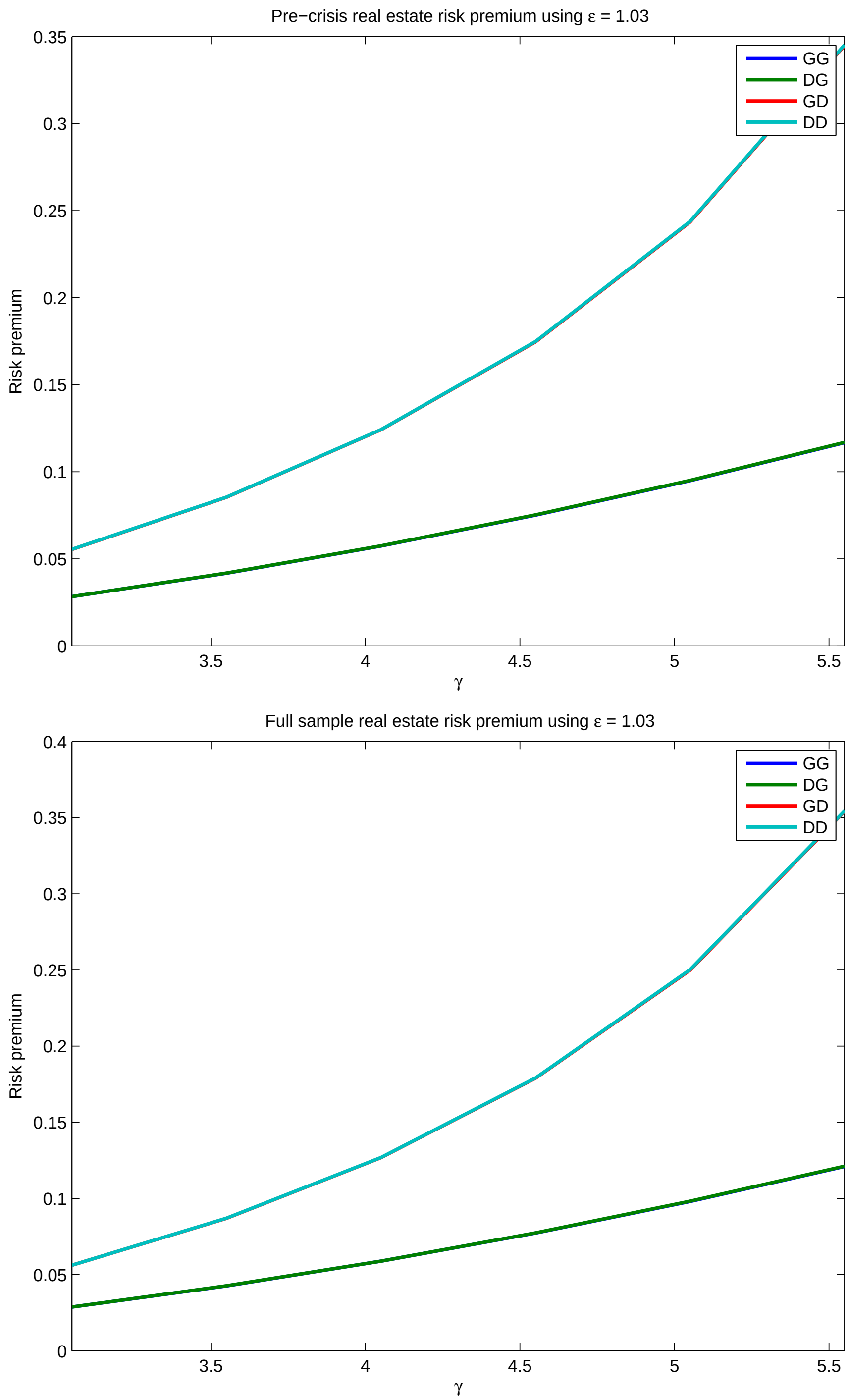
Figure 12: **Equity index risk premium**





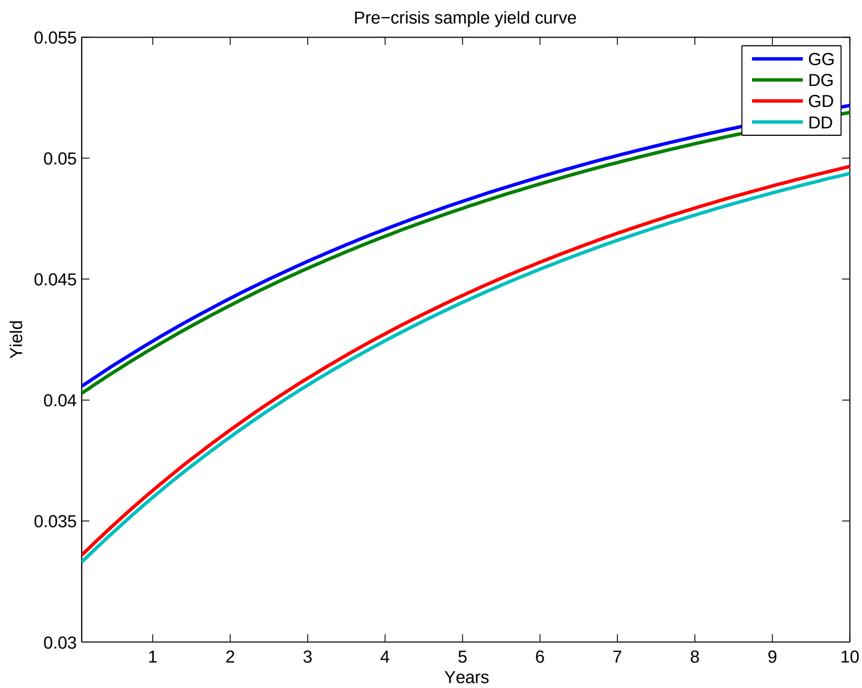
This figure displays the model-implied annualized risk premium associated with the value-weighted CRSP equity index as a function of γ for all our housing consumption models and for the standard CCAPM (CCAPM^{NDS}) and Barro and Jin (2011)'s one-good single power law model (BJ^{NDS}). In our housing consumption models, the dividend structure assumes the functional form described in (27) whereas for CCAPM^{NDS} and BJ^{NDS}, we calibrate the dividend structure to the constrained version of (27), where $k_z = 0$ and consumption means the standard non-durables and services aggregate. The graph uses the parameters of non-housing consumption and expenditure ratio dynamics estimated over the full sample. We have used $\varepsilon = 1.03$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

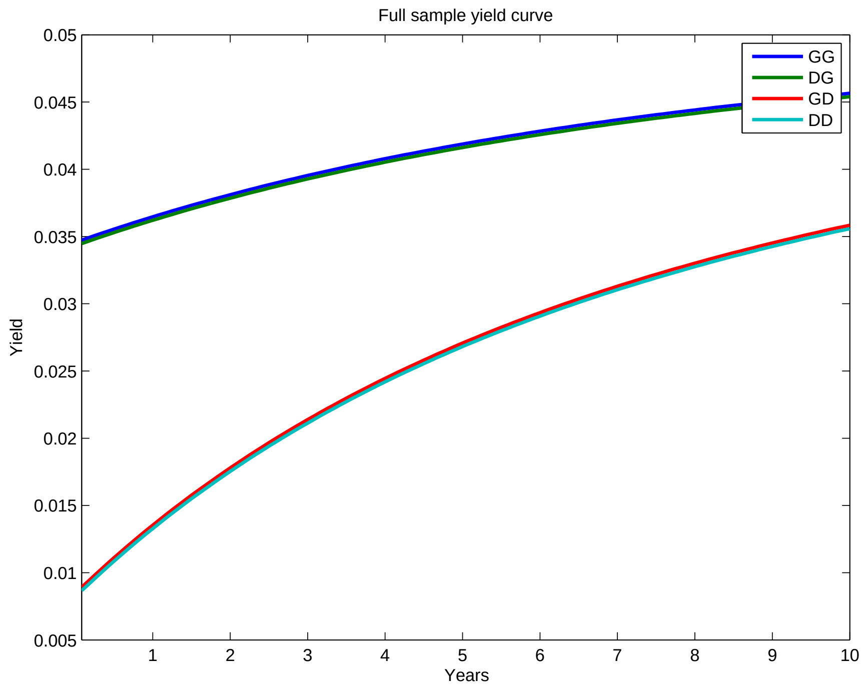
Figure 13: **Real estate risk premium**



This figure displays the model-implied annualized risk premium associated with real estate as a function of γ. The graph uses the parameters of non-housing consumption and expenditure ratio dynamics estimated over the full sample. We have used $\varepsilon = 1.03$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

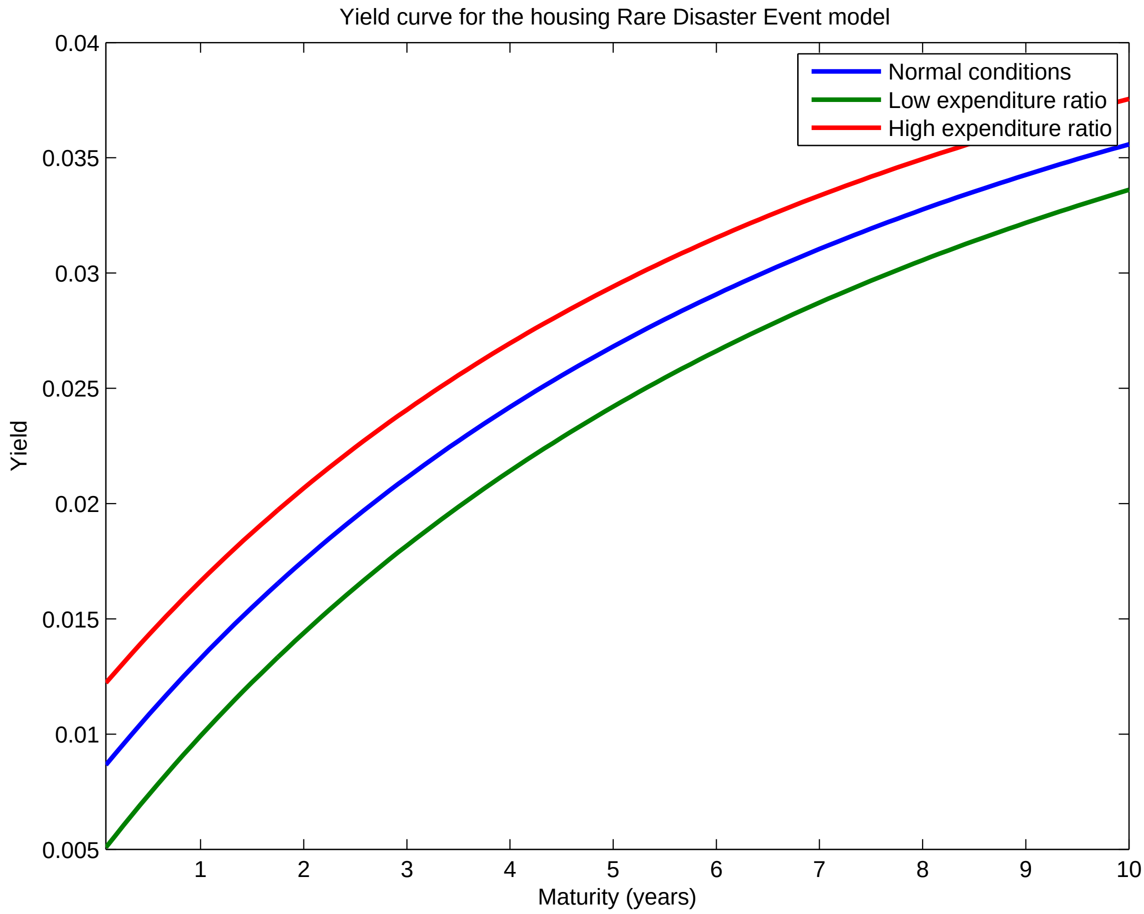
Figure 14: **Yield curve during normal economic conditions**





This figure plots the model-implied yield curve during normal economic conditions, i.e. when $\log z_t = \mu_z$. Yields are annualized. We have used $\varepsilon = 1.03$ and $\gamma = 3$. Acronyms GG, DG, GD and DD refer to the various versions of our housing consumption models, where the first letter describes the distribution of non-housing consumption and the second letter describes the expenditure ratio distribution. The letter G stands for a Gaussian distribution whereas D refers to the Disaster distribution, i.e. a crash for non-housing consumption and a boom or a crash for the expenditure ratio.

Figure 15: **Yield curve implied by the housing Rare Disaster Event Model for various levels of the expenditure ratio**



This figure plots the model-implied yield curve when we assume crashes in non-housing consumption and booms/crashes in the expenditure ratio (DD model). Yields are annualized. We consider three economic conditions: normal conditions where $\log z_t = \mu_z$, a low expenditure ratio condition where $\log z_t = \mu_z - \frac{1}{2}\sigma_z$, and high expenditure ratio condition where $\log z_t = \mu_z + \frac{1}{2}\sigma_z$. We have used $\varepsilon = 1.03, \gamma = 3$. The parameters of the non-housing consumption and expenditure ratio dynamics are based on the full sample estimation.