International SDFs in Segmented Markets*

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Abstract

We characterize international stochastic discount factors (SDFs) under various degrees of market segmentation in a preference-free approach. Our methodology minimizes SDF dispersion subject to international pricing constraints and allows for a factorization into permanent and transitory components. We find that large permanent SDF components are necessary to jointly reconcile well-known exchange rate puzzles, including the low exchange rate volatility, the exchange rate cyclicality and deviations from uncovered interest rate parity. At the same time, segmented stock markets are needed to avoid implausibly large SDF dispersions. These findings demonstrate a trade-off between financial market segmentation, SDF variability and the amount of tradeable exchange rate risk when addressing international asset pricing puzzles.

Keywords: stochastic discount factor, exchange rates, market segmentation, market incompleteness.

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This paper studies the link between international stochastic discount factors (SDFs) and financial market segmentation with a focus on three asset pricing puzzles in international finance: the low exchange rate volatility documented by Obstfeld and Rogoff (2001) and Brandt, Cochrane, and Santa-Clara (2006), the counter-cyclicality puzzle of Kollmann (1991) and Backus and Smith (1993), and the forward premium anomaly of Hansen and Hodrick (1980) and Fama (1984).

Scores of papers have studied these puzzles theoretically and empirically, either separately or jointly. One strand of the recent literature assumes complete markets under various specifications of preferences and consumption dynamics. A second strand addresses the determination of exchange rates under different forms of market segmentation.[1] Finally, a third strand of the literature studies the properties of international stochastic discount factors in segmented markets with a model-free approach. Recently, Lustig and Verdelhan (2016), however, conclude that the three exchange rate puzzles cannot be jointly explained in an international consumption CAPM setting even under incomplete spanning: while the volatility and cyclicality of exchange rates can be matched, this comes at the cost of lowering currency risk premia well below the level observed in the data.

Motivated by the inability of incomplete spanning alone to address these puzzles, we study the implications of financial market segmentation for exchange rate puzzles within more general arbitrage-free economies, in which international SDFs can be decomposed into transitory and permanent components. The common understanding in international financial economics is that whenever markets are incomplete, the real exchange rate, expressed as units of domestic goods per unit of foreign good, is equal to the ratio between foreign and domestic pricing kernels multiplied by a stochastic wedge as in Backus, Foresi, and Telmer (2001). In this paper, we depart from this premise along two dimensions. First, we decompose stochastic discount factors into permanent and transient components. Second, we theoretically show that in markets where

[1] The complete market assumption is used in the habit model of Stathopoulos (2017), among others, to generate sizable currency risk premia. Similarly, Colacito and Croce (2013) employ recursive preferences with highly correlated international martingale components in a two-country complete market setting. Farhi and Gabaix (2016) rely on a complete market economy with time-additive preferences and a time-varying probability of rare consumption disasters. Gabaix and Maggiori (2015) provide a theory of exchange rate determination based on capital flows in incomplete financial markets, while Chien, Lustig, and Naknoi (2015) propose a two-country stochastic growth model with segmented markets that generates smooth exchange rates and highly volatile stochastic discount factors.

foreign and domestic investors can trade the same set of assets, exchange rates exactly equal the ratio of minimum entropy SDFs independent of the amount of market incompleteness and the wedge is zero. More generally, our approach suggests that stochastic wedges can be seen as measures of the amount of untraded risk in international financial markets. Using this framework, we then empirically show that when domestic and foreign investors cannot trade the same set of assets, i.e. in segmented markets, the classic exchange rate puzzles can be resolved, however, at the cost of highly variable SDFs.

We develop these ideas in an economy where domestic and foreign investors have access to three types of assets: short- and long-term bonds and stocks. In order to characterize international SDFs without committing to a particular asset pricing model, we estimate different projections of international SDFs on the space of tradeable returns for domestic and foreign investors. Given the multitude of SDFs pricing returns in incomplete markets, we explore minimum dispersion SDFs, which minimize different notions of variability, e.g., the Hansen and Jagannathan (1991) SDF when we minimize the SDF variance. Additionally, we focus on minimum entropy SDF projections, as well as on a third SDF projection that minimizes the SDF Hellinger divergence. Each of the dispersion measures has appealing features that can be used in our empirical exercise. First, we show that minimum Hellinger divergence SDFs place a direct sharp dispersion bound on the first moment of transient SDF components. Second, we prove that minimum entropy SDFs always imply the validity of the market view of exchange rates in symmetric international markets, irrespective of the unknown degree of market incompleteness. This is not necessarily true for the minimum variance SDF which is not invariant with respect to a change in numéraire. Third, we demonstrate that minimum variance SDFs characterize in a natural way the tradeable component of exchange rate risk in symmetric international markets, giving rise to a model-free interpretation of the resulting exchange rate wedge as untradeable exchange rate risk.

Using these three dispersion measures, we allow for different degrees of market segmentation, from fully disconnected domestic and foreign markets to highly integrated markets, in which (risk-free) short- and long-term bonds and stocks are traded internationally. Hence, we are able to quantify the trade-off between a larger domestic and foreign SDF dispersion necessary to price a wider set of returns and the three exchange rate puzzles. For our empirical analysis, we adopt the insights of Bansal and Lehmann (1997) and Alvarez and Jermann (2005) and identify the transient component of domestic and foreign SDFs using long-term bonds. In this way, we force our preference-free SDF projections to correctly price the returns of long-maturity bonds in the local currency, which is the most natural way to empirically identify the short- and long-run SDF components.

In the empirical study we consider eight benchmark currencies, namely the US dollar, the British pound, the Swiss franc, the Japanese yen, the euro (Deutsche mark before the introduction of the euro), the Australian dollar, the Canadian dollar and the New Zealand dollar. The resulting seven exchange rates are expressed with respect to the US dollar as the domestic currency and the sample period spans January 1975 to December 2015. We summarize our empirical findings as follows.

Firstly, we notice that our main empirical results are largely independent of the particular choice of SDF projection used in segmented international markets. Independent of the degree of market segmentation, we find that permanent (martingale) components of domestic and foreign SDFs across markets are all highly volatile, to the point that they actually dominate the overall SDF variability. This feature is consistent with previous evidence for the US market in, e.g., Alvarez and Jermann (2005) and is necessary in order to generate both a high equity premium and a low term premium. Moreover, the co-movement between transient and permanent SDF components is negative and it is essential to match the typically negative local risk premia of long term bonds.

Under the assumption of fully segmented markets, we find, as expected, that the three exchange rate puzzles cannot be explained jointly. This exercise also allows us to better understand the underlying reasons of this failure. In line with Lustig and Verdelhan (2016), we find that a strong market segmentation helps to explain the low exchange rate volatility by means of a volatile Backus, Foresi, and Telmer (2001)-type wedge between exchange rates and the ratio of foreign and domestic SDFs. This setting is also able to address the cyclicality puzzle of Backus and Smith (1993), because cross-country differences in observable transient SDF components are only weakly related to exchange rates returns. However, the minimum dispersion SDFs derived under autarky are unable to match the observed currency risk premia, since international trading is not allowed. Indeed, we document that significant exchange rate pricing errors arise, especially for the funding currencies (the Japanese yen and the Swiss franc), suggesting that in order to match the currency risk premia once domestic investors can trade foreign assets, minimum dispersion SDFs need to be more volatile. Finally, we find that the permanent components in fully segmented markets are only weakly co-moving. In fact, in the data we find that correlations between domestic and foreign minimum dispersion SDFs under autarky are usually less than 20% and never larger than 65%.

Moving from the autarky case to an economy where investors are allowed to trade internationally the riskless short-term bonds, we find that the ensuing minimum dispersion SDFs jointly address the three exchange rate puzzles. The exchange rate volatility and the Backus-Smith (1993) puzzle are explained by qualitatively similar mechanics as under autarky, with wedges that are on average comparably volatile. Carry trade premia are also in line with those observed in the data, because the international pricing constraints on risk-free bonds effectively force domestic and foreign SDFs to correctly reproduce the cross-section of currency risk premia. As expected, we find that by extending the set of traded assets relative to the autarky setting, the SDF dispersion increases in each market. The increase in SDF dispersion is more pronounced for the funding currencies. Although the SDF co-movement increases, it never exceeds 65%. In summary, an economy with segmented long-term bond and stock markets may explain the three exchange rate puzzles if wedges are as volatile as international SDFs.

When successively opening international markets to long-term bond and stock trading, the above exchange rate puzzles are still explained, by construction. Additionally, the cross-section of international risk premia on these assets is also matched. This feature creates a stronger SDF dispersion trade-off than the one implied by the three standard exchange rate puzzles. In parallel, we find that less asymmetric international markets naturally induce less volatile exchange rate wedges. Therefore, we obtain an increase in SDF dispersion that typically entails a larger co-movement between international SDF permanent components.

We find that the SDF dispersion in integrated long-term bond markets is significantly larger than under integrated short-term bond markets alone, mainly due to both a higher dispersion of the permanent component and a smaller absolute co-movement between permanent and transitory components. These two forces are necessary in order to match the cross-section international long-term bond risk premia, which is a steep function of the short-term interest rate differential. For instance, compared to the case of integrated short-term bond markets, the increase in minimum variance SDFs dispersion is on average 25%, the highest values being encountered especially for the funding currencies. These large SDF dispersions suggest that it may be difficult to explain exchange rate puzzles using structural asset pricing models with traded domestic equity in fully integrated international long-term bond markets. The dispersion of the exchange rate wedges is on average similar to the one of the SDF, whereas the correlation between the domestic and foreign pricing kernels increases significantly, reaching a maximum of 86%.

Finally, we provide a model-free assessment of the asset market view of exchange rates in fully integrated, but incomplete, international bonds and stock markets. We find that in such settings the untradable component of exchange rate risk explaining deviations from the asset market view is small, generating a negligible fraction of exchange rate volatility. Therefore, minimum dispersion SDFs are internationally highly correlated, at the cost of an even larger average SDF dispersion. To illustrate, compared to the autarky setting, the average increase in the minimum volatility is around 60%. Hence, some form of international bond or stock market segmentation appears appropriate for structural models of exchange rate determination, in which case deviations from the asset market view will arise. This finding re-emphasizes the trade-off in international finance between SDF dispersions, the degree of financial market segmentation, and the corresponding amount of tradeable exchange rate risk to jointly address all exchange rate puzzles.

After a literature review, the rest of the paper is organized as follows. Section 1 provides the theoretical framework for our model-free selection of minimum dispersion SDFs in international financial markets. Section 2 describes our data and our main empirical findings, under various benchmark assumptions about the degree of international market segmentation. Section 3 concludes and discusses directions for future research.

Literature Review: Our paper contributes to the literature that studies the ability of market incompleteness to address various puzzles in international finance. Bakshi, Cerrato, and Crosby (2015) and Lustig and Verdelhan (2016) study preference-free SDFs in incomplete markets to address the weak link between exchange rates and macroeconomic fundamentals. The former impose "good deal" bounds on international SDFs to study economies with a low amount of risk sharing and economically motivated pricing errors. Lustig and Verdelhan (2016) introduce a stochastic wedge between foreign and domestic SDFs and conclude that incomplete markets cannot jointly address the three exchange rate puzzles under a Consumption-CAPM framework.[2] In our paper, we describe the properties of international SDFs without making any particular distributional assumptions while allowing them to be factorized into a transient and a permanent component. We demonstrate that this decomposition is key to reconcile stylized facts about exchange rates, especially the Backus-Smith puzzle. We further highlight the distinct economic roles of minimum entropy and minimum variance SDFs with respect to the asset market view in symmetric incomplete international markets.

Another strand of the literature studies structural models of exchange rate determination under different assumptions about market segmentation. Chien, Lustig, and Naknoi (2015) show that while limited stock market participation can reconcile highly correlated international SDFs with a low correlation in consumption growth, it is less successful in addressing the Backus and Smith puzzle. Alvarez, Atkeson, and Kehoe (2009) explain the Backus and Smith puzzle in a general equilibrium model with financial frictions and endogenous market participation. We contribute to this literature by documenting with a model-free approach the large dispersion trade-offs implied by settings with fully integrated international financial markets. This framework is a special case of international symmetric markets, where investors can trade the same assets both domestically and foreign. We also quantify the implications of various degrees of market segmentation in international bond and stock markets. Theoretically, we show that the asset market view for exchange rates always holds for minimum entropy SDFs in symmetric international markets, that is, their ratio equates the change in exchange rates, independent of the degree of incompleteness. As a byproduct, we empirically find that minimum dispersion domestic and foreign SDFs are always highly correlated in symmetric, even if incomplete, market settings. Following Burnside and Graveline (2012), the asset market view of exchange rates does not hold in general for minimum variance SDFs. We provide a general economic interpretation of this feature, by decomposing the wedge between exchange rates and minimum variance SDF ratios in terms of tradable and untradable exchange rate

[2]Using more general settings, the authors derive preference-free results using an entropy-based measure of risk, which however poses a significant computational challenge in terms of the cyclicality of exchange rates.

risks in domestic and foreign asset markets. Moreover, we show that the asset market view holds with respect to the minimum entropy SDFs.

Maurer and Tran (2016) construct minimum variance SDF projections on excess returns in incomplete continuous-time market settings, showing that the asset market view holds if and only if exchange rate risks can be disentangled in symmetric domestic and foreign asset markets, i.e. only in absence of jump risk. Our approach is different and explicitly considers various relevant SDFs projections. This allows us to show that the asset market view always holds for minimum entropy SDFs in symmetric economies, irrespective of the degree of market incompleteness or additional assumptions on the distribution of asset returns. Similarly, considering different admissible SDF projections is key to sharply decompose exchange rate risks in tradable and untradable components from the perspective of domestic and foreign investors.

The SDF factorization in permanent and transient components has been employed previously in various studies of international asset pricing under a complete markets assumption. Chabi-Yo and Colacito (2015) make use of co-entropies to characterize the horizon properties of SDFs co-movement. Lustig, Stathopoulos, and Verdelhan (2016) examine the international bond premia and conclude that the bond return parity condition holds when nominal exchange rates are stationary. The large co-movement of permanent components in their studies is a natural consequence of the underlying completeness assumption. We show that a large co-movement of permanent SDFs components emerges in all symmetric international economies in our study, regardless of their different degree of market incompleteness. For minimum entropy SDFs, this is a direct implication of the fact that the asset market view theoretically holds in this case. For minimum variance SDFs, this follows from the fact that the estimated fraction of untradable exchange rate risk in symmetric economies is rather small. However, we also find that the underlying SDF dispersion when bonds or stocks are traded internationally is probably difficult to explain using structural complete market models.

1 Preference-Free SDFs in Incomplete International Markets

In this section, we introduce our model-free methodology for identifying minimum dispersion SDFs in incomplete domestic and foreign financial markets. One motivation for using minimum dispersion SDFs relies on the fact that they can be understood as optimal SDF projections generated by traded asset returns, which also naturally bound the welfare attainable by marginal investors. In this sense, minimum dispersion SDFs constrain the best deals attainable by domestic and foreign investors. Additionally, minimum dispersion SDFs directly imply model-free constraints on the distribution of asset returns, such as asset pricing bounds on expected (log) returns and Sharpe ratios. These model-free constraints need to be satisfied by any admissible international asset pricing model.

We focus on three distinct families of SDFs, which are obtained by minimizing the entropy, the variance and the Hellinger divergence of the SDF, respectively. We show that in symmetric markets, entropy SDFs feature an appealing property: their ratio equals the exchange rate changes and there is no stochastic wedge left, independent of the amount of incompleteness. In other words, whenever international markets are symmetric, the market view of exchange rates holds with respect to the minimum entropy SDFs. However, this result is not true in general for other minimum dispersion SDFs, as they are not numéraire invariant.

1.1 Minimum-Dispersion SDFs

Consider a domestic and a foreign economy with SDFs M_d and M_f, respectively. For these economies, the vectors $\mathbf{R}_d = (R_{d0}, \ldots, R_{dK_d})'$ and $\mathbf{R}_f = (R_{f0}, \ldots, R_{fK_f})'$ include the set of gross returns priced by M_d and M_f, where R_{d0} and R_{f0} denote the risk-free returns in the domestic and foreign market, respectively. In our empirical analysis, we take the United States (US) as the domestic market and the United Kingdom (UK), Switzerland (CH), Japan (JP), the European Union (EU), Australia (AU), Canada (CA) or New Zealand (NZ) as the foreign markets.[3] For each market $i = d, f$, we study the minimum-dispersion SDF that solves for parameter $\alpha_i \in \mathbb{R}$ the following optimization problem:

$$\begin{aligned} &\min_{M_i} \frac{\log E[M_i^{\alpha_i}]}{\alpha_i(\alpha_i - 1)} \ , \\ &\text{s.t. } E[M_i \mathbf{R}_i] = \mathbf{1} \ ; \ M_i > 0 \ . \end{aligned} \tag{1}$$

The pricing restriction $E[M_i\mathbf{R}_i] = \mathbf{1}$ in equation (1), where $\mathbf{1}$ is a $(K_i + 1) \times 1$ vector of ones, ensures that the SDF satisfies the given pricing constraints, while the positivity constraint $M_i >$

[3] Prior to the introduction of the EURO, we take Germany in its place.

0 ensures that it is indeed an admissible SDF.[4] The formulation in equation (1) depends on parameter α_i, which subsumes various SDF choices in incomplete markets economies. Different values of α_i weigh differently higher order moments of the asset return distribution. For example, for $\alpha_i = 2$, we obtain the well-known minimum variance bounds of Hansen and Jagannathan (1991), while $\alpha_i = 0$ ($\alpha_i = 0.5$) corresponds to entropy (Hellinger) bounds. The latter two also allow us to study the robustness of our findings with respect to the higher-order moments of the asset return distribution. In line with Almeida and Garcia (2012), among others, the minimum-entropy ($\alpha_i = 0$) SDF delivers a tight upper bound on the maximal expected log return with respect to the available traded returns, while the minimum-variance ($\alpha_i = 2$) SDF delivers a tight upper bound on the maximal Sharpe ratio.

More generally, every minimum dispersion SDF corresponds to a different set of tight constraints on the moments of traded asset returns. Since asset returns are observable but SDFs are not, we can conveniently restate the minimum objective function in equation (1) to the maximum objective function in the following dual portfolio problem (see e.g., Orlowski, Sali, and Trojani (2016), Proposition 4):

$$\begin{aligned} &\max_{\lambda_i} -\frac{\log E\left[R_{\lambda_i}^{\alpha_i/(\alpha_i-1)}\right]}{\alpha_i} \ , \\ &\text{s.t. } R_{\lambda_i} > 0 \ , \end{aligned} \tag{2}$$

where $R_{\lambda_i} = \sum_{k=1}^{K_i} \lambda_{ik} R_{ik} + (1 - \sum_{k=1}^{K_i} \lambda_{ik}) R_{i0}$ and λ_{ik} denotes the portfolio weight of asset k in market $i = d, f$. The first-order conditions (FOCs) associated with minimization problem (2) read:

$$E_i \left[R_{\lambda_i^*}^{-1/(1-\alpha_i)} (R_{ik} - R_{i0}) \right] = 0 \tag{3}$$

Using the optimal return $R_{\lambda_i^*}$ in this portfolio problem, we can now derive the minimum dispersion SDF explicitly.

[4] As the risk-free return R_{i0} is traded, an equivalent formulation of problem (1) is:

$$\begin{aligned} &\min_{M_i} \frac{\log E[(M_i/E[M_i])^{\alpha_i}]}{\alpha_i(\alpha_i - 1)} \ , \\ &\text{s.t.} \quad E[M_i \mathbf{R}_i] = \mathbf{1} \ ; \ M_i > 0 \ . \end{aligned}$$

In this formulation, the minimum dispersion features of problem (1) are directly apparent and a consequence of Jensen's inequality; see also Orlowski, Sali, and Trojani (2016) for a more general treatment of dispersion measures in arbitrage-free markets.

Proposition 1. *The minimum dispersion SDF in international financial markets is given by:*

$$M_i^* = R_{\lambda_i^*}^{-1/(1-\alpha_i)} / E[R_{\lambda_i^*}^{-\alpha_i/(1-\alpha_i)}], \tag{4}$$

where $R_{\lambda_i^*}$ are the optimal portfolio returns which satisfy the optimization problem in (2).

Moreover, due to the duality relation between problems (1) and (2), we have that:

$$\frac{\log E[M_i^{*\alpha_i}]}{\alpha_i(\alpha_i - 1)} = -\frac{\log E[R_{\lambda_i^*}^{\alpha_i/(\alpha_i-1)}]}{\alpha_i}. \tag{5}$$

Using different values of α_i, we can now easily derive different minimum dispersion SDFs. Denote by $M_i^*(\alpha_i)$ the optimal minimum dispersion SDF for dispersion parameter α_i.

Example 1. For $\alpha_i = 0$, we obtain the minimum entropy SDF, which is given by:

$$M_i^*(0) = 1/R_{\lambda_i^*}.$$

The minimum Hellinger SDF is obtained for $\alpha_i = 1/2$:

$$M_i^*(1/2) = E(R_{\lambda_i^*})/R_{\lambda_i^*}^2.$$

And for $\alpha_i = 2$, we retrieve the minimum variance SDF:

$$M_i^*(2) = R_{\lambda_i^*}/E(R_{\lambda_i^*}^2).$$

Based on these SDFs, we can now turn our attention to the derivation of a set of bounds. Specifically, for $\alpha_i \in \mathbb{R}$ we derive the following bound on the distribution of any traded portfolio return R_{λ_i}:[5]

$$\frac{\log E[M_i^{\alpha_i}]}{\alpha_i(\alpha_i - 1)} \geq -\frac{\log E[R_{\lambda_i}^{\alpha_i/(\alpha_i-1)}]}{\alpha_i}. \tag{6}$$

Consistently with the above example, for $\alpha_i = 0$ and $\alpha_i = 2$ we obtain the entropy and the variance bounds. In addition, in our empirical study we consider the parameter choice $\alpha_i = 1/2$, which directly implies for any traded return R_{λ_i} the Hellinger bound:

$$\log E\left[M_i^{1/2}\right] \leq \frac{\log E[R_{\lambda_i}^{-1}]}{2} \ , \tag{7}$$

[5] When there is no duality gap between the primal and dual solutions, i.e. for the optimal portfolio return $R_{\lambda_i^*}$, we retrieve Equation (5).

the tightest bound being obtained for the optimal minimum dispersion SDF $M_i^{*\alpha_i}$. Kitamura, Otsu, and Evdokimov (2013) emphasize the optimal robustness features of Hellinger-type dispersion measures. Importantly, we show in the next section that Hellinger bounds naturally induce tight constraints on the first moment of transitory SDF components.

1.2 SDF Components and Exchange Rate Wedges

One aim of this paper is to propose a parsimonious framework to study implications of market incompleteness and segmentation for various exchange rate puzzles. In the following, we first decompose international SDFs into martingale and transient components. We then use these SDFs and allow for different degrees of market segmentation in incomplete markets by restricting the number of assets that investors can trade.

A key feature of the minimum dispersion problem given in (1), is that it applies without loss of generality when we decompose the SDF into permanent and transient components in a model-free way:

$$M_i = M_i^P M_i^T. \tag{8}$$

In line with Alvarez and Jermann (2005), M_i^P is identifiable by normalization $E[M_i^P] = 1$, while $M_i^T := 1/R_{i\infty}$, where $R_{i\infty}$ is the return of the infinite maturity bond.[6] With this parameterization, the normalization of the permanent component is easily ensured, simply by requiring the return on the infinite maturity bond to be priced by the SDF $M_i = M_i^P/R_{i\infty}$, i.e., by defining $R_{i\infty}$ to be one of the components of return vector $\mathbf{R}_i$ in problem (1). Tradeability of $R_{i\infty}$ obviously impacts the form of minimum dispersion SDFs and increases the SDF variability.

Since inequality (7) is valid for any traded return, we obtain the following constraint on the expected transient SDF component:

$$\log E\left[M_i^{1/2}\right] \leq \frac{\log E[R_{\lambda_{i\infty}}^{-1}]}{2} \ , \tag{9}$$

where the tightest bound is given by the optimal SDF $M_i^{*1/2}$. Therefore, the Hellinger minimum dispersion SDF directly produces information about the average size of transient SDF

[6] For instance, in the long run risk model with recursive preferences, the transient component is a function of consumption growth alone, while the permanent (martingale) component is a function of the return of the claim to total future consumption.

components, and vice-versa. In the following sections, we apply factorization (8) to quantify in a model-free way the relative importance of international transient and persistent SDF components for explaining salient features of exchange rates.

A second useful property of minimum dispersion problem (1), is that it can freely accommodate different assumptions on the degree of international market segmentation. We achieve this by adjusting accordingly the set of available returns $\mathbf{R}_d$ and $\mathbf{R}_f$ that can be traded from the perspective of domestic and foreign investors. It is well-known that whenever international markets are complete, domestic and foreign SDFs are uniquely defined. As a consequence, also all minimum dispersion SDFs are identical. In this case, from the Euler equation pricing restrictions, the exchange rate return is uniquely given by the ratio of the foreign and domestic SDFs.

More broadly, when domestic or foreign markets are incomplete, differences between minimum dispersion SDFs can arise and a wedge η between exchange rate returns and the ratio of foreign and domestic SDFs emerges (see e.g., Backus, Foresi, and Telmer (2001)):

$$\frac{M_f}{M_d}\exp(\eta) = X, \tag{10}$$

where X is the (gross) return of the exchange rate, which is defined as the domestic currency price of one unit of the foreign currency. As X increases, the domestic currency depreciates. Obviously, $\eta = 0$ when markets are complete. We systematically address the relation between exchange rate wedges and minimum dispersion SDFs in economies where traded returns $\mathbf{R}_d$ and $\mathbf{R}_f$ reflect different degrees of financial market segmentation, from settings of fully segmented markets to economies where domestic investors can trade foreign bonds and stocks by resorting to exchange rate markets. In doing so, we exploit the fact that factorization (8) is applicable independently of the form of international financial market segmentation, which gives rise to a systematic way for quantifying exchange rate wedges and persistent international SDF components in the decomposition:

$$\frac{M_f^P}{M_d^P}\frac{R_{d\infty}}{R_{f\infty}}\exp(\eta) = X. \tag{11}$$

Intuitively, when we extend the set of tradeable assets from the perspective of domestic or foreign investors, the set of pricing constraints in problem (1) widens, the dispersion of optimal

SDFs increases and the size of the wedge usually shrinks, as markets tend to exhibit a lesser degree of segmentation. Crucially, we show that whenever domestic and foreign investors share the same set of assets, which we label as a symmetric market case (see e.g. Definition 1), the wedge vanishes for the minimum entropy SDF and is quantitatively small for the Hellinger and minimum variance SDFs.

Conclusively, our approach allows a natural quantification of the asset pricing trade-offs between international financial markets segmentation, exchange rate puzzles and SDF dispersion.

1.3 Minimum Dispersion SDFs and Changes of Numéraire

It is natural to think that the properties of minimum dispersion SDFs are particularly sensitive to a change of numéraire given our international markets environment. Therefore, in the following, we explore the effect of different numéraires on international SDFs.

First, given a foreign SDF M_f for return vector $\mathbf{R}_f$, it is always the case that $M_d^e := M_f 1/X$ is a SDF for the domestic currency-converted return vector $\mathbf{R}_d^e := \mathbf{R}_f X$. Symmetrically, $M_f^e := M_d X$ is a SDF for the foreign currency-converted return vector $\mathbf{R}_f^e := \mathbf{R}_d 1/X$. Therefore, M_f (M_d) is a foreign (domestic) SDF for return vector $\mathbf{R}_f$ ($\mathbf{R}_d$) if and only if M_d^e (M_f^e) is a SDF for domestic- (foreign-) currency return vector $\mathbf{R}_d^e$ ($\mathbf{R}_f^e$). In other words, the numéraire transformation $\mathcal{N}_d^f : (M_d, \mathbf{R}_d) \longmapsto (M_f^e, \mathbf{R}_f^e)$ defines again a SDF when changing the numéraire from domestic- to foreign-currency returns. Similarly, $\mathcal{N}_f^d : (M_f, \mathbf{R}_f) \longmapsto (M_d^e, \mathbf{R}_d^e)$ defines a new SDF under a change of numéraire from foreign- to domestic-currency returns.

Second, these numéraire transformations do not preserve in general the minimum dispersion property of a SDF, e.g., if M_d^* is a minimum dispersion SDF for return vector $\mathbf{R}_d$ then in general it does not follow that M_f^{*e} is a minimum dispersion SDF for $\mathbf{R}_f^e$. However, there exist market structures and dispersion measures for which the SDF minimum dispersion property is numéraire invariant. One obvious such situation emerges under complete markets. Indeed, in this case domestic and foreign SDFs are uniquely defined and identical to the optimal SDFs under any dispersion criterion. Hence, it follows that M_d^{*e} (M_f^{*e}) is a uniquely defined SDF for return vector $\mathbf{R}_d^e$ ($\mathbf{R}_f^e$) and it is therefore also the minimum dispersion SDF.

As the complete market assumption is too restrictive for our analysis, we now address more general properties of numéraire invariant minimum dispersion SDFs in incomplete markets.

1.3.1 Minimum Entropy SDFs

Without imposing any particular assumptions on the underlying market structure, there always exists a single numéraire invariant minimum dispersion SDF, i.e. the minimum entropy SDF ($\alpha_i = 0$). From equation (5), this SDF takes the form $M_i^* = R_{\lambda_i^*}^{-1}$, with optimal portfolio weight λ_i^* that uniquely solves for $k = 1, \ldots, K_i$ the first-order conditions of optimization problem (2):

$$E[R_{\lambda_i^*}^{-1}(R_{ik} - R_{i0})] = 0 \ . \tag{12}$$

The numéraire invariance of minimum entropy SDFs is easily derived. For the foreign minimum entropy SDF $M_f^* = R_{\lambda_f^*}^{-1}$, it immediately follows: $M_d^{*e} = R_{\lambda_f^*}^{-1} 1/X = (R_{\lambda_d^*}^e)^{-1}$, where $R_{\lambda_d^*}^e := R_{\lambda_f^*} X$ is a domestic portfolio return. By construction, $(R_{\lambda_d^*}^e)^{-1}$ uniquely solves for $k = 1, \ldots, K_d$ the first order conditions:

$$E[(R_{\lambda_d}^e)^{-1}(R_{dk}^e - R_{d0}^e)] = 0 \ . \tag{13}$$

Therefore, $(R_{\lambda_d^*}^e)^{-1}$ is the minimum entropy SDF for return vector $\mathbf{R}_d^e$. Symmetric arguments show that $M_f^{*e} = (R_{\lambda_f^*}^e)^{-1}$ is the minimum entropy SDF for $\mathbf{R}_f^e$. Note that due to the numéraire invariance property of minimum entropy SDFs, the optimal foreign and domestic portfolio weights are the same. Summarizing, we obtain the following equivalence result:

Proposition 2. *$R_{\lambda_f^*}^{-1}$, with optimal weights λ_f^* uniquely solving the first order conditions $E[R_{\lambda_f^*}^{-1}(R_{fk} - R_{f0})] = 0$ is a minimum entropy pricing kernel for the foreign return vector $\mathbf{R}_f$ if and only if $(R_{\lambda_d^*}^e)^{-1}$, with optimal weights uniquely solving $E[(R_{\lambda_d}^e)^{-1}(R_{dk}^e - R_{d0}^e)] = 0$, is a minimum entropy SDF for the foreign domestically converted return vector $\mathbf{R}_d^e$.*

A key implication of the numéraire invariance of minimum entropy SDFs is that they are always consistent with the asset market view of exchange rates when international financial returns are symmetrically traded, where symmetry is defined as:

Definition 1. *International financial markets are said to be* ***symmetric*** *if the span of returns in the domestic economy coincides with the span of returns in the foreign economy, translated in domestic terms:*

$$\text{span}(\mathbf{R}_d) = \text{span}(\mathbf{R}_d^e). \tag{14}$$

Result 1. *Whenever international financial markets are symmetric, the asset market view always holds for the optimal minimum dispersion SDFs:*

$$X = \frac{M_f^*}{M_d^*} \ . \tag{15}$$

Importantly, among all minimum dispersion SDFs, domestic and foreign minimum entropy SDFs are the only ones that imply property (15) without additional constraints on the structure of financial markets. Moreover, they are both given by the same transformation of a common linear combination of returns, denominated in domestic and foreign currency, respectively. In this sense, these SDFs always imply a perfect international risk sharing in symmetric international financial markets. Notably, however, this risk sharing is not attainable by portfolios of traded returns, because minimum entropy SDFs are nonlinear transformations of returns.

1.3.2 Minimum Variance SDFs

The single minimum dispersion SDF that is tradeable with portfolios of asset returns is the minimum variance SDF ($\alpha_i = 2$). From equation (5), minimum variance SDFs take the form $M_i^* = R_{\lambda_i^*}/E[R_{\lambda_i^*}^2]$, with portfolio weight λ_i^* that uniquely solves for $k = 1, \ldots, K_i$ the first-order conditions in optimization problem (2):

$$E[R_{\lambda_i^*}(R_{ik} - R_{i0})] = 0 \ . \tag{16}$$

It is an immediate consequence of these first-order conditions that minimum variance SDFs are not in general numéraire invariant. Indeed, if $M_f^* = R_{\lambda_f^*}/E[R_{\lambda_f^*}^2]$ is the foreign minimum variance SDF, then $M_d^{*e} = M_f^* 1/X$ cannot be written in general as a linear combination of returns in vector $\mathbf{R}_d^e$ and thus is not going to be in general a minimum variance SDF for $\mathbf{R}_d^e$. Symmetric arguments imply that M_f^{*e} is not in general the minimum variance SDF for $\mathbf{R}_f^e$.

One key implication of these findings is that minimum variance SDFs are not in general consistent with the asset market view of exchange rates in symmetric international markets.

Precisely, when $\text{span}(\mathbf{R}_d) = \text{span}(\mathbf{R}_f)$, the above finding imply in general $M_d^* \neq M_d^{*e}$ and $M_f^* \neq M_f^{*e}$, i.e., a violation of the asset market view of exchange rates:

$$X \neq \frac{M_f^*}{M_d^*} \ . \tag{17}$$

The origins of this violation are further understood by exploiting the numéraire invariance of minimum entropy SDFs displayed in Subsection 1.3.1.

Result 2. *From property (15) of minimum entropy SDFs, we obtain the following decomposition of the exchange rate:*

$$X = \frac{M_f^*(2)}{M_d^*(2)} \cdot \frac{1 + [M_f^*(0) - M_f^*(2)]/M_f^*(2)}{1 + [M_d^*(0) - M_d^*(2)]/M_d^*(2)} \ . \tag{18}$$

Importantly, this decomposition can be derived from asset returns alone. It clarifies that inequality (17) is determined by the ratio of the relative projection errors of foreign and domestic minimum entropy SDFs on the space of foreign and domestic returns. Thus, a violation of the asset market view for minimum variance SDFs is the result of particular unspanned exchange rate risks, which are reproduced by the component of minimum entropy SDFs that is unspanned by asset returns. Similarly, the market view holds with respect to the minimum variance SDFs whenever minimum entropy SDFs are tradeable in domestic and foreign markets, simply because in this case, they are identical.

Deviations from the asset market view for minimum variance SDFs are also naturally characterized using the optimal returns in optimization problem (2).

Result 3. *For dispersion parameter α_i, in terms of optimal returns $R_{\lambda_i^*}(\alpha_i)$, property (15) of minimum entropy SDFs yields:*

$$X = \frac{R_{\lambda_d^*}(2)}{R_{\lambda_f^*}(2)} \cdot \frac{1 + [R_{\lambda_d^*}(0) - R_{\lambda_d^*}(2)]/R_{\lambda_d^*}(2)}{1 + [R_{\lambda_f^*}(0) - R_{\lambda_f^*}(2)]/R_{\lambda_f^*}(2)} \ . \tag{19}$$

Recalling that $R_{\lambda_i^*}(2)$ and $R_{\lambda_i^*}(0)$ are the returns of maximum Sharpe ratio and maximum growth portfolios in market $i = d, f$, this identity characterizes exchange rates in terms of the tradeable risk return trade-offs in international financial markets. The exchange rate is larger when the domestic maximum Sharpe ratio return is higher than the foreign maximum Sharpe ratio return. This effect is produced by the first quotient on the RHS of equation (19) and can be interpreted as a tradeable exchange rate effect due to the relative mean-variance trade-off between domestic and foreign markets. The exchange rate is also higher when the excess return of the domestic maximal growth return relative to the maximum Sharpe ratio return is larger than the corresponding foreign excess return. This effect is summarized by the second quotient on the RHS of equation (19) and it more directly quantifies the risk-return trade-offs between domestic and foreign markets due to the higher moments of returns.

2 Empirical Analysis

Using our model-free method, we can now characterize and quantify empirically the key properties of international SDFs under different assumptions about the degree of segmentation between domestic and foreign arbitrage-free financial markets. The highest degree of segmentation arises when investors can trade only in domestic assets: the risk-free return, the aggregate equity return and the long-term bond return.[7] This setting defines a natural initial benchmark, allowing us to quantify the minimal dispersion properties of international SDFs, when exchange rate returns and wedges may not be priced by these SDFs.

In our approach, exchange rate returns and wedges are priced by minimum dispersion SDFs whenever the domestic (foreign) risk-free return is tradeable for foreign (domestic) investors through currency markets. We introduce this second benchmark setting to isolate two additional important aspects of international SDFs. First, the relation between SDF dispersion and the cross-section of exchange rate risk premia. Second, the link between transient and permanent SDF components, exchange rate wedges and exchange rate puzzles.

We finally extend the intuition from the first two benchmark cases by studying two more general economies in which investors can additionally trade internationally long-term bonds and aggregate equity. These last two settings correspond to economies characterized by a smaller degree of segmentation, in which the cross-sections of currency-converted bond and equity risk premia are matched and exchange rates wedges are smaller. Thus, they are more appropriate for directly quantifying the SDF dispersion trade-offs implied by the assumption of more integrated international markets. Put differently, enlarging the set of traded assets

[7] This last tradeability condition effectively implies factorization (8).

available to investors acts towards the integration of the markets, at a cost of having more dispersion in the underlying SDFs.

2.1 Data

We use monthly data between January 1975 and December 2015 from Datastream. We compute equity returns from the corresponding MSCI country indices' prices and risk-free rates from one-month LIBOR rates. We follow Alvarez and Jermann (2005) and proxy transient SDF components by the inverse of the bond return with the longest maturity available, i.e., the ten-year (government) bonds in our case; see also Lustig, Stathopoulos, and Verdelhan (2016).[8] We start by studying eight benchmark currencies: the US dollar (USD), the British pound (GBP), the Swiss franc (CHF), the Japanese yen (JPY), the euro (EUR) (Deutsche mark (DM) before the introduction of the euro), the Australian dollar (AUD), the Canadian dollar (CAD) and the New Zealand dollar (NZD).[9] The resulting seven exchange rates are expressed with respect to the USD as the domestic currency.

We provide in Table 1 summary statistics for the different time-series. Panel A reports bond market summary statistics. We find that the CHF and the JPY feature low interest rates, in line with the intuition that they act as funding currencies in the carry trade, whereas the remaining ones can be regarded as investment currencies. Cross-sectional differences across countries arise with respect to unconditional long-term bond risk premia. To illustrate, (nominal) long-term risk premia in all countries are negative, but while in Japan and Switzerland they are -0.3% and -1.02%, respectively, in the remaining countries they range between -2.07% (EU) and -6.06% (Australia). The fact that nominal returns on long-term bonds in local currencies are negative has been documented also in Lustig, Stathopoulos, and Verdelhan (2016). There are cross-sectional differences also with respect to unconditional equity premia, especially in the case of Japan relative to all other countries, which exhibits a substantially lower average equity premium of 3.49% per year. New Zealand registers virtually the lowest cross-sectional average

[8]In order to study whether the ten-year bond return is a valid proxy for the (unobservable) infinite maturity bond return, a modeling assumption is needed, e.g., a family of affine term structure models on countries' yields. Lustig, Stathopoulos, and Verdelhan (2016) do not obtain significant differences between the yields of a hypothetical infinite maturity bond and a ten-year bond in such a setting.

[9]Throughout the paper, the sample period ranges between January 1990 to December 2015 for New Zealand, due to data availability on the long-term bonds.

equity premia, but this is also an artifact of the restricted sample period. Switzerland features the lowest market volatility with 15.42%, while the Euro-zone has the largest one (20.08%). These numbers imply a Sharpe ratio of 48% for Switzerland which is close to the one in the US and a much lower Sharpe ratio for Japan, 19%.[10] The unconditional average returns on exchange rates against the US dollar also display cross-sectional variation. The highest (positive) average return is obtained for the Swiss Franc (+2.96%), while the lowest (negative) average return follows for the Australian dollar (−0.86%). The cross-section of unconditional exchange rate volatilities does not exhibit significant variation, even though funding currencies, i.e., the Swiss franc and the Japanese yen, feature a higher volatility (12.12% and 11.32%, respectively), whereas the lowest is encountered for the Canadian dollar (6.78%). The last Panel reports inflation statistics for the countries in our sample. The highest average inflation rates are observed in New Zealand (5.57%) and in the UK (4.74%), while the lowest ones are those for Japan (1.57%) and Switzerland (1.76%).[11] In our empirical study, we deflate all domestic returns and exchange rates by the corresponding domestic Consumer Price Index, in order to obtain real returns and exchange rates.

The rich cross-sectional properties of international asset returns posit a challenge on domestic and foreign SDFs, which need to be consistent with observed exchange rate regularities. Using our model-free methodology, we quantify in the following sections the key trade-offs implied by different degrees of financial market segmentation in explaining these salient features.

2.2 Fully Segmented Domestic and Foreign Markets

Portfolio autarky can be regarded as a ban on trading international assets; see Cole and Obstfeld (1991). We take this setting as our initial benchmark case, since it reflects the highest degree of international market segmentation, in which domestic investors are not allowed to trade any foreign asset. Explicitly, the vector of tradeable real gross returns in market $i = d, f$ reads $\mathbf{R}_i = (R_{i0}, R_{i1}, R_{i\infty})'$, where R_{i0} is the risk-free rate, R_{i1} is the aggregate equity return, and $R_{i\infty}$ the return on the ten-year bond.

[10] Using the whole sample period in the case of New Zealand would yield a Sharpe ratio close to the Japanese one.

[11] Note that the sample includes observations associated with The Great Inflation of the 1970s and early 1980s, also known as stagflation, when markets in general exhibited large inflation rates.

Table 1
Data Summary Statistics

The table provides descriptive statistics for nominal domestic returns, exchange rates and CPI inflation for Switzerland, the Euro-zone (Germany before the introduction of the euro), the United Kingdom, Japan, the US, Australia, Canada and New Zealand. The sample period spans January 1975 to December 2015 (January 1990 to December 2015 for New Zealand) and the sampling frequency is monthly. Returns and inflation rates are annualized and displayed in percentages. In Panel A we report the annualized average returns for one-month risk-free bonds and ten-year government bonds. Panel B reports mean excess returns on equity, their volatility and the corresponding Sharpe ratios, computed as the ratio between the excess return and the return standard deviation. Panel C reports the annualized mean and standard deviation of exchange rates returns with respect to the US dollar. Panel D reports the average CPI inflation and its standard deviation.

	CHF	**EUR**	**GBP**	**JPY**	**USD**	**AUD**	**CAD**	**NZD**
				Panel A: Bonds				
1M	2.81	4.33	7.39	2.61	5.36	8.25	6.31	6.68
10Y	1.79	2.26	3.23	2.31	1.91	2.19	2.04	3.94
				Panel B: Excess stock returns				
Mean	7.39	6.89	6.23	3.49	7.08	5.71	5.15	0.84
Std	15.42	20.08	16.99	18.31	15.71	17.76	16.77	18.23
SR	48	34	37	19	45	32	31	5
				Panel C: Exchange rates				
Mean	2.96	0.03	-0.65	2.85		-0.86	-0.48	0.76
Std	12.12	10.56	10.20	11.32		10.93	6.78	11.92
				Panel D: Inflation				
Mean	1.76	2.22	4.74	1.57	3.69	4.83	3.71	5.57
Std	1.24	1.60	2.12	1.78	1.28	1.22	1.45	1.71

For power parameters $\alpha = 0, 0.5, 2$, we estimate the minimum dispersion SDF given in equation (1) from the empirical version of the dual problem (2):

$$\max_{\lambda_i} - \frac{\log \hat{E}_i \left[R_{\lambda_i}^{\alpha_i/(\alpha_i-1)} \right]}{\alpha_i} ,$$
$$\text{s.t. } R_{\lambda_i} > 0 , \tag{20}$$

where $\hat{E}_i[\cdot]$ denotes the expectation operator under the empirical distribution of return observations $\{\mathbf{R}_{i,t+1} : t = 0, \ldots, T-1\}$. We denote the time series of estimated optimal return observations in problem (20) by

$$R_{\hat{\lambda}_i^*,t+1} := R_{i0,t+1} + \hat{\lambda}_{i1}^*(R_{i1,t+1} - R_{i0,t+1}) + \hat{\lambda}_{i2}^*(R_{i\infty,t+1} - R_{i0,t+1}) . \tag{21}$$

Based on these returns, the time series of estimated minimum dispersion SDFs is obtained in closed-form from equation (5):

$$\hat{M}_{i,t+1}^* = \frac{R_{\hat{\lambda}_i^*,t+1}^{-1/(1-\alpha_i)}}{\hat{E}_i \left[R_{\hat{\lambda}_i^*,t+1}^{-\alpha_i/(1-\alpha_i)} \right]} , \tag{22}$$

where estimated portfolio weights in equation (21) are the unique solution of the exactly identified set of empirical moment conditions:

$$\hat{E}_i \left[R_{\hat{\lambda}_i^*}^{-1/(1-\alpha_i)} (R_{ik} - R_{i0}) \right] = 0 , \tag{23}$$

with $k = 1, ..., K_i$ and $K_d = K_f = 2$ in this case. We estimate parameter vector $\hat{\lambda}_i^*$ in equation (23) using the exactly identified (generalized) method of moments.

2.2.1 Minimum Dispersion SDFs

Since the domestic (foreign) risk-free rate, bond return and equity return are all priced by the minimum dispersion SDF M_d (M_f), the risk premia of these returns in domestic (foreign) currency are also all matched by construction. In Table 2, we report summary statistics for the SDFs using different dispersion measures. As expected, since the risk free return R_{i0} is priced by the minimum dispersion SDF M_i^*, average minimum dispersion SDFs are virtually identical across dispersion measures. Minimum dispersion SDF sample volatilities are also similar across dispersion measures and are by construction lowest for $\alpha_i = 2$. Japan features consistently the least volatile SDF, while the Australian SDF displays the highest volatility.

Overall, these volatilities align with the smallest maximal Sharpe ratios in each economy and in particular with the lowest equity Sharpe ratio for Japan given in Table 1.

Table 2
Properties of SDFs (Fully Segmented Markets)

The table reports joint sample moments of the SDF and its components. Panel A reports statistics with respect to the minimum-entropy SDFs ($\alpha_i = 0$), Panel B for Hellinger SDFs ($\alpha_i = 0.5$) and Panel C for minimum variance SDFs ($\alpha_i = 2$). We use monthly data from January 1975 to December 2015, except for New Zealand for which monthly data starts in January 1990.

	US	**UK**	**CH**	**JP**	**EU**	**AU**	**CA**	**NZ**
	Panel A: $\alpha = 0$ (minimum entropy)							
$E[M_i]$	0.983	0.974	0.990	0.990	0.979	0.966	0.974	0.957
$\text{Std}(M_i)$	0.603	0.681	0.578	0.198	0.481	0.775	0.495	0.323
$\text{Std}(M_i^T)$	0.120	0.122	0.061	0.091	0.068	0.107	0.111	0.091
$\text{Std}(M_i^P)$	0.701	0.783	0.604	0.232	0.530	0.884	0.624	0.415
$\sqrt{\text{Entropy}}(M_i)$	0.515	0.538	0.496	0.193	0.452	0.687	0.437	0.316
$\text{corr}(M_i^T, M_i^P)$	-0.593	-0.591	-0.373	-0.521	-0.693	-0.762	-0.797	-0.948
$\text{corr}(M_i, M_j)$		0.107	0.186	0.113	0.645	0.564	0.411	0.455
	Panel B: $\alpha = 0.5$ (Hellinger)							
$E[M_i]$	0.983	0.974	0.990	0.990	0.979	0.967	0.974	0.957
$\text{Std}(M_i)$	0.571	0.609	0.548	0.200	0.472	0.733	0.474	0.321
$\text{Std}(M_i^T)$	0.120	0.122	0.061	0.091	0.068	0.107	0.111	0.091
$\text{Std}(M_i^P)$	0.664	0.708	0.574	0.232	0.520	0.836	0.593	0.412
$\sqrt{\text{Hellinger}}(M_i)$	0.528	0.554	0.508	0.199	0.457	0.698	0.446	0.318
$\text{corr}(M_i^T, M_i^P)$	-0.624	-0.650	-0.390	-0.504	-0.699	-0.804	-0.833	-0.954
$\text{corr}(M_i, M_j)$		0.132	0.188	0.127	0.621	0.523	0.443	0.439
	Panel C: $\alpha = 2$ (minimum variance)							
$E[M_i]$	0.983	0.974	0.990	0.990	0.979	0.967	0.974	0.957
$\text{Std}(M_i)$	0.549	0.568	0.528	0.197	0.463	0.706	0.457	0.318
$\text{Std}(M_i^T)$	0.120	0.122	0.061	0.091	0.068	0.107	0.111	0.091
$\text{Std}(M_i^P)$	0.633	0.655	0.554	0.233	0.510	0.796	0.561	0.407
$\text{corr}(M_i^T, M_i^P)$	-0.658	-0.723	-0.419	-0.548	-0.847	-0.887	-0.888	-0.969
$\text{corr}(M_i, M_j)$		0.136	0.153	0.139	0.572	0.478	0.467	0.421

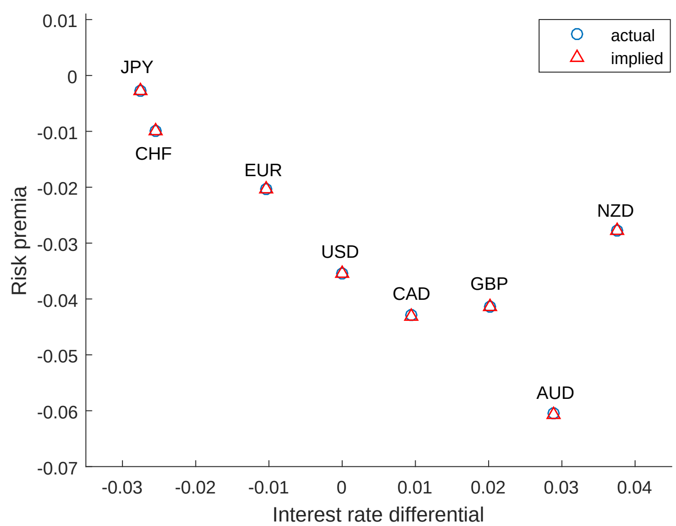
To understand in more detail the structure of the SDF volatility, we decompose the SDFs into their transitory and permanent components and compute the volatility of each component, together with their correlation. The largest fraction of the SDF volatility is generated by the permanent component, regardless of the country considered. For instance, the ratio of permanent over transient SDF volatility in Panel C (minimum variance) ranges between 9.08

for Switzerland to 2.56 for Japan. Due to the negative co-movement between permanent and transient components, the volatility of the permanent component always exceeds the total SDF volatility. This evidence is consistent with previous empirical evidence for US data; see e.g., Alvarez and Jermann (2005).

2.2.2 Local and international premia

The direction of the co-movement between transient and permanent SDF components is related to the (real) long-term bond risk premia in local currency, reported in Figure 1.

Figure 1. Local long-term bond risk premia ($\alpha_d = 0$)

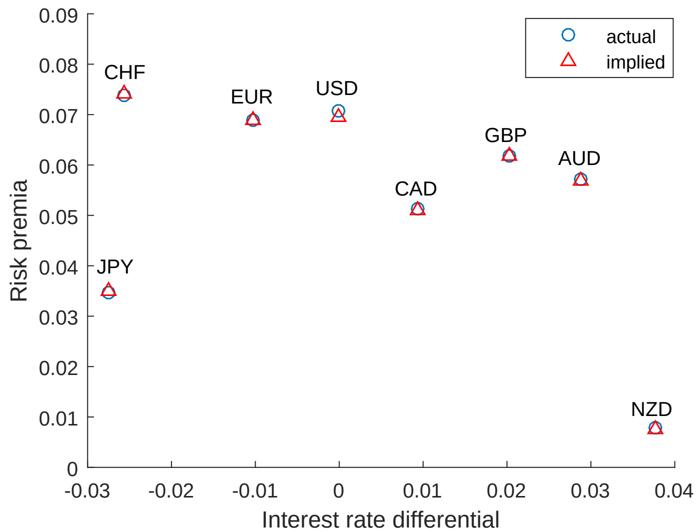


The figure plots the annualized observed local real long-term bond risk premium $E[R_{d\infty,t+1}] - E[R_{d0,t+1}]$ and the risk premium $-\text{cov}(M_{d,t+1}/E[M_{d,t+1}], R_{d\infty,t+1})$, under the minimum entropy SDF $M_{d,t+1}$, against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. Investors are allowed to trade only the domestic risk-free asset, long-term bond and aggregate equity. The currencies considered are USD, GBP, CHF, JPY, EUR, AUD, CAD and NZD.

The risk premia in Figure 1 are all negative, implying that the correlation between permanent SDF components and long-term bond returns is positive. The fact that permanent and transient components are negatively correlated can be intuitively explained by using Stein's Lemma arguments (i.e., (log)normally distributed returns and SDFs), since the transient component is a decreasing function of long-term bond returns.

We report in Figure 2 the local real equity premia. The values are on average aligned around 7%, with the exception of the Japanese market, which exhibits a substantially lower equity premia and the New Zealand, probably due to the shorter sample period considered for this economy. Since the minimum entropy SDF correctly prices returns on the equity indices, we observe that the actual and implied premia are exactly matched.

Figure 2. Local equity risk premia ($\alpha_d = 0$)



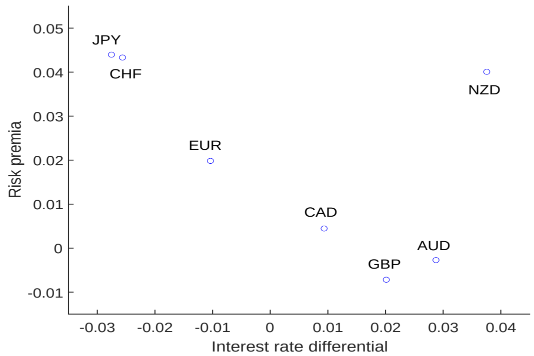
The figure plots the annualized observed local real equity risk premium $E[R_{d1,t+1}] - E[R_{d0,t+1}]$ and the risk premium $-\text{cov}(M_{d,t+1}/E[M_{d,t+1}], R_{d1,t+1})$, under the minimum entropy SDF $M_{d,t+1}$, against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. Investors are allowed to trade only the domestic risk-free asset, long-term bond and aggregate equity. The currencies considered are USD, GBP, CHF, JPY, EUR, AUD, CAD and NZD.

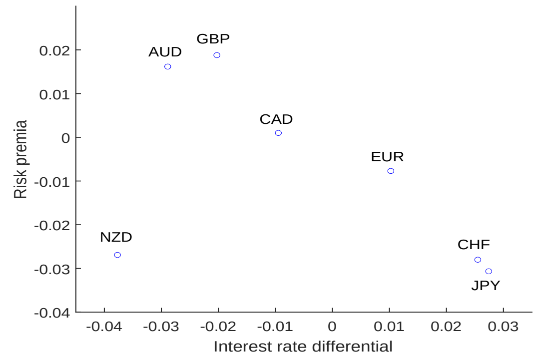
To have a better understanding in terms of the dispersion properties of international SDFs, we plot in Figure 3 for the countries examined in our empirical study, the actual currency risk premia, the international long-term bond risk premia and the international equity premia, both from the perspective of the US domestic investor and the foreign investor. The last two premia naturally embed the local premia and the currency premia. As expected, funding currencies (the Japanese yen and the Swiss franc) entail positive exchange rate premia, whereas investment currencies (the Australian dollar and the British pound) exhibit slightly negative premia, when analyzing from the domestic investor's standpoint. Interestingly to notice, the currency risk premia appears to be monotonically decreasing in the interest rate differential.[12] Similar conclusions follow for the foreign investor and there is an intuitive symmetric tendency

[12] Again, with the exception of the New Zealand, however this might be a consequence of the shorter sample period associated with this country.

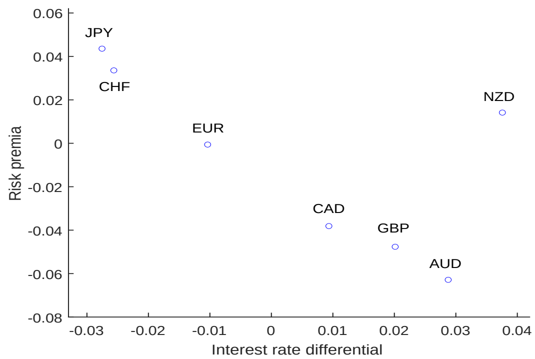
Figure 3. Observed risk premia

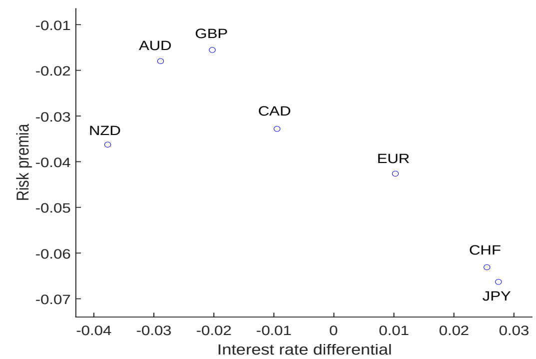
Panel A: Currency risk premia



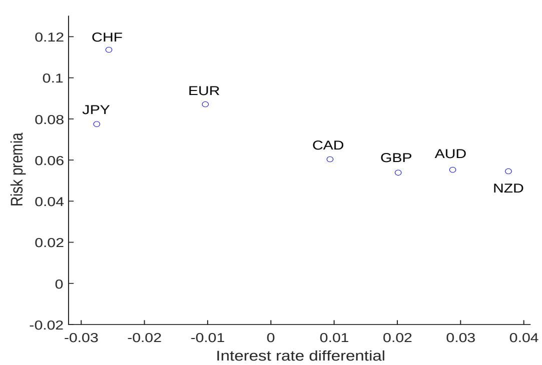


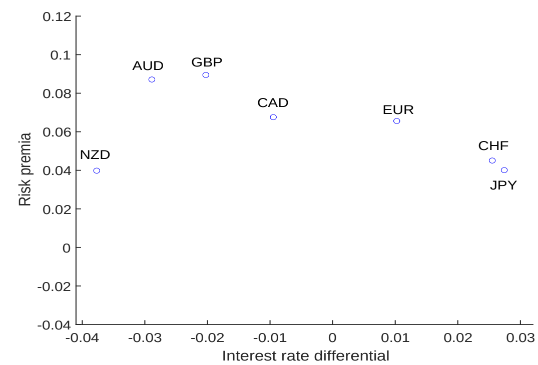
Panel B: International long-term bond risk premia





Panel C: International equity risk premia





(a) Domestic investor (b) Foreign investor

The figure plots for $i = d, f$ the observed risk premium $E[R_{i,t+1}] - E[R_{i0,t+1}]$ against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. Panel A reports the currency risk premium, Panel B the international long-term bond risk premia, whereas Panel C the international equity risk premia. The domestic investor ($i = d$) is reported on the left and the foreign one ($i = f$) on the right. The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD.

between the left and right plots, in the sense that the ranking of currency premia is preserved. Panel B of Figure 3 reports the international long-term bond risk premia, which is a byproduct of the local bond premia and the currency premia. The decreasing pattern previously observed is depicted also in this case. Positive risk premia are again associated with funding currencies, whereas investment currencies typically imply negative premia. A foreign investor who buys the US long-term bond earns a negative risk premium, more so for the funding currencies. The last panel outline the international equity premia, which compared with the previous two premia, do not exhibit a rich cross-sectional variation. In fact, the premia are rather flat for investment currencies and around 6%, while small differences arise with respect to funding currencies. The latter display significant premia, mainly stemming from the exchange rate risk and its co-movement with the equity returns.

Overall, Figure 3 hints at the challenges that asset pricing models face in order to be able to match the risk premia on various class of assets, especially since some of them suggest a decreasing pattern in the interest rate differential, whilst others are flat. We argue that these features naturally induce SDFs which are highly volatile and allowing additional assets to be traded internationally will further increase the underlying dispersion.

2.2.3 Exchange Rate Volatility and Wedges

We now address the low exchange rate volatility puzzle in Brandt, Cochrane, and Santa-Clara (2006), who show that under the assumption of complete markets, international SDFs need to be almost perfectly correlated to match the low volatility of exchange rates. The correlations in Table 2 between minimum dispersion SDF in the US and the foreign markets are on average low. For example, in Panel C correlations are less than 16% for the funding currencies and UK, while they are typically higher for the investment currencies, reaching a level of about 60% only with respect to the Euro SDF. This feature implies a large volatility of the ratio of foreign and domestic (USD) SDFs M_f and M_d, which can be consistent with the low exchange rate volatilities only in the presence of a wedge between this ratio and the exchange rate. Consistent with identity (10), the wedge resulting from the estimated minimum dispersion SDFs is such that:

$$\frac{S_{t+1}}{S_t}\exp(-\eta_{t+1}) = \frac{R_{\hat{\lambda}^*_d,t+1}^{1/(1-\alpha_d)} R_{\hat{\lambda}^*_f,t+1}^{-1/(1-\alpha_f)}}{\hat{E}_d\left[R_{\hat{\lambda}^*_d,t+1}^{-\alpha_d/(1-\alpha_d)}\right]^{-1} \hat{E}_f\left[R_{\hat{\lambda}^*_f,t+1}^{-\alpha_f/(1-\alpha_f)}\right]}\ , \tag{24}$$

where S_t denotes the exchange rate in domestic currency terms of one unit of foreign currency at time t.

Figure 4 plots the time-series of exchange rates and the ratios of minimum entropy SDFs estimated under full financial market segmentation.[13] Not surprisingly, the wedge between SDF ratios and exchange rate returns is large on average and highly time-varying, especially during periods of market turmoils, such as Black Monday in October 1987. Recall that theoretically the wedge disappears in complete arbitrage-free markets, independent of the dispersion measure applied, while it vanishes for minimum entropy SDFs in markets with symmetric international trading. Therefore, we can naturally take the wedges in fully segmented markets as a conservative benchmark to quantify the implications of extreme market segmentation for exchange rate puzzles.

Table 3 reports summary statistics of exchange rate wedges implied by estimated minimum-dispersion SDFs in fully segmented markets. As expected, the wedge in this setup is highly

Table 3
Wedge Summary Statistics (Fully Segmented Markets)

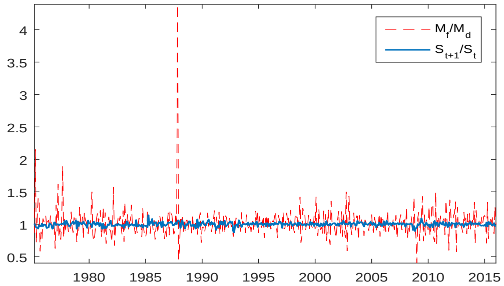
The table reports sample mean, standard deviation, skewness and kurtosis of the wedge in equation (10) ($\eta_{t+1} = \log((M_{d(t+1)}S_{t+1})/(M_{f(t+1)}S_t))$), under fully segmented markets and for dispersion measures $\alpha_i = 0, 0.5, 2$.

	$\alpha = 0$				$\alpha = 0.5$				$\alpha = 2$			
	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)
UK	-0.002	0.649	-0.570	11.705	-0.001	0.676	-0.205	6.441	0.024	0.785	1.074	9.240
CH	0.025	0.620	-0.044	8.258	0.025	0.642	0.039	5.075	0.023	0.708	0.026	3.294
JP	-0.078	0.515	2.020	14.452	-0.079	0.528	1.286	8.418	-0.091	0.562	0.122	3.943
EU	-0.018	0.446	0.239	4.380	-0.019	0.467	0.034	3.324	-0.026	0.513	-0.321	3.544
AU	0.093	0.604	-0.203	5.576	0.094	0.633	0.010	4.821	0.112	0.717	0.941	6.325
CA	-0.036	0.467	0.074	9.800	-0.036	0.483	0.098	5.798	-0.039	0.532	0.458	5.578
NZ	-0.013	0.416	0.518	4.549	-0.012	0.432	0.388	3.778	-0.012	0.458	0.059	3.137

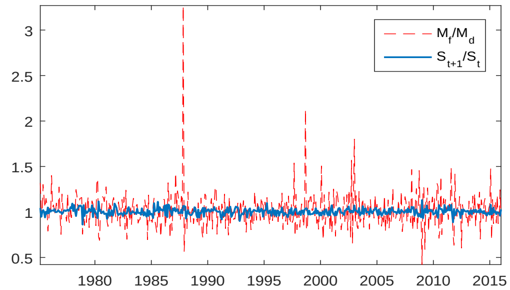
volatile, regardless of the dispersion measure applied and is quantitatively similar to the volatil-

[13] The plot is similar for all dispersion measures.

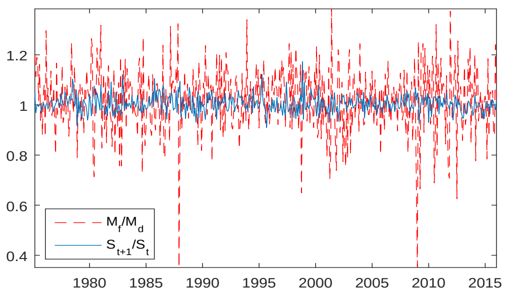
Figure 4. Exchange rate return and SDF ratio in equation (10); $\alpha_i = 0$.



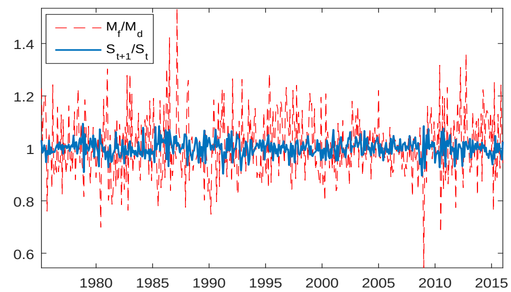
(a) USDGBP



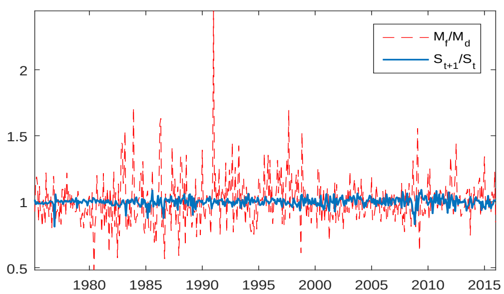
(b) USDCHF



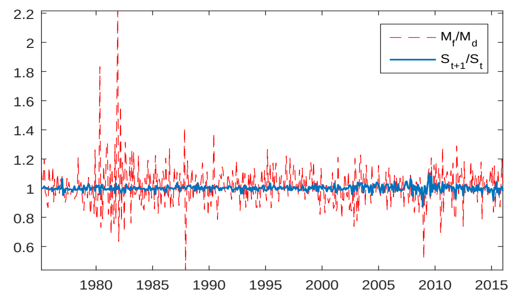
(c) USDJPY



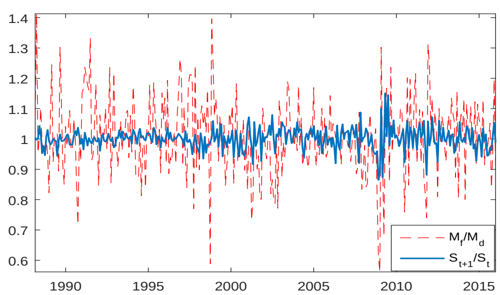
(d) USDEUR



(e) USDAUD



(f) USDCAD



(g) USDNZD

This figure plots the times series of real exchange rate returns S_{t+1}/S_t and minimum entropy ($\alpha_i = 0$) SDF ratios $M_{f(t+1)}/M_{d(t+1)}$, using the USD as the domestic currency and the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD as foreign currencies, respectively.

ities of the corresponding SDFs. Interestingly, in order to match the low volatility of the exchange rates in the data, the wedge is more volatile whenever the dispersion of the SDFs is smaller, especially for the minimum-variance pricing kernels.[14] Moreover, relevant cross-sectional differences arise with respect to the wedge skewness and kurtosis, reflecting an empirically relevant non-normality in exchange rate returns in line with earlier literature; see e.g., Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2015). Importantly, the average wedge is not zero and usually negative, with the exception of the Swiss Franc and Australian dollar. As we discuss in more detail in Section 2.2.4, this feature follows from the fact that minimum dispersion SDFs in fully segmented markets are not ensured to price the exchange rate return correctly, since domestic investors are prevented from trading any foreign assets. In the data, this feature induces a systematic average underestimation of USD exchange rate risk premia across funding currencies and an overestimation with respect to investment currencies (see, e.g., Figure 5 below).

Lustig and Verdelhan (2016) argue that in order to match the low exchange rate volatility using equation (10), the wedge needs to covary positively (negatively) with the domestic (foreign) SDF, i.e., it needs to be pro-cyclical. While this obviously has to be the case for the wedges associated with minimum dispersion SDFs, ex-ante it is less clear whether this pro-cyclicality holds with respect to both the permanent and the transient SDF components, although we would expect this to be the case for the permanent component, since it is the component driving the SDF.

We address this issue in Table 4, where we compute the correlations between exchange rate wedges and the different components of minimum dispersion SDFs. In accordance with the above intuition, all sample correlations in Table 4 between exchange rate wedges and domestic (foreign) SDFs are positive (negative) and usually large in absolute value. Importantly, this pro-cyclicality is almost entirely induced by the wedge co-movement with the permanent components. Indeed, all sample correlations between exchange rate wedges and permanent SDF components in Table 4 are quantitatively identical to the correlations obtained between exchange rate wedges and SDFs. In contrast, the correlations with the transient components

[14]These volatilities are comparable to the wedge volatilities under a simple Gaussian Consumption CAPM framework, inferred by Lustig and Verdelhan (2016), who illustrate that if the domestic and foreign SDF volatilities are about 0.5 and the SDF correlation is low, then the wedge volatility is around 0.7.

Table 4
Correlation Between Wedge and SDFs (Fully Segmented Markets)

This table reports the correlation between the wedge η, the (log) domestic and foreign minimum entropy SDFs ($\alpha_i = 0$), as well as the log permanent and transient components of minimum entropy SDFs. Log SDFs are denoted by $m_i := \log M_i$ and log SDF components by $m_i^U := \log M_i^U$ ($i = d, f$ and $U = T, P$). The domestic currency is the US and the foreign ones the UK, CH, JP, EU, AU, CA and NZ. Standard errors (SE) are computed using a circular block bootstrap of size 10 with 10,000 simulations and reported in square brackets. *,** and *** denote significance at the 10%, 5% and 1% level, respectively.

	$\text{corr}(\eta, m_i)$	SE	$\text{corr}(\eta, m_i^P)$	SE	$\text{corr}(\eta, m_i^T)$	SE
US	0.642***	[0.034]	0.634***	[0.032]	-0.345***	[0.051]
UK	-0.654***	[0.043]	-0.633***	[0.044]	0.332***	[0.056]
US	0.679***	[0.031]	0.681***	[0.033]	-0.413***	[0.039]
CH	-0.575***	[0.071]	-0.562***	[0.073]	0.104*	[0.054]
US	0.910***	[0.039]	0.903***	[0.042]	-0.502***	[0.058]
JP	-0.227***	[0.055]	-0.142***	[0.057]	-0.134**	[0.058]
US	0.537***	[0.047]	0.518***	[0.051]	-0.234***	[0.048]
EU	-0.327***	[0.086]	-0.357***	[0.912]	0.404***	[0.052]
US	0.255***	[0.062]	0.203***	[0.065]	0.092*	[0.052]
AU	-0.688***	[0.049]	-0.713***	[0.044]	0.735***	[0.027]
US	0.573***	[0.077]	0.576***	[0.076]	-0.353***	[0.055]
CA	-0.451***	[0.129]	-0.493***	[0.115]	0.581***	[0.072]
US	0.696***	[0.032]	0.641***	[0.043]	-0.098	[0.072]
NZ	-0.303***	[0.083]	-0.325***	[0.080]	0.391***	[0.071]

usually imply an opposite sign, except for the JPY, and are on average smaller in absolute value, apart from the investment currencies AUD, CAD and NZD. Intuitively, this is explained by the fact that negative bond risk premia are best incorporated by SDFs with negatively correlated permanent and transient components.

2.2.4 Additional Exchange Rate Puzzles

The previous empirical evidence might suggest that a setting with fully segmented financial markets could address the low exchange rate volatility puzzle via a volatile wedge that positively (negatively) co-moves with the permanent component of domestic (foreign) SDFs. As the SDF dispersion is dominated by the dispersion of the permanent component in highly incomplete markets, one might argue that such a framework could produce implications broadly compatible also with the Backus and Smith (1993) puzzle, i.e., the low or negative correlation of consumption growth and real exchange rate returns. To support this observation quanti-

Table 5
Correlation Between Transient SDFs Ratio and Exchange Rate Return (Fully Segmented Markets)

The table reports the correlation between the ratio of the transient SDF components and (real) exchange rate returns. The domestic currency is the USD. Since the transient component is the inverse of the observable return of the long maturity bond, these correlations are independent of the dispersion criterion used (i.e., parameter α_i). Standard errors (SE) are computed using a circular block bootstrap of size 10 with 10000 simulations and reported in square brackets.

	UK	CH	JP	EU	AU	CA	NZ
corr $\left(\frac{M^T_{f,t+1}}{M^T_{d,t+1}}, \frac{S_{t+1}}{S_t}\right)$	-0.059	0.050	-0.064	0.043	-0.002	-0.017	-0.005
SE	[0.051]	[0.050]	[0.061]	[0.045]	[0.051]	[0.054]	[0.059]

tatively, Table 5 reports the sample correlations between exchange rate returns and transient SDF component ratios in our data. These correlations are indeed all small or moderately negative and statistically not significant, in a way that is broadly consistent with the puzzle. Table 6 further supports these findings, by reporting the point estimates of Backus-Smith (1993)-type regressions. We obtain low and insignificant coefficients when regressing log differences of transient SDF components on real exchange rate returns. Point estimates based on permanent SDF components are larger in absolute value, but on average not significant, except for Canada and New Zealand, where they are close to 1, while for the EU they are even negative. Similar point estimates are obtained in regressions involving log differences of minimum dispersion SDFs. While the Backus-Smith (1993)-type regression findings using transient SDF components are independent of the dispersion measure used and the degree of market segmentation, the results using permanent components are not. However, Table 6 indicates that the findings for the permanent SDF components under fully segmented markets are fairly consistent across dispersion measures.

We conclude the study of exchange rate puzzles in the benchmark economy with fully segmented markets by addressing the violations of uncovered interest rate parity. With this

Table 6
Backus-Smith (1993)-type regressions (Fully Segmented Markets)

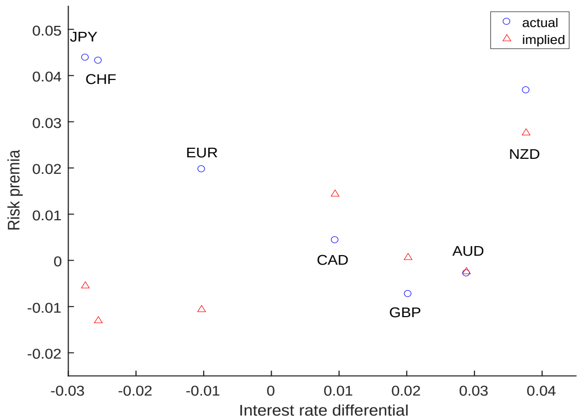
This table reports the point estimates of a linear regression of the log difference between foreign and domestic SDFs on the log change in the real exchange rate: $m_{f,t+1} - m_{d,t+1} = \delta + \beta\Delta s_{t+1} + u_{t+1}$, where $\Delta s_{t+1} = s_{t+1} - s_t$ and small-cap letters denote quantities in logs. We additionally report point estimates of a linear regression of the log difference of each component of the SDF on the log change in the real exchange rate: $m^j_{f,t+1} - m^j_{d,t+1} = \delta^j + \beta^j\Delta s_{t+1} + u^j_{t+1}$, where $j = P, T$ for permanent and transitory components, respectively. Standard errors are reported in square brackets. *, ** and *** highlight significance at the 10%, 5% and 1% level, respectively.

	Panel A: US/UK			Panel B: US/CH		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.323	0.293	0.603*	0.147	0.156	0.219
	[0.278]	[0.290]	[0.338]	[0.227]	[0.236]	[0.261]
$\boldsymbol{\beta^P}$	0.407	0.377	0.687*	0.097	0.106	0.170
	[0.320]	[0.332]	[0.378]	[0.251]	[0.259]	[0.284]
$\boldsymbol{\beta^T}$	-0.084	-0.084	-0.084	0.049	0.049	0.049
	[0.068]	[0.068]	[0.068]	[0.044]	[0.044]	[0.044]
	Panel C: US/JP			Panel D: US/EU		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	-0.157	-0.168	-0.051	-0.553***	-0.593***	-0.506**
	[0.199]	[0.205]	[0.220]	[0.173]	[0.182]	[0.204]
$\boldsymbol{\beta^P}$	-0.074	-0.085	0.030	-0.597***	-0.637***	-0.550**
	[0.226]	[0.231]	[0.246]	[0.202]	[0.210]	[0.232]
$\boldsymbol{\beta^T}$	-0.083	-0.083	-0.083	0.044	0.044	0.044
	[0.053]	[0.053]	[0.053]	[0.046]	[0.046]	[0.046]
	Panel E: US/AU			Panel F: US/CA		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	-0.282	-0.318	-0.194	0.936***	0.952***	1.194***
	[0.234]	[0.245]	[0.281]	[0.298]	[0.308]	[0.339]
$\boldsymbol{\beta^P}$	-0.277	-0.312	-0.188	0.964***	0.979***	1.222***
	[0.264]	[0.275]	[0.310]	[0.371]	[0.382]	[0.412]
$\boldsymbol{\beta^T}$	-0.005	-0.005	-0.005	-0.028	-0.028	-0.028
	[0.049]	[0.049]	[0.049]	[0.089]	[0.089]	[0.089]

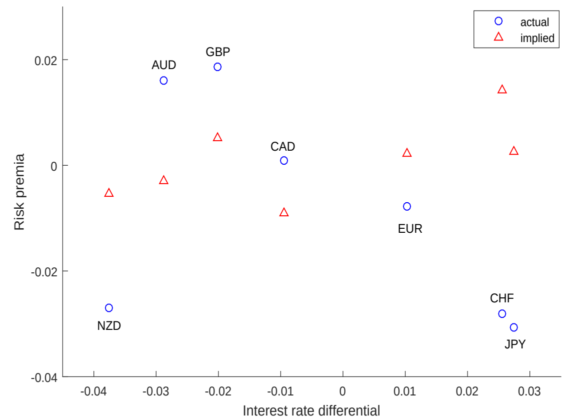
	Panel F: US/NZ		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1.2365***	1.274***	1.437**
	[0.187]	[0.194]	[0.205]
$\boldsymbol{\beta^P}$	1.242***	1.279***	1.443***
	[0.210]	[0.216]	[0.227]
$\boldsymbol{\beta^T}$	-0.006	-0.006	-0.006
	[0.037]	[0.037]	[0.037]

goal in mind, we estimate for $i = d, f$ the unconditional real exchange rate risk premium $E[R^e_{i0,t+1}] - E[R_{i0,t+1}]$ and the risk premium $-\text{cov}\left(M_{i,t+1}/E[M_{i,t+1}], (R^e_{i0,t+1} - R_{i0,t+1})\right)$ implied by the estimated minimum entropy SDF $M_{i,t+1}$.[15] Figure 5 plots the cross-section of exchange rate risk premia estimated in these two ways against the average cross-country interest rate differentials, computed as the difference between foreign and domestic nominal one-month LIBOR rates. For the domestic investor, we find that while the average cross-sectional

Figure 5. Currency risk premium ($\alpha_i = 0$)



(a) Panel A: Domestic investor



(b) Panel B: Foreign investor

The figure plots for $i = d, f$ the observed exchange rate risk premium $E[R^e_{i0,t+1}] - E[R_{i0,t+1}]$ and the risk premium $-\text{cov}\left(M_{i,t+1}/E[M_{i,t+1}], (R^e_{i0,t+1} - R_{i0,t+1})\right)$ under the minimum dispersion SDF $M_{i,t+1}$ against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. Panel A reports the currency risk premium for the domestic investor ($i = d$), whereas Panel B for the foreign one ($i = f$). The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD.

exchange risk premium is 1.97%, minimum dispersion risk premia are typically biased downwards and imply a cross-sectional average premium of only 0.15%, where the largest biases arise whenever the interest rate differential is negative and especially for the funding currencies CHF and JPY. For the investment currencies, the biases are smaller on average, however there is a tendency to overestimate the observed risk premia. For the AUD currency, the risk premium is exactly matched, but it might be an artifact of the low premium in the first place. Actual exchange rate risk premia also suggest a rather large degree of cross-sectional variability, with estimated premia that range from a minimum of -0.7% to a maximum of

[15] Again, similar findings are obtained for other dispersion measures.

4.4%. In contrast, minimum dispersion risk premia only range from −0.4% to 2.6%. Large biases emerge also from the perspective of the foreign investor. The pattern is in line with the one from the domestic perspective: positive (negative) interest rate differentials entail an overestimation (underestimation) of the true premia, the only exception being the NZD, probably as a result of the shorter sample period. Overall, there is a predilection to symmetry in the two plots, in the sense that the ranking of the currency pairs is preserved between domestic and foreign investors. Importantly, the cross-sectional differences of pricing errors suggest that the SDFs in less segmented markets will naturally induce a larger increase in SDF dispersion for the funding currencies, in order to match their large actual risk premia, i.e. the dispersion trade-off is more significant when biases are large. Moreover, differences arise across domestic and foreign investors, i.e. for EURUSD pair, the pricing error is higher (lower) in Panel A (B), suggesting that the US SDF dispersion will increase more than that of the EU SDF.

In summary, minimum dispersion SDFs in fully segmented international markets are not consistent with the exchange rate risk premium puzzle. To address this puzzle, it is necessary to price exchange rate returns in partially integrated markets. However, the broader pricing constraints induced by a stronger market integration increase the SDF variability and create a trade-off between SDF dispersion, market segmentation and exchange rate wedges. We characterize this trade-off quantitatively in the subsequent sections.

2.3 Domestic Investors Trade Foreign Risk-Free Bonds

We extend the autarky setting in the previous sections, by considering international minimum dispersion SDFs that additionally price various relevant sets of foreign assets through the exchange rate market. We start by addressing the case where risk free bonds are traded internationally. Hence, the vector of tradeable real gross returns in the domestic (US) market reads $\mathbf{R}_d = (R_{d0}, R_{d1}, R_{d\infty}, R^e_{d0})'$, where $R^e_{d0,t+1} := R_{f0,t+1}S_{t+1}/S_t$ is the domestic currency return of the foreign risk free asset. Similarly, the vector of tradeable real gross returns in the foreign market reads $\mathbf{R}_f = (R_{f0}, R_{f1}, R_{f\infty}, R^e_{f0})'$, where $R^e_{f0,t+1} := R_{d0,t+1}S_t/S_{t+1}$ is the foreign currency return of the domestic risk free asset.

The estimation of minimum dispersion SDFs in this economy is again achieved by solving optimization problem (20), with one additional pricing constraint (the one induced by R^e_{i0})

that needs to be satisfied, i.e., $K_d = K_f = 3$ in the set of moment conditions (23). Hence, the estimated optimal portfolio return in market $i = d, f$ reads:

$$R_{\hat{\lambda}_i^*,t+1} = R_{i0,t+1} + \hat{\lambda}_{i1}^*(R_{i1,t+1} - R_{i0,t+1}) + \hat{\lambda}_{i2}^*(R_{i\infty,t+1} - R_{i0,t+1}) + \hat{\lambda}_{i3}^*(R_{i0,t+1}^e - R_{i0,t+1}) \; .$$

The closed-form expression for the estimated minimum dispersion SDF follows as in the autarky case, simply by plugging this optimal return into equation (22).

2.3.1 Minimum Dispersion SDFs

Different from the autarky case, we obtain an estimated minimum dispersion US (domestic) SDF for each bilateral trade against the foreign currency. Besides matching the risk premia on the domestic returns in the autarky case, these SDFs are able to match also the risk premium on returns R_{d0}^e and R_{f0}^e. Therefore, they will match the exchange rate risk premium in the data. Table 7 documents how the enlarged set of tradeable assets affects the properties of minimum dispersion SDFs.

As under autarky, the pricing constraint on the domestic risk free return implies the normalization of the SDF first sample moment to the average of the inverse risk free rate in the data. Due to the additional pricing restriction on returns R_{d0}^e and R_{f0}^e, the variability of the minimum dispersion SDF increases relative to the autarky case. For instance, while the SDF variability in the UK, EU, Australian and Canadian markets is virtually unchanged, Switzerland exhibits a 19%, Japan a 82% and New Zealand a 15% higher volatility of the minimum variance SDF. The apparent high increase for the Japanese SDF is also due to the fact that under autarky this pricing kernel exhibited the lowest volatility, consistent with its market Sharpe Ratio, and despite this sharp rise, the level is still below than the one for the remaining currencies. Moreover, while the variability of US minimum dispersion SDFs across different bilateral trades in Table 7 is similar to the autarky case for the UK, EU, Australian, Canadian and New Zealand markets, it is about 24% (27%) higher with respect to the Japanese (Swiss) market. This last finding reflects the more pronounced dispersion trade-offs of SDFs that are required to match the exchange rate risk premia of funding currencies relative to the USD. Indeed, the CHF/USD and JPY/USD parities are those associated with the largest biases in the risk premia under autarky in Figure 5, both in domestic and foreign terms, yielding the

Table 7

Properties of SDFs (Trading in Foreign Short-Term Bonds)

The table reports joint sample moments of the SDF and its components. Panel A reports statistics with respect to the minimum-entropy SDFs ($\alpha_i = 0$), Panel B for Hellinger SDFs ($\alpha_i = 0.5$) and Panel C for minimum variance SDFs ($\alpha_i = 2$), $i = d, f$, $j = d, f$, $i \neq j$. We use monthly data from January 1975 to December 2015.

	US	**UK**	**US**	**CH**	**US**	**JP**	**US**	**EU**	**US**	**AU**	**US**	**CA**	**US**	**NZ**
							Panel A: $\alpha = 0$ (minimum entropy)							
E[M_i]	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.979	0.982	0.966	0.983	0.973	0.983	0.956
Std(M_i)	0.611	0.723	0.769	0.674	0.722	0.364	0.645	0.487	0.603	0.821	0.634	0.514	0.539	0.380
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.707	0.824	0.843	0.698	0.800	0.379	0.732	0.535	0.702	0.931	0.729	0.641	0.585	0.466
$\sqrt{\text{Entropy}}(M_i)$	0.520	0.550	0.659	0.598	0.666	0.362	0.575	0.461	0.515	0.702	0.533	0.457	0.486	0.359
corr(M_i^T,M_i^P)	-0.586	-0.567	-0.503	-0.310	-0.527	-0.280	-0.578	-0.680	-0.593	-0.726	-0.566	-0.781	-0.372	-0.846
corr(M_i,M_j)		0.122		0.374		0.460		0.646		0.585		0.396		0.536
							Panel B: $\alpha = 0.5$ (Hellinger)							
E[M_i]	0.983	0.973	0.983	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.577	0.631	0.730	0.650	0.702	0.363	0.625	0.479	0.572	0.762	0.592	0.494	0.525	0.372
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.668	0.728	0.803	0.673	0.776	0.377	0.709	0.527	0.664	0.864	0.682	0.612	0.567	0.457
$\sqrt{\text{Hellinger}}(M_i)$	0.533	0.568	0.676	0.610	0.674	0.363	0.587	0.466	0.528	0.717	0.547	0.466	0.496	0.363
corr(M_i^T,M_i^P)	-0.617	-0.640	-0.526	-0.318	-0.540	-0.272	-0.593	-0.683	-0.623	-0.780	-0.602	-0.814	-0.377	-0.862
corr(M_i,M_j)		0.154		0.351		0.469		0.627		0.538		0.430		0.523
							Panel C: $\alpha = 2$ (minimum variance)							
E[M_i]	0.983	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.555	0.581	0.699	0.630	0.681	0.359	0.608	0.471	0.551	0.728	0.568	0.474	0.512	0.366
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.637	0.666	0.766	0.651	0.747	0.376	0.684	0.518	0.634	0.816	0.648	0.579	0.549	0.446
corr(M_i^T,M_i^P)	-0.652	-0.719	-0.554	-0.340	-0.564	-0.301	-0.617	-0.716	-0.656	-0.829	-0.638	-0.866	-0.393	-0.886
corr(M_i,M_j)		0.166		0.276		0.492		0.586		0.480		0.451		0.503

clearly higher dispersion of both domestic (US) and foreign (JP and CH) SDFs. In line with the intuition regarding the pricing errors under autarky, the volatility of the minimum variance EU SDF increases by only 1.7%, whereas the one of the corresponding US SDF by 10%.

Consistently with the autarky case, permanent SDF components are in all cases more volatile than minimum dispersion SDFs and they are negatively correlated with the transient SDF components.[16] However, all such correlations are lower in absolute value than under autarky, in a way that is clearly more pronounced for bilateral trades related to the funding currencies CHF and JPY, as well as with respect to the investment currency NZD. This is a natural consequence of the fact that the increased dispersion of the permanent SDF components in these cases imply a less negative correlation with the transient component in order to match the long-term bond risk premia in local currencies. To have a better understanding about the mechanism, note that the correlation between the transient and permanent SDF components for an SDF M can be expressed as:

$$\mathrm{Corr}(M^T, M^P) = \frac{E[M] - E[M^T]}{\sqrt{\mathrm{Var}(M^T)}\sqrt{\mathrm{Var}(M^P)}}, \tag{25}$$

and since the numerator and the distribution of the transient SDF component are fixed regardless of the degree of market segmentation and of the dispersion measure used, the correlation decreases whenever the volatility of the permanent SDF component increases.

Finally, all correlations between domestic and foreign SDFs are below 0.65, still indicating a fairly imperfect SDF co-movement.

2.3.2 Exchange Rate Volatility and Wedges

After inserting the time series of the optimal returns $R_{\hat{\lambda}^*_{d,t+1}}$ and $R_{\hat{\lambda}^*_{f,t+1}}$ of Section 2.3 in definition (24), we obtain the exchange rate wedge in the presence of internationally tradeable short term bonds. Figure 11, reported in the Appendix for interest of space, documents that the wedge is still highly volatile, which is consistent with domestic and foreign minimum dispersion SDFs that reflect a substantial degree of residual market segmentation.

[16] Recall that the annualized standard deviation of the transient component is unchanged, since it is determined by the standard deviation of the (inverse) of the long-maturity bond (gross) return.

Table 8 reports summary wedge statistics. When risk-free bonds are tradeable internationally, the average wedge is typically not zero, and the wedge volatility is similar to the one under autarky. As expected, the highest wedge is obtained for the JPY and AUD currencies, known to have experienced crashes. Moreover, the wedge's higher moments reflect the non-normality of exchange rate returns and a quite evident sensitivity on the choice of the admissible SDFs in incomplete international markets, especially for UK, Japan, EU and AU markets, with, e.g., fatter tails and opposite signs for skewness for $\alpha_i = 2$ and $\alpha_i = 0, 0.5$. This is a consequence of the fact that compared with the other dispersion measures, the minimum variance is not able to capture extreme movements since it does not account for higher moments.

Table 8

Wedge Summary Statistics (Trading in Foreign Short-Term Bonds)

The table reports sample mean, standard deviation, skewness and kurtosis of the wedge in equation (10) ($\eta_{t+1} = \log((M_{d(t+1)}S_{t+1})/(M_{f(t+1)}S_t))$), for dispersion measures $\alpha = 0, 0.5, 2$. The optimal derived SDFs account for the fact that domestic investors can trade the short-term foreign risk-free bond.

	$\alpha = 0$				$\alpha = 0.5$				$\alpha = 2$			
	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)
UK	0.003	0.636	-0.646	13.55	0.005	0.665	-0.207	6.847	0.042	0.814	1.074	9.239
CH	-0.006	0.682	-0.367	6.270	-0.007	0.713	-0.174	4.647	-0.021	0.826	-0.019	3.724
JP	-0.123	0.545	1.446	8.938	-0.124	0.554	1.053	6.713	-0.149	0.612	-0.259	5.417
EU	-0.048	0.439	0.265	4.026	-0.048	0.455	0.058	3.629	-0.059	0.517	-0.554	5.011
AU	0.104	0.581	-0.181	5.573	0.106	0.614	0.050	4.898	0.129	0.716	1.051	6.714
CA	-0.036	0.490	0.148	9.963	-0.036	0.507	0.093	5.655	-0.040	0.561	0.305	5.082
NZ	-0.020	0.413	0.362	4.556	-0.021	0.419	0.265	4.045	-0.029	0.442	0.178	3.834

The wedge cyclicality properties summarized in Table 9 are roughly consistent with those under autarky: the pro-cyclicality is almost completely explained by a pronounced positive (negative) correlation with the permanent components of the domestic (foreign) SDF. The co-movement with the transient component is instead typically weaker and of opposite sign. With the exception of the JPY/USD and the NZD/USD wedges, these correlations are also quantitatively similar to those obtained under autarky.

Table 9
Correlation Between Wedge and SDFs (Trading in Foreign Short-Term Bonds)

This table reports the correlation between the wedge η, the (log) domestic and foreign minimum entropy SDFs ($\alpha = 0$), as well as the log permanent and transient components of minimum entropy SDFs. Log SDFs are denoted by $m_i := \log M_i$ and log SDF components by $m_i^U := \log M_i^U$ ($i = d, f$ and $U = T, P$). Standard errors (SE) are computed using a circular block bootstrap of size 10 with 10000 simulations and reported in square brackets. *** denotes significance at the 1% level.

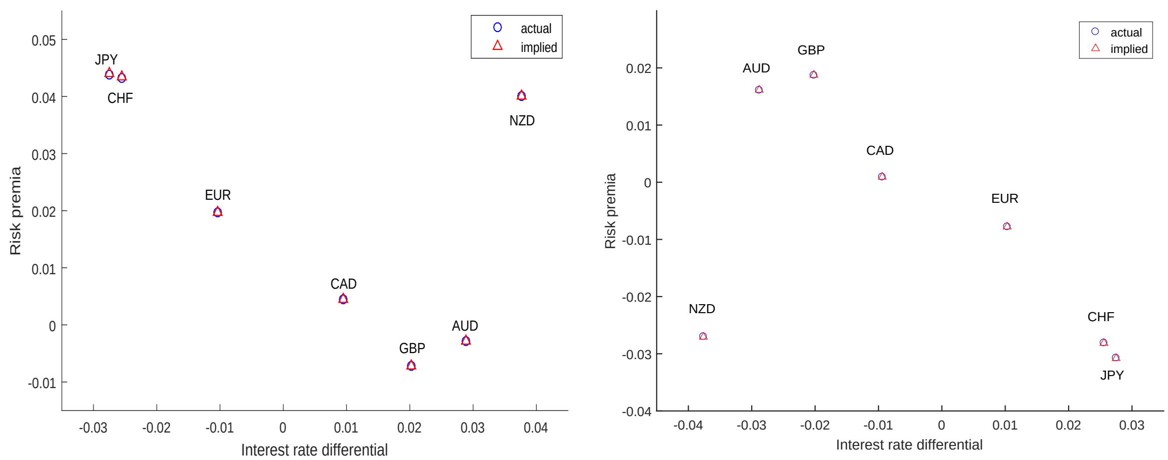
	$\text{corr}(\eta, m_i)$	SE	$\text{corr}(\eta, m_i^P)$	SE	$\text{corr}(\eta, m_i^T)$	SE
US	0.658***	[0.039]	0.651***	[0.039]	-0.356***	[0.049]
UK	-0.617***	[0.052]	-0.602***	[0.057]	0.338***	[0.053]
US	0.541***	[0.026]	0.569***	[0.029]	-0.431***	[0.038]
CH	-0.594***	[0.051]	-0.585***	[0.053]	0.077	[0.054]
US	0.728***	[0.039]	0.759***	[0.042]	-0.532***	[0.058]
JP	-0.201***	[0.054]	-0.200***	[0.056]	-0.029	[0.061]
US	0.552***	[0.047]	0.546***	[0.050]	-0.273***	[0.058]
EU	-0.296***	[0.084]	-0.324***	[0.909]	0.402***	[0.052]
US	0.257***	[0.062]	0.204***	[0.065]	0.087	[0.057]
AU	-0.685***	[0.049]	-0.714***	[0.044]	0.756***	[0.030]
US	0.606***	[0.077]	0.605***	[0.076]	-0.342***	[0.055]
CA	-0.426***	[0.120]	-0.470***	[0.111]	0.565***	[0.069]
US	0.523***	[0.031]	0.441***	[0.040]	0.278***	[0.039]
NZ	-0.465***	[0.075]	-0.508***	[0.071]	0.606***	[0.057]

2.3.3 Additional Exchange Rate Puzzles

According to the empirical evidence in Table 5, transient SDF components and exchange rate returns are only weakly correlated. Within the class of asset pricing models which feature a non negligible martingale component, this property is convenient to motivate a low correlation between cross-sectional differences in consumption growth and exchange rate returns. We quantify these relations in more detail in Table 10, which reports the point estimates of a set of Backus-Smith (1993)-type regression of log differences in minimum dispersion SDFs and martingale SDF components on real exchange rate returns.

In a complete market, the population point estimate from these regressions based on the overall SDFs is exactly 1. More generally, when risk-free returns are traded internationally, Lustig and Verdelhan (2016) show that the same finding holds also in incomplete markets. Therefore, the setting with internationally tradeable risk-free bonds clearly implies different implications from those obtained in Table 6 under autarky. Importantly, since the SDF variability in Table 7 is dominated by the permanent component, similar implications hold for regressions using the persistent components of minimum dispersion SDFs. Indeed, Table 10 shows that all regression point estimates are significantly different from 0 and never significantly different from the target value of one. While these findings concretely highlight the role of permanent martingale components in Backus-Smith-type regressions, it is important to realize that they are naturally consistent with the Backus-Smith (1993) puzzle, which can be motivated in our setting by the low correlation between transient SDF components and exchange rate returns; see again Table 5.

Figure 6. Currency risk premia, $\alpha_i = 0$



(a) Panel A: Domestic investor (b) Panel B: Foreign investor

The figure plots for $i = d, f$ the observed exchange rate risk premium $E[R^e_{i0,t+1}] - E[R_{i0,t+1}]$ and the risk premium $-\text{cov}\left(M_{i,t+1}/E[M_{i,t+1}], (R^e_{i0,t+1} - R_{i0,t+1})\right)$ under the minimum dispersion SDF $M_{i,t+1}$ against the average interest rate differential. Panel A reports the currency risk premium for the domestic investor ($i = d$), whereas Panel B for the foreign one ($i = f$). The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD.

We finally address the cross-section of exchange rate risk-premia in our model-free setting with internationally traded risk-free bonds. As M_d (M_f) prices return R^e_d (R^e_f) via the corresponding Euler equation, we expect the cross-section of exchange rate risk premia to be perfectly explained when risk-free returns are traded internationally. Indeed, Figure 6 Panel

Table 10
Backus-Smith (1993)-Type Regressions
(Trading in Foreign Short-Term Bonds)

This table reports the point estimates of a linear regression of the log difference between foreign and domestic SDFs on the log change in the real exchange rate: $m_{f,t+1} - m_{d,t+1} = \delta + \beta \Delta s_{t+1} + u_{t+1}$, where $\Delta s_{t+1} = s_{t+1} - s_t$ and small-cap letters denote quantities in logs. We additionally report point estimates of a linear regression of the log difference of each component of the SDF on the log change in the real exchange rate: $m^j_{f,t+1} - m^j_{d,t+1} = \delta^j + \beta^j \Delta s_{t+1} + u^j_{t+1}$, where $j = P, T$ for permanent and transitory components, respectively. Standard errors are reported in square brackets. Labels ** and *** highlight significance at the 5% and 1% level, respectively.

	Panel A: US/UK			Panel B: US/CH		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.801***	0.918***	1.0683***	0.709***	0.849***	1.059***
	[0.274]	[0.287]	[0.352]	[0.253]	[0.265]	[0.307]
$\boldsymbol{\beta^P}$	0.886***	1.003***	1.153***	0.660**	0.799***	1.009***
	[0.316]	[0.329]	[0.391]	[0.276]	[0.288]	[0.329]
	Panel C: US/JP			Panel D: US/EU		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.937***	1.003***	1.114***	0.935***	0.975***	0.994***
	[0.229]	[0.222]	[0.245]	[0.184]	[0.191]	[0.217]
$\boldsymbol{\beta^P}$	1.019***	1.086***	1.197***	0.892***	0.931***	0.949***
	[0.249]	[0.253]	[0.276]	[0.213]	[0.219]	[0.245]
	Panel E: US/AU			Panel F: US/CA		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.981***	1.031***	1.087***	0.974***	1.039***	1.062***
	[0.231]	[0.244]	[0.284]	[0.312]	[0.323]	[0.357]
$\boldsymbol{\beta^P}$	0.986***	1.037***	1.093***	1.002***	1.067***	1.089**
	[0.262]	[0.275]	[0.315]	[0.383]	[0.395]	[0.429]

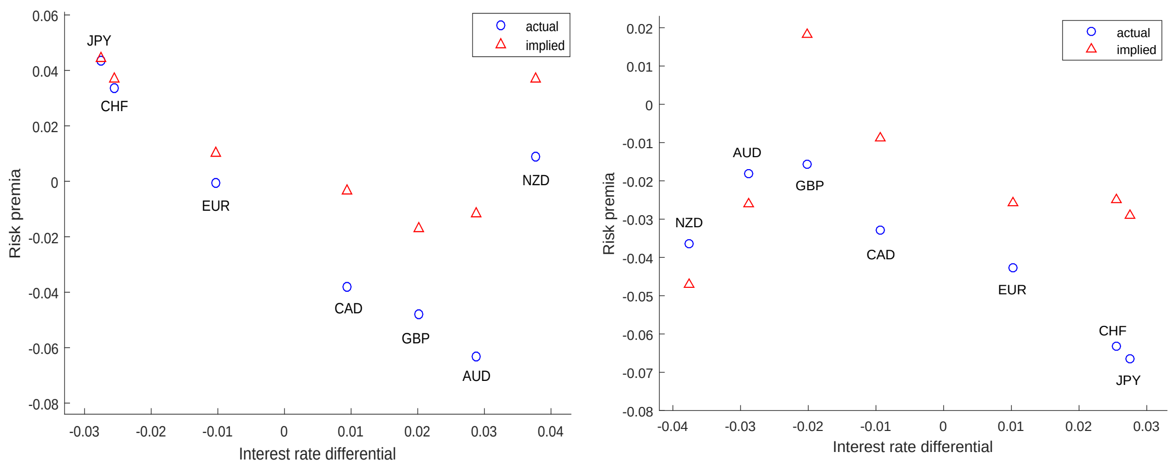
	Panel F: US/NZ		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.875***	0.933***	1.045**
	[0.186]	[0.189]	[0.199]
$\boldsymbol{\beta^P}$	0.881***	0.939***	1.051**
	[0.202]	[0.205]	[0.215]

A (B) shows that the cross-section of domestic USD (foreign) exchange rate risk premia is exactly matched by the negative covariance between the normalized domestic SDF M_d (M_f) and real exchange rate returns.

2.3.4 *International long-term bond risk premia*

It is useful to study to which extent the minimum dispersion SDFs of the setting with internationally tradeable risk-free bonds is able to price the international risk premia of additional assets, such as long-term bonds or equities. Figure 7 reports the cross-sections of (real) international long-term bond risk premia in the data, together with the risk premia implied by the minimum entropy SDFs.

Figure 7. International long-term bond risk premia ($\alpha_i = 0$)



(a) Panel A: Domestic investor

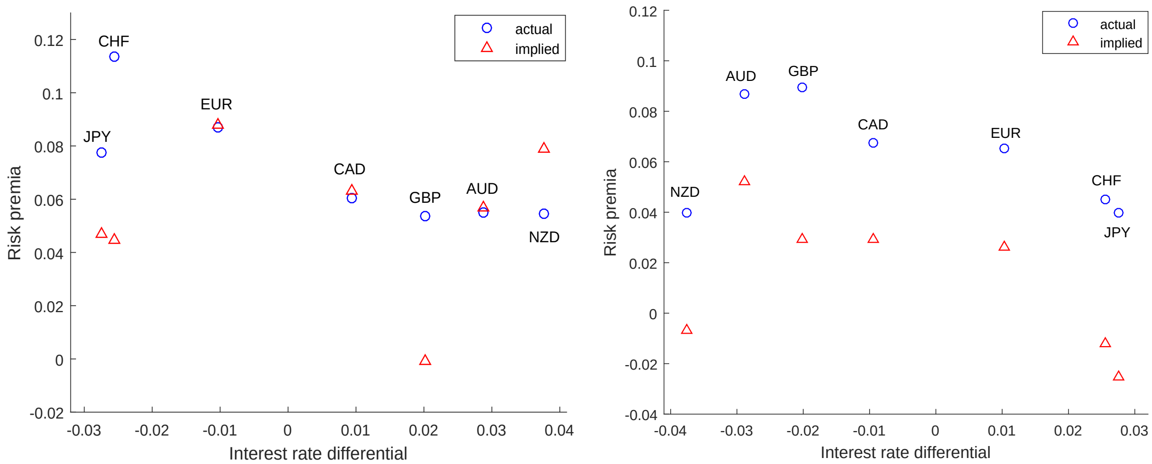
(b) Panel B: Foreign investor

The figure plots for $i = d, f$ the observed international long-term bond risk premium $E[R^e_{i\infty,t+1}] - E[R_{i0,t+1}]$ and the risk premium $-\text{cov}\left(M_{i,t+1}/E[M_{i,t+1}], (R^e_{i\infty,t+1} - R_{i0,t+1})\right)$ under the minimum dispersion entropy SDF $M_{i,t+1}$ against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. Panel A reports the long-term bond risk premium for the domestic investor ($i = d$), whereas Panel B for the foreign one ($i = f$). The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD.

In our sample, international domestic bond risk premia are monotonically decreasing in the average interest rate differential, except for the New Zealand dollar. The domestic bond risk premia implied by minimum dispersion SDFs when only risk-free bonds are internationally traded are also similarly monotonic and well aligned to the empirical risk premia whenever the interest rate differential is negative. Specifically, it is clear from Panel A that the US SDF dispersion with respect to CH and JP is not going to increase when investors can trade additionally the foreign long term bond. However, the implied risk premia generally overestimates the actual one, especially in the case of investment currencies for which the interest rate differential is positive. On the other hand, the long term bond risk premia from the foreign investor's perspective are all negative and the fit worsens, with implied quantities being on average higher than their observed counterparts, investment currencies (AUD and NZD) exhibiting the smallest pricing errors. For the remaining currency pairs, the biases suggest a pronounced SDF dispersion trade-off between markets with internationally traded risk-free bonds and markets with internationally traded risk-free and long-term bonds. Interestingly, the risk premia of the international long-term bonds preserve the ranking of the currency risk premia illustrated in Figure 6, as well as the symmetry tendency when going from domestic to foreign standpoints. Although there is a similar pattern with the exchange rate risk premia, it seems that as the interest rate differential increases, the actual international long-term bond risk premia decreases faster. Overall, it seems that the implied risk premia in both panels of Figure 7 are able to capture the direction, but not the size of the actual risk premia on international long-term bonds. Moreover, since the implied values correspond to the currency risk premia, we find that there is a close link between the two premia, with the former being typically higher than the one for the international bonds, highlighting once again the negative local bond premia. Ultimately, these findings anticipate a further increase in SDF dispersion to be able to match also the cross section of international equity premia. For completeness, we display in Figure 8 the actual international equity premia versus the one implied by the minimum entropy SDF when investors can trade international risk-free bonds only. From the domestic perspective, the implied risk premia underestimates on average the real one, and provides a quite accurate description for the EUR, CAD and AUD. On the contrary, large downward biases arise from the foreign investor's perspective, implying that the foreign SDF dispersion is going to increase more than the associated domestic one in order to match the international equity premia.

Lastly, we examine the international term premia, i.e. the expected return on the foreign (domestic) long-term bond minus the expected return on the short term bond, both converted

Figure 8. International equity risk premia ($\alpha_i = 0$)



(a) Panel A: Domestic investor

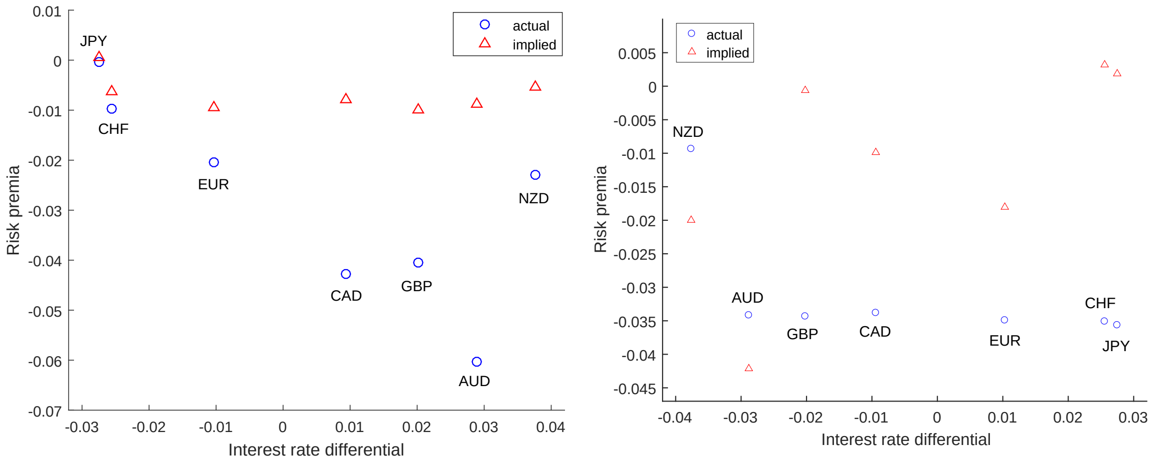
(b) Panel B: Foreign investor

The figure plots for $i = d, f$ the observed international equity risk premium $E[R^e_{i1,t+1}] - E[R_{i0,t+1}]$ and the risk premium $-\text{cov}\left(M_{i,t+1}/E[M_{i,t+1}], (R^e_{i1,t+1} - R_{i0,t+1})\right)$ under the minimum dispersion entropy SDF $M_{i,t+1}$ against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. Panel A reports the equity risk premium for the domestic investor ($i = d$), whereas Panel B for the foreign one ($i = f$). The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD.

in domestic (foreign) units. We find that this is simply the difference between the international long-term bond premia and the currency risk premium. The results are displayed in Figure 9. From the domestic US investor's perspective, we observe that the implied premia suggest a rather flat term structure, which is not present in the data. The actual term premia appears to be monotonically decreasing in the interest rate differential, the exception being the New Zealand dollar, probably due to the shorter sample period. Interestingly, from the foreign investor's point of view, the actual term premia are flat, the exception being the NZD, for the same previously mentioned reason, however the implied premia exhibit biases as large as the slope. These findings suggest that in incomplete market settings, US long term bonds translated in different currencies yield on average similar returns to foreign investors, whereas bonds denominated in foreign currencies do not deliver the same dollar returns.

In summary, minimum dispersion SDFs in segmented markets with internationally traded risk-free returns are consistent with well-known exchange rate puzzles. The cross-section of minimum dispersion SDFs is dominated by large permanent SDF components, which nega-

Figure 9. International term premia ($\alpha_i = 0$)



(a) Panel A: Domestic investor

(b) Panel B: Foreign investor

The figure plots for $i = d, f$ the observed international term-structure bond risk premium $E[R^e_{i\infty,t+1}] - E[R^e_{i0,t+1}]$ and the risk premium $-\text{cov}\left(M_{i,t+1}/E[M_{i,t+1}], (R^e_{i\infty,t+1} - R^e_{i0,t+1})\right)$ under the minimum dispersion entropy SDF $M_{i,t+1}$ against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. Panel A reports the term-structure bond risk premium for the domestic investor ($i = d$), whereas Panel B for the foreign one ($i = f$). The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD.

tively co-move with the transient components and are cross-sectionally imperfectly correlated. Exchange rate wedges are similarly volatile as minimum dispersion SDFs and their distribution does not resemble a normal distribution. Finally, minimum dispersion SDFs imply a quite loose description of the cross-section of international long-term bond risk premia in the data, which naturally preserves the ranking, but overestimates them.

2.4 Domestic Investors Trade Short- and Long-Term Foreign Bonds

We now address a less segmented market setting, in which short- and long-term bonds are traded internationally. The tradeable vectors of returns in markets $i = d, f$ read $\mathbf{R}_i = (R_{i0}, R_{i1}, R_{i\infty}, R^e_{i0}, R^e_{i\infty})'$, where $R^e_{d\infty,t+1} := R_{f\infty,t+1}S_{t+1}/S_t$ ($R^e_{f\infty,t+1} := R_{d\infty,t+1}S_t/S_{t+1}$) is the domestic (foreign) currency return of the foreign (domestic) long-term bond. Compared to the previous section, the minimum dispersion SDFs in optimization problem (20) follow with the additional pricing constraint induced by return $R^e_{i\infty}$, i.e., $K_d = K_f = 4$ in the set of moment conditions (23). The estimated optimal portfolio returns reads:

$$\begin{aligned} R_{\hat{\lambda}^*_i,t+1} &= R_{i0,t+1} + \hat{\lambda}^*_{i1}(R_{i1,t+1} - R_{i0,t+1}) + \hat{\lambda}^*_{i2}(R_{i\infty,t+1} - R_{i0,t+1}) \\ &\quad + \hat{\lambda}^*_{i3}(R^e_{i0,t+1} - R_{i0,t+1}) + \hat{\lambda}^*_{i4}(R^e_{i\infty,t+1} - R_{i0,t+1}) \; . \end{aligned} \quad (26)$$

Thus, the estimated minimum dispersion SDFs follow again in closed-form from the empirical moment conditions in equation (22).

2.4.1 Minimum dispersion SDFs

In contrast to the previous market setting, the minimum dispersion SDFs in this section price exactly also the cross-section of international long-term bond risk premia, which is reported in Figure 13 in the Appendix. As anticipated, this feature induces an additional increase in dispersion, which is illustrated in Table 11. In order to match the bond risk premia in local currencies, transient and permanent SDF components are again negatively related. Therefore, the increase in SDF dispersion is induced in most cases by a significant dispersion of the permanent component, which in compliance with identity (25) leads to a smaller absolute co-movement between permanent and transient components. Relative to the findings in Table 7, Table 11 also documents an increase in the co-movement between domestic and foreign minimum dispersion SDFs and permanent SDF components. Overall, this evidence confirms the foreseen SDF dispersion trade-offs between markets with internationally traded risk-free bonds and markets with internationally traded risk-free and long-term bonds.

2.4.2 Exchange Rate Volatility and Wedges

The increased cross-sectional co-movement of the permanent SDF components above can be understood by rearranging terms in equation (11) into the following identity

$$\frac{S_{t+1}}{S_t}\frac{R_{f\infty,t+1}}{R_{d\infty,t+1}} = \frac{M^P_{f,t+1}}{M^P_{d,t+1}} e^{\eta_{t+1}} \; , \quad (27)$$

where the wedge η_{t+1} is computed according to definition (24) with the optimal returns given in equation (26). The low variability of the LHS of this identity in the data can be explained either by a low variability of both the ratio of permanent SDF components and the wedge, by a

Table 11

Properties of SDFs (Trading in Foreign Short- and Long-Term Bond)

This table reports the annualized mean and volatility of the SDFs and their components, as well as the correlation between domestic (US) and foreign SDFs and the correlation between transient and permanent components within the same SDF. The SDFs are derived when domestic investors can trade additionally the risk-free and the long maturity foreign bonds.

	US	**UK**	**US**	**CH**	**US**	**JP**	**US**	**EU**	**US**	**AU**	**US**	**CA**	**US**	**NZ**
Panel A: $\alpha = 0$ (minimum entropy)														
E$[M_i]$	0.982	0.974	0.982	0.990	0.982	0.991	0.982	0.979	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.664	0.709	0.779	0.751	0.722	0.500	0.690	0.517	0.888	0.804	0.713	0.567	0.636	0.395
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.757	0.801	0.852	0.772	0.800	0.514	0.774	0.564	0.995	0.912	0.808	0.693	0.677	0.476
$\sqrt{\text{Entropy}}(M_i)$	0.575	0.611	0.662	0.677	0.666	0.488	0.604	0.485	0.715	0.706	0.611	0.498	0.580	0.384
corr(M_i^T, M_i^P)	-0.551	-0.588	-0.500	-0.283	-0.527	-0.210	-0.549	-0.648	-0.422	-0.742	-0.517	-0.727	-0.319	-0.832
corr(M_i, M_j)		0.483		0.548		0.651		0.814		0.856		0.841		0.784
Panel B: $\alpha = 0.5$ (Hellinger)														
E$[M_i]$	0.982	0.974	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.635	0.666	0.738	0.733	0.702	0.495	0.660	0.509	0.800	0.759	0.681	0.544	0.618	0.392
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.722	0.755	0.810	0.753	0.776	0.508	0.740	0.554	0.883	0.860	0.768	0.658	0.655	0.471
$\sqrt{\text{Hellinger}}(M_i)$	0.589	0.624	0.679	0.689	0.674	0.491	0.617	0.491	0.735	0.719	0.627	0.509	0.5885	0.386
corr(M_i^T, M_j^P)	-0.575	-0.621	-0.522	-0.287	-0.540	-0.205	-0.571	-0.653	-0.471	-0.784	-0.541	-0.761	-0.326	-0.839
corr(M_i, M_j)		0.515		0.528		0.663		0.807		0.840		0.856		0.785
Panel C: $\alpha = 2$ (minimum variance)														
E$[M_i]$	0.982	0.974	0.982	0.990	0.982	0.991	0.982	0.979	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.613	0.633	0.705	0.711	0.681	0.488	0.638	0.499	0.757	0.729	0.654	0.524	0.597	0.387
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.690	0.711	0.771	0.730	0.747	0.501	0.711	0.543	0.819	0.816	0.728	0.620	0.630	0.464
corr(M_i^T, M_i^P)	-0.605	-0.676	-0.551	-0.305	-0.564	-0.227	-0.597	-0.687	-0.509	-0.829	-0.573	-0.812	-0.342	-0.856
corr(M_i, M_j)		0.514		0.460		0.682		0.773		0.821		0.851		0.782

strong negative co-movement between the ratio of permanent SDF components and the wedge, or by a combination of these effects. A direct implication of these properties is that in complete markets (i.e., $\eta_{t+1} = 0$) permanent components need to be almost perfectly positively related. In contrast, in an incomplete market setting a trade-off emerges between the co-movement of the permanent SDF components and the long-run cyclicality of exchange rate wedges. While this trade-off is partly visible also in Tables 7 and 9, it becomes even more apparent when long-term bonds are internationally traded, because the wedge variability tends to decrease while the permanent SDF variabilities increase in less segmented markets.

Table 12, which reports the summary statistics of the wedge in the setting with internationally traded long-term bonds, confirms the above intuition. Indeed, the wedge volatility decreases in all panels compared to the findings in Table 8, where lower wedge volatilities typically arise for exchange rate parities with the strongest correlations between permanent SDF components in Table 11. Similar to the previous market settings, the higher-order moments of the wedge still imply a skewed and fat tailed distribution of exchange rate returns. However, these higher-moment features are clearly more sensitive to the particular selection of minimum dispersion SDFs in incomplete markets, especially for the funding currencies CHF and JPY and for the investment currencies CAD and NZD, known to have experienced extreme movements.

Table 12
Wedge Summary Statistics
(Trading in Foreign Short- and Long-Term Bond)

This table reports the annualized mean, standard deviation, skewness and kurtosis of the wedge $\eta_{t+1} = \log\left(\frac{S_{t+1}M_{d,t+1}}{S_t M_{f,t+1}}\right)$ for $\alpha_i = 0, 0.5, 2$. Minimum dispersion SDFs account for the fact that domestic investors can trade both short- and long-term foreign bonds.

	$\alpha = 0$				$\alpha = 0.5$				$\alpha = 2$			
	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)
UK	0.009	0.533	-0.414	8.865	0.009	0.555	-0.063	5.090	-0.040	0.565	0.237	3.856
CH	0.043	0.618	-0.523	5.376	0.042	0.647	-0.290	4.409	0.034	0.764	0.245	4.201
JP	-0.069	0.456	0.865	7.015	-0.070	0.461	0.574	5.366	-0.091	0.513	-0.310	4.585
EU	-0.053	0.324	0.192	6.329	-0.055	0.342	-0.179	5.453	-0.065	0.412	-0.935	7.059
AU	-0.017	0.364	0.003	4.131	-0.018	0.391	0.062	3.263	-0.022	0.484	-0.022	5.501
CA	-0.062	0.295	0.743	5.775	-0.062	0.305	0.318	3.835	-0.071	0.349	-0.283	4.055
NZ	-0.061	0.337	0.506	4.199	-0.062	0.341	0.362	3.789	-0.105	0.466	-4.539	53.581

Despite the lower variability compared to the previous market settings, the wedge still generates an important fraction of the total exchange rate variation when long-term bonds are traded internationally.[17] Therefore, a nontrivial wedge cyclicality arises again, which is addressed in Table 13.

Table 13
Correlation Between Wedge and SDFs
(Trading in Foreign Short- and Long-Term Bond)

This table reports the correlation between the wedge η, the (log) domestic and foreign minimum entropy SDFs ($\alpha_i = 0$), as well as the log permanent and transient components of minimum entropy SDFs. Log SDFs are denoted by $m_i := \log M_i$ and log SDF components by $m_i^U := \log M_i^U$ ($i = d, f$ and $U = T, P$). Standard errors (SE) are computed using a circular block bootstrap of size 10 with 10,000 simulations and reported in square brackets. *** denotes significance at the 1% level.

	$\text{corr}(\eta, m_i)$	SE	$\text{corr}(\eta, m_i^P)$	SE	$\text{corr}(\eta, m_i^T)$	SE
US	0.454***	[0.062]	0.397***	[0.067]	0.045	[0.067]
UK	-0.485***	[0.063]	-0.431***	[0.069]	0.000	[0.072]
US	0.372***	[0.036]	0.341***	[0.034]	-0.012	[0.044]
CH	-0.589***	[0.050]	-0.576***	[0.051]	0.001	[0.054]
US	0.545***	[0.061]	0.493***	[0.060]	0.038	[0.058]
JP	-0.213***	[0.066]	-0.209***	[0.066]	-0.029	[0.051]
US	0.502***	[0.064]	0.446***	[0.068]	0.029	[0.052]
EU	-0.075	[0.099]	-0.075	[0.094]	0.053	[0.051]
US	0.286***	[0.068]	0.256***	[0.070]	0.067	[0.071]
AU	-0.259***	[0.087]	-0.237***	[0.081]	0.049	[0.055]
US	0.566***	[0.086]	0.497***	[0.088]	0.084	[0.067]
CA	0.066	[0.096]	0.054	[0.094]	0.007	[0.073]
US	0.578***	[0.039]	0.546***	[0.045]	0.054	[0.039]
NZ	-0.013	[0.078]	-0.017	[0.074]	0.032	[0.059]

Consistently with intuition and with the previous findings, the wedge is pro-cyclical with respect to both the overall SDFs and the permanent SDF components, the only exception being the Canadian SDF which is virtually uncorrelated with the associated wedge. This pro-cyclicality is weaker than in Table 9, which was obtained under the assumption that only risk-free bonds can be traded internationally. As discussed above, this evidence is a natural

[17] We document this feature in more detail with the plots in Appendix A, where we illustrate the time series of exchange rate returns together with the time series of ratios of minimum dispersion SDFs when long-term bonds are traded internationally.

consequence of the low volatility of the LHS of identity (27) in the data and the larger co-movement of permanent SDF components when long-term bonds are traded internationally. Finally, no apparent cyclicality between wedges and long-term bond returns is left in Table 13, despite the fact that Table 11 still implies a substantial negative co-movement of permanent with transient SDF components, i.e., a positive co-movement with long-term bond returns.

2.4.3 Additional Exchange Rate Puzzles

Note that as the set of internationally traded assets in this section contains the traded assets in Section 2.3, our minimum dispersion SDFs of economies with internationally traded long-term bonds are all consistent with the cyclicality properties of exchange rates and with the deviations from the UIP.[18] Therefore, segmented markets with internationally traded short- and long-term bonds imply minimum dispersion SDFs that can address the well-known exchange rate puzzles. The dispersion of these SDFs is slightly higher and even more clearly dominated by large martingale SDF components than in settings where only short-term bonds are traded internationally. The lower wedge dispersion comes in parallel with a higher cross-sectional co-movement of permanent SDF components, a slightly weaker wedge pro-cyclicality with respect to these components and no co-movement with the transient SDF components.

2.5 Unrestricted International Trading

We finally address the least segmented market setting in which investors can trade without restrictions domestic and foreign bonds and stocks. The tradeable vector of returns in market $i = d, f$ reads $\mathbf{R}_i = (R_{i0}, R_{i1}, R_{i\infty}, R^e_{i0}, R^e_{i\infty}, R^e_{i1})'$, where $R^e_{d1,t+1} := R_{f1,t+1}S_{t+1}/S_t$ ($R^e_{f1,t+1} := R_{d1,t+1}S_t/S_{t+1}$) is the domestic (foreign) currency return of the foreign (domestic) aggregate equity return. The estimated optimal portfolio return in market $i = d, f$ reads:

$$\begin{aligned} R_{\hat{\lambda}^*_i,t+1} &= R_{i0,t+1} + \hat{\lambda}^*_{i1}(R_{i1,t+1} - R_{i0,t+1}) + \hat{\lambda}^*_{i2}(R_{i\infty,t+1} - R_{i0,t+1}) \\ &\quad + \hat{\lambda}^*_{i3}(R^e_{i0,t+1} - R_{i0,t+1}) + \hat{\lambda}^*_{i4}(R^e_{i\infty,t+1} - R_{i0,t+1}) + \hat{\lambda}^*_{i5}(R^e_{i1,t+1} - R_{i0,t+1}) \ , \end{aligned}$$

[18] For completeness, Appendix A collects the plots of the corresponding cross-sections of exchange rate risk premia, while Appendix B reports the implied coefficient estimates of Backus-Smith-type regressions.

and there are $K_d = K_f = 5$ moment conditions that need to be satisfied in the Euler equations (23). As before, the estimated closed-form minimum dispersion SDFs follow from the empirical moment conditions (22).

Note that as this setting implies symmetrically traded international returns, the market view of exchange rates holds by construction for the minimum entropy SDFs. There are two additional market structures in which we can construct symmetrically traded international returns, namely when investors can trade the domestic and foreign risk-free bonds only, or when they can invest in both risk-free and long-term bonds domestically and abroad. As these frameworks are rather restrictive, especially the former as it does not allow for the SDF factorization into transient and permanent components, and since usually one would like to trade the equity, at least domestically, we report the results in the supplementary appendix.

2.5.1 Minimum dispersion SDFs

The summary statistics on minimum dispersion SDFs under unrestricted trading are reported in Table 22. Compared to the previous findings, we document a very large average increase in SDF dispersion relative to an economy under autarky. For instance, the increase in the minimum volatility between Table 22 and Table 2 is as large as 32.8%, 60%, 334%, 34.3%, 12%, 43.3% and 68.2% for the UK, CH, JP, EU, AU, CA and NZ SDFs. Again the high percentage increase for the Japan SDF is due to the fact that under autarky, the volatility of the SDF was low relative to the other economies. In parallel, the average increase in the minimum volatility for the US SDF is 30%.[19] As expected, a dominating fraction of the higher minimum SDF dispersion is generated by a clearly higher dispersion of the permanent SDF component. Indeed, the increase in the minimum volatility of the permanent SDF components between Table 22 and Table 2 is 25.8%, 54%, 287.5%, 65.1%, 9.8%, 31% and 46.2% for the UK, CH, JP, EU, AU, CA and NZ SDFs. The average increase in the minimum volatility for the permanent component of the US SDF is 38.8%. The residual increase in SDF volatility follows from the less negative co-movement of permanent and transient SDF components in the more integrated economy with unrestricted trading. This lower co-movement balances the

[19]For comparison, the volatility increase in the economy with internationally traded short-term bonds (short- and long term bonds) was only 3%, 19.3%, 82.2%, 1.7%, 3.1%, 3.7% and 15.1% (11.4%, 34.7%, 147%, 7.8%, 3.3%, 14.7% and 21.7%) for the UK, CH, JP, EU, AU, CA and NZ SDFs. The average increase in the minimum volatility for the US SDF was 8.6% (20.9%).

Table 14
Properties of SDFs (Unrestricted Trading)

This table reports the annualized mean and volatility of the SDFs and their components, as well as the correlation between domestic (US) and foreign SDFs and the correlation between transient and permanent components within the same SDF. The SDFs are derived when international trading is unrestricted.

	US	UK	US	CH	US	JP	US	EU	US	AU	US	CA	US	NZ
							Panel A: $\alpha = 0$ (minimum entropy)							
E[M_i]	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.841	0.872	0.979	0.926	0.740	0.694	0.690	0.681	0.919	0.951	0.726	0.720	0.639	0.557
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.917	0.948	1.048	0.951	0.814	0.707	0.774	0.725	1.029	1.065	0.823	0.827	0.681	0.625
$\sqrt{\text{Entropy}}(M_i)$	0.684	0.703	0.795	0.753	0.687	0.636	0.604	0.585	0.732	0.702	0.618	0.616	0.581	0.519
corr(M_i^T,M_i^P)	-0.454	-0.498	-0.407	-0.233	-0.519	-0.155	-0.549	-0.502	-0.411	-0.636	-0.506	-0.607	-0.317	-0.634
corr(M_i,M_j)		0.992		0.989		0.989		0.985		0.992		0.994		0.981
							Panel B: $\alpha = 0.5$ (Hellinger)							
E[M_i]	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.784	0.805	0.925	0.880	0.720	0.677	0.661	0.647	0.823	0.843	0.688	0.682	0.620	0.547
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.856	0.882	0.991	0.903	0.791	0.688	0.741	0.687	0.908	0.943	0.775	0.779	0.657	0.612
$\sqrt{\text{Hellinger}}(M_i)$	0.706	0.725	0.823	0.780	0.695	0.646	0.599	0.599	0.752	0.768	0.634	0.631	0.590	0.526
corr(M_i^T,M_i^P)	-0.483	-0.534	-0.428	-0.243	-0.531	-0.152	-0.570	-0.524	-0.461	-0.716	-0.533	-0.641	-0.324	-0.646
corr(M_i,M_j)		0.990		0.989		0.989		0.984		0.990		0.994		0.980
							Panel C: $\alpha = 2$ (minimum variance)							
E[M_i]	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.739	0.754	0.873	0.834	0.699	0.658	0.639	0.622	0.776	0.791	0.659	0.655	0.600	0.535
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.803	0.824	0.930	0.853	0.763	0.670	0.711	0.659	0.839	0.874	0.733	0.735	0.632	0.595
corr(M_i^T,M_i^P)	-0.517	-0.587	-0.455	-0.268	-0.552	-0.169	-0.597	-0.564	-0.500	-0.775	-0.566	-0.683	-0.340	-0.665
corr(M_i,M_j)		0.989		0.988		0.989		0.984		0.988		0.993		0.979

increased SDF dispersion in order to still reproduce the cross-section of domestic risk premia on long term bonds in local currencies.

Table 22 finally documents a virtually perfect co-movement of domestic and foreign minimum dispersion SDFs, since the lowest correlation between SDFs is as high as 97.9%. We find that this very large co-movement is completely determined by permanent SDF components that are almost perfectly positively related, giving rise to a lowest correlation of 96.5% across exchange rate parities (see e.g. Table 16).

2.5.2 Exchange Rate Volatilities and Wedges

The almost perfect co-movement of permanent SDFs components in the setting with unrestricted trading can be understood using the same basic logic put forward in Section 2.4.2 with identity (27). Indeed, the large SDF dispersions in Table 22 can be empirically consistent with identity (27) only in presence of a sufficiently large wedge dispersion or when permanent SDF components are strongly positively correlated. As the least segmented market setting with unrestricted trading implies much higher SDF dispersions and a naturally lower wedge dispersion than in the setting of Section 2.4.2, the co-movement between SDF components in Table 22 needs indeed to be much larger than in Table 11.

Summary statistics of the wedge in the setting with unrestricted trading are reported in Table 15. Here, we focus on minimum variance and minimum Hellinger SDFs alone, because the wedge resulting from minimum entropy SDFs vanishes by construction. This feature is illustrated in Figure 10, where exchange rate returns are highlighted using circles since otherwise they would be indistinguishable from minimum entropy SDF ratios.

Table 15
Wedge Summary Statistics (Unrestricted Trading)

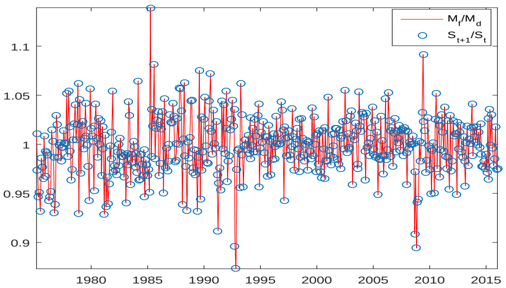
This table reports the annualized mean, standard deviation, skewness and kurtosis of the wedge η, for $\alpha \in \{0.5, 2\}$. The domestic currency is the US dollar. The wedge is $\eta_{t+1} = \log\left(\frac{S_{t+1}M_{d,t+1}}{S_t M_{d,t+1}}\right)$. The minimum dispersion SDFs account for the fact that domestic investors can trade any foreign asset.

	$\alpha = 0.5$				$\alpha = 2$			
	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)
UK	0.000	0.022	-0.395	6.336	-0.007	0.059	-0.259	11.62
CH	-0.001	0.026	-1.277	8.426	-0.019	0.120	-6.146	68.40
JP	0.000	0.023	-1.286	7.608	-0.009	0.083	-4.483	36.18
EU	0.000	0.021	-0.065	5.814	0.000	0.064	-1.130	11.47
AU	0.000	0.019	0.424	8.003	0.005	0.075	2.110	24.82
CA	0.000	0.013	-0.488	6.080	-0.001	0.034	-0.538	5.772
NZ	-0.001	0.023	-2.695	17.23	-0.031	0.216	-15.61	268.3

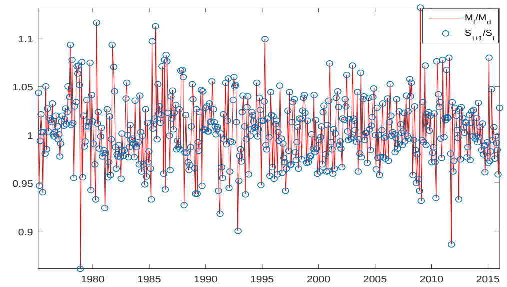
Consistent with the above intuition, we obtain a very small wedge dispersion in all cases. The smallest wedges arise for the minimum Hellinger divergence SDFs, which are those resembling most the minimum entropy SDFs. While the wedge implied by minimum variance SDFs is more volatile and larger on average, it is clearly less so than in all previous market settings, to the point that it generates a quite small fraction of exchange rate volatility in identity (27). Given this largely negligible wedge dispersion, the wedge cyclicality properties are also not particularly interesting in the setting with unrestricted trading.[20]

[20]We report for completeness in Appendix B the correlation between the wedge obtained for $\alpha = 2$ and the corresponding minimum variance SDFs.

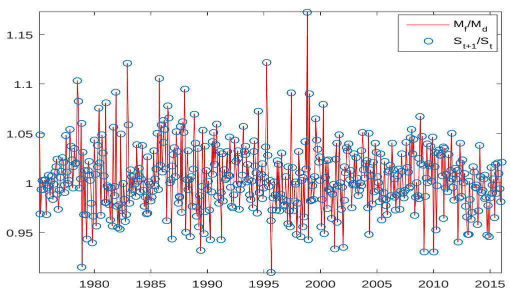
Figure 10. Real exchange rate change against ratio of SDFs; $\alpha_i = 0$



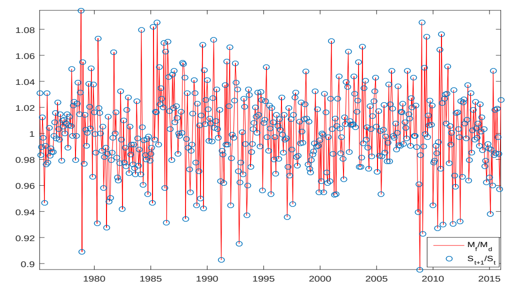
(a) USDGBP



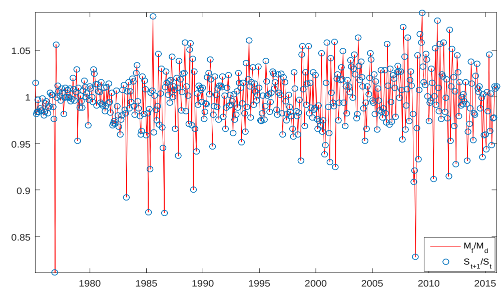
(b) USDCHF



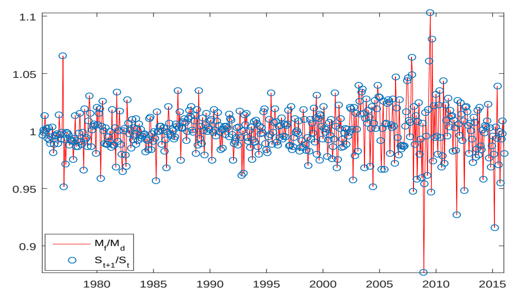
(c) USDJPY



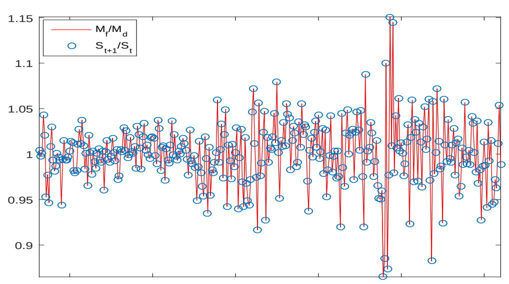
(d) USDEUR



(e) USDAUD



(f) USDCAD



(g) USDNZD

This figure reports the real exchange rate return against the ratio of the foreign and domestic minimum entropy SDFs ($\alpha_i = 0$). The domestic currency is the USD, whereas the foreign currencies are the GBP, CHF, JPY, EUR, AUD, CAD and the NZD.

2.5.3 *Additional Exchange Rate Puzzles*

Coherently with the findings in Section 2.4.3, minimum dispersion SDFs of economies with unrestricted trading are consistent with the cyclicality properties of exchange rates and the UIP deviations.[21] Therefore, they can in principle address the well-known exchange rate puzzles, within an economy that additionally satisfies the market view of exchange rates and has almost perfectly co-moving martingale SDF components across markets. However, the extraordinarily large increases in SDF dispersions in this economy, relative to a setting under autarky, highlight a trade-off between international SDF dispersion, the degree of financial market integration and well-established exchange rate puzzles in the literature. We study further important aspects of this key trade-off in the next section.

2.6 *How Much International Market Segmentation?*

The findings in the previous sections imply that an economy with internationally traded short-term bonds and nontrivial SDF martingale components can explain the three exchange rate puzzles in the literature, whenever a wedge dispersion similar to the dispersion of domestic and foreign SDFs is acceptable.

Similar broader economies with less segmented markets can be considered, in dependence of the trade-off they imply for the dispersion of exchange rate wedges and permanent SDF components. According to our findings and the summary statistics in Table 16, economies with less segmented markets tend to imply a smaller dispersion of exchange rate wedges, together with a larger dispersion and a stronger positive co-movement of permanent SDF components. In these economies, the largest incremental effects on dispersion and co-movement arise by sequentially opening domestic long term bond markets and stock markets to foreign investment. On the other hand, the correlation between transitory components of domestic and foreign SDFs in Table 17 is weaker and independent of both the dispersion measure and the degree of market segmentation considered.

[21] For completeness, we plot in Appendix A the corresponding cross-sections of exchange rate risk premia. In Appendix B, we report the implied coefficient estimates of Backus-Smith-type regressions.

Table 16

Correlation of Permanent SDF Components Across Exchange Rate Parities

This table reports the correlation between permanent components of domestic and foreign SDFs. The domestic SDF is the US one, whereas the foreign SDFs are those for the UK, CH, JP, EU, AU, CA and NZ. Standard errors are computed using a circular block bootstrap of size 10 with 10000 simulations and reported in square brackets. Labels ** and *** denote significance at the 5% and 1% level, respectively.

		UK	CH	JP	EU	AU	CA	NZ
Autarky								
$\mathrm{corr}(M_d^P, M_f^P)$	$\alpha = 0$	0.112***	0.179***	0.147***	0.618***	0.562***	0.362***	0.493***
		[0.041]	[0.053]	[0.036]	[0.049]	[0.061]	[0.057]	[0.069]
	$\alpha = 0.5$	0.137***	0.183***	0.161***	0.596***	0.525***	0.389***	0.481***
		[0.043]	[0.055]	[0.038]	[0.049]	[0.060]	[0.055]	[0.064]
	$\alpha = 2$	0.152***	0.157***	0.177**	0.554***	0.485***	0.416***	0.464***
		[0.045]	[0.057]	[0.039]	[0.050]	[0.055]	[0.050]	[0.061]
Trading in foreign short-term bonds								
$\mathrm{corr}(M_d^P, M_f^P)$	$\alpha = 0$	0.121***	0.351***	0.395***	0.622***	0.583***	0.394***	0.561***
		[0.044]	[0.054]	[0.037]	[0.053]	[0.064]	[0.058]	[0.069]
	$\alpha = 0.5$	0.152***	0.328***	0.403***	0.604***	0.538***	0.379***	0.547***
		[0.046]	[0.059]	[0.038]	[0.054]	[0.061]	[0.053]	[0.066]
	$\alpha = 2$	0.174***	0.258***	0.424***	0.567***	0.485***	0.405***	0.523***
		[0.049]	[0.060]	[0.044]	[0.058]	[0.056]	[0.051]	[0.064]
Trading in foreign bonds								
$\mathrm{corr}(M_d^P, M_f^P)$	$\alpha = 0$	0.535***	0.583***	0.682***	0.834***	0.864***	0.834***	0.777***
		[0.043]	[0.090]	[0.043]	[0.050]	[0.062]	[0.053]	[0.064]
	$\alpha = 0.5$	0.572***	0.566***	0.695***	0.829***	0.856***	0.852***	0.779***
		[0.0451]	[0.0683]	[0.0430]	[0.0521]	[0.057]	[0.049]	[0.061]
	$\alpha = 2$	0.581***	0.504***	0.712***	0.801***	0.842***	0.853***	0.776***
		[0.041]	[0.052]	[0.041]	[0.055]	[0.052]	[0.050]	[0.057]
Unrestricted international trading								
$\mathrm{corr}(M_d^P, M_f^P)$	$\alpha = 0$	0.972***	0.984***	0.981***	0.978***	0.987***	0.976***	0.968***
		[0.019]	[0.007]	[0.004]	[0.007]	[0.006]	[0.005]	[0.006]
	$\alpha = 0.5$	0.969***	0.984***	0.980***	0.976***	0.984***	0.975***	0.968***
		[0.013]	[0.004]	[0.004]	[0.006]	[0.005]	[0.005]	[0.005]
	$\alpha = 2$	0.968***	0.982***	0.977***	0.974***	0.98***	0.973***	0.965***
		[0.009]	[0.003]	[0.004]	[0.006]	[0.005]	[0.004]	[0.005]

Table 17
Correlation of Transitory SDF Components Across Exchange Rate Parities
This table reports the correlation between transitory components of domestic and foreign SDFs. The domestic SDF is the US one, whereas the foreign SDFs are those for the UK, CH, JP, EU, AU, CA and NZ. Standard errors are computed using a circular block bootstrap of size 10 with 10000 simulations and reported in square brackets. Labels ** and *** denote significance at the 5% and 1% level, respectively.

Correlation transitory components							
	UK	CH	JP	EU	AU	CA	NZ
corr(M_d^T,M_f^T)	0.157***	0.266***	0.235***	0.422***	0.406***	0.250***	0.593***
SE	[0.043]	[0.052]	[0.083]	[0.046]	[0.064]	[0.038]	[0.048]

A natural question is how much dispersion of the permanent SDF components can be realistically generated by structural SDF specifications calibrated to macroeconomic data. In this respect, the minimum SDF dispersions obtained for the setting with unrestricted trading might already generate quite a challenge.

However, the issue might be even more demanding, due to the potentially weak empirical coherence of domestic minimum dispersion SDFs in bilaterally nearly integrated markets. This feature is quantified in Table 18, showing that the correlations between minimum dispersion domestic SDFs across exchange rate parities are relatively low in settings with unrestricted international trading. Based on this evidence, we conclude that specifications combining large permanent SDF components and a relevant degree of international stock market segmentation might give rise to more plausible empirical descriptions of international asset returns.

Table 18
Correlation Between Domestic SDFs (Unrestricted Trading)

This table reports the correlation between the seven US minimum dispersion SDFs implied by exchange rate parities with respect to the UK, CH, JP, EU, AU, CA and NZ currencies. Standard errors are computed using a circular block bootstrap of size 10 with 10000 simulations and reported in square brackets. Label *** denotes significance at the 1% level.

		Unrestricted international trading					
		US/CH	US/JP	US/EU	US/AU	US/CA	US/NZ
$\alpha = 0$	US/UK	0.649***	0.527***	0.688***	0.469***	0.664***	0.629***
		[0.055]	[0.052]	[0.038]	[0.088]	[0.049]	[0.100]
	US/CH		0.615***	0.757***	0.316***	0.431***	0.422***
			[0.037]	[0.036]	[0.039]	[0.068]	[0.079]
	US/JP			0.734***	0.494***	0.579***	0.601***
				[0.024]	[0.068]	[0.068]	[0.085]
	US/EU				0.551***	0.684***	0.679***
					[0.049]	[0.045]	[0.070]
	US/AU					0.553***	0.712***
						[0.117]	[0.063]
	US/CA						0.679***
							[0.072]
		US/CH	US/JP	US/EU	US/AU	US/CA	US/NZ
$\alpha = 0.5$	US/UK	0.665***	0.564***	0.665***	0.515***	0.687***	0.619***
		[0.051]	[0.048]	[0.039]	[0.077]	[0.049]	[0.087]
	US/CH		0.646***	0.749***	0.389***	0.463***	0.448***
			[0.035]	[0.033]	[0.039]	[0.064]	[0.069]
	US/JP			0.759***	0.527***	0.597***	0.600***
				[0.023]	[0.065]	[0.067]	[0.076]
	US/EU				0.611***	0.713***	0.684***
					[0.044]	[0.041]	[0.061]
	US/AU					0.562***	0.716***
						[0.078]	[0.058]
	US/CA						0.667***
							[0.073]
		US/CH	US/JP	US/EU	US/AU	US/CA	US/NZ
$\alpha = 2$	US/UK	0.651***	0.597***	0.641***	0.538***	0.685***	0.626***
		[0.047]	[0.045]	[0.046]	[0.047]	[0.049]	[0.062]
	US/CH		0.673***	0.721***	0.454***	0.483***	0.484***
			[0.032]	[0.029]	[0.039]	[0.067]	[0.067]
	US/JP			0.778***	0.547***	0.620***	0.611***
				[0.023]	[0.058]	[0.067]	[0.066]
	US/EU				0.647***	0.743***	0.695***
					[0.044]	[0.043]	[0.056]
	US/AU					0.588***	0.713***
						[0.057]	[0.047]
	US/CA						0.664***
							[0.047]

3 Conclusion

In this paper, we estimate model-free minimum dispersion SDFs to understand the asset pricing implications of different degrees of market segmentation in international financial markets. Since markets are incomplete and there potentially exists a host of different SDFs, we explore various SDFs that minimize different measures of SDF dispersion: variance, entropy and Hellinger divergence. At the same time, we allow for a factorization of international SDFs into permanent (martingale) and transient components and for the potential presence of a stochastic wedge between exchange rates and the ratio of foreign and domestic SDFs.

Theoretically, we show that the different international minimum dispersion SDFs entail intuitive economic interpretation. Specifically, minimum Hellinger divergence SDFs place a sharp bound on the first moment of transient SDF components. Minimum entropy SDFs, on the other hand, always imply the validity of the market view of exchange rates in symmetric international markets, irrespective of the degree of market incompleteness. Finally, we prove that minimum variance SDFs characterize in a natural way the tradeable component of exchange rate risk in symmetric international markets, which gives rise to an intuitive interpretation of the resulting exchange rate wedge as untradeable exchange rate risk.

Using a cross-section of developed countries, we document the following novel findings. In order to jointly explain the exchange rate puzzles, it is critical for the SDF to be factorized into a permanent and a transient component. While we find that permanent SDF components induce in all cases the largest fraction of SDF dispersion, we document that they are necessary to imply an exchange rate cyclicality that is compatible with the Backus and Smith (1993) puzzle, i.e., the small or negative correlation between international consumption growth differentials and exchange returns. We then show that international minimum dispersion SDFs jointly explain the three exchange rate puzzles whenever domestic and foreign risk-free bonds can be traded internationally.

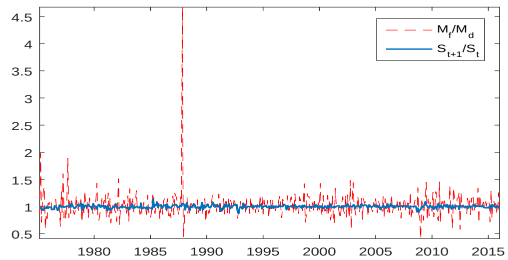
To study the consequences of different degrees of market segmentation, we benchmark our results to the autarky case, in which domestic and foreign financial markets are fully segmented, entailing the highest degree of incompleteness. This allows us to quantify the trade-off between the larger degree of SDF dispersion needed to price a wider set of traded assets, the ability of international SDFs to jointly explain exchange rate puzzles, and the stochastic properties of the resulting wedge between exchange rates and SDF ratios.

We find that the implied SDF dispersion when domestic equity is tradeable together with international risk-free and long-term bonds is not excessive. In parallel, the resulting exchange rate wedge dispersion may be compatible with conservative estimates found in the literature. In contrast, the SDF dispersion of symmetric international markets where aggregate stock returns are internationally tradeable may be implausibly large. In this context, the ratio of minimum entropy SDFs equals the exchange rate return by construction. Moreover, the fraction of tradeable exchange rate risk measured by minimum variance SDFs is large, which implies a small fraction of untradeable exchange risk. Based on this evidence, we conclude that there is a trade-off in international economies, between SDF dispersions, the degree of financial market integration, the amount of tradeable exchange rate risk and the validity of the market view of exchange rates.

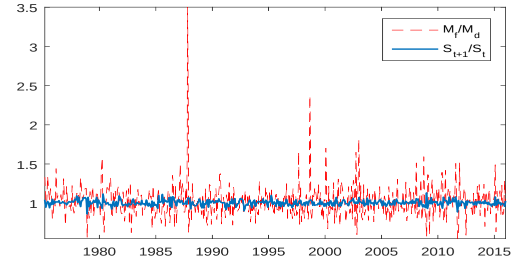
Appendix

A Figures

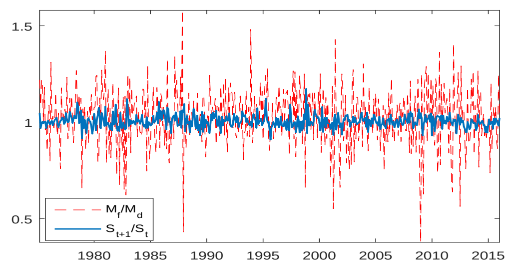
Figure 11. Exchange rate return and SDF ratio in equation (10); $\alpha_i = 0$



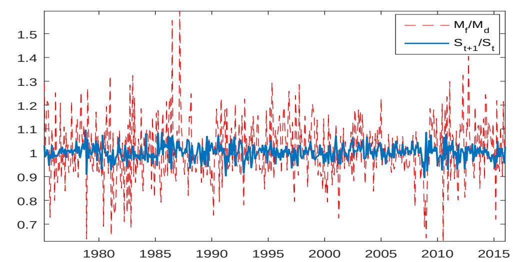
(a) USDGBP



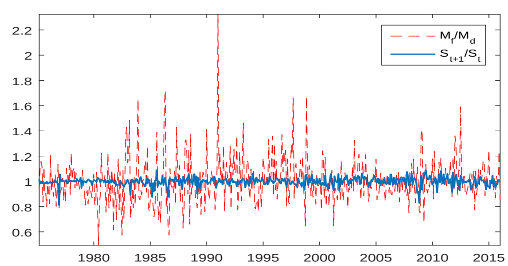
(b) USDCHF



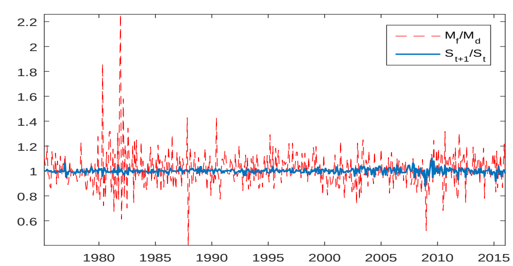
(c) USDJPY



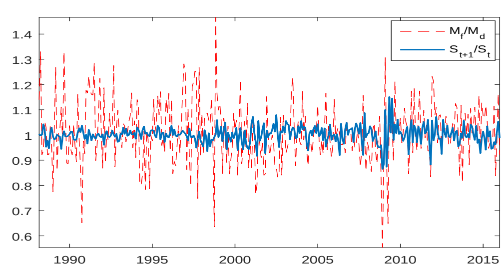
(d) USDEUR



(e) USDAUD



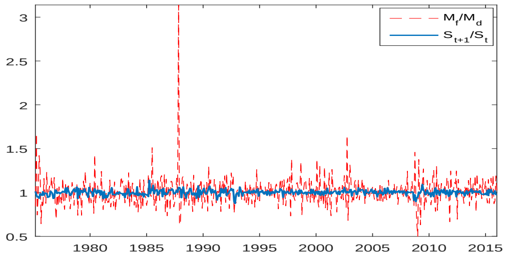
(f) USDCAD



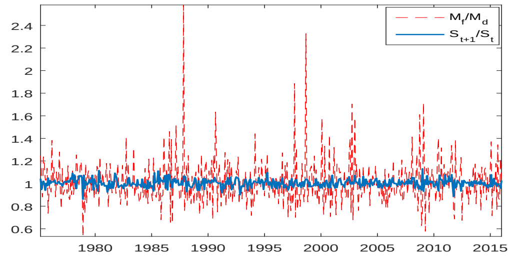
(g) USDNZD

This figure plots the time series of real exchange rate returns S_{t+1}/S_t and minimum entropy ($\alpha_i = 0$) SDF ratios $M_{f(t+1)}/M_{d(t+1)}$, using the USD as the domestic currency and the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD as foreign currencies, respectively.

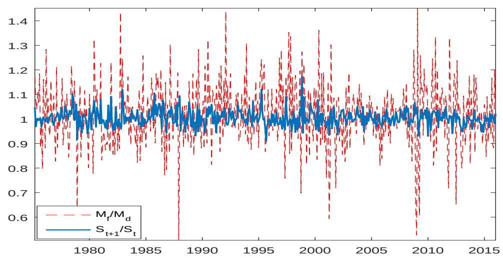
Figure 12. Real Exchange Rate Return Against Ratio of SDFs; $\alpha_i = 0$ (Trading in Foreign Short- and Long-Term Bond)



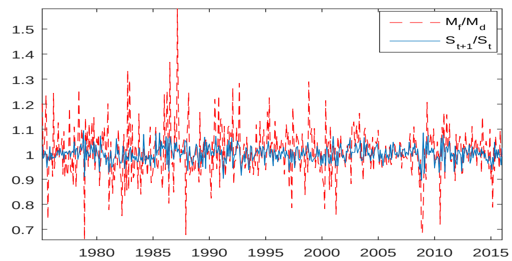
(a) USDGBP



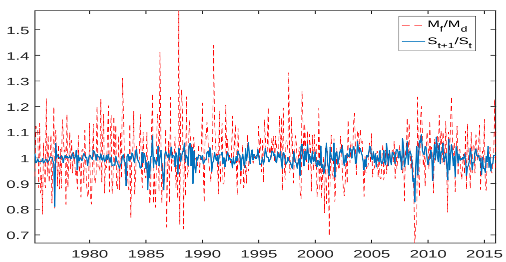
(b) USDCHF



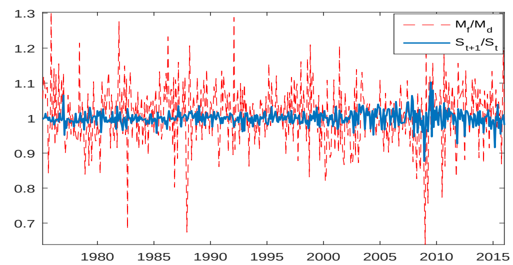
(c) USDJPY



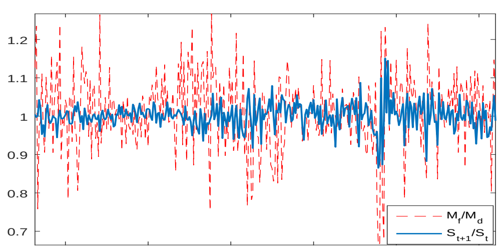
(d) USDEUR



(e) USDAUD



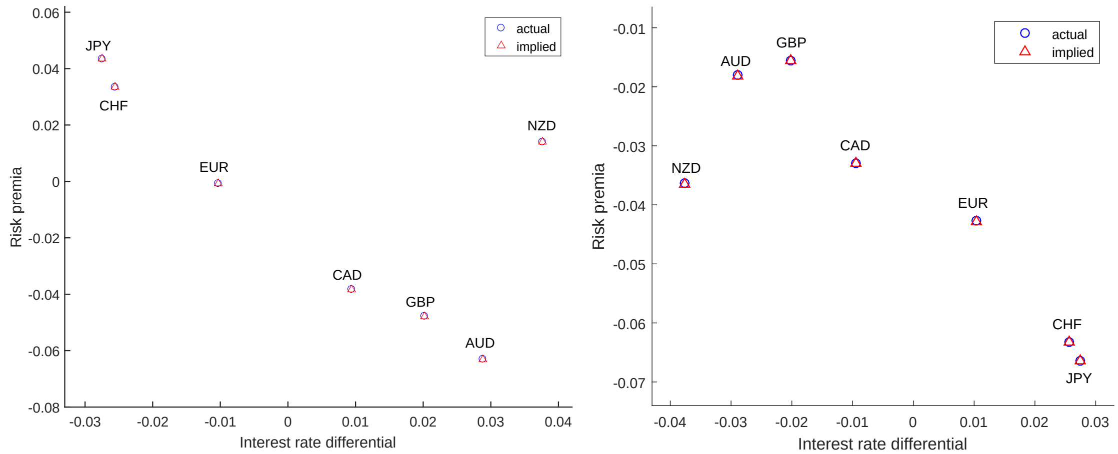
(f) USDCAD



(g) USDNZD

This figure reports the real exchange rate return against the ratio of the foreign and domestic minimum entropy SDFs ($\alpha_i = 0$). The domestic currency is the USD, whereas the foreign currencies are the GBP, CHF, JPY, EUR, AUD, CAD and the NZD.

Figure 13. International long-term bond risk premia ($\alpha_i = 0$)

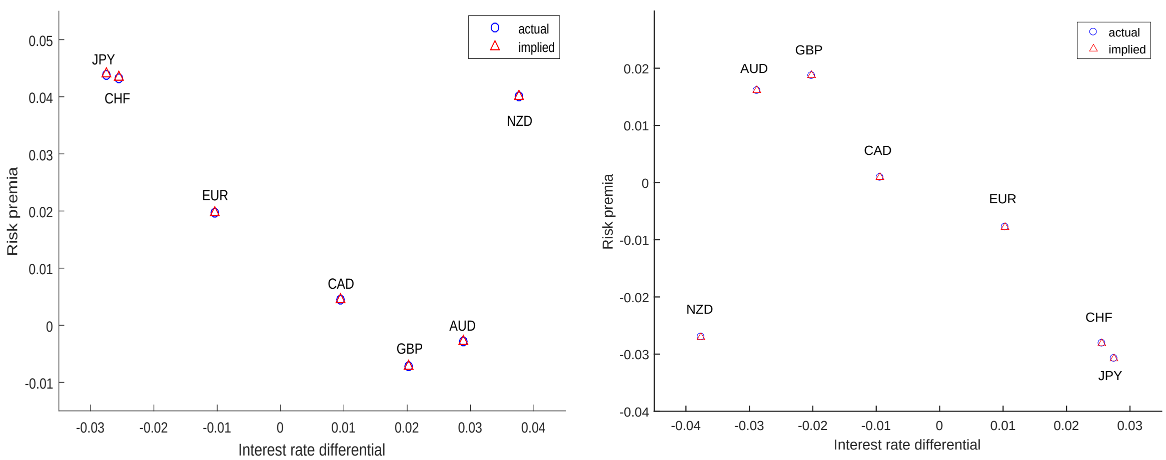


(a) Panel A: Domestic investor

(b) Panel B: Foreign investor

The figure plots for $i = d, f$ the observed international long-term bond risk premium $E[R^e_{i\infty,t+1}] - E[R_{i0,t+1}]$ and the risk premium $-\text{cov}\left(M_{i,t+1}/E[M_{i,t+1}], (R^e_{i\infty,t+1} - R_{i0,t+1})\right)$ under the minimum dispersion SDF $M_{i,t+1}$ against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. Panel A reports the long-term bond risk premium for the domestic investor ($i = d$), whereas Panel B for the foreign one ($i = f$). The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD.

Figure 14. Currency risk premia; $\alpha_i = 0$ (Trading in Foreign Short- and Long-Term Bond)



(a) Panel A: Domestic investor

(b) Panel B: Foreign investor

The figure plots for $i = d, f$ the observed exchange rate risk premium $E[R^e_{i0,t+1}] - E[R_{i0,t+1}]$ and the risk premium $-\text{cov}\left(M_{i,t+1}/E[M_{i,t+1}], (R^e_{i0,t+1} - R_{i0,t+1})\right)$ under the minimum dispersion SDF $M_{i,t+1}$ against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. Panel A reports the currency risk premium for the domestic investor ($i = d$), whereas Panel B for the foreign one ($i = f$). The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD.

B Tables

Table 19
Backus-Smith (1993)-Type Regressions (Trading in Foreign Short- and Long-Term Bond)

This table reports the point estimates of a linear regression of the log difference between foreign and domestic SDFs on the log real exchange rate return: $m_{f,t+1} - m_{d,t+1} = \delta + \beta \Delta s_{t+1} + u_{t+1}$, where $\Delta s_{t+1} = s_{t+1} - s_t$ and small-cap letters denote quantities in logs. We additionally report point estimates of a linear regression of the log difference of the permanent component of the SDF on the log real exchange rate return: $m^P_{f,t+1} - m^P_{d,t+1} = \delta^P + \beta^P \Delta s_{t+1} + u^P_{t+1}$. Standard errors are reported in square brackets. Label *** highlights significance at the 1% level.

	Panel A: US/UK			Panel B: US/CH		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.776***	0.891***	0.937***	0.816***	0.920***	1.050***
	[0.230]	[0.239]	[0.249]	[0.229]	[0.241]	[0.284]
$\boldsymbol{\beta^P}$	0.860***	0.976***	0.986***	0.766***	0.871***	1.001***
	[0.238]	[0.248]	[0.294]	[0.234]	[0.245]	[0.288]
	Panel C: US/JP			Panel D: US/EU		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.969***	1.030***	1.123***	0.972***	0.989***	0.971***
	[0.183]	[0.185]	[0.205]	[0.136]	[0.144]	[0.173]
$\boldsymbol{\beta^P}$	1.052***	1.113***	1.206***	0.927***	0.945***	0.927***
	[0.188]	[0.191]	[0.213]	[0.144]	[0.151]	[0.180]
	Panel E: US/AU			Panel F: US/CA		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1.081***	1.086***	1.113***	0.939***	0.997***	0.996***
	[0.145]	[0.156]	[0.192]	[0.188]	[0.195]	[0.222]
$\boldsymbol{\beta^P}$	1.086***	1.091***	1.118***	0.966***	1.025***	1.023**
	[0.152]	[0.163]	[0.199]	[0.203]	[0.210]	[0.237]

	Panel F: US/NZ		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.813***	0.903***	1.423***
	[0.152]	[0.154]	[0.209]
$\boldsymbol{\beta^P}$	0.818***	0.908***	1.429***
	[0.155]	[0.157]	[0.208]

Table 20
Correlation between Wedge and SDFs (Unrestricted Trading)

This table reports the correlation between the wedge η, the (log) domestic and foreign minimum variance SDFs ($\alpha_i = 2$), as well as the log permanent and transient components of minimum variance SDFs. Log SDFs are denoted by $m_i := \log M_i$ and log SDF components by $m_i^U := \log M_i^U$ ($i = d, f$ and $U = T, P$). Standard errors (SE) are computed using a circular block bootstrap of size 10 with 10000 simulations and reported in square brackets. Labels *, ** and *** denote significance at the 10%, 5% and 1% level, respectively.

	$\text{corr}(\eta, m_i)$	SE	$\text{corr}(\eta, m_i^P)$	SE	$\text{corr}(\eta, m_i^T)$	SE
US	0.043	[0.062]	0.034	[0.067]	0.034	[0.067]
UK	0.072	[0.063]	0.105	[0.069]	-0.233***	[0.072]
US	0.391***	[0.099]	0.381***	[0.102]	-0.071	[0.062]
CH	0.248***	[0.094]	0.246	[0.096]	-0.021	[0.053]
US	0.339***	[0.103]	0.320***	[0.103]	-0.029	[0.085]
JP	0.232**	[0.098]	0.216**	[0.097]	0.099**	[0.042]
US	0.077	[0.095]	0.070	[0.096]	-0.002	[0.069]
EU	-0.041	[0.092]	-0.055	[0.092]	0.159***	[0.054]
US	-0.061	[0.103]	-0.057	[0.099]	-0.001	[0.054]
AU	-0.173*	[0.104]	-0.165*	[0.099]	0.062	[0.051]
US	0.079	[0.065]	0.078	[0.065]	-0.030	[0.065]
CA	0.021	[0.063]	0.016	[0.063]	0.018	[0.051]
US	0.602**	[0.252]	0.587**	[0.236]	-0.005	[0.088]
NZ	0.338*	[0.180]	0.324*	[0.177]	-0.114	[0.086]

Table 21
Backus-Smith (1993)-Type Regressions (Unrestricted Trading)

This table reports the point estimates of a linear regression of the log difference between foreign and domestic SDFs on the log real exchange rate return: $m_{f,t+1} - m_{d,t+1} = \delta + \beta\Delta s_{t+1} + u_{t+1}$, where $\Delta s_{t+1} = s_{t+1} - s_t$ and small-cap letters denote quantities in logs. We additionally report point estimates of a linear regression of the log difference of the permanent component of the SDF on the log real exchange rate return: $m^P_{f,t+1} - m^P_{d,t+1} = \delta^P + \beta^P\Delta s_{t+1} + u^P_{t+1}$. Standard errors are reported in square brackets. Label *** highlights significance at the 1% level.

	Panel A: US/UK			Panel B: US/CH		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1***	1.060***	1.0216***	1***	1.079***	1.233***
	[0.000]	[0.009]	[0.0261]	[0.000]	[0.009]	[0.043]
$\boldsymbol{\beta^P}$	1.085***	1.145***	1.0653***	0.951***	1.030***	1.183***
	[0.068]	[0.067]	[0.0742]	[0.044]	[0.045]	[0.064]
	Panel C: US/JP			Panel D: US/EU		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1***	1.056***	1.107***	1***	1.050***	1.055***
	[0.000]	[0.009]	[0.033]	[0.000]	[0.008]	[0.027]
$\boldsymbol{\beta^P}$	1.083***	1.139***	1.189***	0.956***	1.006***	1.011***
	[0.053]	[0.053]	[0.065]	[0.046]	[0.046]	[0.056]
	Panel E: US/AU			Panel F: US/CA		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1***	1.042***	1.117***	1***	1.044***	1.031***
	[0.000]	[0.007]	[0.029]	[0.000]	[0.008]	[0.022]
$\boldsymbol{\beta^P}$	1.005***	1.047***	1.122***	1.027***	1.072***	1.059***
	[0.049]	[0.049]	[0.059]	[0.089]	[0.090]	[0.093]

	Panel F: US/NZ		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1***	1.055***	1.409***
	[0.000]	[0.009]	[0.095]
$\boldsymbol{\beta^P}$	1.006***	1.061***	1.415***
	[0.038]	[0.038]	[0.098]

Table 22

Properties of SDFs (Trading in foreign short-term bonds and equity)

This table reports the annualized mean and volatility of the SDFs and their components, as well as the correlation between domestic (US) and foreign SDFs and the correlation between transient and permanent components within the same SDF. The SDFs are derived when investors can trade the foreign risk-free bond and the equity.

	US	**UK**	**US**	**CH**	**US**	**JP**	**US**	**EU**	**US**	**AU**	**US**	**CA**	**US**	**NZ**
							Panel A: $\alpha = 0$ (minimum entropy)							
E$[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.777	0.821	0.964	0.859	0.739	0.574	0.645	0.632	0.600	0.955	0.631	0.654	0.560	0.564
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.865	0.909	1.034	0.894	0.813	0.585	0.732	0.676	0.699	1.091	0.726	0.759	0.606	0.564
$\sqrt{\text{Entropy}}(M_i)$	0.604	0.657	0.791	0.680	0.687	0.542	0.575	0.552	0.515	0.744	0.534	0.588	0.507	0.495
corr(M_i^T, M_i^P)	-0.477	-0.517	-0.411	-0.251	-0.519	-0.185	-0.578	-0.537	-0.595	-0.610	-0.568	-0.657	-0.356	-0.614
corr(M_i, M_j)		0.701		0.842		0.846		0.830		0.712		0.672		0.875
							Panel B: $\alpha = 0.5$ (Hellinger)							
E$[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.694	0.751	0.913	0.796	0.719	0.564	0.625	0.601	0.574	0.847	0.592	0.630	0.546	0.538
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.775	0.836	0.979	0.822	0.790	0.574	0.709	0.643	0.667	0.947	0.681	0.726	0.588	0.609
$\sqrt{\text{Hellinger}}(M_i)$	0.624	0.678	0.818	0.705	0.694	0.548	0.587	0.564	0.528	0.768	0.547	0.599	0.516	0.506
corr(M_i^T, M_i^P)	-0.529	-0.560	-0.431	-0.267	-0.531	-0.181	-0.593	-0.558	-0.621	-0.713	-0.602	-0.684	-0.362	-0.645
corr(M_i, M_j)		0.740		0.872		0.848		0.824		0.707		0.699		0.879
							Panel C: $\alpha = 2$ (minimum variance)							
E$[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.652	0.701	0.863	0.752	0.699	0.553	0.608	0.581	0.553	0.790	0.568	0.610	0.532	0.521
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.723	0.774	0.921	0.773	0.763	0.564	0.685	0.620	0.636	0.872	0.648	0.692	0.568	0.584
corr(M_i^T, M_i^P)	-0.571	-0.621	-0.459	-0.293	-0.552	-0.199	-0.617	-0.596	-0.655	-0.777	-0.637	-0.722	-0.379	-0.676
corr(M_i, M_j)		0.746		0.868		0.847		0.811		0.688		0.700		0.877

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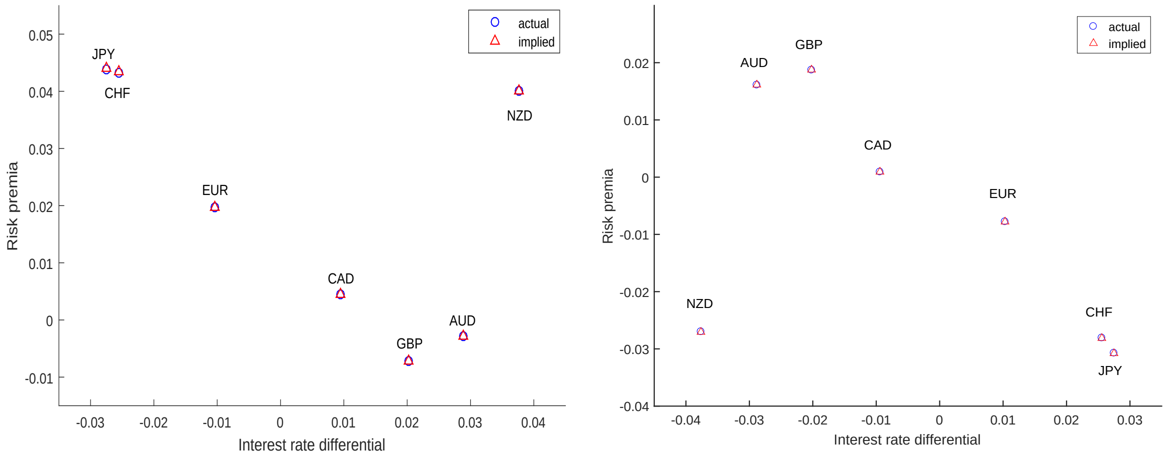
Supplementary Appendix

S. A Figures

Figure S1. Currency risk premia; $\alpha_i = 0$ (Unrestricted Trading)



(a) Panel A: Domestic investor (b) Panel B: Foreign investor

The figure plots for $i = d, f$ the observed exchange rate risk premium $E[R^e_{i0,t+1}] - E[R_{i0,t+1}]$ and the risk premium $-\text{cov}\left(M_{i,t+1}/E[M_{i,t+1}], (R^e_{i0,t+1} - R_{i0,t+1})\right)$ under the minimum dispersion SDF $M_{i,t+1}$ against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. Panel A reports the currency risk premium for the domestic investor ($i = d$), whereas Panel B for the foreign one ($i = f$). The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD.

S. B Tables

Table S1
Properties of SDFs (Symmetric Trading: Short- and Long-Term Bonds)

The table reports annualized joint sample moments of the SDF and its components. Panel A reports statistics with respect to the minimum-entropy SDFs ($\alpha_i = 0$), Panel B for Hellinger SDFs ($\alpha_i = 0.5$) and Panel C for minimum variance SDFs ($\alpha_i = 2$), $i = d, f$, $j = d, f$, $i \neq j$. We use monthly data from January 1975 to December 2015.

	US	**UK**	**US**	**CH**	**US**	**JP**	**US**	**EU**	**US**	**AU**	**US**	**CA**	**US**	**NZ**
Panel A: $\alpha = 0$ (minimum entropy)														
$\mathrm{E}[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
$\mathrm{Std}(M_i)$	0.437	0.465	0.495	0.411	0.512	0.439	0.412	0.379	0.608	0.627	0.496	0.494	0.477	0.386
$\mathrm{Std}(M_i^T)$	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
$\mathrm{Std}(M_i^P)$	0.539	0.586	0.589	0.440	0.597	0.453	0.514	0.436	0.690	0.745	0.606	0.636	0.516	0.468
$\sqrt{\mathrm{Entropy}}(M_i)$	0.406	0.436	0.471	0.392	0.503	0.429	0.404	0.373	0.568	0.585	0.421	0.418	0.456	0.375
$\mathrm{corr}(M_i^T, M_i^P)$	-0.761	-0.802	-0.705	-0.500	-0.694	-0.238	-0.807	-0.836	-0.596	-0.904	-0.686	-0.783	-0.414	-0.843
$\mathrm{corr}(M_i, M_j)$		0.975		0.977		0.984		0.965		0.983		0.989		0.975
Panel B: $\alpha = 0.5$ (Hellinger)														
$\mathrm{E}[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
$\mathrm{Std}(M_i)$	0.429	0.458	0.487	0.407	0.506	0.436	0.410	0.377	0.592	0.610	0.463	0.461	0.469	0.383
$\mathrm{Std}(M_i^T)$	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
$\mathrm{Std}(M_i^P)$	0.529	0.574	0.579	0.435	0.589	0.449	0.508	0.433	0.671	0.723	0.569	0.588	0.506	0.463
$\sqrt{\mathrm{Hellinger}}(M_i)$	0.412	0.442	0.475	0.396	0.504	0.431	0.406	0.374	0.574	0.591	0.431	0.429	0.460	0.377
$\mathrm{corr}(M_i^T, M_i^P)$	-0.773	-0.814	-0.715	-0.500	-0.700	-0.232	-0.812	-0.833	-0.609	-0.928	-0.727	-0.841	-0.419	-0.850
$\mathrm{corr}(M_i, M_j)$		0.972		0.976		0.982		0.962		0.982		0.987		0.974
Panel C: $\alpha = 2$ (minimum variance)														
$\mathrm{E}[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
$\mathrm{Std}(M_i)$	0.420	0.447	0.479	0.401	0.499	0.430	0.405	0.373	0.577	0.594	0.442	0.439	0.459	0.378
$\mathrm{Std}(M_i^T)$	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
$\mathrm{Std}(M_i^P)$	0.515	0.556	0.565	0.429	0.579	0.444	0.501	0.428	0.650	0.696	0.536	0.546	0.495	0.456
$\mathrm{corr}(M_i^T, M_i^P)$	-0.800	-0.857	-0.737	-0.524	-0.717	-0.256	-0.830	-0.867	-0.631	-0.967	-0.774	-0.912	-0.438	-0.868
$\mathrm{corr}(M_i, M_j)$		0.983		0.976		0.983		0.964		0.982		0.987		0.972

Table S2
Wedge Summary Statistics
(Symmetric Trading: Short- and Long-Term Bonds)

The table reports annualized sample mean, standard deviation, skewness and kurtosis of the wedge in equation (10) ($\eta_{t+1} = \log((M_{d(t+1)}S_{t+1})/(M_{f(t+1)}S_t))$), for dispersion measures $\alpha = 0.5, 2$. For $\alpha = 0$, the wedge vanishes. The optimal derived SDFs account for the fact that domestic investors can trade the short-term domestic and foreign risk-free bonds only.

	$\alpha = 0.5$				$\alpha = 2$			
	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)
UK	0.000	0.019	-0.699	5.460	0.002	0.053	1.074	9.240
CH	0.000	0.018	-1.083	5.962	-0.003	0.055	-2.419	14.45
JP	0.000	0.021	-0.987	5.590	-0.003	0.056	-2.238	15.48
EU	0.000	0.018	0.274	4.061	0.000	0.048	-0.206	5.049
AU	0.000	0.016	0.202	5.210	0.001	0.050	1.948	16.54
CA	0.000	0.011	-0.324	4.721	0.000	0.027	-0.632	6.554
NZ	0.000	0.018	-2.495	15.22	-0.009	0.095	-10.37	139.7

Table S3
Correlation between Wedge and SDFs
(Symmetric Trading: Short- and Long-Term Bonds)

This table reports the correlation between the wedge η, the (log) domestic and foreign minimum variance SDFs ($\alpha_i = 2$), as well as the log permanent and transient components of minimum variance SDFs. Log SDFs are denoted by $m_i := \log M_i$ and log SDF components by $m_i^U := \log M_i^U$ ($i = d, f$ and $U = T, P$). Standard errors (SE) are computed using a circular block bootstrap of size 10 with 10000 simulations and reported in square brackets. Labels *, ** and *** denote significance at the 10%, 5% and 1% level, respectively.

	$\text{corr}(\eta, m_i)$	SE	$\text{corr}(\eta, m_i^P)$	SE	$\text{corr}(\eta, m_i^T)$	SE
US	-0.046	[0.112]	-0.035	[0.107]	-0.011	[0.072]
UK	-0.150	[0.123]	-0.155	[0.117]	0.147	[0.086]
US	0.256***	[0.090]	0.234***	[0.087]	-0.067	[0.062]
CH	0.146*	[0.088]	0.129	[0.086]	0.054	[0.044]
US	0.224**	[0.098]	0.204**	[0.096]	-0.041	[0.074]
JP	0.129	[0.094]	0.101	[0.093]	0.119**	[0.052]
US	0.068	[0.065]	0.060	[0.066]	-0.017	[0.061]
EU	-0.041	[0.070]	-0.060	[0.069]	0.161***	[0.063]
US	0.002	[0.055]	0.007	[0.052]	-0.030	[0.053]
AU	-0.092	[0.066]	-0.086	[0.064]	0.051	[0.055]
US	0.078	[0.078]	0.075	[0.079]	-0.041	[0.079]
CA	0.027	[0.077]	0.018	[0.075]	0.023	[0.066]
US	0.439**	[0.203]	0.422**	[0.184]	-0.046	[0.062]
NZ	0.267*	[0.161]	0.204	[0.148]	-0.086	[0.082]

Table S4
Backus-Smith (1993)-Type Regressions
(Symmetric Trading: Short- and Long-Term Bonds)

This table reports the point estimates of a linear regression of the log difference between foreign and domestic SDFs on the log real exchange rate return: $m_{f,t+1} - m_{d,t+1} = \delta + \beta\Delta s_{t+1} + u_{t+1}$, where $\Delta s_{t+1} = s_{t+1} - s_t$ and small-cap letters denote quantities in logs. We additionally report point estimates of a linear regression of the log difference of the permanent component of the SDF on the log real exchange rate return: $m^P_{f,t+1} - m^P_{d,t+1} = \delta^P + \beta^P\Delta s_{t+1} + u^P_{t+1}$. Standard errors are reported in square brackets. Label *** highlights significance at the 1% level.

	Panel A: US/UK			Panel B: US/CH		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1***	1.035***	0.981***	1***	1.031***	1.042***
	[0.000]	[0.008]	[0.023]	[0.000]	[0.007]	[0.021]
$\boldsymbol{\beta^P}$	1.085***	1.120***	1.066***	0.951***	0.981***	0.993***
	[0.068]	[0.067]	[0.074]	[0.044]	[0.045]	[0.051]
	Panel C: US/JP			Panel D: US/EU		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1***	1.044***	1.022***	1***	1.030***	0.985***
	[0.000]	[0.008]	[0.026]	[0.000]	[0.007]	[0.020]
$\boldsymbol{\beta^P}$	1.083***	1.127***	1.105***	0.956***	0.986***	0.941***
	[0.053]	[0.053]	[0.060]	[0.046]	[0.046]	[0.053]
	Panel E: US/AU			Panel F: US/CA		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1***	1.025***	1.035***	1***	1.027***	0.979***
	[0.000]	[0.006]	[0.021]	[0.000]	[0.007]	[0.017]
$\boldsymbol{\beta^P}$	1.005***	1.031***	1.040***	1.027***	1.055***	1.007***
	[0.049]	[0.049]	[0.054]	[0.089]	[0.090]	[0.093]

	Panel F: US/NZ		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1***	1.033***	1.141***
	[0.000]	[0.008]	[0.042]
$\boldsymbol{\beta^P}$	1.006***	1.039***	1.146***
	[0.038]	[0.038]	[0.055]