Information Flows across the Futures Term Structure: Evidence from Crude Oil Prices

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January 31, 2017

Abstract

We apply the concepts of mutual information and information flows and we built directed graphs to investigate empirically the propagation of price fluctuations across a futures term structure. We focus on price relationships for North American crude oil futures because this key market experienced several structural shocks between 2000 and 2014: financialization (starting in 2003), infrastructure limitations (in 2008-2011) and regulatory changes (in 2012-2014). We find large variations over time in the amount of information shared by contracts with different maturities. The mutual information increased substantially starting in 2004 but fell back sharply in 2012-2014. In the crude oil space, our findings point to a possible re-segmentation of the futures market by maturity in 2012-2014. This raises questions about the causes of market segmentation. In addition, although on average short-dated contracts (up to 6 months) emit more information than backdated ones, a dynamic analysis reveals that, after 2012, similar amounts of information flow backward as flow forward along the futures maturity curve. Moreover, the directions of the transfers between pairs of maturities become drastically different. This has implications for the Samuelson effect.

Keywords: Mutual information, Market integration, Information flows, Shocks Propagation, Directed graphs, Term structure, Futures, WTI

JEL Classification Codes: G10, G13, G14, Q49

[*]The authors thank Scott Mixon, Casey Petroff, Jonathan Wallen, participants at the Seventh International Forum on Financial Risks (Institut Louis Bachelier, Paris, March 2014) and especially the discussant, Patrice Poncet, for useful comments on an early draft. Thanks also go to the participants at the seminar at the Department of Energy (December 2016), and to Gautier Boucher and Pierre-Alain Reigneron for research assistance. Lautier and Raynaud gratefully acknowledge support from the Chair Finance and Sustainable Development and the FIME Research Initiative. Robe contributed to this project during a period when he was also doing consulting work for the U.S. Department of Energy (DOE) and for the U.S. Commodity Futures Trading Commission (CFTC). No compensation was received from, and no resources were used at, either the DOE or the CFTC for this project. The opinions expressed in this paper are those of the authors only, and not those of the DOE or the CFTC. Errors and omissions, if any, are the authors' sole responsibility.

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1 Introduction

Commodity futures markets fulfill the key economic functions of allowing for hedging and price discovery. In these markets, two important questions arise.

First, are futures prices interconnected across the maturity curve? Theory suggests that they should be linked through the cost-of-carry relationship. In practice, however, such market integration requires cross-maturity arbitrage. Büyükşahin, Harris, Overdahl and Robe (2009) document that even the three largest U.S. commodity futures markets did not witness substantial activity in longer-dated derivatives until 2003 (crude oil) or later (corn and natural gas). This empirical reality suggests the possibility of changes in cross-maturity informational linkages in the past decade.

Second, assuming that different-maturity futures prices are interconnected, where in the term structure do prices shocks originate, and which other parts of the term structure do they reach? Is the direction of the shocks' propagation stable over time? Theoretically, the physical market is the place for the absolute price to emerge as a function of the supply and demand for the underlying asset. In turn, the derivative market allows for relative pricing: futures prices derive from the spot price. Under normal circumstances, the prices shocks should thus spread from the underlying asset to the derivative instrument. Yet, amid a massive increase in far-dated commodity futures trading after 2003, might one not expect to also observe prices shocks spreading from the far end to the short (physical) end of the futures curve?

We apply concepts derived from the theory of information of Shannon (1948) and we built directed graphs to answer these questions empirically. The New York Mercantile Exchange's (Nymex) West Texas Intermediate (WTI) sweet crude oil futures market provides an ideal setting for our analysis. Among all commodity markets, the WTI futures market boasts the highest level of activity together with the greatest number of far-out delivery dates (up to seven years).

The theory of information, first proposed by Shannon (1948), aims at quantifying information. In this article, we associate information to unexpected futures prices' returns. Mutual information is a key concept in this context. Given that two variables are interdependent (as is the case, for example, for two times series of futures prices' return for two different maturities), the mutual information gives the amount of uncertainty that is reduced, compared to the case where the two variables are independent. The mutual information can thus be used as a proxy for market integration. Compared with correlations, this probabilistic approach does not depend on any assumption on the variables under consideration.

We find substantial variations over time in the amount of mutual information shared by futures contracts with different delivery dates. Intermediate-maturity contracts (6 months to 2 years) share relatively more mutual information than other contracts. For all contracts, cross-maturity integration increased dramatically after 2003 (amid tight oil supply conditions and the onset of commodity markets' financialization) but fell back sharply in 2012 (to pre-2005 levels) and fell further in 2013 and 2014 (to pre-2002 levels).Taken together, our term structure findings point to a possible re-segmentation of the futures market by maturity in 2012-2014

To investigate the propagations of prices shocks, we then rely on the concept of transfer entropy. Such measures allows for dynamic analyses and introduce directions. They allows to answer the following question: does a shock on the return of a given maturity τ at time t create a shock at time t+1 on an another maturity? Directions are important to know whether prices shocks evolve from short-term to long-term maturities, or vice and versa. This idea is closely linked to the Granger causality, in a non parametric world.

On average across our 2000-2014 sample period, we find that the nearby contract sends more information than any other maturity and that short-dated contracts (maturities up to 6 months) emit more information than backdated ones – a pattern consistent with the typical functioning of futures market. A dynamic analysis, however, reveals that the amount of information flows originating in the far end of the curve increased as cross-maturity integration progressed. Nowadays, similar amounts of information flow from the near and far ends of the maturity curve but the directions of the shocks (from near- to far-dated contracts or vice-versa) is less stable.

Finally, using these non parametric measures in the framework of the graph theory gives us the possibility to sort among futures contracts, according to their maturity. A key reason underlying our choice of graph theory is that, insofar as all the futures prices that we are studying create a system, then this system is a complex one: it is made of many components that may interact in various ways through time. On any sample day year, after having discarded illiquid maturities, there remain 33 different delivery dates in the crude oil market, so we have 1056 pairs of maturities to examine after accounting for directionality. Moreover, such linkages may change through time as a result of evolving trading practices in the past 15 years. Finally, chances are few that the relationships between different maturities are always linear.

A graph gives a representation of pairwise relationships within a collection of discrete entities. Each point of the graph constitutes a node (or vertex). In the present article, a node corresponds to the time series of prices returns of a futures contract for a given maturity. The links (or edges) of the graph can then be used in order to describe the relationships between the nodes. More precisely, the graph can be weighted in order to take into account the intensities and/or the directions of the connections. We do both on the basis of information theory.

There are several ways to enrich the information contained in a graph through its links. In finance, for example, the connections between the nodes can be related to the correlations of price returns or to the positions of market operators. Here, we rely on the theory of information in order to enrich the links of the graph in two ways: first, to determine the intensities of the links; second, to obtain their direction. To the best of our knowledge, such an application is unprecedented in studies of futures term structures and of commodity markets.

The use of the graph theory in this context allows us to examine precisely where the entropy is transferred in the prices system. Even more, we are able like to know whether or not a prices shock hitting the first-month maturity has a chance to propagate along the whole prices curve, up to the last maturity, or if is gradually amortized before reaching that point.

We show that on average on the period, the directed graphs support the conventional view of how a futures markets operates: specifically that prices shocks are thought to form in the physical market, here represented by the short maturities, and transmit to the paper market, here made up of further-out maturities. Moreover, transfers from the shortest maturities along the whole prices curve are far from negligible. The same is not true for shocks born in the far end extremity of the curve: they are amortized more rapidly. Finally, we underline that this behavior changes in the end of the period. These results have implications for Samuelson's (1965) hypothesis regarding the term structure of futures volatilities

The paper proceeds as follows. Section 2 summarizes our contribution to the literature. Section 3 outlines our methodology, which is based on mutual information and transfer entropy. Section 4 presents the data and our empirical results. Section 5 concludes.

2 Literature

The present paper contributes to three literatures: on term structures and market segmentation, on causality, and on the use of graph theory in the context of financial markets.

The theoretical literature on the term structure of futures prices for commodities in general, and crude oil in particular, includes many distinguished contributions such as those of Schwartz (1997), Routledge, Seppi and Spatt (2000), Casassus and Collin-Dufresne (2005), Casassus, Collin-Dufresne and Routledge (2007), Carlson, Khokher and Titman (2007), Kogan, Livdan and Yaron (2009), Liu and Tang (2010) and Baker (2014). The models proposed in those papers, however, do not deal with the possibility that market frictions may prevent different parts of the price curve from moving in sync.[1]

Questions related to the information contained in a term structure of prices and the possible implications of market imperfections for segmentation date back to the works of Culbertson (1957) and Modigliani and Sutch (1966) on "preferred habitats" in bond markets. Spurred in part by interest rate behaviors during the 2008-2011 financial crisis and the so-called Great Recession, the past ten years have seen a resurgence of theoretical and empirical work on segmentation. The latter is defined as a situation in which different parts of the price curve are disconnected from each other. Gürkaynak and Wright (2012), who review this still-growing literature, conclude that "the preferred habitat approach (has) value, especially at times of unusual financial market turmoil" (p. 360). For example, D'Amico and King (2013) document the existence of a "local supply" effect in the yield curve in 2009 during the U.S. Federal Reserve's unprecedented program to purchase \$300 billion of U.S. Treasury securities.

Research on possible term structure segmentation in commodity futures markets deals almost exclusively with the crude oil market, which boasts the highest trading volumes and (in the United States) contract maturities extending up to seven years. In contrast to interest rate markets, prior work in the WTI futures space suggests that the Lehman crisis and its direct aftermath did not witness an increase in market segmentation. Granted, on the basis of the informational value of

[1] A different part of the literature on commodity price formation analyzes the role of spot markets in revealing trader information. That body of work comprises theoretical work by Stein (1987) and Smith, Thompson and Lee (2014), as well as empirical work by Ederington, Fernando, Holland and Lee in the specific case of crude oil. We focus instead on information flows across the futures term structure. Those papers highlight the role played by inventories. Our paper instead contributes to the literature investigating the extent to which, or the potential mechanisms through which, information travels within futures markets.

futures prices, Lautier (2005) finds cross-maturity segmentation during the 1990's. She argues, however, that this phenomenon had become less strong by the end of her sample period in 2002. Indeed, using recursive cointegration techniques, Büyükşahin *et al.* (2011) document that WTI cross-maturity linkages became statistically significant in 2003-2004 and remained so through at least May 2011 (the end of their sample period).[2]

We complement this prior work in several ways: we quantify the information shared by different contracts according to their maturity, assess the direction of the information flows between maturities, and document how these measures have evolved through time. Our analysis, based on different techniques, confirms the prior finding of increasing market integration until 2011. We show, however, that when discussing market segmentation one must distinguish between forward *vs.* backward flows of information, as both types of flows are not equally impacted by segmentation. Furthermore, in sharp contrast to the cross-maturity integration that characterized the 2004-2011 period, we show that different parts of the WTI term structure became much less integrated in 2012-2014.

Insofar as it focuses on price relationships, the present paper belongs to a vast literature on prices linkages. If the spatial dimension of market integration has been analyzed in depth elsewhere for equities and currencies as well as for commodities,

cross-maturity linkages have not. Other than the two articles discussed above, prior work on information flows in the crude oil market abstracts from term structure issues and investigates instead the relationship between spot and futures prices.[3] In that context, a central empirical question is whether price discovery takes place on the futures or the spot market (Garbade and Silber, 1983). While early studies tend to rely on Granger causality to provide an answer, a number of papers apply other techniques in an attempt to tease out causality when the relationship between prices might be non-linear.[4]

The methodological choices in the present paper are likewise motivated not only by concerns

[2] Using a comprehensive, trader-level dataset of end-of-day futures positions and trader type, Büyükşahin *et al.* document that this market development can be attributed to what has been dubbed the "financialization" of commodity markets (specifically, the increased market activity by commodity swap dealers, hedge funds and other financial traders). See Büyükşahin and Robe (2011, 2014) for further evidence regarding the financialization of energy (2011) and other (2014) commodity markets; see Cheng and Xiong (2014) for a review of the financialization literature.

[3] Kawamoto and Hamori (2011) are an exception. These authors look at WTI futures contracts with maturities up to nine months. Their focus, however, is on market efficiency and unbiasedness.

[4] See, e.g., Silvapulle and Moosa (1999), Switzer and El-Khoury (2007), Alzahrani, Masih and Al-Titi (2014) and references cited in those papers.

about possible non-linearities but also by the sheer size of the system we consider (1056 daily pairs of maturities, accounting for directionality). A number of papers use graph theory to investigate prices connections. See, e.g., Haigh and Bessler (2004), Bryant, Bessler and Haigh (2006), Wang (2010), Lautier and Raynaud (2012), Dimpfl and Peter (2014) and references cited therein. Compared with this body of work, our article utilizes another type of graph relying on the theory of information. This allows us to study all the possible connections between the different maturities of the North American crude oil market, in a non parametric world.

3 Methodology

In order to study the interdependence of, and the directionality of price movements for different futures contracts, we rely on the theory of information based on the notion of entropy proposed by Shannon (1948). This Section first presents the concept of "mutual information" that quantifies the dependency between two random variables and that we use as a proxy for market integration. Unlike correlations, the mutual information measure captures non-linear relationships between variables; however, it does not allow for studying the propagation of information. On that purpose, we rely on measures of transfer entropy.

3.1 Mutual information

In what follows, we consider a time series of a futures prices' returns for a given maturity τ as a discrete random variable R_τ with a probability distribution $p(r_\tau)$. Before enlarging to a large scale study, we first want to study the interdependence of two series corresponding to the maturities τ_1 and τ_2. This measure of the interdependence, also called "mutual information", rely on the use of several quantities: the "information entropy" of one variable, as well as the conditional and joint entropies of two variables. These different measures are linked to each others.

A first step is to consider the "information entropy" $H(R_\tau)$ of the futures prices' return. This quantity captures the degree of uncertainty associated to the variable R_τ. In other words, it measures how much we ignore about the futures prices' returns for a given maturity. To illustrate this idea, let us omit temporarily the maturity dimension τ and assume that we can observe all the possible values of the variable R: $r_1, r_2, r_3, ...r_n$. Let us denote by $p(r_i)$ the probability that R

is equal to r_i: $P(R = r_i) = p(r_i)$. If the probabilities associated with all realizations are such as $p(r_1) = p(r_2) = p(r_3) = ... = p(r_n) = \frac{1}{n}$, then all realizations are equally likely and $H(R)$ is maximum.

Formally, the information entropy $H(R_\tau)$ is defined as:

$$H(R_\tau) = -r_\tau \sum_p (r_\tau) \log p(r_\tau) \tag{1}$$

where $\sum_p$ is the sum over all the possible values of R_τ.Note that this quantity increases with the number of possible values for the random variable.

Next let us consider the case of two maturities τ_1 and τ_2, and the interdependency between R_{τ_1} and R_{τ_2}. Let us denote by $p(r_{\tau_1}, r_{\tau_2})$ the joint probability distribution of the two variables. What remains unknown of R_{τ_1} if the values of R_{τ_2} are known is captured by the notion "conditional entropy", ie the entropy of R_{τ_1} conditionally on R_{τ_2}:

$$H(R_{\tau_1}|R_{\tau_2}) = -\sum_{r_{\tau_1}, r_{\tau_2}} p(r_{\tau_1}, r_{\tau_2}) \log \frac{p(r_{\tau_2})}{p(r_{\tau_1}, r_{\tau_2})} \tag{2}$$

Using the conditional probability distribution of the two variables $p(r_{\tau_1}|r_{\tau_2})$, the conditional entropy can be rewritten:

$$H(R_{\tau_1}|R_{\tau_2}) = -\sum_{r_{\tau_1}, r_{\tau_2}} p(r_{\tau_1}, r_{\tau_2}) \log p(r_{\tau_1}|r_{\tau_2}) \tag{3}$$

Another interesting quantity directly linked to the preceding one is the joint information entropy $H(R_{\tau_1}, R_{\tau_2})$. It quantifies the amount of information revealed by evaluating R_{τ_1} and R_{τ_2} simultaneously. This symmetric measure is related to the conditional entropy as follows:

$$H(R_{\tau_1}, R_{\tau_2})) = H(R_{\tau_1}|R_{\tau_2})) + H(R_{\tau_2})) = H(R_{\tau_2}|R_{\tau_1})) + H(R_{\tau_1}) = H(R_{\tau_2}|R_{\tau_1})) \tag{4}$$

On the basis of these definitions, it is now possible to express the mutual information $M(R_{\tau_2}, R_{\tau_1}))$. This quantity measures the amount of information obtained about one variable through the other. Given that the two variables are *interdependent*, the mutual information gives the amount of uncertainty that is reduced, compared to the case where the two variables are *independent*. There are two (equivalent ways) to compute the mutual information $M(R_{\tau_2}, R_{\tau_1}))$. One possibility is to rely on equation 4:

$$M(R_{\tau_1}, R_{\tau_2}) = H(R_{\tau_1}, R_{\tau_2}) - H(R_{\tau_1}|R_{\tau_2}) - H(R_{\tau_2}|R_{\tau_1}) \tag{5}$$

This expression shows intuitively that the mutual information is a symmetric quantity. Moreover, as a consequence of the properties of the joint entropy, it is equivalently possible to write:

$$\begin{aligned} M(R_{\tau_1}, R_{\tau_2}) &= H(R_{\tau_1}) - H(R_{\tau_1}|R_{\tau_2}) \\ &= H(R_{\tau_1}) + (H(R_{\tau_2}) - H(R_{\tau_1}, R_{\tau_2})) \end{aligned}$$

Relying on equations 1 and 2, another possible expression is:

$$M(R_{\tau_1}, R_{\tau_2}) = \sum_{r_{\tau_1}, r_{\tau_2}} p(r_{\tau_1}, r_{\tau_2}) \log \frac{p(r_{\tau_1}, r_{\tau_2})}{p(r_{\tau_1})p(r_{\tau_2})} \tag{6}$$

In this article, we use the mutual information as a measure of the integration of the crude oil futures market in the maturity dimension. This quantity indeed includes synchronous correlations between pairs of futures prices' return, due to the common history of the returns, and/or to common shocks. Compared with correlations however, this probabilistic approach does not rely on any assumption regarding the behavior of futures returns.

3.2 Transfer entropy

If we are now interested in the propagation of shocks along the futures prices curve, there is a need first for dynamic measures, second for directions. Dynamic measures aim to answer the question: does a shock on the return of a given maturity τ at time t create a shock at time $t+1$ on an another maturity? Directions are important to know whether prices shocks evolve from short-term to long-term maturities, or vice and versa. This idea is closely linked to the Granger causality, in a non parametric world.

Starting from information entropy, one natural way to proceed is to rely on the notion of transfer entropy (Schreiber, 2000). This measure quantifies how much information entropy (or uncertainty) is *transported* between dates t and $t+1$ from one variable to another (in our case, from one futures maturity to another). It relies on transition probabilities rather than on static probabilities.

Relying on the definition of conditional entropy as described by equation 3, and introducing time in the analysis allows for defining a new quantity: the entropy rate h. Two kind of rates can be distinguished, according to the type of dependance under consideration. The entropy rate $h(R_{\tau_1})$ quantifies the uncertainty on the next value of R_{τ_1}, if only the previous state of R_{τ_1} matters:

$$h(R_{\tau_1}) = -\sum p(r_{\tau_1}^{t+1}, r_{\tau_1}^{t}, r_{\tau_2}^{t}) \log p(r_{\tau_1}^{t+1} | r_{\tau_1}^{t}) \tag{7}$$

The entropy rate $h(R_{\tau_1}|R_{\tau_2})$ quantifies the uncertainty on the next value of R_{τ_1}, if both the previous states of R_{τ_1} and R_{τ_2} have an influence:

$$h(R_{\tau_1}|R_{\tau_2}) = -\sum p(r_{\tau_1}^{t+1}, r_{\tau_1}^{t}, r_{\tau_2}^{t}) \log p(r_{\tau_1}^{t+1} | r_{\tau_1}^{t}, r_{\tau_2}^{t}) \tag{8}$$

Once these dynamic measures defined, it is possible to introduce directions in the analysis: the "transfer entropy" T from R_{τ_2} to R_{τ_1} is the difference between the two rates:

$$T_{R_{\tau_2} \to R_{\tau_1}} = h(R_{\tau_1}) - h(R_{\tau_1}|R_{\tau_2}) = \sum p(r_{\tau_1}^{t+1}, r_{\tau_1}^{t}, r_{\tau_2}^{t}) \log \frac{p(r_{\tau_1}^{t+1} | r_{\tau_1}^{t}, r_{\tau_2}^{t})}{p(r_{\tau_1}^{t+1} | r_{\tau_1}^{t})} \tag{9}$$

Likewise, the transfer from R_{τ_1} to R_{τ_2} can be written as follows:

$$T_{R_{\tau_1} \to R_{\tau_2}} = h(R_{\tau_2}) - h(R_{\tau_2}|R_{\tau_1}) = \sum p(r_{\tau_2}^{t+1}, r_{\tau_1}^{t}, r_{\tau_2}^{t}) \log \frac{p(r_{\tau_2}^{t+1} | r_{\tau_1}^{t}, r_{\tau_2}^{t})}{p(r_{\tau_2}^{t+1} | r_{\tau_2}^{t})} \tag{10}$$

This transfer entropy measure is equivalent to Granger causality in the case of a linear dependency between two Gaussian random variables (Barnett, Barrett and Seth, 2009). Transfer entropy presents however the advantages of being model-free and of holding in the case of non-linearity.

Equipped with the above definitions, we are able to provide an insightful analysis of the propagation of prices shocks along the term structure, for all maturities and all directions. In what follows, we first present the data and the way they are manipulated. Then we present our empirical results.

4 Data

Our dataset consists of the daily settlement prices for Nymex's WTI light, sweet crude oil futures contracts from the 21^{st} of January 2000 to the 25^{th} of February 2014. We construct 33 time series of futures prices. The first 28 are for the 28 shortest-dated contract maturities (i.e., contracts with 1 to 28 months until expiration). The last five time series correspond, respectively, to contract maturities of 30, 36, 48, 60 or 72 months. Futures roll dates are calendar-based.[5]

Our empirical analyses use daily futures returns. We compute daily futures returns as the logarithm of the price difference: $r_\tau = (\ln F_\tau(t) - \ln F_\tau(t - \Delta t))/\Delta t$, where $F_\tau(t)$ is the price of the futures contract with maturity τ at t and Δt is the time interval between consecutive sample days.

Figure 1 depicts the evolution of WTI futures prices and returns in our sample period. For readability, we focus on the nearby, one- and two-year contracts. The *right* panel of the figure shows that the returns' volatility is lower for the two backdated contracts than for the nearby futures, an empirical fact consistent with the Samuleson (1965) hypothesis that volatility should increase as a futures contract's maturity nears.[6]

Quite obvious in the *left* panel of Figure 1 are the sharp oil price rise in 2007-2008 and the consequent precipitous price decrease after August 2008. Equally notable, and especially relevant to the present study, is the difference in the relative behaviors of nearby *vs.* longer-dated contracts at the beginning (before 2004) and at the end (2012-2014) of our sample. As previously documented in Lautier (2005) for pre-2002 data and Büyükşahin *et al.* (2011) for the May 2000 to May 2011 period, Figure 1 shows that the one-year (*two-year*) futures prices did not move in sync with the nearby price prior to 2003 (*2004*) but started doing so soon thereafter. Figure 1 extends these prior empirical findings by showing that, starting in late 2011 and accelerating in 2012, a disconnect has reappeared between short- and further-dated WTI futures.

All dynamic analyses are made on the basis of rolling windows having a length of 500 trading days.

[5]That is, we use the Nymex calendar to determine which contract has (for example) a one- *vs.* two-month maturity and when the first-deferred contract becomes the nearby contract.

[6]We obtain the same volatility ranking with futures rolled based on the preponderance of the open interest rather than calendar dates. Bessembinder, Coughenour, Seguin and Monroe Smoller (1996) give an elegant theoretical analysis of conditions under which the Samuelson (1965) effect holds, such as asset markets in which spot price changes include a temporary component (so that investors expect mean-reversion) or, alternatively, the assumption that information revelation about spot prices is systematically clustered around futures expiration dates (as in Anderson and Danthine, 1983). For more recent analyses of the Samuelson effect, see Jaeck and Lautier (2016).

5 Empirical Results

We present empirical evidence on the information shared by futures contracts of different maturities and its evolution through time. Next, we introduce directionality and examine information flows. Finally, we study the stability of the directions of transfer entropy.

5.1 Mutual Information: A Proxy for Market Integration

Changes in the WTI market during the period under consideration can be characterized through the lens of mutual information. Recall the mutual information measure quantifies how much we know about the return R_{τ_1} once the return R_{τ_1} is known. In other words, this quantity captures the synchronous moves in prices. In what follows, we distinguish between the mutual information shared by all the futures contracts under consideration, and the mutual information attached to one specific maturity.

5.1.1 Mutual information shared by all maturities

On the basis of equation 6, that defines the mutual information for a pair of maturities τ_1 and τ_2, we can extend the analysis to the case of the average mutual information shared at date t by all T futures prices' returns, M_t^T:

$$M_t^T = < M_t(R_{\tau_i}, R_{\tau_j}) >_{i,i>j} \tag{11}$$

where i) T is the total number of maturities, ii) the $M_t(R_{\tau_i}, R_{\tau_j})$ are the elements of the $(T \times T)$ matrix of mutual information computed on day t using daily returns for the Two prior years (500 trading days), iii) $<>_{i,i>j}$ denotes the averaging operator over the relevant contract maturities i.

Figure 2 depicts the dynamic behavior of the mutual information shared by all maturities over the period. The high values of M_t^T between 2004 and the beginning of 2011 are evidence of strong cross-maturity linkages.[7] As an increase in M_t^T can be interpreted in terms of greater integration of the futures market for crude oil during that period, this finding complements the cointegration-

[7] To provide a visual reference for the statistical significance of the measure, we generate a benchmark by "shuffling" (i.e., using permutations of) the time index of each analyzed dataset. The resulting "shuffled" time series have the same statistical properties (mean, variance and higher moments) as the original ones but temporal relationships are removed. The resulting benchmark (i.e., the "shuffled" mutual information) is plotted in red in Figure 2. Unlike the actual series (M_t^T, left-hand scale in black), the counterfactual (right-hand scale) is close to 0 and does not display any systematic pattern over time.

based study of this phenomemon by Büyükşahin *et al.* (2011). Figure 2, however, also shows that the mutual information M_t^T decreases sharply after 2011 and drops further, to a very low level, in 2013-2014. This finding is novel. It provides formal support for our interpretation of Figure 1 in the previous subsection: starting in 2012, different parts of the term structure of WTI futures prices have become much less integrated.

This apparent re-segmentation of the WTI market across futures delivery dates follows a partial geographic segmentation of world crude oil markets after the Summer of 2008 as evidenced by the emergence of a large differential between the prices of North American (WTI) and world (Brent) benchmark crudes. This large Brent-WTI price differential has been linked by Fattouh (2010), Pirrong (2010) and Büyükşahin *et al.* (2013) to two developments in the physical market for crude oil: infrastructure constraints at the delivery point for WTI futures in Cushing, OK, and a divergence in supply-side conditions in North America *vs.* the rest of the world. In contrast, rather than physical-oil market fundamentals, Büyükşahin *et al.* (2011) show that the WTI futures market's cross-maturity integration in 2004-2011 stemmed from a large increase in (cross-maturity) spread trading by hedge funds and other financial institutions amid what has been dubbed the "financialization of commodities." Intriguingly, the 2012-2014 re-segmentation apparent in Figure 2 itself has been taking place amid a massive growth in North American oil production, export restrictions and transportation constraints, as well as new derivatives regulations in the United States. Interdependency, though, need not imply causation. Our findings therefore raise the important question of the respective roles played by fundamentals *vs.* trading activity and (dis)incentives in the WTI market's re-segmentation.

5.1.2 Mutual information for each contract maturity

Figure 3 gives more insight into the developments identified in Figure 2 by depicting the mutual information for *each* contract maturity over the course of our sample period. For each maturity τ_i and each day of the sample period t, the level of mutual information that a maturity shares with all others is as follows:

$$M_t(R_{\tau_i}) =< M_t(R_{\tau_i}, R_{\tau_j}) >_{j,j\neq i}$$

This measure is plotted in a color ranging from blue (very low) to green, yellow, orange or red (very high).

Figure 3 shows that, for all maturities, the mutual information is much higher in $2004-2011$ than at other times. Figure 3 also provides evidence that all futures prices do not have the same level of mutual information with the others. To wit, at each point in time, the graph "temperature" is typically cooler at the short end of the term structure (situated on the top of the plot) than at the far end of the curve (on the bottom of the graph), showing that near-dated contracts usually contain less mutual information. This finding is consistent with the notion that short-dated crude oil futures prices are more volatile.[8] In contrast, there is a lot of mutual information for middle maturities: at any given time, the two extremities of the futures maturity curve share less mutual information with the others than the intermediate one. Overall, these results suggest that the WTI futures term structure consists of three main segments: from the first to the third months, from the 4th to the 30th months, and finally the furthest-out delivery dates (those beyond the 30th month).

An important development took place in $2012-2014$, a period when the graph temperature becomes much cooler across the entire spectrum and is coolest for backdated contracts (those with maturities greater than three years). Figure 3 shows that the middle part of the maturity curve, where the amount of mutual information is the highest, is also larger in 2004-2011 than before or after. In other words, Figure 3 establishes that the integration phenomenon observed in Figure 2 comes principally from what happens at intermediate maturities.

Figure 4 provides additional evidence of this fact by averaging, over the sample period, the information that a specific maturity shares with all others. Each point on the curve gives the average mutual information for each maturity. The bars around each point show the average variance of the measure over the sample period.

Figure 4 shows that the average mutual information is a hump-shaped function of contract maturity. It reaches its maximum near the 18 months maturity. Again, there is more mutual information at the back end of the curve (which we capture up to 6 years out) than at the front end (up to 3 months out). Whereas the shortest maturities are mostly influenced by shocks emerging in the physical market, the behavior of the longest maturities might be related to other factors such as expectations of future supply and demand for the commodity, technological changes, future

[8]See, e.g., Robe and Wallen (2014) for evidence on the term structure of WTI implied volatilities in 2000-2014.

discoveries, or possibly a lack of liquidity in the WTI futures markets.[9]

5.2 Transfer entropy between maturities

A key component of the analysis of cross-maturity linkages is the examination of transfer entropy between different maturities. To answer the question of which side of the term structure is the shock transmitter and which one is the receiver, we first perform a static analysis across the whole sample period and then carry out a dynamic analysis using rolling windows of two years (500 trading days).

5.2.1 Static analysis: the average transfer entropy associated to each maturity

We start by computing the transfer entropy over the entire sample period. This approach gives us a picture of the "average" behavior of the system, i.e., if one maturity sends on average more than what it receives, and *vice-versa*. Starting from equation (10), that focuses on the pairwise transfer between maturities τ_1 and τ_2, we can extend this measure and compute the total amount of entropy sent, on average on the period, from the futures prices' return with maturity i to all other maturities $j \neq i$:

$$T^S_{R_{\tau_i}} =< T_{R_{\tau_i} \to R_{\tau_j}} >_{j, j \neq i} \tag{12}$$

Similarly the quantity received on average by the maturity i from all others is:

$$T^R_{R_{\tau_i}} =< T_{R_{\tau_i} \leftarrow R_{\tau_j}} >_{j, j \neq i} \tag{13}$$

where $<>_{j,j\neq i}$ denotes the average over all the contract maturities other than i.

Figure 5 depicts the average transfer entropy associated to each maturity for our entire 2000-2014 sample period. The black line corresponds to the total amount of information entropy emitted by each maturity on average over the period, given by Equation 12; the red line shows the total amount of information entropy received by each contract on average over the period, computed using Equation 13. The bars represent, for each maturity, the average variance recorded for the measure; they are particularly large for the entropy received on the long-term maturities.

Figure 5 shows that maturities up to one and one half years (precisely up to and including the 19-month contract) emit more than they receive. Most of the information entropy is sent to

[9] For a discussion of these likely explanatory factors, see e.g. Cortazar and Schwartz (2003).

the far-out maturities. Once contract maturities extend beyond 6 months, the information entropy emitted decreases with the maturity although the pattern levels off beyond the two-year mark. The information entropy received exhibits a different pattern: it is high for the first three maturities, lowest for maturities ranging from 6 to 18 months and highest for maturities of 27 months and beyond (with the maximum value reached at the back end of the term structure). Intuitively, these static results imply that market participants whose “preferred habitat” (Modigliani and Sutch, 1966) is the back end of the maturity curve are more likely to be the object of a shock than to be the source of one.

5.2.2 Dynamic analysis: forward and backward information flows

In what follows, we propose an analysis of the way the transfer entropy has changed over time. To link the analysis with the Samuelson effect, we restrain our focus on two sub-sets of the transfer entropy. The later indeed measures the entropy emitted (received) by *one* maturity, whatever the direction of this emission (reception). If we want to have insight into prices shocks propagation and the direction of these propagations from the short to the long term (or vice and versa), then we want to restrain ourselves to what is emitted in one direction only, but from *any* maturity. To this end, we propose the notions of "forward" and "backward information flows".

The forward flow ϕ_f is the sum of the transfers of entropy from *any* maturity τ_i to *all higher* maturities τ_j:

$$\phi_f = \sum_{i<j} T_{R_{\tau_i} \to R_{\tau_j}} \tag{14}$$

Similarly the backward flow ϕ_b is given by the sum of the transfers of entropy from *any* maturity τ_i to *all smaller* maturities τ_j:

$$\phi_b = \sum_{i>j} T_{R_{\tau_i} \to R_{\tau_j}} \tag{15}$$

In other words, the forward flows capture the propagation of shocks in the direction of the long-term maturities, whereas the backward flows measure the propagations in the direction of the short-term maturities.

Figures 6 exhibits the forward and backward flows on each day from February 2002 to February 2014. The values of ϕ_f and ϕ_b are computed using daily futures returns for the prior two years (500 trading days). This figure highlights big changes. Both forward and backward flows decrease from 2000 to 2009 (backward flows) or 2010 (forward flows). This pattern reflects the progressive cross-maturity integration of the WTI futures market in the last decade. In $2012 - 2014$, however, both flows increase.[10]

In $2000 - 2009$, the forward flow is almost always much stronger than the backward flow. Thereafter, however, the amplitude of the two information flows is comparable. Put differently, whereas in the beginning of our sample period, the term structure of futures prices was generally more prone to influence from shocks arising at the near end of the maturity curve, this is not true after 2010: the short-term prices (and, hence, physical prices) can be influenced by price fluctuations moving backward from far-dated contracts. In other words, the driving forces of price movements seem to have become comparable all along the term structure, and prices shocks nowadays propagate as easily in the forward direction as in the backward direction.

These findings raise questions regarding the Samuelson effect. Samuelson (1965) hypothesized that futures prices volatility should increase as futures contracts approach their maturity. In theoretical models of the Samuelson effect, such as Bessembinder et al (1996), the volatility of futures prices stems from shocks that arise in the physical market and are transmitted to the paper market. The term structure of volatilities is therefore downward-sloping: the direction for the propagation of shocks is forward (i.e. from short to long-term maturities) with a progressive absorption as contract maturity increase. However, our empirical results show that, in the later part of our sample period, there are price shocks coming from the far end of the futures term structure that spread to shorter maturities (and hence, arguably, to the physical market). In other words, it is nowadays possible to have backward propagation of prices shocks from the far end of the prices curve to the physical market. This results strongly contrasts what is usually said about the Samuelson effect in commodity markets.

[10] To provide a visual benchmark for the statistical significance of the changes depicted by the red and black curves, we generate benchmark counterfactual forward and backward flow measures, $\phi_{f,shuffle}$ and $\phi_{b,shuffle}$, by permutating ("shuffling") the time index of each analyzed dataset. The resulting time series have the same statistical properties (mean, variance and higher moments) as the original ones but temporal relationships are removed. The resulting information flows are shown in green (forward flow $\phi_{f,shuffle}$) and blue (backward flow $\phi_{b,shuffle}$). Unlike the actual forward and backward flows (depicted in black and red, respectively), the counterfactual information flows fluctuate very little. Crucially, $\phi(t)_{f,shuffle}$ and $\phi(t)_{b,shuffle}$ are almost equal for all t

5.3 Pair-wise analysis: the net transfer entropy between two maturities

The notion of transfer entropy shows that certain maturities send more than the others. As exhibited by Figure 5, this is the case, on average over the study period, for the first-month futures contract. To go further in this direction, it would be interesting to see precisely where the entropy is transferred. Even more, we would like to know whether or not a prices shock hitting the first-month maturity has a chance to propagate along the whole prices curve, up to the 72^{th} maturity, or if is gradually amortized before reaching that point.

In order to investigate such questions, we propose a pairwise analysis of the directions of prices shocks. The direction is determined by the *net* amount of entropy transported from one maturity to one other [11]. Such a pairwise study however requires the examination of 1056 links. This leads us to rely on the graph theory, that is especially suited for large scale analyses.

In what follows, we exploit the non parametric measures presented before in the framework of the graph theory. We first built a filtered directed graph that exhibits not only the directions of the pairwise transfer entropy, but also their strength. This filtered graph retains only the most important connections between maturities. We then analyze the direction of the net transfer entropy, expressed in percentage, maturity by maturity, in a static as well as in a dynamic way. Finally we assess the stability of the directions in the whole prices system.

5.3.1 The directionality index

A graph is defined by its nodes and links. Quite intuitively, we attribute one time series of futures prices' returns to each node of the graph (*i.e.*, one node per maturity, with a total of 33 nodes). In order to enrich the links of the graph with directions as well as with an information about the importance of the connection between two nodes, we construct an index of directionality $D_{R_{\tau_i} R_{\tau_j}}$ that combines equations (9) and (10):

$$D_{R_{\tau_i} R_{\tau_j}} = \frac{T_{R_{\tau_i} \to R_{\tau_j}} - T_{R_{\tau_j} \to R_{\tau_i}}}{T_{R_{\tau_i} \to R_{\tau_j}} + T_{R_{\tau_j} \to R_{\tau_i}}} \tag{16}$$

$D_{R_{\tau_i} R_{\tau_j}}$ gives first the direction between two nodes. It is bounded by -1 and 1. When $D_{R_{\tau_i} R_{\tau_j}}$ is greater than 0, the net transfer entropy is positive and the link is directed from R_{τ_i} to R_{τ_j};

[11] Figure 5 shows that the first month futures contracts sends a lot; but it receives also a large amount.

otherwise, it is directed from R_{τ_j} to R_{τ_i}. Finally, the level of the index expresses the *strength* of the connection between the two returns. This expression first divides by two the dimensionality of the graph: we are left with 528 connections instead of 1056. Second, it allows for comparison between all contracts.

5.3.2 The filtered directed graph

We rely on this index to built a directed a graph. We first retain a static and full connected graph: we compute the values of the index over the entire $2000 - 2014$ sample period and generate the $(T \times T)$ matrix of directionality $\bar{D}_{R_{\tau_i} R_{\tau_j}}$. Such graph, with its 528 connections, is however difficult to read. Hence we filter it according to the strength of the connection between to nodes. As depicted by Figure 7, we distinguish between four level of strength: directionality indexes lower than 0.3 are not exhibited; the black arrows stand for indexes between 0.3 and 0.4, the blue for the range 0.5 to 0.6, and the red for values between 0.6 and 0.7 (there are no values higher than 0.7 in our graph).

Let us come back to the question of the extent of shocks propagation raised by Figure 5. As depicted by Figure 7, shocks arising at the very short end of the curve, on the first-month maturity, go up to the far end extremity of the curve, on the 72^{th} maturity. More importantly, as illustrated by the red color of the link connecting the two nodes, such transfers are far from negligible on average on the period. The same is true for the 6th, 9th and 10th months. Conversily, what is received from the short-term maturities, as illustrated by maturities 1 to 3, does not come from far end maturities, and is not very strong. So it seems that forward shocks propagate themselves on the whole term structure, whereas backward shocks are amortized more rapidly.

More generally, this figure shows that on average over the period, the far-out maturities, which for readability we have positioned at the center of the graph, are those to which the links point. The most important nodes, in terms of information received, are the 48th and 60th. So this filtered graph provides us with a reference case, not only in that it represents what happens over the entire sample period but also in that it appears to support the conventional view of how a futures markets operates (specifically that prices shocks are thought to form in the physical market, here represented by the short maturities, and transmit to the paper market, here made up of further-out maturities).

5.3.3 Outgoing links associated to each maturity

The directionality index also allows for dynamic analyses. On the basis of on one-year rolling windows, we compute, at each date t, the instantaneous directionality matrix $D_{R_{\tau_i}R_{\tau_j}}(t)$. This allows to built daily directed graphs, to examine their properties and their evolution over time.

Figure 8 exhibits the relative proportion, for each maturity, of outgoing and ongoing links, on average and on the sample period. It shows, for example, that approximately 60% of the links attached to the one-month maturity are outgoing links (40% are ongoing). The same observation can be done for the six-months maturity. However, the dispersion around this average, measured through the variance, is strong for the first month maturity, and low for the six-months;

More generally, this analysis shows that: i) the proportion of outgoing links is a decreasing function of the maturities; ii) the first 18 maturities are characterized by a proportion of outgoing links that is higher than 50%. This figure is stable, except for the three first maturities; iii) in contrast, the deferred maturities are characterized by a large proportion of ongoing links (up to 80% on average on the sample for certain ones), with quite large deviations around the average.

This result is consistent with the conventional view of the functioning of a futures market, as with the conclusions drawn through Figures 5 and 7. Up to now however, the specific behavior of the one-, two- and three-month maturities was not observable.

5.3.4 Stability and survival ratios

A second point of interest regarding the properties of daily directed graphs is their stability: do the directions in the graph evolve during the period? How? To answer these questions, we rely on the reference case: the static full directed graph built on the basis of the matrix of directionality $\bar{D}_{R_{\tau_i}R_{\tau_j}}$. We then compare the directions of the links in the reference case and in the daily directed graphs.

In order to give evidence of the distance between the static and the daily graph, we compute survival ratios, ratio $\bar{S}_R(t)$, as follows:

$$\bar{S}_R(t) = \frac{1}{N} D_{R_{\tau_i}R_{\tau_j}}(t) \cap \bar{D}_{R_{\tau_i}R_{\tau_j}} \tag{17}$$

This ratio quantifies the similarities in the two directionality matrices, and express them as a percentage of the total number of elements. If $\bar{S}_R(t) = 1$, the two matrices are identical, the two graphs are the same. At the other extreme, if $\bar{S}_R(t) = 0$, then the set of directed links is totally different.

Figure 9 shows that, from 2000 until the end of 2010 and with the exception of a six-month period at the end of 2005, the survival ratio is generally higher than 70%. This results indicates that during this period, day after days, most of the directed links remained in the same state as in the benchmark case. Thereafter, the ratio displays some variations but generally decreases, sometimes up to 30%– a finding suggesting a profound change in the pattern of shocks propagation in recent years.

6 Conclusion

We apply the notions of mutual information and transfer entropy to empirically investigate the nature of price relationships across a futures term structure. The Nymex's WTI crude oil futures market, a large market that experienced a number of participatory and regulatory changes between February 2000 and February 2014, provides an ideal setting for our analysis.

We find substantial variations over time in the amount of information shared by futures with different delivery dates. The common share increased dramatically starting in 2004 (amid tight oil supply conditions and the onset of commodity markets' financialization) but fell back sharply in 2012 (to pre-2005 levels) and fell further in 2013 and 2014 (to pre-2002 levels). On average over the entire sample period, short-dated contracts (maturities up to 3 months) emit more information than longer-dated ones. While this pattern seems consistent with the typical functioning of futures market, a dynamic analysis using rolling windows reveals that the information flows originating at the back end of the maturity curve have increased over time. Nowadays, similar amounts of information flow from both ends of the curve. Notably, the directionality of information flows (from near- to far-dated contracts or vice-versa) is less stable after 2005. These results suggest two natural venues for further research.

First, our term structure findings raise a theoretical question regarding the Samuelson (1965) hypothesis that a futures contract price's volatility should be inversely related with the contract's time to maturity. A theoretical model is needed to determine whether physical fundamentals or paper market conditions may be responsible for such a pattern of information flows in a term structure.

Second, our analysis establishes that the WTI market was segmented until 2003, integrated between 2004 and 2011, and segmented once more in 2012-2014. Questions regarding the possible presence of a market segmentation affecting different parts of a term structure date back to Modigliani and Sutch (1966). In the present paper we document, that the WTI market has since 2011 once again become segmented – a calamity for market participants who use backdated futures to hedge long-term price risk. The analysis of Büyükşahin *et al.* (2011) suggests that the unprecedented WTI market integration across the term structure in 2003-2011 was due to a combination of tight oil supply conditions and the onset of commodity markets' financialization. One therefore wonders if, in a similar vein, physical-market developments or a pullback by financial institutions from participating at the far end of the WTI futures term structure explains the apparent post-2011 re-segmentation of the market. Answering this question requires access to non-public trader-level trading data: we therefore leave it for further research.

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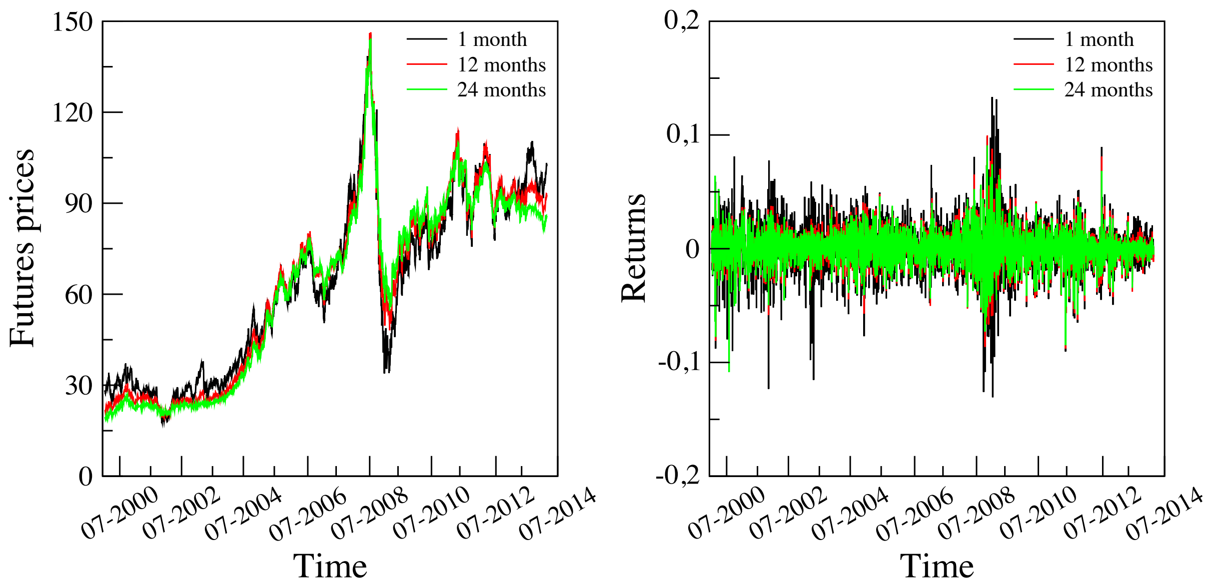

Figure 1: WTI crude oil futures prices and returns, 2000-2014

Note: Figure 1 depicts the evolution of the nearby, one- and two-year out WTI crude oil futures prices and returns in our sample period (January 2000 to February 2014; Source: Nymex). Prices are Nymex end-of-day settlement values. Futures roll dates are calendar-based. Daily futures returns are computed as the daily logarithm price differential r_τ, with: $r_\tau = (\ln F_\tau(t) - \ln F_\tau(t - \Delta t)) / \Delta t$, where $F_\tau(t)$ is the price of the futures contract with maturity τ at time t and Δt is the time interval between two consecutive sample days.

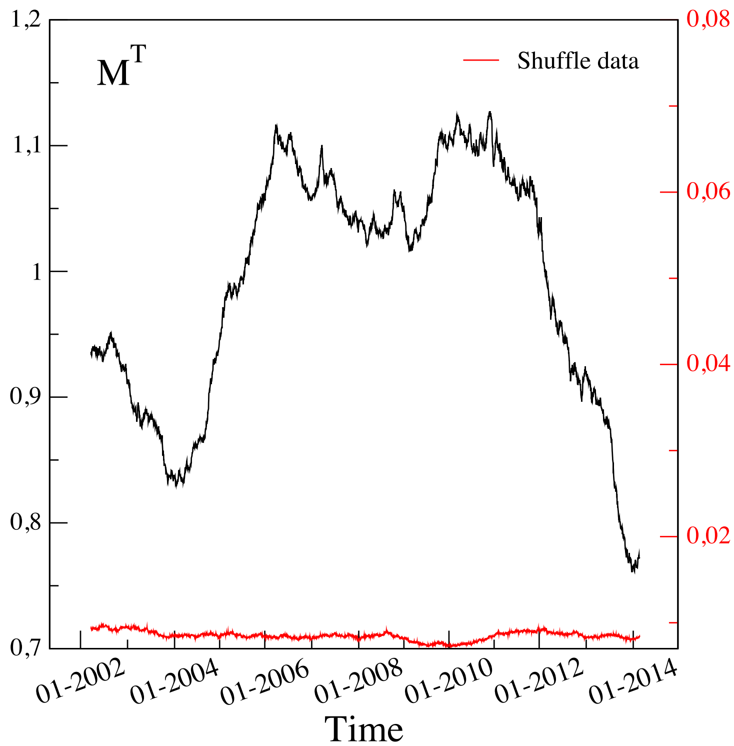

Figure 2: Average mutual information M_t^T shared at date t by all maturities, 2000-2014

Note: Figure 2 plots the evolution over time of the average mutual information shared at date t by all WTI futures contract maturities, $M_t^T = < M_t(R_{\tau_i}, R_{\tau_j}) >_{i,i>j}$ where the $M_t(R_{\tau_i}, R_{\tau_j})$ are the elements of the $(T \times T)$ matrix of mutual information computed on day t using daily returns for the prior two years (500 trading days) and $<>_{i,i>j}$ denotes the averaging operator over the relevant contract maturities i. The increase of M^T from 2004 until the end of 2010 shows that cross-maturity linkages are becoming more and more intense and can be interpreted as a higher integration of the futures market for crude oil. To provide a visual reference for the statistical significance of the changes depicted by the black curve, we generate a benchmark by "shuffling" (i.e., using permutations of) the time index of each analyzed dataset. The resulting "shuffled" time series have the same statistical properties (mean, variance and higher moments) as the original ones but temporal relationships are removed. The resulting benchmark (i.e., the "shuffled" mutual information) is plotted in red. Unlike the actual series (M_t^T, left-hand scale in black), the counterfactual (right-hand scale in red) is close to 0 and does not display any systematic pattern over time.

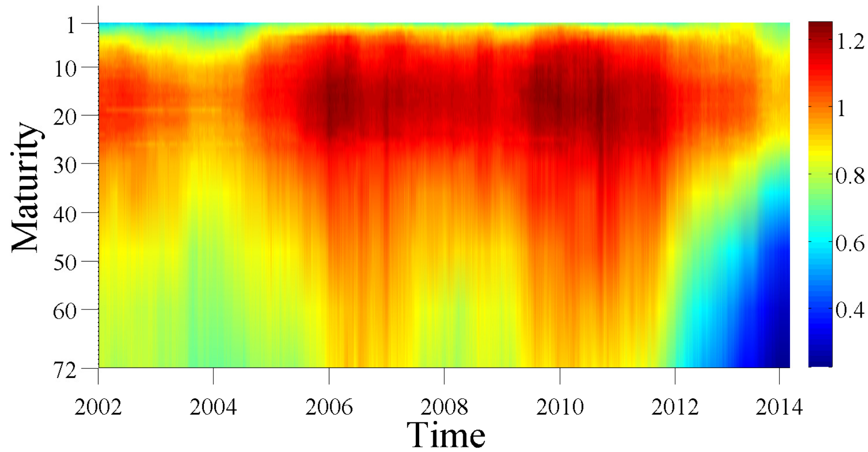

Figure 3: Mutual information shared at date t by each contract with all others, 2000-2014

Note: Figure 3 depicts the mutual information for futures returns over the course of our 2000-2014 sample period. For every maturity $i = 1, 2, ..., 72$ and every day between February 2002 and February 2014, the level of mutual information that one maturity share with all others is computed using daily returns of the previous 500 trading days and is displayed in a color ranging from blue (very low) to green, yellow, orange or red (very high).

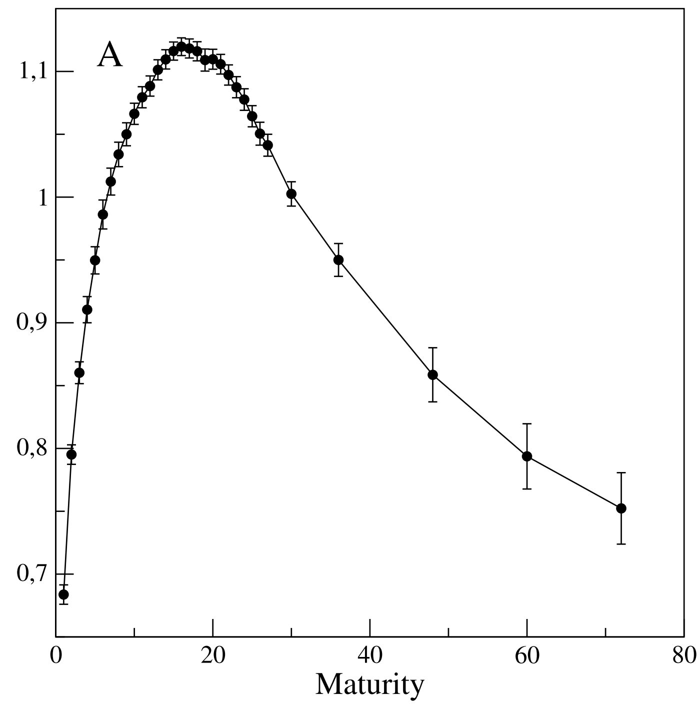

Figure 4: Mutual information shared by one maturity with all others averaged over the period, 2000-2014

Note: Figure 4 shows the average A, computed over the entire 2000-2014 sample period, of the mutual information that a specific maturity shares with all others. Each point on the curve gives the average mutual information for each maturity. The bars around each point show the average variance of the measure over the whole sample period.

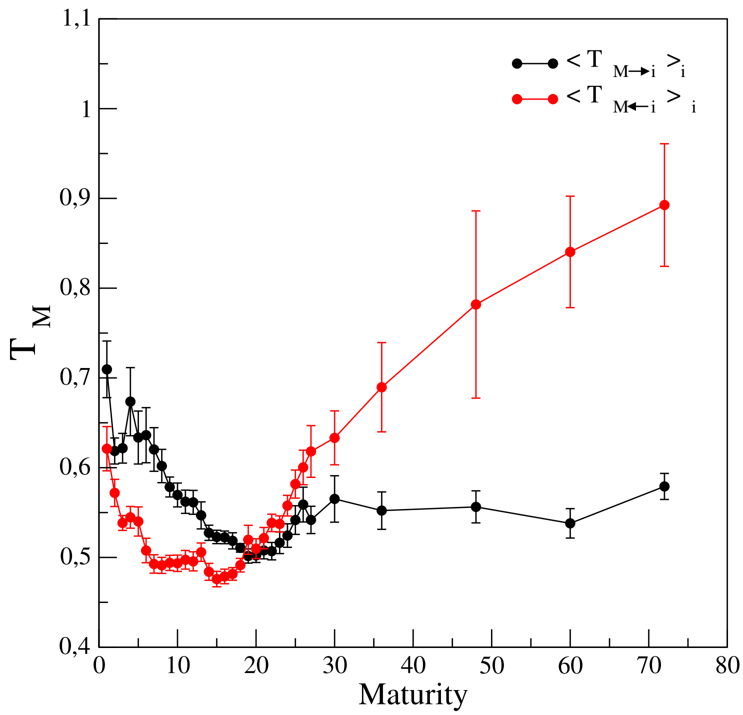

Figure 5: Average transfer entropy associated to each maturity over the period 2000-2014

Note: Figure 5 depicts the average transfer entropy associated to each maturity for our entire 2000-2014 sample period. The black line corresponds to the total amount of information entropy emitted by each maturity on average over the period, given by Equation 12; the red line shows the total amount of information entropy received by each contract on average over the period, computed using Equation 13. The bars represent, for each maturity, the variance recorded for the measure.

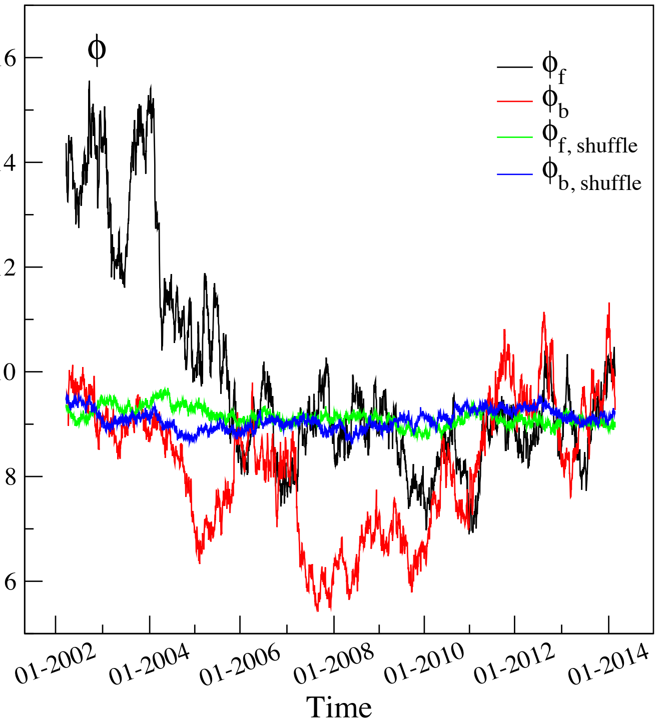

Figure 6: Daily forward and backward information flows between maturities, 2000-2014

Note: Figure 6 illustrates what is emitted, daily, by all maturities in the direction of longer maturities (forward flows ϕ_f, black curve) and what is received by all of them from shorter maturities (backward flows ϕ_b, red curve). On each day from February 2002 to February 2014, we compute the values of ϕ_f and ϕ_b applying, respectively, Equations 14 or 15 to daily futures returns for the prior year (500 trading days). To provide a visual benchmark for the statistical significance of the changes depicted by the red and black curves, we generate two benchmarks, $\phi_{f,shuffle}$ and $\phi_{b,shuffle}$, by "shuffling" (i.e., using permutations of) the time index of each analyzed dataset. The resulting "shuffled" time series have the same statistical properties (mean, variance and higher moments) as the original ones but temporal relationships are removed. The resulting benchmarks (i.e., the "shuffled" information flows) are plotted in green (forward flow $\phi_{f,shuffle}$) and blue (backward flow $\phi_{b,shuffle}$). Unlike the actual forward and backward flows, the counterfactual flows fluctuate very little and are very close.

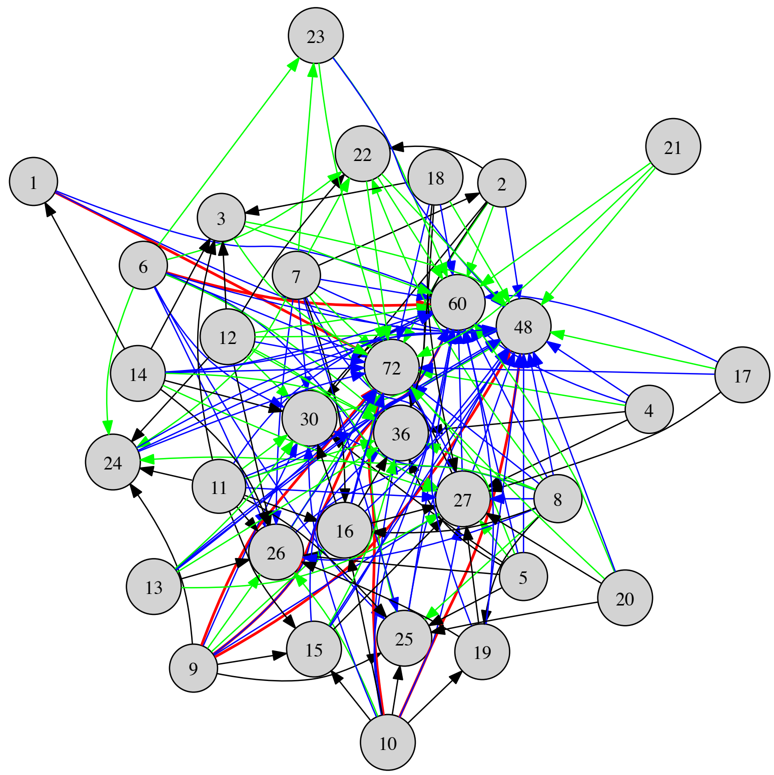

Figure 7: Pair-wise analysis - filtered directed graph, benchmark case, 2000-2014

Note: Figure 7 presents the filtered directed graph extracted from the directionality matrix $\bar{D}_{R_{\tau_i}R_{\tau_j}}$ computed in the static case (i.e., for the entire $2000-2014$ sample period). The graph's links are oriented according to $\bar{D}_{R_{\tau_i}R_{\tau_j}}$, which measures the strength of the net transfer entropy: for a couple of nodes (R_{τ_i}, R_{τ_j}), if the transfer entropy $T_{R_{\tau_i}\rightarrow R_{\tau_j}}$ is greater than $T_{R_{\tau_j}\rightarrow R_{\tau_i}}$, then the edge is oriented from maturity τ_i to maturity τ_j, otherwise from τ_j to τ_i. The graph is filtered: only directionality indexes higher that 0.3 are retained. Besides, the color of the links indicates intensity with colors ranging from black (directionality index between 0.3 and 0.4), to green (0.4 to 0.5), to blue (0.5 to 0.6) to red (0.6 to 0.7).

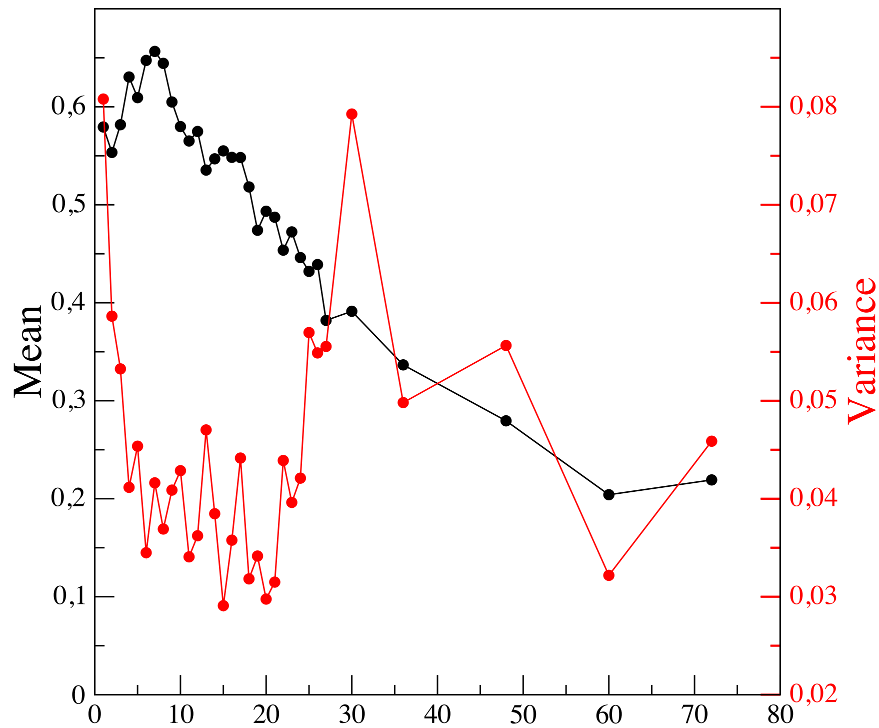

Figure 8: Mean and variance of outflows, static analysis

Note: Figure 8 summarizes our connectivity analysis. Whereas Figure 7 identifies, for each WTI futures contract maturity, which other WTI futures are associated with that maturity, Figure 8 simply plots the mean and variance of the percentage of outer links (i.e., the fraction of all contracts in the WTI term structure) associated with each contract maturity. Sample period: February 2000 to February 2014.

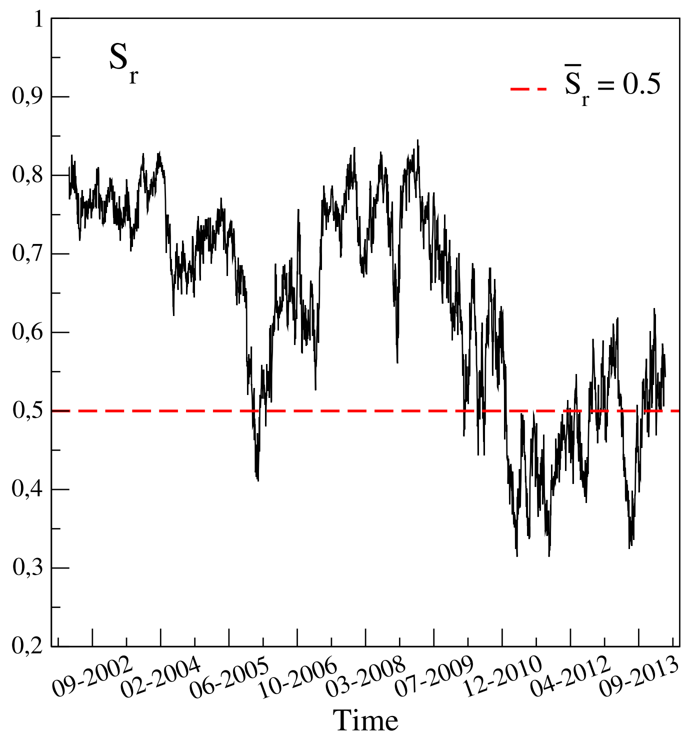

Figure 9: Survival ratios

Note: Figure 9 provides insight into the distance between the static directed graph, used as a reference, and daily directed graphs, by plotting the survival ratio $\bar{S}_R(t)$. We measure $\bar{S}_R(t)$ as the number of element of same sign in $\frac{1}{N} D_{R_{\tau_1} R_{\tau_2}}(t) \cap \bar{D}_{R_{\tau_1} R_{\tau_2}}$. On each day t from February 2001 to February 2014, the values are computed using daily returns from the prior year ($N = 500$ trading days). At one extreme, if $\bar{S}_R(t) = 1$, then the two graphs are identical, the pattern of shocks propagation has remained stable. At the other extreme, if $\bar{S}_R(t) = 0$, then the set of directed links has been completely rearranged.