Risk aversion, order placement strategies and price formation in a dynamic limit order book: the perspective of European carbon markets.

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Abstract

An order placement model is proposed to examine the nature of trading costs and of order flow in a dynamic limit order book where traders are risk averse. The adverse selection costs of uninformed sellers (*resp.* buyers) are found to be positively (*resp.* negatively) related to the arrival rates of market buy (*resp.* sell) orders. Next, the bid-ask spread is decomposed in three factors: the differences in risk-adjusted asset valuations, the two adverse selection costs of buyers and sellers if an equivalent arrival rate of buy and sell market orders is expected. Analyzing European carbon futures data confirms these two findings and precises that the arrival of same side market orders at the origin of a *diagonal effect* is due to adverse selection considerations at opening hours, then by the increasing influence of order splitting strategies.

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1. Introduction

If the question formulated in Glosten's (1994) celebrated paper: "Is the electronic order book inevitable?" sound provocative nearly a quarter of century ago, it now seems to be answered. Hence, more than half of the world's financial markets have adopted an electronic limit order book (LOB hereafter) to facilitate trading (Moinas, 2008; Gould et *al.*, 2013)[3].

A central feature of a LOB is the absence of market makers that provide continuous bid and ask quotations. Instead, traders can place, modify, and cancel a buy or sell limit order for which a price and quantity is specified. Alternatively, traders can submit a market order, without any price. A transaction occurs when an incoming order or an existing order's price is modified so that the order crosses the spread of best bid and best ask prices. The unexecuted orders form the consolidated source of liquidity and the bid ask spread serves as a proxy of trading costs[4].

With the widespread use of LOBs, the theoretical literature studying the determinants of order strategies and the nature of trading costs has witnessed a spurt in interest (Moinas, 2008). Glosten (1994) builds a static model where risk neutral traders place market orders when they are better informed on the payoff of the risky asset. His model allows to determine the bid-ask spread in equilibrium but imposes an exogenous order choice for all traders. Foucault (1999) proposes a dynamic model where traders' order strategies depend on their asset valuation and the best limit prices offered. Foucault (1999) demonstrates that if the value of the traded asset fluctuates after the disclosure of public information, a limit order may be executed in the case of adverse information. It is said to be picked off and the trading profit may become negative. Since higher volatility creates a higher picking-off risk, traders increase the reservation prices of limit orders widening the bid-ask spread. The order mix then shifts in favor of limit orders, but fewer are executed and market orders are more expensive. Liu (2009) extends the Foucault (1999)'s model allowing for order cancellations and revisions. He shows that if monitoring the information flow contained in the LOB is not too costly, uninformed traders actively revise their orders to reduce picking-off and non-execution risks. An alternative view of asymmetric information is espoused by Handa et *al.* (2003)[5]. They incorporate another type of risk borne by limit order traders: the adverse selection risk due to the presence of informed counterparty traders that place market orders to benefit from their short lived private information. Then, Handa et *al.* (2003) derive a bid-ask spread as a function of the differences in traders' valuations

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[3] According to Moinas (2008), most of financial markets are order driven markets and have mainly adopted LOB systems with sometimes distinct features with regard to priority of order execution, competition or transparency.

[4] For exchanges and regulators, larger bid-ask spreads are a signal of lower liquidity and greater inefficiencies. Brokers must offer a competitive bid-ask spread to their clients while traders track the evolution of quoted bid-ask spreads because they directly impact on the profitability of their limit order strategies.

[5] The existence of trading frictions makes the Foucault (1999)'s model resemble an asymmetric information model. Like Goettler et *al.* (2005), Foucault (1999) hypothesizes that the fundamental value of the risky asset varies according to a binomial tree with the same up and down probabilities. Afterwards, the order choice (limit or market order) and the trading directions (buy or sell side) both depend on the traders' private valuation.

and of adverse selection. Interestingly, the size of the spread is found to be greater in balanced markets than in unbalanced markets with unequal numbers of (high private value) buyers or (low private value) sellers. This result also predicted by Foucault (1999) is confirmed with data related to CAC40 stocks. Goettler et *al*. (2005) relax some assumptions made by Foucault (1999)[6] but retain the idea that both changes in traders' valuations and the consensus value on the traded asset determine order strategies[7]. Their results suggest that the *diagonal effect* namely the tendency that the same order types tend to follow each other (Biais et *al*., 1995[8]) is an equilibrium property of LOB due to the persistence of its informational state and the waiting time required by risk neutral traders to take advantage of stale limit orders.

At present, there is no reason to believe that all the above-mentioned results tested with stock markets data, remain valid in LOB markets where traders exhibit risk averse preferences. Departure from the usual assumption of risk neutrality is relatively rare in modelling order strategies. In a mean-variance setting, Kovaleva and Iori (2012) study the impact of a random delay in the limit order execution on the selling strategy of a risk-averse trader who cannot revise their orders. They find that the bid-ask spread increases as his risk aversion increases. On the empirical side, Marshall et *al.* (2011) document that the bid–ask spreads in commodity markets increase with volatility as Foucault (1999) predicts, but this relationship varies significantly across commodity families given the different levels of risk aversion[9].

The purpose of our study is to embed the models of Foucault (1999) and Handa et *al.* (2003) in a richer framework where risk averse uninformed traders monitor their LOB screens to extract public information before submitting orders. Our framework nests the approach of these two authors to model two types of information asymmetries. We interpret the picking-off risk in the sense of adverse execution due to the arrival of public information, consistent with Foucault (1999) and the adverse selection risk due to the inability of uninformed traders to discern if their counterparties hold private signals on the asset value as for Handa et *al*. (2003).

As a first step toward the study of trading costs, the order strategies of uninformed traders are analyzed in a fairly structured dynamic environment where they interact with informed and noise traders. Next, we propose a reduced form of our model in which uninformed traders homogeneously recognize a similar arrival rate of buy and sell market orders. In both models, uninformed traders interact with informed traders and noise traders. They are the only ones to

[6] Foucault (1999) derives closed form solutions at the cost of two restrictive assumptions. First, limit orders are valid for one trading period and an equal proportion of buyers and sellers is assumed for the general case.

[7] For Foucault (1999), the difference in asset valuations generate necessary gain opportunities for allowing trades whereas Handa et *al.* (2003) assumes that it is an outcome of taxes, liquidity shocks or portfolio considerations.

[8] Biais et *al.* (1995) provide evidence of the *diagonal effect* on Paris Bourse for CAC 40 stocks. They put forward three possible explanations: *(i)* traders split large orders, *(ii)* they follow what other traders are doing, *(iii)* they react similarly to the same events ("herding").

[9] Marshall et *al*. (2011) show that energy bid-ask spreads are less sensitive to return volatility even if energy returns are considerably more volatile than other commodities (e.g., agricultural and precious metals).

place either market orders or limit orders to trade a risky asset. An additional feature of our model is the incorporation of an endogenous noise affecting public information signals that uninformed traders extract after monitoring their LOB screens as in Bloomfield et *al.* (2009)[10]. By making variable the precision of these signals, we bridge the gap between two extreme cases: a perfectly known precision or a completely noisy precision. On the basis of these signals, uninformed traders conjecture the arrival rates of market orders and consider interdependence of buyers' and sellers' decisions to revise limit orders or place new orders. We solve the equilibrium of our model and obtain corresponding bid and ask prices by optimizing the uninformed traders' order strategies as Kovaleva and Iori (2012) do.

The main objective of our model is to provide a novel bid-ask spread decomposition[11] more adapted to commodity markets and for which the predictions of Foucault (1999) and Handa et *al.* (2003) may be tested. Important related questions that we address include the following: How do uninformed traders react when they capture changes in the arrival of market orders or a rise in the asset volatility? Why do adverse selection costs constitute the main components of the bid-ask spread as it is often reported (e.g., Marshall et *al*., 2011)? Do buyers' adverse selection costs differ from sellers' ones and vary according to the level of risk aversion? In which circumstances does the bid-ask spread achieve its maximum (minimum) size?

Concerning our objective to deliver valuable insights on the composition of the bid-ask spread traded in a LOB where traders are risk averse, we obtain the following theoretical results:

- First, we verify that the price improvement offered by limit orders serves to compensate uninformed traders for their risks of adverse selection and of being picked off.
- Second, the adverse selection risk is found to be dependent on the rate of market orders and on the degree of precision and concentration of public information flow.
- Third, the bid-ask spread is decomposed in three factors: differences between buyers' and sellers' reservation values, the two adverse selection costs of uninformed buyers and sellers if an equivalent arrival rate of buy and sell market orders is expected. In such context, we validate the Handa et *al.* (2003)'s prediction since the spread achieves a maximum (*resp.* minimum) at the most balance (*resp.* imbalance) value of market competition measure.
- Fourth, a numerical analysis of our model reveals that adverse selection costs of uninformed buyers (*resp.* sellers) are positively related to the degree of their risk aversion and the volatility of reservation values but not in a linear form as in Foucault (1999).

[10] Experimental results of Bloomfield et *al.* (2009) suggest that the order strategies of noise traders allow uninformed traders to reduce their adverse selection risks so that the bid-ask spread should decrease.

[11] If spread estimators using transaction data (e.g., Madhavan et *al*., 1997) perform poorly to estimate adverse selection costs, assessing their performance is made more difficult for commodities given that spreads are often unobservable. In addition, Van Ness et *al*. (2001) point out that these estimators assume unit quantity and equally-spaced trades. Therefore, the asymmetric information (*i.e.* adverse selection) components of the bid-ask spread are often overestimated, which is confirmed by Kalaitzoglou and Ibrahim (2016) for European carbon futures.

We further exploit order and trade data of ECX that concentrates 90% of European carbon futures trading (Mizrach and Otsubo, 2014) to test the four implications of our model. The so-called EU Emission Allowances (EUA) futures market is viewed as an emerging commodity market where the levels of risk aversion (Chevallier, 2012), of information asymmetry and of futures volatility are significantly high (Kalaitzoglou and Ibrahim, 2016)[12].

Overall, our empirical tests confirm the merits of our model and contribute to the market microstructure literature on carbon markets in three ways. We find that the buyers and sellers' adverse selection costs represent on average 70% of the bid-ask spread but evolve in line with seasonal variations in the level of information asymmetry as reported by Medina et *al.* (2014) and Mizrach and Otsubo (2014). Moreover, the bid–ask spread and adverse selection costs of uninformed sellers (*resp.* buyers) follow a U-shaped (*resp.* inverted U-shaped) pattern while the bid-ask spread component due to traders' beliefs heterogeneity is rather constant along the trading session. Finally, we detect that the *diagonal effect* is significant in the European carbon market and is an LOB equilibrium property (Goettler et *al*., 2005). Interestingly, this market feature is explained by adverse selection considerations at the opening hours and then by the increasing influence of order splitting strategies[13] along a quicker liquidity replenishment.

The rest of the paper is organized as follows. The next section sets out the model and describes the order strategies of uninformed traders. Section 2 analyzes the equilibrium of our model and its implications in terms of price formation and bid-ask spread. In Section 3, we perform a numerical analysis of our model that delivers further implications. Section 4 presents empirical tests and our findings using European carbon (EUA) futures data. Our conclusions and an outlook for further research appear in Section 5. All proofs are gathered in the appendix.

2. A new order placement model set in a CARA-normal framework

We first focus on the four features of our order placement model set in a CARA-normal framework before describing the optimal order strategies that uninformed traders can follow.

2.1. Characteristics of the model

(1) Market structure. We consider the following competitive model of asset trading. Traders have access to two assets[14]: a single risky asset x with stochastic terminal value X_T, liquidated

[12] The EU Emissions Trading Scheme (EU ETS) created a new sort of commodity: a EUA (or quota) freely allocated that needed to be surrendered for a tonne of CO_2 emitted by one of the 13 000 installations covered.

[13] Biais et *al.* (1995) put forward three explanations for the diagonal effect: (*i*) traders split large orders, (*ii*) traders follow what other traders are doing, (*iii*) traders react similarly to the same events ("herding").

[14] The choice of a portfolio including a bond is motivated by two considerations. First, in times of higher uncertainty, traders should rebalance their portfolios toward less risky assets (e.g., bonds) as their risk aversion increases. Second, Chevallier (2012) found a 3% return with a stand deviation (risk) ≤ 0.06 for a portfolio including energy (EUA, oil, gas, coal) futures, weather, ECB 5-year benchmark bond, equities, T-bills. He concludes that carbon, gas, coal and bond assets share the best properties to form an optimal portfolio.

within a pre-specified time horizon T and a riskless bond with perfectly elastic supply paying out a certain payoff $R_T \geq 1$ at time T. Only the risky asset is traded over a span of t trading times with one unit limit or one unit market order in a pure LOB trading system.

The market operates in discrete time, each time step being characterized by new submission of orders or limit order revisions. The usual price/time priority for order execution applies[15]. If the order does not result in a trade, it is added to the LOB as a limit order. Trade is executed at the value of the best price *i.e.* the highest priced bid (*resp.* lowest priced ask) quote.

(2) Market participants. Three groups of market participants trade the risky asset which are:

- *Informed traders* (I) are rational agents who only use market orders[16] to benefit from their short lived private information about the risky asset fundamental value (Handa et *al*., 2003).
- *Noise/sentiment traders* (N) can either behave as irrational traders who act as if they have private information, or follow feedback speculative strategies (De Long et *al*., 1990; Bloomfield et *al*., 2009). They use markets orders to obtain immediate order execution.
- *Uninformed traders* (U) are rational agents without private information that trade by means of limit and market orders to maximize their terminal wealth. Two profiles of uninformed traders with different beliefs on the value of the risky asset are considered. For the buyer (*resp*. seller) group, $\mu_{b,x}$ (*resp*. $\mu_{s,x}$) is perceived as the reservation price of variance $\sigma^2_{b,x}$ (*resp*. σ^2_{sx}) for the true value of the risky asset, for which they are likely to buy (*resp*. sell) one unit of the risky asset. When an uninformed buyer (*resp*. seller) trades for a share of the risky asset at a specified price P_{bid} (*resp.* P_{ask}), he expects a terminal wealth ω_T is a random variable given by $W_T = X_{b,t} + (W_1 - P_{bid}) \times R_T$ (*resp.* $\omega_T = (W_1 + P_{ask}) \times R_T - X_{s,t}$.

All of these traders are risk averse. For instance, uninformed traders are supposed to formulate their order strategies to maximize the expectation of utility of W_T: $E(u(W_T)) = - \exp(\varphi(W))$. This utility takes the form of a negative exponential function that depends on a constant absolute risk aversion (CARA) parameter φ such as in the framework of Kovaleva and Iori (2012).

(3) Endogenous noise affecting public information. We conform to the method of Berkman and Koch (2008) to estimate an endogenous noise affecting public information. Since the activity of noise traders is not directly observable, we consider the daily net initiated order flow by B brokers through which they trade on t denoted by random variables $\{n_t; b=1,\ldots, B\}$ that are *iid* distributed with mean zero and variance σ_t^2. If each broker has an equal market share *i.e.*

[15] In a LOB, a limit order is executed given time and price priority rules. Those posted earlier are further ahead in the queue (time priority) and are executed if no other orders have price priority and an incoming trader is willing to be a counterparty. A marketable limit order above the ask executes at the ask, is considered as a market order.

[16] It is a classical assumption made by the theoretical literature related to order submission strategies. For instance, Handa et *al*. (2003) assume that informed traders only submit market orders because of short-lived private information. In equilibrium of his model, Rosu (2009) finds that the patient informed trader who have long-lived private information also post market orders when prices deviate far from the fundamental asset value.

N/B noise traders trade through each broker, the $\{n_t\}$ variables are aggregated measures of net initiated order flow (OF) across B groups of N/B noise traders which are therefore *iid* distributed with mean zero and variance $(N/B)\times\sigma_t^2$. In either case, the net initiated order flow per broker: **OF/B** becomes a consistent estimate of an endogenous noise proportional to the net initiated order flow across noise traders provided that there are enough active brokers.[17]

(4) Public information and order strategy. By monitoring the information flow through their LOB screens, uninformed traders become aware of new public information (that arrives randomly) and may learn news before others. Here, the informational state of the LOB contains real-time information signals that are publicly visible. Further, uninformed traders are supposed to extract and interpret these signals affected by endogenous noise to place new orders or revise ones. As a result, their order strategies depend on their posterior beliefs about the fundamental value of the risky asset and the precision of the new (noisy) public information received.

We denote L(Θ) the information set of the LOB state that involves real time and public information. Its interpretation differs from uninformed buyers and sellers (Liu, 2009). In particular, we assume that the uninformed buyers (*resp.* sellers) extract the noisy signal $\tilde{Z}_{b,1}$ (*resp.* $Z_{s,1}$) from the same information set L(Θ) perceived from the initial time of trading t =1.

Let us consider the case in which uninformed buyers are seeking to predict the fundamental value of the risky asset $X_{b,T}$ after interpreting a noisy signal $\tilde{Z}_{b,1}$, which verifies $\tilde{Z}_{b,1} = X_{b,T} + \varepsilon_{b,1}$ where $\varepsilon_{b,1}$ is an idiosyncratic shock, independent of $X_{b,T}$, with mean 0 and variance $\frac{OF}{B}\cdot\sigma^2_{b,\varepsilon,1}$. Assuming that $X_{b,T}$ and $\tilde{Z}_{b,1}$ have a joint normal distribution, we exploit the Projection theorem to update their conjectures on the buying reservation prices[18] as follows:

$$\begin{pmatrix} X_{b,T} \\ \tilde{Z}_{b,1} \end{pmatrix} \sim N_1\left[\begin{pmatrix} \mu_{b,x} \\ \mu_{b,x} \end{pmatrix}, \begin{pmatrix} \sigma^2_{b,x} & \sigma^2_{b,x} \\ \sigma^2_{b,x} & \sigma^2_{b,x} + \frac{OF}{B}\cdot\sigma^2_{b,\varepsilon,1} \end{pmatrix}\right] \text{ at the first time of trading t= 1;}$$

$$\begin{pmatrix} X_{b,T} \\ \tilde{Z}_{b,1} \end{pmatrix} \sim N_t\left[\begin{pmatrix} \mu_{b,x} \\ \mu_{b,x} \end{pmatrix}, \begin{pmatrix} t\sigma^2_{b,x} & t\sigma^2_{b,x} \\ t\sigma^2_{b,x} & t\sigma^2_{b,x} + t\frac{OF}{B}\cdot\sigma^2_{b,\varepsilon,1} \end{pmatrix}\right] \text{ at the time of trading t ;}$$

[17] Berkman and Koch (2008) confirm the merits of their proxy that we have denoted **OF/B** by showing that their daily variations are positively (*resp.* negatively) correlated with the arrival rate of uninformed traders, the trading volume and the market depth (*resp*. the bid-ask spread and the probability of informed trading).

[18] One reason to use normal distributions is this popularity amongst the order placement models and its properties: expectation, variance, projection theorem of joint normal distributions and integral calculus. Another reason is that coupling exponential utility functions with normal distribution allows to control the effects of informed trading and risk aversion on prices. A first effect leads to a reduction of price information disclosure whereas the second effect contribute to increase it by diminishing the impact of noise trading. With normal distributions and exponential utility, Biais and Foucault (1993) find that these two effects are exactly offset.

Where: $\rho^2_{b,x} = \dfrac{\sigma^2_{b,x}}{\sigma^2_{b,x} + \frac{OF}{B}\cdot\sigma^2_{b,\varepsilon,1}}$ is the degree of uninformed buyers' projection of fundamental value $X_{b,T}$ onto the noisy signal $\tilde{Z}_{b,1}$extracted from the LOB informational state L(Θ).

Likewise, uninformed sellers are assumed to homogeneously observe a noisy signal after monitoring the LOB informational set. Based on this signal, they forecast the fundamental value of the risky asset $X_{s,T}$ after interpreting a noisy signal $\tilde{Z}_{b,1}$, which corresponds to $\tilde{Z}_{s,1} = X_{s,T} + \varepsilon_{s,1}$ where $\varepsilon_{s,1}$ is an idiosyncratic shock, independent of $X_{s,T}$ with mean 0 and variance $\frac{OF}{B}\cdot\sigma^2_{s,\varepsilon,1}$. Assuming that $X_{s,t}$ and $\tilde{Z}_{s,1}$ are jointly normal distributed, their projections are as follows:

$$\begin{pmatrix} X_{s,T} \\ \tilde{Z}_{s,1} \end{pmatrix} \sim N_1\left[\begin{pmatrix} \mu_{s,x} \\ \mu_{s,x} \end{pmatrix}, \begin{pmatrix} \sigma^2_{s,x} & \sigma^2_{s,x} \\ \sigma^2_{s,x} & \sigma^2_{s,x} + \frac{OF}{B}\sigma^2_{s,\varepsilon,1} \end{pmatrix}\right] \text{ at the first time of trading t= 1;}$$

$$\begin{pmatrix} X_{s,T} \\ \tilde{Z}_{s,1} \end{pmatrix} \sim N_t\left[\begin{pmatrix} \mu^s_x \\ \mu^s_x \end{pmatrix}, \begin{pmatrix} t\sigma^2_{s,x} & t\sigma^2_{s,x} \\ t\sigma^2_{s,x} & t\sigma^2_{s,x} + t\frac{OF}{B}\cdot\sigma^2_{s,\varepsilon,1} \end{pmatrix}\right] \text{ at the time of trading t;}$$

Where: $\rho^2_{s,x} = \dfrac{\sigma^2_{s,x}}{\sigma^2_{s,x} + \frac{OF}{B}\cdot\sigma^2_{s,\varepsilon,1}}$ is the degree of uninformed sellers' projection of fundamental value $X_{s,T}$ onto the noisy signal $\tilde{Z}_{s,1}$extracted from the LOB informational state L(Θ).

We introduce a specific public information structure for two distinct reasons. First, prices and order flow of informed traders aggregate dispersed private information and could provide further information about the fundamental value of the risky asset. Second, the net order flow of informed traders initiated by their brokers give some indications about the realised amount of noise trading which can allow uninformed traders to partly hedge noise trader risk (to which market orders is exposed). For $\rho^2_b = \rho^2_s$ = [0;1), these two informational roles of price and order flow are present. Conversely, the only visible signal is this held by informed traders when $\rho^2_b = \rho^2_s = 1$ (perfect correlation). Besides, as ρ^2_b and ρ^2_s go down, the noise components of prices and (order book) public information become increasingly uncorrelated across traders.

The above-mentioned model features **(1); (2); (3); (4)** have important consequences on the order strategies of risk averse uninformed traders that we describe in the next paragraph.

2.2. The uninformed trader's limit order strategies

Order placement strategies is a dynamic process that involves not only the submission of market and limit orders but also the revision of limit orders. In our setting, uninformed traders are the only limit order submitters. We make the classical assumption that a limit order expires at the end of the trading session if it is not cancelled before (e.g., Handa et *al*., 2003).

Uninformed traders can trade with different counterparty traders without restriction: informed traders who arrive with a probability p_I, noise traders with a probability p_N and uninformed traders with a probability p_U. In addition, we suppose that uninformed buyers (*resp.* sellers) homogeneously recognize the arrival rate of market sell (*resp.* buy) orders: k_s^M (*resp.* k_b^M) which are submitted to urgently sell (*resp.* buy) orders. From Fig. 1, we observe that a limit buy order may be: **(1)** executed against informed sellers (C_I), **(2)** executed against noisy sellers (C_N), **(3)** executed against uninformed sellers (C_U), or **(4)** unexecuted (C_{RE}). A market buy order is immediately executed with an uninformed limit sell order. The probability to face informed sellers' counterpart is $p_I \times k_s^M$ while $P_N \times k_s^M$, $p_U \times k_s^M$, and $(1- k_s^M)$ are the probability to face noise sellers, aggressive uninformed sellers and patient uninformed sellers (Glosten, 1994) respectively. In addition, Fig. 1 displays all possible configurations for which uninformed sellers can execute a limit sell order or alternatively a market sell order. Accordingly, their probability to be confronted to informed buyers is $p_I \times k_b^M$ while $P_N \times k_b^M$, $p_U \times k_b^M$, $1- k_b^M$ are the probability to be confronted to noise buyers, aggressive uninformed buyers and patient uninformed buyers respectively (Glosten, 1994).

<Fig. 1 is inserted about here>

Henceforth, we are able to determine four specific profit functions for an uninformed trader submitting a limit order and executing (or not) it at time t given the profile of counterpart trader met (C_I, C_N, C_U,C_{RE}). For the four scenarios presented below, the subscript b and s indicate the buy and sell sides respectively, L denotes an uninformed trader's limit order strategy, E (W_T) stands for his expected terminal wealth at the end of the trading session T.

(1) Informed traders at the opposite side. For Glosten (1994) and Handa et *al.* (2003), limit order traders are confronted to an adverse selection problem when they meet informed counterparty traders. Both Foucault (1999) and Liu (2009) consider that uninformed traders who place a buy (*resp.* sell) limit order write a free put (*resp.* call) option of the execution price as the best bid price P_{bid} (*resp.* the best ask price P_{ask}) to informed traders. For Foucault (1999), this option may be exercised when volatility is higher, triggering a potential limit order harmful execution. By placing a limit buy (*resp.* sell) order conditioning on informed trades (C_I) given the implied probability density function $f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1}, C_I\right)$(*resp.* $f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1}, C_I\right)$), an uninformed trader with a terminal wealth W_T expects the following utility:

- <u>Buy side:</u> $E^L_{b,t}\left[u(W_T)\middle|\tilde{Z}_{b,1}, C_I\right] = \int_{\infty}^{R_T P_{bid}} \left[1 - \exp\left[-\phi\left(X_{b,t} + W_1 - P_{bid}\right)\right]\right] f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1}, C_I\right) dX_{b,T}$
- <u>Sell side:</u> $E^L_{s,t}\left[u(W_T)\middle|\tilde{Z}_{s,1}, C_I\right] = \int_{R_T P_{ask}}^{\infty} \left[1 - \exp\left[-\phi\left(-X_{s,t} + W_1 + P_{ask}\right)\right]\right] f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1}, C_I\right) dX_{s,T}$

(2) Noise traders (N) at the opposite side. Market microstructure literature often postulates that noise traders do not hold private information. They can use market orders to satisfy immediate liquidity or portfolio rebalancing needs. Also, their speculative feedback strategies explain their aggressive trading (de Long et *al*., 1990). Theoretically speaking, noise sellers (*resp.* buyers) trade the altruistic price P_{bid} (*resp.* P_{ask}) with limit order traders to get immediate order execution. Accordingly, the actions of noise traders beget a bid-ask spread reduction, allowing uninformed traders to reduce their adverse selection losses but hinder the adjustment of prices to the fundamental asset value if the market is less efficient (Bloomfield et *al*., 2009). Then, execution of limit orders become uncertain because the fundamental asset value is more volatile or the picking-off risk may increase (Foucault, 1999) if adverse public information arrives. By posting a limit buy (*resp.* sell) order conditioning on trading with a noise trader counterpart (C_N) given the implied probability density function $f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1},C_N\right)$(*resp.* $f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1},C_N\right)$, an uninformed trader with a terminal wealth W_T expects the following utility:

- Buy side:

$$\begin{aligned}E^L_{b,t}\left[u(W_T\middle|\tilde{Z}_{b,1},C_N\right] &= \int_{-\infty}^{P_{bid}R_T}\left[1-\exp\left[-\phi\left(X_{b,t}+R_T(W_1-P_{bid})\right)\right]\right]f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1},C_N\right)dX_{b,T}\\ &+\int_{P_{bid}R_T}^{\infty}\left[1-\exp\left[-\phi\left(X_{b,t}+R_T(W_1-P_{bid})\right)\right]\right]f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1},C_N\right)dX_{b,T}\\ &=\int_{-\infty}^{\infty}\left[1-\exp\left[-\phi\left(X_{b,t}+R_T(W_1-P_{bid})\right)\right]\right]f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1},C_N\right)dX_{b,T}\end{aligned}$$

- Sell side:

$$\begin{aligned}E^L_{s,t}\left[u(W_T)\tilde{Z}_{s,1},C_N\right] &= \int_{-\infty}^{P_{ask}R_T}\left[1-\exp\left[-\phi\left(-X_{s,t}+R_T(W_1+P_{ask})\right)\right]\right]f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1},C_N\right)dX_{s,T}\\ &+\int_{P_{ask}R_T}^{\infty}\left[1-\exp\left[-\phi\left(-X_{s,t}+R_T(W_1+P_{ask})\right)\right]\right]f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1},C_N\right)dX_{s,T}\\ &=\int_{-\infty}^{\infty}\left[1-\exp\left[-\phi\left(-X_{s,t}+R_T(W_1+P_{ask})\right)\right]\right]f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1},C_N\right)dX_{s,T}\end{aligned}$$

Notice that the left hand side term of the above equations represents the price improvement while the right hand side term accounts for the picking-off risk of a limit order.

(3) Uninformed traders at the opposite side. Unlike Foucault (1999) and Handa et *al. (*2003), we also consider the case for which an uninformed market order trader is the counterpart. He is supposed to behave like a liquidity trader (Goettler et *al*., 2005; Bloomfield et *al*., 2009) in submitting market orders to gain immediacy or because they estimate higher waiting costs due to existing aggressive price limit orders in the same side (Rosu, 2009). Given the arrival of an uninformed market order counterparty trader, an uninformed order trader can realize trading gains from price improvement but also can face risk of being picked off as is the case previously with noise traders. By submitting a limit buy (sell) order conditioning on uninformed trade (C_U) given the implied probability density function $f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1},C_U\right)$ (*resp.* $f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1},C_U\right)$), an uninformed trader with a terminal wealth W_T expects the following utility:

- Buy side: $E^L_{b,t}\left[u(W_T)\middle|\tilde{Z}_{b,1},C_U\right]=\int_{-\infty}^{\infty}\left[1-\exp\left[-\phi\left(X_{b,t}+R_T(W_1-P_{bid})\right)\right]\right]f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1},C_U\right)dX_{b,T}$
- Sell side: $E^L_{s,t}\left[u(W_T)\middle|\tilde{Z}_{s,1},C_U\right]=\int_{-\infty}^{\infty}\left[1-\exp\left[-\phi\left(-X_{s,t}+R_T(W_1+P_{ask})\right)\right]\right]f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1},C_U\right)dX_{s,T}$

(4) Limit orders of the opposite side. In the opposite LOB side, there are several reasons for an absence of incoming market orders. First, as informed traders obtain new information which is desirable, they can submit same side market orders. Second, the opposite side noise traders may have no incentives demand liquidity in submitting place market orders. Third, uninformed traders of the opposite side may decide to place new limit order or undercut existing ones. In these three possible configurations, any limit order trader will lose the opportunity of price improvement and run the risks of uncertain limit order execution. To this respect, we assume that an uninformed trader is rational in a sense that if he expects that a limit order cannot be executed before the end of period T, he will prefer to invest in the bond from which he earns a fixed payoff R_T (Liu, 2009). By placing a limit buy (*resp.* sell) order conditioning on the risk of non-execution (C_{NE}) given the implied probability density function $f_{b,t}\left(X_{b,T} \middle| \tilde{Z}_{b,1}, C_{NE}\right)$(*resp.* $f_{s,t}\left(X_{s,T} \middle| \tilde{Z}_{s,1}, C_{NE}\right)$), an uninformed trader with a terminal wealth W_T has the expected utility:

- Buy side: $E^L_{b,t}\left[u(W_T) \middle| \tilde{Z}_{b,1}, C_{NE}\right] = \int_{-\infty}^{\infty} 1 - \exp[-\phi(-R_T(W_1))] f_{b,t}\left(X_{b,T} \middle| \tilde{Z}_{b,1}, C_{NE}\right) dX_{b,T}$
- Sell side: $E^L_{s,t}\left[u(W_T) \middle| \tilde{Z}_{s,1}, C_{NE}\right] = \int_{-\infty}^{\infty} [1 - \exp[-\phi(-R_T(W_1))]] f_{s,t}\left(X_{s,T} \middle| \tilde{Z}_{s,1}, C_{NE}\right) dX_{s,T}$

2.3. The uninformed trader's market order strategies

The uninformed buyer who has a higher valuation for the risky asset faces a decision tree displayed in the upper panel of Fig. 1. If the competition for order execution makes their limit order strategy unprofitable given insignificant non-execution risks and waiting costs (Rosu, 2009), uninformed buyers can instead submit a market order strategy denoted M. By placing a market buy (sell) order at time 1, given the implied probability density function $f_{b,t}\left(X_{b,T} \middle| \tilde{Z}_{b,1}\right)$ (*resp.* $f_{s,t}\left(X_{s,T} \middle| \tilde{Z}_{s,1}\right)$), an uninformed trader with a terminal wealth W_T has the expected utility:

- Buy side: $E^M_{b,1}\left[u(W_T) \middle| \tilde{Z}_{b,1}\right] = \int_{-\infty}^{\infty} \left[1 - \exp\left[-\phi\left(X_{b,t} + R_T(W_1 - P_{ask})\right)\right]\right] f_{b,1}\left(X_{b,T} \middle| \tilde{Z}_{b,1}\right) dX_{b,T}$
- Sell side: $E^M_{s,1}\left[u(W_T) \middle| \tilde{Z}_{s,1}\right] = \int_{-\infty}^{\infty} \left[1 - \exp\left[-\phi\left(R_T(W_1 + P_{bid}) - X_{s,t}\right)\right]\right] f_{s,1}\left(X_{s,T} \middle| \tilde{Z}_{b,1}\right) dX_{s,T}$

3. The equilibrium of the model and optimal prices

3.1. The equilibrium of our model and the optimal bid and ask prices

We infer the equilibrium of our model to make optimal the uninformed trader's order strategy who faces uncertain market conditions: absence of knowledge upon the precision of (noisy) public information and no identification of counterparty traders among others. In previous sections, we have seen that the order strategy of uninformed traders involves two steps:

- either submit a buy (*resp.* sell) limit order or a buy (*resp.* sell) market order ;

- if a limit order strategy is followed, determine or revise the bid or ask price at which the order is posted. We begin by transposing the optimal order strategy of an uninformed buyer.

After normalizing the payoff of the uninformed trader to zero if the limit order expires in the case that it is not executed, we write his expected utility for an order strategy combining a limit buy order placed at a price P_{bid} and a buy market order executed at P_{ask} as follows:

$$\begin{aligned}
E_{b,t}[u(W_T)] &= \int_{-\infty}^{\infty}\left[1-\exp\left[-\phi\left(X_{b,t}+R_T(W_1-P_{ask})\right)\right]\right] f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1}\right) dX_{b,T} \\
&= k_s^M . p_I \int_{\infty}^{P_{bid}}\left[1-\exp\left[-\phi\left(X_{b,t}+R_T(W_1-P_{bid})\right)\right]\right] f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1},C_I\right) dX_{b,T} \\
&+ k_s^M .(p_N+p_U)\int_{-\infty}^{\infty}\left[1-\exp\left[-\phi\left(X_{b,t}+R_T(W_1-P_{bid})\right)\right]\right] f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1},C_\theta\right) dX_{b,T} \\
&+ (1-k_s^M)\int_{-\infty}^{\infty}\left[1-\exp\left[-\phi\left(W_1R_T\right)\right]\right] f_{b,t}\left(X_{b,T}\middle|\tilde{Z}_{b,1},C_{NE}\right) dX_{b,T}
\end{aligned} \qquad (1)$$

With: $\theta \in \{N, U\}$ and the respective probabilities which both verify $p_I + p_N + p_U = 1$.

The uninformed trader observes the LOB informational state before placing (or revising) his buy order at time t. For the sake of tractability, the probability density functions conditioning on each of his counterparty trader are assumed identical. Using a Taylor expansion, we obtain the linear buy side equilibrium figured out in Eq. (2) where its left hand side represents the expected price improvement of a limit order over a market order while the right hand side aggregates the expected risks of adverse selection, of picking off and of non-execution. **Proposition 1.1.** summarizes the implications in terms of price formation.

Proposition 1.1. The uninformed buyer aims at producing a sufficient price improvement to cover the adverse selection costs due to the presence of informed traders and minimize the risks of non-execution and of picking-off due to the arrival of (noisy) public information signals.

$$\underbrace{k_s^M(P_{ask}-P_{bid})}_{\text{Price improvement}} = \frac{1}{R_T}\left[\underbrace{k_s^M \times LOSS_b^{AS,RA}}_{\substack{\text{Adverse selection risk}\\ \text{due to private information}}} + \underbrace{k_s^M \times \phi\sqrt{\frac{B}{OF}}(t-1)\cdot\left(1-\rho_b^2\right)\cdot\sigma_{b,x}^2}_{\text{Picking-off risk due to adverse public information}} + \underbrace{\left(1-k_s^M\right)\times\left(\overline{F}_b - P_{ask}R_T\right)}_{\text{Non-execution risk}}\right] \qquad (2)$$

With the following conditions : $t > 1$ and $0 \le \rho_b^2 \le 1$

- $\rho_b = \dfrac{\sigma_{b,x}^2}{\sigma_{b,x}^2 + \frac{OF}{B}\cdot\sigma_{b,x}^2}$ is the degree of an uninformed buyer's projection of fundamental asset value;
- $LOSS_b^{AS,RA} = \dfrac{1}{\phi}\times p_I \times\left(1-N(\tilde{V}_b)-(1-N(\tilde{Y}_b)\right)\times\left[1-\exp\left(-\phi(R_T(W_1-P_{bid})+\overline{F}_b\right)\right]$ is the adverse selection loss of risk averse buyer. It is a nonlinear-implicit function of: the arrival rate of market sell orders, his probability to trade with an informed trader (P_I), his revised fundamental value of the risky asset: $\overline{F}_b = \rho_b^2\tilde{Z}_{b,1} + (1-\rho_b^2)\cdot\left(\mu_{b,x} + \frac{1}{2}\phi\sigma_{b,x}^2\right)$ and the following two random variables:

$$\tilde{V}_b = \frac{\left(P_{bid}R_T - \rho_b^2\tilde{Z}_{b,1} + (1-\rho_b^2)\cdot\mu_{b,x}\right)}{\sqrt{\frac{OF}{B}t\cdot\sigma_{b,x}^2}} \quad \text{and} \quad \tilde{Y}_b = \frac{\left(P_{bid}R_T - \rho_b^2 Z_{b,1} - (1-\rho_b^2)\cdot\mu_{b,x} + \phi(1-\rho_b^2)\frac{OF}{B}t\cdot\sigma_{b,x}^2\right)}{\sqrt{\frac{OF}{B}t\cdot\sigma_{b,x}^2\left(1-\rho_b^2\right)}}$$

Appendix A.1 shows the proof.

□

Similarly, the uninformed seller observes the informational state of LOB at time t-1 to submit (or revise) his limit sell order accordingly at time t. He is indifferent between placing a limit order or a market order if his expected utility from executing a market order at the bid price P_{bid} equals that from trading with a limit order submitted at the ask price P_{ask}:

$$\begin{aligned} E_{s,t}[u(W_T)] &= \int_{-\infty}^{\infty}\left[1-\exp\left[-\phi\left(R_T(W_1+P_{bid})-X_{s,t}\right)\right]\right]f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1}\right)dX_{s,T} \\ &= k_b^M.p_I\int_{P_{ask}}^{\infty}\left[1-\exp\left[-\phi\left(R_T(W_1+P_{ask})-X_{s,t}\right)\right]\right]f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1},C_I\right)dX_{s,T} \\ &+ k_b^M.(p_N+p_U)\int_{-\infty}^{\infty}\left[1-\exp\left[-\phi\left(R_T(W_1+P_{ask})-X_{s,t}\right)\right]\right]f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1},C_\theta\right)dX_{s,T} \\ &+ (1-k_b^M)\int_{-\infty}^{\infty}\left[1-\exp\left[-\phi\left(W_1R_T(\omega_1)\right)\right]\right]f_{s,t}\left(X_{s,T}\middle|\tilde{Z}_{s,1},C_{NE}\right)dX_{s,T} \end{aligned} \quad (3)$$

With $\theta\in\{N,U\}$ and the respective probabilities verify $p_I+p_N+p_U=1$.

We now derive the linear equilibrium for the sell side as made for the buy side. The left hand side of Eq. (4) represents the price improvement expected from a limit order, while the right hand side encapsulates the expected risks of adverse selection loss, of picking-off and of non-execution. **Proposition 1.2.** summarizes the implications in terms of price formation.

Proposition 1.2. The uninformed seller aims at producing a sufficient price improvement to cover the adverse selection costs due to the presence of informed traders and minimize the risks of non-execution and of picking-off due to the arrival of (noisy) public information signals.

$$\underbrace{(1-k_b^M)(P_{ask}-P_{bid})}_{\text{Price improvement}} = \frac{1}{R_T}\left[\underbrace{k_b^M\times LOSS_s^{AS,RA}}_{\substack{\text{Adverse selection risk} \\ \text{due to private information}}} + \underbrace{k_b^M\times\varphi\sqrt{\frac{B}{OF}}\cdot(t-1)\cdot\left(1-\rho_s^2\right)\sigma_{s,x}^2}_{\text{Picking-off risk due to adverse public information}} + \underbrace{\left(1-k_b^M\right)\times\left(P_{bid}R_T-\overline{F}_S\right)}_{\text{Non-execution risk}}\right] \quad (4)$$

$$\text{With the following conditions}: t>1 \text{ and } 0\le\rho_s^2\le 1$$

- $\rho_s=\dfrac{\sigma_{s,x}^2}{\sigma_{s,x}^2+\dfrac{OF}{B}\cdot\sigma_{s,x}^2}$ is the degree of an uninformed seller's projection of fundamental asset value;
- $LOSS_s^{AS,RA}=\dfrac{1}{\phi}\times p_I\times\left(1-N(\tilde{V}_s)-(1-N(\tilde{Y}_s)\right)\times\left[1-\exp\left(-\phi(R_T(W_1+P_{ask})+\overline{F}_s\right)\right]$ is the adverse selection loss of a risk averse seller. It is a nonlinear-implicit function of: the arrival rate of market buy orders, his probability to trade with an informed trader (P_I), his revised fundamental value of the risky asset: $\overline{F}_s=\rho_s^2\tilde{Z}_{s,1}+(1-\rho_s^2)\cdot\left(\mu_{s,x}+\dfrac{1}{2}\phi\sigma_{s,x}^2\right)$ and the following two random variables:

$$\tilde{V}_s=\frac{\left(P_{ask}R_T-\rho_s^2\tilde{Z}_{s,1}+(1-\rho_s^2)\mu_{s,x}\right)}{\sqrt{\frac{OF}{B}t\sigma_{s,x}^2}} \quad\text{and}\quad \tilde{Y}_s=\frac{\left(P_{ask}R_T-\rho_s^2 Z_{s,1}-(1-\rho_s^2)\cdot\mu_{s,x}+\phi(1-\rho_s^2)\frac{OF}{B}t\cdot\sigma_{s,x}^2\right)}{\sqrt{\frac{OF}{B}t\sigma_{s,x}^2\left(1-\rho_s^2\right)}}$$

Appendix A.1 shows the proof.

□

We are now able to conclude on several points. First, the optimal limit prices should generate enough profits to cover the direct costs implied by adverse selection (Handa et *al.*, 2003) and picking-off risks (Foucault, 1999). We find that the picking-off risk is positively related to the volatility of the risky asset in line with the prediction made by Foucault (1999). Second, the uninformed trader adjusts his buy and sell reservation asset values given the level of volatility, his coefficient of risk aversion, and his public (noisy) information signals received. Third, a lower execution risk at each price under the best quote induce less incoming aggressive counterpart orders and higher waiting costs for execution (Rosu, 2009). Finally, in a context of imperfect information and market uncertainty, a fully revealing equilibrium in which informed traders' signals are fully revealed into prices is unlikely (Foster and Viswanathan, 2016)[19].

Instead, **Proposition 2** focuses on a signal-revealing, complete equilibrium solved from the partial buy and sell sides' equilibrium strategies that are determined in Eqs. (2) and (4).

Proposition 2: There is a unique and signal revealing equilibrium of price quotation for a given trading time t which involves the following optimal bid (P_{bid}) and ask (P_{ask}) prices:

$$\begin{aligned} P_{bid} = {} & \frac{1}{R_T}\left[\frac{\left(1-k_b^M\right)}{\left(1-k_s^M \cdot k_b^M\right)}\times\overline{F}_s\right]+\frac{1}{R_T}\left[\frac{\left(1-k_s^M\right)\cdot k_b^M}{\left(1-k_s^M k_b^M\right)}\left\{\overline{F}_b-\mathrm{LOSS}_b^{AS,RA}-\frac{1}{2}\phi\sqrt{\frac{B}{OF}}\cdot(t-1)\left(1-\rho_s^2\right)\sigma_{b,x}^2\right\}\right] \\ & +\frac{1}{R_T}\left[\frac{k_b^M}{\left(1-k_s^M \cdot k_b^M\right)}\left\{\left(\mathrm{LOSS}_b^{AS,RA}-\mathrm{LOSS}_s^{AS,RA}\right)+\frac{1}{2}\phi\sqrt{\frac{B}{OF}}\cdot(t-1)\left[\left(1-\rho_b^2\right)\sigma_{b,x}^2-\left(1-\rho_s^2\right)\sigma_{s,x}^2\right]\right\}\right] \text{ for } t\geq 1 \end{aligned} \tag{5}$$

$$\begin{aligned} P_{ask} = {} & \frac{1}{R_T}\left[\frac{\left(1-k_s^M\right)}{\left(1-k_s^M \cdot k_b^M\right)}\times\overline{F}_b\right]+\frac{1}{R_T}\left[\frac{\left(1-k_b^M\right)\cdot k_s^M}{\left(1-k_s^M \cdot k_b^M\right)}\times\left\{\overline{F}_s+\mathrm{LOSS}_s^{AS,RA}+\frac{1}{2}\phi\sqrt{\frac{B}{OF}}\cdot(t-1)\left(1-\rho_s^2\right)\sigma_{s,x}^2\right\}\right] \\ & +\frac{1}{R_T}\left[\frac{k_s^M}{\left(1-k_s^M k_b^M\right)}\left\{\left(\mathrm{LOSS}_b^{AS,RA}-\mathrm{LOSS}_s^{AS,RA}\right)+\frac{1}{2}\phi\sqrt{\frac{B}{OF}}\cdot(t-1)\cdot\left[\left(1-\rho_b^2\right)\sigma_{b,x}^2-\left(1-\rho_s^2\right)\sigma_{s,x}^2\right]\right\}\right] \text{ for } t\geq 1 \end{aligned} \tag{6}$$

In Eq. (5), the first part of the bid price is related to the asset valuations of market sell order traders and limit buy order traders. For a risk-averse market sell order trader, the reservation value is adjusted to asset volatility $\sigma_{s,x}^2$ and by the discount rate: $\phi\sqrt{\frac{B}{OF}}\left(1-\rho_s^2\right)$. For a risk-averse limit buy (uninformed) trader, the reservation value depends on the asset volatility, the discount rate $\phi\sqrt{\frac{B}{OF}}\left(1-\rho_b^2\right)$ and the expected loss of adverse selection: $\mathrm{LOSS}_b^{AS,RA}$. In other words, an uninformed buyer can improve his price to increase its trading profits with noisy sellers and aggressive uninformed sellers. Otherwise, he faces the risk of adverse selection expressed in the second term of Eq. (5), this of being picked-off, and of non-execution expressed in the third term of Eq. (5). According to the rule of competition applied to order execution, the preference of uninformed buyers for placing limit orders generates an increase

[19] Foster and Viswanathan (1996) show that the lower (*resp.* higher) the correlation of private signals about the fundamental asset value held by informed traders is, the less (*resp.* more) traded prices are informative.

of P_{bid} reducing *de facto* their expected utility. Ultimately, while the expected utility of limit buy order at time t equals this of market buy order, uninformed buyers are indifferent between trading via limit order and market order (Kovaleva and Iori, 2012).

The first term of Eq. (6) makes the ask price dependent to the asset valuations of market buy order traders and limit sell order traders. The rationale used to explain the escalation of bid price also apply to explain the escalation of the ask price. Additionally, the bid and ask prices are together related to the difference between discounted adverse selection losses of buyers and sellers: $\mathrm{LOSS}_b^{AS,RA} - \mathrm{LOSS}_s^{AS,RA}$ from the third term of Eqs. (5) and (6). If $\mathrm{LOSS}_s^{AS,RA}$ is lower (*resp.* higher) than $\mathrm{LOSS}_b^{AS,RA}$, bid and ask prices simultaneously would increase (*resp.* fall). Other things being equal, uninformed sellers can expect a rise (*resp.* a fall) in their trading profits whereas uninformed buyers can expect an increase (*resp.* decrease) of their trading profits. Therefore, the evolution of adverse selection risks borne by uninformed traders are directly related to the arrival rate of market orders. **Proposition 3** focuses on the implied correlations between variation in adverse selection risks and variation in the arrival rate of market orders.

Proposition 3. The expected adverse selection losses recognized by uninformed traders are directly affected by the arrival rates of market buy and sell orders.

(a) The expected adverse selection costs recognized by uninformed buyers (*resp.* sellers) are negatively (*resp.* positively) associated with the arrival rates of market buy orders, implying: $\partial \mathrm{LOSS}_b^{AS,RA} / \partial k_b^M < 0$ (*resp.* $\partial \mathrm{LOSS}_s^{AS,RA} / \partial k_s^M > 0$)

(b) The expected adverse selection costs recognized by uninformed sellers (*resp.* buyers) are negatively (*resp.* positively) associated with the arrival rates of market sell orders, implying: $\partial \mathrm{LOSS}_b^{AS,RA} / \partial k_b^M > 0$ (*resp.* $\partial \mathrm{LOSS}_s^{AS,RA} / \partial k_s^M < 0$)

Proof of Proposition 3 is given in **Appendix A.2**.

□

3.2. A novel bid-ask spread decomposition (reduced form of the model)

As a first step toward the study of price formation, the order strategy of uninformed trader is scrutinized in a fairly structured dynamic environment. In the following paragraph, we propose a reduced form of our model in which uninformed traders homogeneously recognize a similar arrival rate of buy and sell market orders. This implies that the parameter k verifies $k = k_b^M$ and $1 - k = k_s^M$ and directly affects the probability of order execution as seen in Fig. 2.

If k tends to unity, most of market participants are considered sellers. To avoid the risk of non-execution, two strategies are possible, either they decrease the bid price of their limit order or alternatively the ask price of their market sell order. Therefore, their revised bid and ask quotations approaches their reservation value. At the opposite side, few numbered buyers have an absolute competitive advantage over outnumbered sellers but the adverse selection risk still concerns uninformed buyers if sellers are informed. If k tends to zero, outnumbered buyers will compete with each other and pushes their prices up. The ask price of market buy orders approaches the reservation buyers' value while the bid price of the limit order is close to the buyers' reservation value minus sellers' loss of adverse selection. Also, uninformed sellers benefit from a competitive advantage even through sellers still suffer from adverse selection risks if counterparty traders are informed.

<Fig. 2 is inserted about here>

Hence, we can infer the buy and sell partial equilibrium uninformed traders' strategies should they expect an equivalent arrival rate of buy and sell market orders. After determining the optimal bid and ask prices, we obtain a novel decomposition of the bid-ask spread in three factors, which is presented in **Proposition 4**.

Proposition 4. If uninformed traders expect an equivalent arrival rate of buy and sell market orders, the equilibrium bid-ask spread π is decomposed in three weighted factors:
- the discounted adverse selection costs of uninformed buyers and sellers respectively;
- the present value of the different risk-adjusted traders' valuations of the risky asset.

$$\pi = \frac{\omega_1}{R_T} \times \left[\overline{RV}_b - \overline{RV}_s\right] + \frac{\omega_2}{R_T} \times ASC_b + \frac{\omega_3}{R_T} \times ASC_s \tag{7}$$

Given the weights: $\omega_1 = \frac{k(1-k)}{1-k\times(1-k)}$; $\omega_2 = \frac{k^2}{1-k\times(1-k)}$; $\omega_3 = \frac{(1-k)^2}{1-k\times(1-k)}$ such that $\omega_1 + \omega_2 + \omega_3 = 1$.

And the following bid-ask spread factors:

- $\overline{RV}_b = \rho_b^2 \tilde{Z}_{b,t} + (1-\rho_b^2)\cdot(\mu_x^b - \frac{1}{2}\phi\sigma_{b,x}^2)$: the risk-adjusted valuation of a buyer;
- $\overline{RV}_s = \rho_s^2 \tilde{Z}_{s,t} + (1-\rho_s^2)\cdot(\mu_x^s + \frac{1}{2}\phi\sigma_{s,x}^2)$: the risk-adjusted valuation of a seller;
- $ASC_b = \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\cdot(1-\rho_b^2)\cdot\sigma_{b,x}^2 + LOSS_b^{AS,RA}$: the discounted adverse selection costs of a buyer;
- $ASC_s = \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\cdot(1-\rho_s^2)\cdot\sigma_{s,x}^2 + LOSS_s^{AS,RA}$: the discounted adverse selection costs of a seller.

Proof of **Proposition 4** is given in **Appendix A.3**.

□

In the case that $\rho_b = \rho_s$, the expected adverse selection loss of sellers (buyers) is only involved in the bid-ask spread composition, generating $\boldsymbol{\omega_3=1}$ (*resp.* $\boldsymbol{\omega_2=1}$) under extreme market competition (**k=0%**) (*resp.* **k=100%**). Ceteris Paribus**,** the size of the bid-ask spread is minimized**.** When **k=50%**, that means a well-balanced market, the bid-ask spread achieves a maximum and the equality $\boldsymbol{\omega_1=\omega_2=\omega_3=1/3}$ is verified. In either case, it is the largest value of $\boldsymbol{\omega_1}$ *i.e.* the weight of the different valuations to reflect traders' heterogeneous beliefs.

Generally, the viability of LOB depends on a constant provision of liquidity at constrained costs, which may be either immediacy costs (for the profitability of market orders) or bid-ask spread (for the profitability of limit orders). From our model, we understand that the bid-ask spread must provide enough price improvement for uninformed traders to reduce their risk of adverse selection and of picking-off. Thus, the bid-ask spread also incorporates the effect of buyer's and sellers' heteregeneous valuations when the fundamental value of the risky asset is unknown. Finally, we determine endogenously the precision of (noisy) public information that is likely to affect the weights corresponding to the three bid-ask spread components.

3.3. The behaviour of bid ask-spread according to the precision of public information

Bloomfield et *al*. (2009) oppose informed traders who hold private signals on the fundamental value to uninformed traders who trade on the basis of noisy prices and information. Similar to these authors, we assume that informed traders trade on the basis of private signals while uninformed traders update their beliefs on the (unknown) risky asset fundamental value at each time of the trading period [0;T] conditioning on the noisy public information observed at t-1. Then, we allow for variations of noisy public information precisions between two extreme cases: a perfect precision ρ=1 or a complete imprecision ρ=0 to study the associated effects on the bid ask-spread with the **Corollary 1 and Corollary 2**.

Corollary 1: In the case of perfect correlation of public information signals which implies that both $\rho_b = \rho_s = 1$ and $\sqrt{\frac{B}{OF}} \rightarrow 1$, the equilibrium bid-ask spread π verifies the following equation:

$$\pi' = \frac{\omega_1}{R_T} \cdot \left(\tilde{Z}_{b,t} - \tilde{Z}_{s,t}\right) + \frac{\omega_2}{R_T} \text{LOSS}_b^{AS} + \frac{\omega_3}{R_T} \text{LOSS}_s^{AS} \tag{8a}$$

Corollary 2: In the case of very low level of correlation of public information signals implying that both ρ_b and $\rho_s = 0$ and $\sqrt{\frac{B}{OF}} \rightarrow 0$, the equilibrium bid-ask spread π is written as follows:

$$\pi'' = \frac{\omega_1}{R_T} (\overline{RV}_b'' - \overline{RV}_s'') + \frac{\omega_2}{R_T} \text{LOSS}_b^{AS} + \frac{\omega_3}{R_T} \text{LOSS}_s^{AS} \tag{8b}$$

Where: $\overline{RV}_b'' = \mu_{b,x} + \frac{1}{2}\phi\sigma_{b,x}^2$ (*resp.* $\overline{RV}_s'' = \mu_{s,x} + \frac{1}{2}\phi\sigma_{s,x}^2$) are the risk adjusted valuation of buyer (*resp.* seller) on the fundamental value of the risky asset.

Appendix A.4 shows the proof.

□

For these two propositions, the first order conditions that give extremums (maximum and minimum) of Eq. (7) are maintained. With an extreme level of order imbalance (**k=0%**) (*resp.* **k=100%**) the expected loss of adverse selection of sellers (buyers) are only involved, the bid-ask spread is minimized and $\boldsymbol{\omega_3 = 1}$ (*resp.* $\boldsymbol{\omega_2 = 1}$). If **k=50%**, we verify the result of Handa et *al*. (2003) *i.e.* the bid-ask spread achieves a maximum with $\boldsymbol{\omega_1 = \omega_2 = \omega_3 = 1/3}$.

These two results confirm the stability of our model inferences in terms of the size and composition of the bid-ask spread in response to changing market conditions.

4. Numerical simulation and implications for the bid-ask spread

Even through our model offers an insightful analysis of price formation and a bid-ask spread decomposition, its unique solution is not in a linear closed form. To transform Eq. (7) in a linear form, we assume that the expected value of the terminal wealth ranged from zero to unity. Based on this assumption, we set up the following basic parameters values for a scenario analysis of our model: $\mu_{b,x}$= 0.12, $\mu_{s,x}$= 0.11, their respective variances: $\sigma_{b,x}$=0.003 and $\sigma_{s,x}$=0.003, *R*=1.01, ϕ=1, the arrival rate of market orders : $k = k_b^M = k_s^M = 0.5$ and the degree of precision for noisy signals $\sqrt{\frac{B}{OF}} \cdot \rho_b = \sqrt{\frac{B}{OF}} \cdot \rho_s$ = 0.2. We thus proceed by a recursive process on the equilibrium conditions to obtain convergent solutions. Hence, we can explore the model implications using numerical tests drawn from independently tuning the parameters.

Fig. 3 presents the effect of a variation in the arrival rate of market buy order given two risk aversion coefficients (ϕ= 1 or ϕ= 1,5) on the adverse selection costs and the spread. We confirm the implications of **Proposition 3**, *i.e.* the uninformed (*resp.* buyers') adverse selection cost are negatively (*resp.* positively) associated with the arrival rates of market buy orders when the size of the spread is rather constant. This result suggests that the first negative effect on adverse selection **(ω_3)** neutralize the second one **(ω_2),** while **ω_1** is unaffected.

<Fig. 3 is inserted about here>

Fig. 4 plots the adverse selection costs and spread as the precision of buyer's and sellers' noisy signals equally vary. The precision of noisy signals descends (*resp.* ascends) the expected adverse selection costs for sellers (*resp.* buyers) whereas the bid ask-spread is stable whatever the level of risk aversion is. This result is not surprising because when uninformed traders hold more precise information, they are more confident on their estimation of their reservation buy and sell values. Therefore, they revise more appropriately the price of their limit orders downward (*resp.* upward) reducing their risk of being picked-off in the event that an adverse (*resp.* favorable) information arrives (Foucault, 1999).

<Fig. 4 is inserted about here>

In Fig. 5, we examine the joint effect of increasing the variance of buy and sell reservation values and the precision of noisy signals on the difference in risk-adjusted valuation (**ω_1**) and on the bid-ask spread. Whatever the variance of buy and sell reservation values is, the percentage contribution of **ω_1** to the bid-ask spread remains stable whereas its absolute level augments as soon as the precision of noisy signals is lowered.

<Fig. 5 is inserted about here>

5. Empirical Analysis

Our empirical analysis aims at first verifying whether the predictions presented in **Propositions 2** and **4**, **Corollaries 1** and **2** are valid. We focus our analysis on an emerging commodity market: the EU Emission Allowance (EUA) carbon market for four main reasons:

- All carbon exchanges use a LOB trading system from market inception until now.
- Some market microstructure studies have explained how informed interact actively with uninformed traders in EUA futures market (Kalaitzoglou and Ibrahim, 2013; 2016).
- The size of the bid-ask spread fluctuates throughout the year and is mainly determined by adverse selection costs while inventory costs play a minor role (Medina et *al.*, 2014).
- A competitive and active brokerage sector facilitates the recourse to block trading and order splitting strategies by informed traders (Frunza, 2010; Ibikunle et *al.*, 2016)[20].
- Chevallier et *al.* (2009) estimate a risk aversion coefficient for EUA futures higher than for equity markets, which depends on the occurrence of annual regulatory (compliance) events.

5.1. Data selection

The data used are drawn from the Thomson Reuters Tick History database (TRTH) that contains the history of order book information available to market participants in real time in the LOB of ECX. Market participants are polluting firms and external investors who can trade if they have a trading account (as a principal) or via their brokers (as a customer).

The LOB of ECX concentrates almost 90% of EUA futures trading activity from 2008 to 2012 (Mizrach and Otsubo, 2014). Also called Webice, it is an order driven market where matched bid and ask quotes are executed based on first price and then time priority. Trading is continuous from Monday to Friday, with trading hours 07:00–17:00 London Time (GMT). Every EUA futures contract, 'lot', corresponds to 1000 EUAs (1 EUA is the right to emit 1 tonne equivalent CO_2). Prices are quoted in Euros and the minimum tick is 0.01€.

We study a sample of five EUA December futures during their latest 254 trading days when they are the most liquid and concentrate investors' attention covering the entire Phase II of EU ETS namely March 1, 2008 to December 31, 2012. For each trade, TRTH reports the futures code, the price and time of execution, the size in lots[21], while for each LOB update, the dataset reports the timestamp to the nearest hundredth of a second, the best five bid and ask prices and their respective quantity demanded (ask size) or offered (bid size).

[20] ECX is held by the Intercontinental Exchange (ICE), a leading platform in the energy derivatives trading. Twelve brokers are very active on ECX: BGC Partners; CantorCO2e; Evolution Markets; GFI Group; ICAP; Marex Spectron; PVM Oil Associates, 42 Financial Services, Tradition Financial Services, Tullet Prebon, all of members of the London Energy Brokers Association (LEBA), and Newedge, Consus. According to Frunza (2010), their average net margins have been reduced due to a high pressure of their customers, the falling prices and the high number of competitors: 1.11 billion euros in 2009, and two years after: 0.63 billion euros.

[21] EUA futures contract are traded per lot ($1000t_{eq}C0_2$) with a tick size of 0.01€ (*i.e.*10€ per lot) on ECX.

We apply several filters to clean trade and order book data. Trades that occurred in ECX during the pre-opening period (6:45 and 7:00) or in the after-hours market are discarded. We remove orders above and below 50 ticks from the best quote to avoid the existence of stale or erroneous orders. In line with prior empirical studies on the bid-ask spread in the EUA futures market (Frino et *al.*, 2010; Medina et al., 2014, Mizrach and Otsubo, 2014) we classify orders executed at the best prevailing ask (*resp.* bid) as buyer-initiated (seller-initiated) trades.

5.2. Bid-ask spreads, timing and size of trades

According the TRTH database, we generate all necessary variables to examine the validity of our model and its implications. We first compute the proportional bid-ask spread:

$$\mathrm{PBAS} = \frac{\mathrm{BestP}_{\mathrm{ask\ t}} - \mathrm{BestP}_{\mathrm{bid\ t}}}{\frac{\mathrm{BestP}_{\mathrm{ask\ t}} + \mathrm{BestP}_{\mathrm{bid\ t}}}{2}}$$

We study the behavior of PBAS as well as the below mentioned variable into 15-minute intervals. This interval is a tradeoff between too much aggregation and noisy a dataset[22].

We calculate the order imbalance as a proxy of our market competition parameter **k**[23]:

$$\mathrm{k} = \frac{\text{number of trades at the ask + limit sell orders submitted}}{\text{number of trades at the ask and bid + limit buy and sell orders submitted}} \times 100\%$$

Given the k parameter, the components of *PBAS* are estimated as follows:

- *Weight of the different valuations* (**ω1**%) = **k**×(1-**k**) / [1- **k**×(1-**k**)]
- *Weight of the buyer's expected loss of adverse selection* (**ω2**%) = **k**² / [1- **k**×(1-**k**)]
- *Weight of the seller's expected loss of adverse selection* (**ω3**%) = (1-**k**)² / [1- **k**×(1-**k**)]

Table 1 presents monthly means for the above variables of interest. Between May and September, the size of the proportional bid-ask spread decreases on average by 1.1% when the information asymmetry decreases (Medina et *al.*, 2014). In contrast, its size increases by 9.7% between December and April when the levels of information asymmetry and of risk aversion are significantly higher (Chevallier et *al.*, 2009).

We follow the method of Handa et *al*. (2003)[24] to explore the linkages between the proportional bid-ask spread (**PBAS**%), its factors and the order imbalance **k** in more details. We divide our sample into two parts where the first part is **k**% larger than 50% and the second part is *k*% smaller than 50%. We observe that in the region where k is greater (less) than 50%, the spread is positively related to **k** and when **k** is closer to 50%, it achieves its highest levels in line with the predictions of **Proposition 4** even if the influence of informed trading merits

[22] As a robustness check, we have estimated the order imbalance k over a 30-minute interval and calculated k with the immediate LOB depth (limit orders posted at the best bid or ask). Our results were qualitatively similar.
[23] Handa et *al.* (2003) use a very similar measure to ours in dividing the trading volume at the ask plus this of limit buys by the sum of the volume of limit buys and sells and the volume of trades.
[24] Handa et *al.* (2003) show that the monthly proportional spreads follow an inverted U-shaped pattern over order imbalance values, *i.e* spreads are lower (*resp.* higher) when the market is unbalanced (*resp.* well-balanced).

further investigation. Indeed, the factors that accounts for adverse selection (**ω_2, ω_3**) follow an opposite pattern, being lower (higher) when the order flow is proportionately more concentrated on sell side (balanced). Moreover, we obtain that the size of the spread is significantly higher for days when the order imbalance k is higher (*resp.* lower) than 50% between December and April (*resp.* May and September) consistent with Mizrach and Otsubo (2014) findings. These two results provide evidence that the size of the bid–ask spread and its three factors (**ω_1, ω_2, ω_3**) are directly related to the order imbalance measure as it specified in our model.

Panel B presents the results of univariate regressions of **PBAS** over three periods of the continuous trading session: 7:00-9:00, 9:00-15:00, 15:00-17:00. Consistent with Medina et *al*. (2014) and Ibikunle et *al*. (2016) findings, monthly bid-ask spreads exhibit an intraday U-shaped pattern. Indeed, they are greater at the opening hours (7:00 to 9:00) in comparison to normal hours (9:00 to 13:00) before they increase during the latest two hours (15:00 to 17:00).

<Table 1 is inserted about here>

Hitherto, our emphasis on the relation between order imbalance and the bid-ask spreads provide results similar to these of Handa et *al.* (2003) obtained with data from stock markets. Because **k**% simultaneously generates **ω_1** *i.e.* the influence of traders' heterogeneous beliefs and the expected adverse selection costs of buyers (**ω_2**%) and sellers (**ω_3**%), it will be relevant to consider **ω_1, ω_2, ω_3** altogether with the bid-ask spread. To this respect, we perform two model regressions on the proportional bid-ask spreads (**PBAS**%) to assess whether the bid-ask spread decomposition remains robust to changing market conditions.

In the first regression model, we consider time intervals as the unique control variable. Medina et *al.* (2014) estimate a larger Probability of Informed trading (PIN: Easley et *al.*, 1996)[25] during morning hours for all EUA December futures studies. Ibikunle et *al.* (2016) assess the influence of block trading strategies of informed traders on bid-ask spreads and adverse selection costs: intense at the open and mild at the close, which partially explain the U-shaped pattern of the bid-ask spread. Kalaitzoglou and Ibrahim (2013) show that private information held by informed traders is incorporated into prices quicker when the trading volume and the trading frequency are increasing. Also, they find that fundamental and uninformed traders narrow their spreads to provide liquidity until the last hour contributing to make prices noisier. In line with these complementary findings, we assume that time intervals can capture the effect of informed trading and noise on spreads.

[25] Easley et *al.* (1996) estimate the probability of informed trading (PIN) in a sequential model where informed traders buy (*resp.* sell) when news is good (*resp.* bad), and do not trade when no news. Uninformed traders do not know the probability of other trader counterparts being informed like in our model. In contrast, they place limit buy and sell orders at a constant rate while in our model they submit buy and sell orders at different rate.

For all of these reasons, we constitute three time intervals INT_j previously determined by Ibikunle et *al*. (2016) that we include as control variables into the following model regression:

$$PBAS_t = \overline{RV}_{b-s} + \sum_{j=1}^{3} \omega_2 \cdot INT_j \cdot \left(ASC_{b,t} - \overline{RV}_{b-s}\right) + \sum_{j=1}^{3} \omega_3 \cdot INT_j \cdot \left(ASC_{s,t} - \overline{RV}_{b-s}\right) + \varepsilon_t \tag{9}$$

Where:

- $\overline{RV}_{b-s} = \overline{RV}_b - \overline{RV}_s$: the difference between risk-adjusted valuations of buyers and sellers;
- $\mathbf{ASC_b}$ (*resp.* $\mathbf{ASC_s}$): the adverse selection costs of buyers (*resp.* sellers) are defined in Eq. (7);
- $\mathbf{INT_1}$ if **PBAS** is between 7:00 and 9:00; $\mathbf{INT_2}$ if PBAS is between 9:00 and 15:00, $\mathbf{INT_3}$ if PBAS between 15:00 and 17:00;
- ε_t is the random error term.

According to Palao and Pardo (2014), traders tend to round their prices to digits ending in 0 or 5 and simultaneously adjust their trades as a multiple of five contracts when liquidity is lower. Moreover, Kalaitzoglou and Ibrahim (2016) show that uninformed traders submit more limit orders when the spread is large, or when informed traders trade large orders, especially at earliest hours. Ibikunle et *al.* (2016) find that trade size is inversely related on the degree of price noisiness and liquidity risks perceived by informed traders so that they prefer block trades to execute large orders at opening hours (7:00 to 9:00). In addition to the above, Kalaitzoglou and Ibrahim (2013) estimate a gradual influence of order splitting strategies on prices, given a higher concentration of medium sized trades at closing hours.

Given all of these evidences provided by the literature, we consider a second regression model where trade size is used to proxy the influence of private information signals and noise. Precisely, we consider three categories of trade size identical to those determined by Frino et *al.* (2010) and Ibikunle et *al.* (2016) to use trade size as dummy variables. In order to test the monotonically increasing relation between these two proxies and adverse selection costs, we suppress $\boldsymbol{\omega_1}$ variable to avoid multicollinearity. Then, we examine this implied relation by running the following panel regression on the proportional bid ask spread for each EUA futures:

$$PBAS_t = \overline{F}_{b-s} + \sum_{i=0}^{3}\sum_{j=1}^{3} \omega_2 \cdot SIZE_i \cdot INT_j \cdot \left(ASC_{s,t} - \overline{F}_{b-s}\right) + \sum_{i=0}^{3}\sum_{j=1}^{3} \omega_3 \cdot SIZE_i \cdot INT_j \left(ASC_{b,t} - \overline{F}_{b-s}\right) + \varepsilon_t \tag{10}$$

Where:

- $\overline{RV}_{b-s}$, $\mathbf{ASC_b}$, $\mathbf{ASC_s}$, $\mathbf{INT_j}$ with j=1,2,3 are computed analagously to the previous Eq. (9).
- $\mathbf{SIZE_1 = 1}$ for a trade size which falls in the range: 1 and 19 contracts; $\mathbf{SIZE_2 = 1}$ for a trade size between 20 et 49 contracts; $\mathbf{SIZE_3 = 1}$ for a trade with more than 50 contracts.

For robustness purposes, we generate a great number of convergent simulation data according to variations in our model parameters. The purpose of this simulation is to verify that regression tests are relevant to analyze the relations with the spread implied by our model. For that purpose, we examine the bid ask spreads involved by simulation data using panel regression based on three time intervals and trade size as follows:

$$\text{PBAS}=\left[\overline{\text{RV}}_{\text{b-s}}\right]+\omega_2\sum_{i=1}^{3}\sum_{j=1}^{3}\rho_{\text{b,i}}\sigma_{\text{b,j}}\left[\text{ASC}_{\text{b},\rho_{\text{b,i}},\sigma_{\text{b,j}}}-\overline{\text{RV}}_{\text{b-s}}\right]+\omega_3\sum_{i=1}^{3}\sum_{j=1}^{3}\rho_{\text{i}}\sigma_{\text{b,j}}\left[\text{ASC}_{\text{s},\rho_{\text{s,i}},\sigma_{\text{s,j}}}-\overline{\text{RV}}_{\text{b-s}}\right]+\varepsilon \qquad (11)$$

Where :

- $\boldsymbol{\rho_{bi}}$ ($\boldsymbol{\rho_{si}}$) is the degree of precision (correlation) of noisy signals at the buy (sell) side and $\boldsymbol{\sigma_{bi}}$ is the volatility of reservation prices of uninformed buyers (sellers) given the following:
  - $\boldsymbol{\rho_{b1} = \rho_{s1} = \sigma_{b1} = \sigma_{s1} = 0,4}$ if the simulation is performed between 7:00 and 8:59:59s ; between 15:00 and 17:00 respectively ;
  - $\boldsymbol{\rho_{b2} = \rho_{s2} = \sigma_{b2} = \sigma_{s2} = 0,3}$ if the simulation is performed between 9:00 and 14:59:59s.

As shown in Table 2, nearly all monthly $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_2}$ and $\boldsymbol{\omega_3}$ coefficients turn out to be significant in view of their t-statistics. If the coefficients of $\boldsymbol{\omega_1}$ are always significant and positive, those of $\boldsymbol{\omega_2}$ and $\boldsymbol{\omega_3}$ are more often negative and insignificant. We note that $\mathbf{ASC_s}$ and $\mathbf{ASC_b}$ coefficients are more significant during the pre-compliance period from November to April, before polluting firms submit and disclose their level of verified carbon emissions. This provides support for the observation of Medina et *al*. (2014) about more severe adverse selection risks during this period where information asymmetry among traders is higher (see also Table 1).

Finally, the F-statistics reported in Panels A and B confirm that the dependencies between $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_2}$ and $\boldsymbol{\omega_3}$ have strong explanatory power on the size of bid-ask spreads. The column *Simul*. reports significant negative $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_2}$ coefficients, suggesting a negative relation between the spread and the following: the adverse selection costs of buyers and sellers, the precision of noisy information signals and asset volatility.

<Table 2 is inserted about here>

In Table 3, we report the value of adverse selection costs calculated from the coefficients $\boldsymbol{\omega_2}$ and $\boldsymbol{\omega_3}$ obtained previously considering $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_1}$ as a constant. On average, the difference in risk adjusted valuations represent 29.4% of the spread, while the adverse selection costs of sellers (*resp.* buyers) account for 36.5 % (*resp.* 34.1%). The aggregated adverse selection costs represent 72.3% of the spread at the opening, then slightly decrease (70.6%) to remain constant during the latest two hours while the bid-ask spread decreases (see Table 1). As a result, the bid–ask spread and adverse selection costs of sellers (*resp.* buyers) follows an intraday mild U-shaped (*resp.* inverted U-shaped) pattern. The increase of bid-ask spread at the approach of closing hours (15:00 to 17:00) is more likely to be caused by a more intense trading activity of uninformed traders rather than a variation in adverse selection. This interpretation of such deadline effects corroborates this of Kalaitzoglou and Ibrahim (2016) who find that uninformed traders submit more market orders later in the day since the order execution risk is lowered.

Overall, the results of Panels B confirm those of Panel A supporting the idea of a random rate of public information arrival, a roughly constant effect of the rounding of order sizes over the trading day (Palao and Pardo, 2014). If the adverse selection costs of sellers are the most important spread component, we observe that adverse selection costs of buyers (*resp.* sellers) decrease (*resp.* increase) of two percentage points between October and April for medium and large trades. This result suggests that both order splitting and block trading strategies initiated by informed traders have gradual influences on traded prices (Kalaitzoglou and Ibrahim, 2013) along the trading session. Indeed, informed traders are intended to split large buy market orders to minimize price impact and for camouflage purposes (Ibikunle et *al*., 2016). As for small trades, we obtain an 8% increase of the component "Differences in risk-adjusted valuations". We interpret this increase as a signal of the uninformed traders' appetite for executing small trades. They contribute by more intense activity on the segment of small trades to make the order imbalance closer to 50% and widen the bid-ask spread.

Given the complexity of bid-ask spread determination and many potential liquidity and information factors, the above results suggest that the spread decomposition in three factors: differences in risk-adjusted valuations, adverse selection of buyers and sellers is suitable.

Market microstructure studies of carbon markets generally use spread estimators to estimate aggregate adverse selection costs, inventory and order processing costs (e.g., Medina et *al*., 2014; Mizrach and Otsubo, 2014). Kalaitzoglou and Ibrahim (2016) build a dynamic asymmetric information pricing model where the responsiveness of price changes to surprises in order flow (information) and changes in trading costs depend on the type of traders who instigate the next trade. This model relies on the strong assumption that only price-relevant information is included in the last trade. Conversely, we consider that uninformed traders monitor the full order book informational state prior to trade. Contrary to the existing literature, we also distinguish the adverse selection costs of buyers and sellers if the expected arrival rate of buy and market orders is equivalent. For these reasons, our model appears to be in a better position to study the composition of trading costs in the European carbon market at least.

<Table 3 is inserted about here>

5.3. Variations in adverse selection costs and the influence of incoming market orders

Although the results of Table 3 are consistent with those found by the literature, our regressions may be potentially affected by spurious correlations with other liquidity variables. For example, let us assume that informed traders trade more when there are larger than usual trading volumes. They may do so in an attempt to camouflage their private information. Alternatively, they trade around information events such as the publication of compliance results by mid-April in the European carbon market. Since these information events often trigger higher trading activities, the inverse correlation between the size of the spread and volatility could reflect the effect of trading volume on spreads (Medina et *al*., 2014). Furthermore, the direct relation between adverse selection costs and market orders seen in

Proposition 3 may differ given the size of the trade. Indeed, the execution of market orders involves an instantaneous quote adjustment when the trade is large (Biais et *al.*, 1995).

To shed light on these issues, we conduct a simultaneous equations model which has two periods, one period before the announcement of compliance results (November-April) and one period post their announcement (May-September). The dependent variables are the proportional bid-ask spreads and the estimated adverse selection costs. The dummy variable indicates the change in liquidity costs around the compliance event and is denoted COMP. The independent variables are the volume of market orders and the autocorrelation of buy and sell market orders. The model we estimate is:

$$PBAS_{i,t} = \overline{RV}_{b-s} + \sum_{i=1}^{3}\sum_{j=1}^{3}\delta_1 INT_j.SIZE_i(ASC_{s,t} - \overline{RV}_{b-s}) + \sum_{i=1}^{3}\sum_{j=1}^{3}\delta_2 INT.SIZE_i\left(ASC_{s,t} - \overline{RV}_{b-s}\right) + \gamma_{1,d}COMP_{t,d} + \varepsilon_{i,t} \quad (12a)$$

$$\begin{cases} ASC_{b,i,t} = \alpha_{1,b} + \beta_{1,b}MO_{b,t} + \beta_{2,b}MO_{s,t} + \beta_{3,b}AUTOCORR_{ask,t} + \beta_{4,b}AUTOCORR_{bid,t} + \mu_{b,t} \\ ASC_{s,i,t} = \alpha_{1,s} + \beta_{1,s}MO_{b,t} + \beta_{2,s}MO_{s,t} + \beta_{3,s}AUTOCORR_{ask,t} + \beta_{4,s}AUTOCORR_{bid,t} + \mu_{s,t} \end{cases} \quad (12b)$$

Where :

- **MO_b** is the volume of buy (sell) market orders executed;
- **COMP**=1 if the observation is between November 1 and April 30, 0 otherwise;
- **$AUTOCORR_b$** (*resp.* **$AUTOCORR_s$**) is the correlation between improvement in bid (*resp.* ask) quotes and the location of next order executed at the best bid or the ask quote.

Since the time-series observations of dependent and independent variables may be subject to spurious regressions, in which autocorrelation indicate a significant relation between them while in fact there is none (Van Ness et *al.*, 2011)[26], we check the first-order autocorrelation of residuals for each regression. Since the Durbin-Watson statistics indicate a mild positive autocorrelation, the two-step transform method of Prais-Winsten to correct for autocorrelation[27]. Table 4 reports the regression results based on the Prais-Winsten method which are qualitatively superior to these obtained without this correction for autocorrelation.

The sign of relations between **MO_b and MO_s** and **ASC_b, ASC_s** are those anticipated in **Proposition 3** for small (Panel A) and medium sized trades (Panel B). In contrast, we see from Panel C that the **MO_b and MO_s** coefficients are more rarely significant at 1% level and their signs are not those expected. This is likely because a large order's size dwarfs the quoted depth, making the bid-ask spread is a less relevant measure of their trading costs. Almost **MO_b and MO_s** coefficients for small (Panel A) and medium sized trades (Panel B are significant at 1% level. Those estimated in post compliance period (May-September) are more significant which may reflect the presence of more competitive limit orders in the opposite LOB side and a

[26] More precisely, we obtain a Durbin-Watson statistic for ASC_b (*resp.* ASC_s) distributed as follows: 2.185 (*resp.* 2.245) for the maximum value and 1.385 (*resp.* 1.485) for the minimum value.

[27] The Prais-Winsten procedure refines the Cochrane-Orcutt procedure by including the first observation of the transformed data. See Beck and Katz (1995) for a detailed discussion of the advantages of this procedure.

quicker incorporation of information into traded prices (Kalaitzoglou and Ibrahim, 2016). This first important result provides evidence against the hypothesis that random order placement mechanically results in frequent limit orders within the spread when the latter is large in line with the findings of Palao and Pardo (2014). Rather, this suggests that when the spread is large after large liquidity shocks, uninformed traders quickly place limit orders within the best quotes to supply liquidity at better prices and gain time priority.

A second important result is that $\mathbf{AUTOCORR_b}$ (*resp.* $\mathbf{AUTOCORR_s}$) coefficients for large orders are higher and only significant at opening hours when the adverse selection cost $\mathbf{ASC_s}$ **(***resp.* $\mathbf{ASC_s}$) is higher. After a large sale (*i.e.* a negative informational signal) that consumes the liquidity at the bid, bid and ask prices are adjusted downward for the next trade. To smooth the price impact associated to large market orders, informed traders are likely to split them as soon as limit orders are more competitive and liquidity is better. Interestingly, the experimental results of Majois (2011) suggest that the *diagonal effect* implying positive serial correlation of market orders essentially reflects the existence of order splitting strategies, as the same informed traders tend to submit the same medium market orders in succession. In our model, this persistent order flow pattern can also appear because a change in the risk-adjusted asset valuations induce uninformed traders to follow similar order strategies (Biais et *al*., 1995).

A third important result related to the higher significance of $\mathbf{AUTOCORR_b}$ and $\mathbf{AUTOCORR_s}$ coefficients with signs identical to $\mathbf{MO_b}$ and $\mathbf{MO_s}$ for medium sizes in normal hours (9:00 to 15:00) and in latest hours (15:00 to 17:00) supports the explanation provided by Majois (2010). This result clearly indicates an increasing frequency of splitted medium market orders at the origin of a pronounced *diagonal effect* when informed traders anticipate a lower price impact because liquidity is more quickly replenished (Ellul et *al*., 2007). Moreover, the fact that $\mathbf{AUTOCORR_s}$ and $\mathbf{MO_s}$ coefficients are significantly higher in normal and latest hours indicates a quicker liquidity replenishment at sell side so that aggressive limit buy orders and sell market orders are more likely late in the day (Kalaitzoglou and Ibrahim, 2016).

Overall, our results suggest that a rise in sellers' adverse selection costs greatly increases (*resp.* decreases) the likelihood of small limit (*resp.* market) sell orders, moderately increases (*resp.* decreases) that of medium limit (*resp.* market) orders, but has little effect on large orders.

<Table 4 is inserted about here>

6. Conclusion

The "raison d'être" of a commodity market depends on whose perspective is considered. The ideal market, for any trader, may be the one in which orders are accommodated with lowest trading costs. For exchanges, the priority is to foster market liquidity in order to maximize their trading commissions. To this end, most of them have adopted an electronic LOB.

Our paper proposes a model which is an extension of the frameworks of Foucault (1999) and Handa et *al.* (2003) to examine trading costs and order book dynamics under the assumption of traders' risk aversion preferences. The novelty of our approach lies in its capacity to make the order strategies of risk averse uninformed traders endogenous to the noisy public information that they capture after monitoring their LOB screens. We proceed in two steps.

We first present a generic version of our model set in a CARA-normal framework from which we develop an optimal order strategy for uninformed traders and derive optimal bid and ask prices. Then, we propose a reduced form of our model where uninformed traders expect an equivalent arrival rate of buy and sell market orders when placing or revising orders.

The main contributions of our model are threefold. First, the bid-ask spread is tactically managed by uninformed traders to compensate for the risks of adverse selection (Glosten, 1994) and of picking-off they bear (Foucault, 1999). Second, our model inferences involve that adverse selection costs are positively (*resp.* negatively) related to the expected arrival rates of market buy (*resp.* sell) orders. Third, we disentangle three factors of the bid-ask spread in the case of the reduced form of the model: the differences of risk-adjusted valuations and the adverse selection costs of uninformed buyers and sellers respectively. In either case, we verify the Handa et *al.* (2003)'s result to the extent that the size of the bid-ask spread achieves a maximum in balanced markets whatever the precision of noisy public information is.

Even if the bid-ask spread decomposition is not an easy task due to numerous potential explanatory factors, our empirical results appear to be particularly encouraging. For the EUA carbon futures market, we find that the aggregated adverse selection costs account for 70% of bid-ask spread consistent with the literature (Medina et *al*., 2014; Mizrach and Otsubo, 2014). We also document that the bid-ask spread (*resp.* adverse selection of buyers) behaves according to a U-shaped (*resp.* inverted U-shaped) intraday pattern. Interestingly, the other bid–ask spread component related to traders' beliefs heterogeneity is rather constant in a context of significant risk aversion and uncertainty about the fundamental value of EUA (Chevallier et *al*., 2009). Moreover, we verify and enrich the findings of Kalaitzoglou and Ibrahim (2016) in relation to the gradual influence of order splitting strategies along the trading session at ECX. We show that the *diagonal effect*, which commands the arrival rate of market orders is a LOB equilibrium property (Goettler et *al*. 2005). It is explained by adverse selection considerations at earliest hours (7:00 to 9:00), an increasing frequency of splitted orders along a quicker liquidity exhaustion-replenishment cycle in normal hours (9:00 to 15:00) and in closing hours (15:00 to 17:00). This result is important because it implies that the market efficiency of the European carbon futures market may be undermined.

Spread decomposition models often assume that the adverse selection component of the spread increases with trade size, because informed traders would prefer to trade via larger orders (e.g., Easley et *al*., 1996). Rather, our results suggest that informed traders trade medium orders combined with order splitting strategies for camouflage motives or to minimize price impact when liquidity is better. An important implication of these results is that uninformed traders do not benefit from being known as uninformed. Besides, uninformed traders might be interested in preannouncing their orders via a flash order facility to advertise on their desire for liquidity before trading. This has not yet been a feature of the LOB of ECX. Consistent with the findings of Skjeltorp et *al.* (2016), executed flash orders could help them reduce their adverse selection costs because informed traders extract less consumer surplus from uninformed orders as prices become less noisy so that the overall market efficiency could also be enhanced.

Avenues for further research may be stretched in several directions.

On the empirical side, developing an algorithm to detect hidden orders (e.g., iceberg orders[28]) may be useful to assess whether the order imbalance measure that we have used in our study is affected or not. Furthermore, our approach to decompose the bid-ask spread could be tested on more mature commodity markets such as energy derivatives markets.

On the theoretical side, we suppose that uninformed traders are the only limit order traders. We could instead consider a setting in which informed traders can choose to place either a limit order or a market order (Bloomfield et *al*., 2009). Consequently, uninformed traders may opt for different order strategies since liquidity conditions will be necessarily affected. A complete analysis of the potential model implications is left for future work.

[28] Traders can place iceberg orders on ECX for which only a fraction of the total order quantity is disclosed. The remaining part is visible when the displayed quantity is executed, keeping price priority but losing time priority.

7. References

Beck, N., Katz, J.N., 1995. What to do (and not to do) with time-series-cross-section data. *American Political Journal Review*, 89, 634–647.

Berkman, H., Koch, P.D., 2008. Noise trading and the price formation process. *Journal of Empirical Finance*. 15(2), 232–250.

Biais, B., Foucault, T., 1993. Asymétries d'information et marchés financiers : une synthèse de la littérature récente. *L'Actualité Economique*, 69, 8–44.

Biais, B., Hillion, P., Spatt, C., 1995. An empirical analysis of the limit order book and the order flow in Paris Bourse. *The Journal of Finance,* 50, 1655–1689.

Bloomfield, R.J., O'Hara, M., Saar, G., 2009. How noise trading affects markets: An experimental analysis? *Review of Financial Studies*, 22, 2275–2302.

Chevallier, J. 2012. Econometric Analysis of Carbon Markets: The European Union Emissions Trading Scheme and the Clean Development Mechanism. Springer.

Chevallier, J., Ielpo, F., Mercier, L., 2009. Risk aversion and institutional information disclosure on the European carbon market: a case study of the 2006 compliance event. *Energy Policy,* 37 (1), 15–28.

De Long, J.B., Shleifer, A., Summers, L.H., Waldmann R.J., 1990. Noise Trader Risk in Financial Markets. *Journal of Political Economy*, 98, 4, 703–738.

Easley, D., Kiefer, N., O'Hara, M., Paperman, J., 1996. Liquidity, information, and infrequently traded stocks. *The Journal of Finance,* 51, 1405–1436.

Ellul, A., Holden, C.W., Jain, P., Jennings, R., 2007. Order dynamics: recent evidence from the NYSE. *Journal of Empirical Finance*, 14 (5), 636–661

Foster, F.D., Viswanathan, S., 1996. Strategic trading when agents forecast the forecasts of others. *The Journal of Finance,* 51(4), 1437–1478.

Frino, A., Kruk, J., Lepone, A., 2010. Liquidity and transaction costs in the European carbon futures market. *Journal of Derivatives and Hedge Funds,* 16(2), 100–115.

Foucault, T., 1999. Order flow composition and trading costs in a dynamic limit order market. *Journal of Financial Markets,* 2, 99–134

Frunza, M.-C., 2010. Carbon allowances: A new financial asset. From market fundamentals to derivatives. Edition *Universitaires Européennes*.

Goettler, R., Parlour, C., Rajan, U., 2005. Equilibrium in a Dynamic Limit Order Market. *The Journal of Finance,* 60, 2149–2192.

Gould, M.D., Porter M.A., Williams, S., McDonald, M., Fenn, D.J., Howison, S.D, 2013. Limit order books. *Quantitative Finance*, 13(11), 1709–1742.

Handa, P., Schwartz, R.A, Tiwari, A., 2003. Quote setting and price formation in an order driven market. *Journal of Financial Markets,* 6, 4, 461–489.

Ibikunle, G., Gregoriou, A., Pandit, N.R., 2016. Price impact of block trades: the curious case of downstairs trading in the EU emissions futures market. *The European Journal of Finance,* 22, 120–42.

Kalaitzoglou, I., Ibrahim, B.M., 2013. Does order flow in the European carbon allowances market reveal information? *Journal of Financial Markets,* 16, 604–635.

Kalaitzoglou, I., Ibrahim, B.M., 2016. Why do carbon prices and price volatility change? *Journal of Banking and Finance,* 63, 76–94.

Liu, W-M., 2009. Monitoring and limit order submission risks. *Journal of Financial Markets*, 12, 107–141.

Madhavan, A., Richardson, M., Roomans, M., 1997. Why do security prices change? A transaction-level analysis of NYSE Stocks. *Review of Financial Studies*, 10, 1035–64.

Marshall, B.R., Nguyen, N., Visaltanachoti, N.H, 2011. Commodity Liquidity Measurement and Transaction Costs. *Review of Financial Studies*, 25(2), 599–638.

Medina, V., Pardo, A., Pascual, R., 2014. The timeline of trading frictions in the European carbon market. *Energy Economics*, 42, 378–394.

Mizrach, B., Otsubo, Y., 2014. The market microstructure of the European climate exchange. *Journal of Banking and Finance*, 39, 107–116.

Majois, C., 2010. Order aggressiveness and the diagonal effect in experimental double auction markets. *Economics Letters*, 107(2), 304-309.

Moinas, S., 2008. Le Carnet d'Ordres: une revue de littérature. *Finance*, 29(1) 81–147.

Palao, F., Pardo, A., 2014. What makes carbon traders cluster their orders? *Energy Economics*, 43, 158–165.

Rosu, I., 2009. A Dynamic Model of the Limit Order Book. *Review of Financial Studies*, 22(11), 4601–4641.

Skjeltorp, J.A., Sojli, E., Tham, W.W., 2016. Flashes of Trading Intent at NASDAQ. *Journal of Financial and Quantitative Analysis*, 51 (1), 165-196.

Van Ness, B.F., Van Ness, R.A., Warr, R.S., 2001. How well do adverse selection components measure adverse selection? *Financial Management*, 30, 5–30.

8. Appendix

Appendix A.1. Proof of Proposition 1.1 and 1.2

$$f_{b,1}(X_{b,T}|\tilde{Z}_{b,1}) = \frac{1}{\sqrt{2\pi(1-\rho_b^2)\sigma_{b,x}^2}} \cdot \exp\left[-\frac{\left[X_{b,T} - \rho_b^2 \cdot (\tilde{Z}_{b,1} - \mu_{b,x}) - \mu_{b,x}\right]^2}{2\cdot(1-\rho_s^2)\sigma_{b,x}^2}\right]$$

$$f_{b,t}(X_{b,T}|\tilde{Z}_{b,1}) = \frac{1}{\sqrt{2\pi \cdot t\frac{OF}{B}(1-\rho_b^2)\sigma_{b,x}^2}} \cdot \exp\left[-\frac{\left[X_{b,T} - \rho_b^2(\tilde{Z}_{b,1} - \mu_{b,x}) - \mu_{b,x}\right]^2}{2\cdot t\frac{OF}{B}(1-\rho_s^2)\sigma_{b,x}^2}\right]$$

are the normal probability density function of the asset value recognized by the uninformed buyer at time 1 and *t*, respectively.

Inserting these terms in Eq. (1) and using the approximate equation of (1.1), we get:

$$1-\exp\left[-\phi\cdot\left(\rho_b^2\tilde{Z}_{b,1}+(1-\rho_b^2)\left(\mu_{b,x}-\frac{1}{2}\phi\sigma_{b,x}^2\right)+R_T(W_1-P_{ask})\right)\right]\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi\cdot t\frac{OF}{B}\sigma_{b,x}^2(1-\rho_b^2)}}\cdot\exp\left[-\frac{\left[X_{b,T}-\rho_b^2(\tilde{Z}_{b,1}-\mu_{b,x})-\mu_{b,x}+\phi(1-\rho_b^2)\sigma_{b,x}^2\right]^2}{2\cdot\sigma_{b,x}^2(1-\rho_b^2)}\right]\partial X_{b,T}$$

$$=k_s^M p_I\int_{-\infty}^{R_T P_{bid}}\frac{1}{\sqrt{2\pi\cdot t\frac{OF}{B}\sigma_{b,x}^2(1-\rho_b^2)}}\cdot\exp\left[-\frac{\left[X_{b,T}-\rho_b^2(\tilde{Z}_{b,1}-\mu_{b,x})-\mu_{b,x}\right]^2}{2\cdot t\frac{OF}{B}\sigma_{b,x}^2(1-\rho_b^2)}\right]\partial X_{b,T} \tag{1.1}$$

$$-\ \exp\left[-\phi\cdot\left(\rho_b^2\tilde{Z}_{b,1}+(1-\rho_b^2)\left(\mu_{b,x}-\frac{1}{2}\phi\sigma_{b,x}^2\right)+R_T(W_1-P_{bid})\right)\right]\int_{-\infty}^{R_T P_{bid}}\frac{1}{\sqrt{2\pi t\sigma_{b,x}^2}}\cdot\exp\left[-\frac{\left[X_{b,T}-\rho_b^2(\tilde{Z}_{b,1}-\mu_{b,x})-\mu_{b,x}\right]^2}{2\cdot\sigma_{b,x}^2(1-\rho_b^2)}\right]\partial X_{b,t}$$

$$+k_s^M(p_N+p_U)\cdot\left(1-\exp\left[-\phi\cdot\left(\rho_b^2\tilde{Z}_{b,1}+(1-\rho_b^2)\left(\mu_{b,x}-\frac{1}{2}\phi\sigma_{b,x}^2\right)+R_T(W_1-P_{bid})\right)\right]\right)\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi t\sigma_{b,x}^2}}\cdot\exp\left[-\frac{\left[X_{b,T}-\rho_b^2(\tilde{Z}_{b,1}-\mu_{b,x})-\mu_{b,x}\right]^2}{2\cdot t\frac{OF}{B}\sigma_{b,x}^2(1-\rho_b^2)}\right]\partial X_{b,t}$$

$$+\left(1-k_s^M\right)\cdot\left(1-\exp[-\phi(W_1R_T)]\right)$$

Assuming that $\tilde{V}_b = \frac{\left(P_{bid}R_T - \rho_b^2\tilde{Z}_{b,1} - (1-\rho_b^2)\mu_{b,x}\right)}{\sqrt{t\cdot\frac{OF}{B}\sigma_{b,x}^2(1-\rho_b^2)}}$ and $\tilde{Y}_b = \frac{\left(P_{bid}R_T - \rho_b^2\tilde{Z}_{b,1} - (1-\rho_b^2)\mu_{b,x} - \phi\cdot t\cdot\frac{OF}{B}\sigma_{b,x}^2(1-\rho_b^2)\right)}{\sqrt{t\cdot\frac{OF}{B}\sigma_{b,x}^2(1-\rho_b^2)}}$

and N (.) is the standard normal distribution cumulative density probability.

Further, we can rewrite (1.1) as follows as:

$$1-\exp\left[-\phi\left(\mu_{b,x}-\frac{1}{2}\phi\sigma_{b,x}^2+R_T(W_1-P_{ask})\right)\right]=k_s^M\cdot\left(1-\exp\left[-\phi\left(\rho_b^2\tilde{Z}_{b,1}+\left(\mu_{b,x}-\frac{1}{2}\phi\cdot t\frac{OF}{B}\sigma_{b,x}^2\right)(1-p_b^2)+R_T(W_1-P_{bid})\right)\right]\right) \tag{1.2}$$
$$+k_s^M p_I\cdot\left[1-N(\tilde{V}_b)-\left(1-N(\tilde{Y}_b)\right)\right]\exp\left[-\phi\left(\rho_b^2\tilde{Z}_{b,1}+\left(\mu_{b,x}-\frac{1}{2}\phi\cdot t\frac{OF}{B}\sigma_{b,x}^2\right)(1-p_b^2)+R_T(W_1-P_{bid})\right)\right]+\left(1-k_s^M\right)\cdot\left(1-\exp[-\phi W_1R_T]\right)$$

In the right hand side of (1.2), the first term represents the expected utility of order execution without the presence of informed trader counterparty. The third term is the expected utility of non-execution. The second term accounts for the expected utility loss due to informed trading since it is related to the probability of informed trading p_I and the negative signs represent the utility losses. To simplify the notation, we then consider that $\overline{F}_b = \rho_b^2\tilde{Z}_{b,1} + (1-\rho_b^2)\left(\mu_{b,x} - \frac{1}{2}\phi\sigma_{b,x}^2\right)$ and that the buyer's expected utility losses of informed trading is:

$$LOSS_b^{AS} = \left(1-N(\tilde{V}_b) - (1-N(\tilde{Y}_b)\right)\times\left[1-\exp\left(-\phi(\overline{F}_b + R_T(W_1 - P_{bid}))\right)\right] \tag{1.3}$$

We now derive the equilibrium of the buy side in indifferent expected value of the uninformed buyers' utility between trading via limit order and trading via market order consistent with the approach of Kovaleva and Iori (2012). Then we can conclude that:

$$\begin{aligned}1-\exp\left[-\phi\left(\overline{F}_b+R_T\left(W_1-P_{ask}\right)\right)\right]&=k_s^M\cdot\left(1-\exp\left[-\phi\left(\overline{F}_b+R_T\left(W_1-P_{bid}\right)\right)\right]\right)\\&\quad+k_s^M p_I\cdot LOSS_b^{AS}+\left(1-k_s^M\right)\cdot\left(1-\exp\left[-\phi\left(W_1R_T\right)\right]\right)\end{aligned}\tag{1.4}$$

In order to transfer the above utility equation (1.4) into the linear equilibrium of the expected terminal wealth, we assume the restriction that the expected value of the terminal wealth at each state is very small and positive.

Then, applying a Taylor expansion for an exponential function gives the below equation:

$$\begin{aligned}\phi\left(\overline{F}_b+R_T\left(W_1-P_{ask}\right)\right)&\approx k_s^M\cdot\phi\left[\left(\mu_{b,x}-\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-p_b^2\right)\sigma_{b,x}^2+R_T\left(W_1-P_{bid}\right)\right)\right]\\&\quad+k_s^M\cdot\frac{1}{\phi}\phi\cdot p_I\cdot LOSS_b^{AS}+\left(1-k_s^M\right)\cdot\phi\left(\overline{F}_b-P_{ask}R_T\right)\end{aligned}\tag{1.5}$$

Assuming that $LOSS_b^{AS,RA}=\frac{1}{\phi}p_I\cdot LOSS_b^{AS}$ we can rewrite (1.5) as follows as :

$$\phi\cdot k_s^M\left(P_{bid}-P_{ask}\right)\approx k_s^M\cdot\phi\left[\left(\mu_{b,x}-\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-p_b^2\right)\sigma_{b,x}^2\right)\right]+k_s^M\phi LOSS_b^{AS,RA}+\left(1-k_s^M\right)\cdot\phi\left(\overline{F}_b-P_{ask}R_T\right)\tag{1.6}$$

Notice that $LOSS_b^{AS,RA}$ which represents the expected losses of adverse selection borne by a risk averse uninformed trader is written in an original non-linear format, due to the difficulty to translate it into an approximate linear format.

If (1.6) is divided by ϕ, we get the linear expected terminal wealth equilibrium for the buy side:

$$k_s^M\left(P_{bid}-P_{ask}\right)\approx k_s^M\cdot\left[\left(\mu_{b,x}-\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-p_b^2\right)\sigma_{b,x}^2\right)\right]+k_s^M\cdot LOSS_b^{AS,RA}+\left(1-k_s^M\right)\cdot\left(\overline{F}_b-P_{ask}R_T\right)\tag{1.7}$$

Likewise, the model equilibrium implies that uninformed sellers are indifferent between via limit order or market order trading. We derive the approximation of the sell side equilibrium as we have done previously for this of the buy side.

$$k_b^M\left(P_{ask}-P_{bid}\right)\approx k_b^M\cdot\frac{1}{R_T}LOSS_s^{AS,RA}+k_b^M\cdot\frac{1}{R_T}\cdot\frac{1}{2}\phi(t-1)(1-p_s^2)\sqrt{\frac{B}{OF}}\sigma_{s,x}^2+(1-k_b^M)\cdot\frac{1}{R_T}\left(P_{bid}R_T-\overline{F}_s\right)\tag{1.8}$$

Where: $V_s=\dfrac{\left(P_{ask}R_T-\rho_s^2\tilde{Z}_{s,1}-(1-p_s^2)\mu_{s,x}\right)}{\sqrt{t\frac{B}{OF}\sigma_{s,x}^2}}$ and $Y_s=\dfrac{\left(P_{ask}R_T-\rho_s^2\tilde{Z}_{s,1}-(1-p_s^2)\mu_{s,x}-\phi\frac{OF}{B}t\cdot\sigma_{s,x}^2(1-p_s^2)\right)}{\sqrt{\frac{OF}{B}t\sigma_{s,x}^2(1-p_s^2)}}$ and

$$\overline{F}_s=\rho_s^2\tilde{Z}_{s,1}+(1-\rho_s^2)\left(\mu_{s,x}-\frac{1}{2}\phi\sigma_{s,x}^2\right)$$

$LOSS_s^{AS,RA}=\frac{1}{\phi}\cdot p_I\cdot\left\{\left[(1-N(V_s)-(1-N(Y_s))\right]\cdot(1-\exp)-\left[\phi\left(R_T\left(W_1+P_{ask}\right)-\overline{F}_s\right)\right]\right\}$ is the expected adverse selection loss faced by a risk averse uninformed seller

Appendix A.2. Proof of Proposition 3

We now attempt to examine how bid and ask prices are affected by the arrival rates of market buy and sell orders. As for the arrival rates of market buy orders, we determine its connection to price quotation mechanisms according to the first order condition: $\partial P_{ask} / k_b^M$ and $\partial P_{bid} / k_b^M$. Taking the derivative on the quotes obtained in Eqs. (5a) and (5b), we get the partial differential equations which are assumed to be positive:

$$\frac{\partial P_{ask}}{\partial k_b^m} = \frac{1}{R_T}\left[\frac{\left(1-k_b^M\right)\cdot k_b^M}{\left(1-k_s^M \cdot k_b^M\right)^2}\times\left\{\overline{F}_b - \left(\mu_{s,x} + \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_s^2\right)\cdot\sigma_{s,x}^2 + LOSS_s^{AS,RA}\right)\right\}\right]$$
$$+\frac{1}{R_T}\left[\frac{k_s^M \cdot k_s^M}{\left(1-k_s^M \cdot k_b^M\right)^2}\times\left\{\left(LOSS_b^{AS,RA} - LOSS_s^{AS,RA}\right) + \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_b^2\right)\cdot\sigma_{b,x}^2 - \left(1-\rho_s^2\right)\cdot\sigma_{s,x}^2\right\}\right] \quad (2.1)$$
$$+\frac{1}{R_T}\left(\frac{k_s^M}{\left(1-k_s^M \cdot k_b^M\right)}\left(\frac{\partial LOSS_b^{AS,RA}}{\partial k_b^M}\right) - \frac{k_b^M \cdot k_s^M}{\left(1-k_s^M \cdot k_b^M\right)}\left(\frac{\partial LOSS_s^{AS,RA}}{\partial k_b^M}\right)\right) > 0$$

$$\frac{\partial P_{bid}}{\partial k_b^M} = \frac{1}{R_T}\left[\frac{\left(1-k_b^M\right)\cdot k_b^M}{\left(1-k_s^M \cdot k_b^M\right)^2}\times\left\{\left(\mu_{b,x} - \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_b^2\right)\cdot\sigma_{b,x}^2 - LOSS_b^{AS,RA}\right) - \overline{F}_s\right\}\right]$$
$$+\frac{1}{R_T}\left[\frac{1}{\left(1-k_s^M \cdot k_b^M\right)^2}\left\{\left(LOSS_b^{AS,RA} - LOSS_s^{AS,RA}\right) + \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_b^2\right)\cdot\sigma_{b,x}^2 - \left(1-\rho_s^2\right)\cdot\sigma_{s,x}^2\right\}\right] \quad (2.2)$$
$$+\frac{1}{R_T}\left(\frac{\left(1-k_b^M\right)\cdot k_b^M}{\left(1-k_s^M \cdot k_b^M\right)}\left(\frac{\partial LOSS_b^{AS,RA}}{\partial k_b^M}\right) - \frac{k_b^M}{\left(1-k_s^M \cdot k_b^M\right)}\left(\frac{\partial LOSS_s^{AS,RA}}{\partial k_b^M}\right)\right) > 0$$

We multiply (2.1) by $\left(1-k_s^M \cdot k_b^M\right)\cdot k_b^M$. After rearranging and substituting the ask price as formulated in Eq. (5a), we obtain the following inequality:

$$k_b^M\left[P_{ask} - \frac{1}{R_T}\left(\mu_{s,x} + \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_s^2\right)\cdot\sigma_{s,x}^2 + LOSS_s^{AS,RA}\right)\right] - \frac{1}{R_T}\left(k_b^M\, \partial ASC_s / \partial k_b^M - \partial LOSS_b^{AS,RA} / \partial k_b^M\right) > 0 \quad (2.3)$$

We also multiply (2.2) by $\left(1-k_s^M \cdot k_b^M\right)\cdot k_b^M$. After rearranging and substituting the bid price as formulated in Eq. (6), we obtain the following inequality:

$$\left[P_{bid} - \left(\frac{1}{R_T}\overline{F}_s\right)\right] - \frac{1}{R_T}k_b^M \cdot \left(k_b^M\, \partial LOSS_s^{AS,RA} / \partial k_b^M - k_s^M \cdot k_b^M\, \partial LOSS_b^{AS,RA} / \partial k_b^M\right) > 0 \quad (2.4)$$

As for the arrival rates of market sell orders, we determine similarly, its connection to the ask price quotation given the following first order conditions, $\partial P_{ask} / k_s^M$ and $\partial P_{bid} / k_s^M$. Taking the derivative on the quotes obtained in Eqs. (5) and (6), we get the partial differential equations and assume them as negative:

$$\frac{\partial P_{ask}}{\partial k_s^M} = \frac{1}{R_T}\left[\frac{\left(1-k_b^M\right)}{\left(1-k_s^M \cdot k_b^M\right)^2}\left\{\overline{F}_b - \left(\mu_{s,x} + \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_s^2\right)\cdot\sigma_{s,x}^2 + LOSS_s^{AS,RA}\right)\right\}\right]$$
$$+\frac{1}{R_T}\left[\frac{1}{\left(1-k_s^M \cdot k_b^M\right)^2}\left\{\left(LOSS_b^{AS,RA} - LOSS_s^{AS,RA}\right) + \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_b^2\right)\cdot\sigma_{b,x}^2 - \left(1-\rho_s^2\right)\cdot\sigma_{s,x}^2\right\}\right] \quad (2.5)$$
$$+\frac{1}{R_T}\left(\frac{k_s^M}{\left(1-k_s^M \cdot k_b^M\right)}\left(\frac{\partial LOSS_b^{AS,RA}}{\partial k_s^M}\right) - \frac{k_b^M \cdot k_s^M}{\left(1-k_s^M \cdot k_b^M\right)}\left(\frac{\partial LOSS_s^{AS,RA}}{\partial k_s^M}\right)\right) < 0$$

$$\frac{\partial P_{bid}}{\partial k_s^M} = \frac{1}{R_T}\left[\frac{\left(1-k_b^M\right)\cdot k_b^M}{\left(1-k_s^M \cdot k_b^M\right)^2}\left\{\left(\mu_{b,x} - \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_b^2\right)\cdot\sigma_{b,x}^2 - LOSS_b^{AS,RA}\right) - \overline{F}_s\right\}\right]$$
$$+\frac{1}{R_T}\left[\frac{k_s^M \cdot k_b^M}{\left(1-k_s^M \cdot k_b^M\right)^2}\left\{\left(LOSS_b^{AS,RA} - LOSS_s^{AS,RA}\right) + \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_b^2\right)\cdot\sigma_{b,x}^2 - \left(1-\rho_s^2\right)\cdot\sigma_{s,x}^2\right\}\right]$$
$$+\frac{1}{R_T}\left(\frac{k_s^M \cdot k_b^M}{\left(1-k_s^M \cdot k_b^M\right)}\left(\frac{\partial LOSS_b^{AS,RA}}{\partial k_s^M}\right) - \frac{k_b^M}{\left(1-k_s^M \cdot k_b^M\right)}\left(\frac{\partial LOSS_s^{AS,RA}}{\partial k_s^M}\right)\right) < 0$$
(2.6)

We multiply (2.5) by $\left(1-k_s^M \cdot k_b^M\right)\cdot k_b^M$. Then, rearranging and substituting the ask price as formulated in Eq. (5) gives the following inequation:

$$k_s^M\left\{\left[\frac{1}{R_T}\left(\mu_{b,x} - \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_b^2\right)\cdot\sigma_{b,x}^2\right) - P_{bid}\right] - \frac{1}{R_T}\left(k_s^M\,\partial LOSS_b^{AS}/\partial k_s^M - \partial LOSS_s^{AS}/\partial k_s^M\right)\right\} > 0 \quad (2.7)$$

We also multiply (2.6) by $\left(1-k_s^M \cdot k_b^M\right)\cdot k_b^M$. After rearranging and substituting the bid price as formulated in Eq. (6), we obtain the following inequation:

$$\left\{\left[\frac{1}{R_T}\left(\overline{F}_b - P_{ask}\right)\right] - \frac{1}{R_T}k_s^M\cdot\left(k_s^M\,\partial LOSS_b^{AS}/\partial k_s^M - k_s^M\cdot k_b^M\,\partial LOSS_s^{AS}/\partial k_s^M\right)\right\} > 0 \quad (2.8)$$

Appendix A.3. Proof of Proposition 4

Eqs. (5) and (6) both determine the optimal prices for the buy and sell side in equilibrium. Assuming that $k = k_s^M$ and $1-k = k_b^M$ we can rewrite Eqs. (5) and (6) in the following manner:

$$P_{ask} = \frac{1}{R_T}\left[\frac{k^2}{\left(1-k(1-k)\right)}\times\left(\overline{F}_s + LOSS_s^{AS,RA} + \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_s^2\right)\sigma_{s,x}^2\right)\right] + \frac{1}{R_T}\left[\frac{(1-k)}{\left(1-k(1-k)\right)}\times\overline{F}_b\right] \quad (3.1)$$
$$+\frac{1}{R_T}\left[\frac{k}{\left(1-k(1-k)\right)}\times\left\{\left(LOSS_b^{AS,RA} - LOSS_s^{AS,RA}\right) + \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_s^2\right)\sigma_{b,x}^2 - \left(1-\rho_b^2\right)\sigma_{s,x}^2\right\}\right]$$

$$P_{bid} = \frac{1}{R_T}\left[\frac{k}{1-k(1-k)}\times(1-\rho_s^2)\left(\mu_{s,x} + \frac{1}{2}\phi\sigma_{s,x}^2\right)\right] + \frac{1}{R_T}\left[\frac{(1-k)^2}{1-k(1-k)}\times\left(\mu_{b,x} - LOSS_b^{AS,RA} - \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_s^2\right)\sigma_{s,x}^2\right)\right] \quad (3.2)$$
$$+\frac{1}{R_T}\left[\frac{1-k}{1-k(1-k)}\times\left(\left[LOSS_b^{AS,RA} - LOSS_s^{AS,RA}\right] + \frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)\left(1-\rho_b^2\right)\sigma_{b,x}^2 - \left(1-\rho_s^2\right)\sigma_{s,x}^2\right)\right]$$

Then, we get the comprehensive equilibrium and the associated optimal bid and ask prices:

$$P_{ask}=\frac{1}{R_T}\left[\frac{(1-k)\cdot k}{(1-k(1-k))}\times\left(\overline{F}_s+LOSS_s^{AS,RA}+\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)(1-\rho_s^2)\cdot\sigma_{s,x}^2\right)\right]+\frac{1}{R_T}\left[\frac{(1-k)}{(1-k\cdot(1-k))}\times\overline{F}_b\right]$$
$$+\frac{1}{R_T}\left[\frac{k}{(1-k\cdot(1-k))}\times\left(\left[LOSS_b^{AS,RA}-LOSS_s^{AS,RA}\right]+\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)(1-\rho_s^2)\sigma_{b,x}^2-(1-\rho_b^2)\sigma_{s,x}^2\right)\right] \quad (3.3)$$

$$P_{bid}=\frac{1}{R_T}\left[\frac{k}{1-k\cdot(1-k)}\times\overline{F}_s\right]+\frac{1}{R_T}\left[\frac{(1-k)^2}{1-k\cdot(1-k)}\times\left(\overline{F}_b-LOSS_b^{AS,RA}-\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)(1-\rho_b^2)\sigma_{b,x}^2\right)\right]$$
$$+\frac{1}{R_T}\left[\frac{1-k}{1-k\cdot(1-k)}\times\left(\left[LOSS_b^{AS,RA}-LOSS_s^{AS,RA}\right]+\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)(1-\rho_b^2)\sigma_{b,x}^2-(1-\rho_s^2)\sigma_{s,x}^2\right)\right] \quad (3.4)$$

Finally, we can simplify (3.3) and (3.3) respectively following the equations:

$$P_{ask}=\delta\cdot\frac{\overline{F}_s}{R_T}+(1-\delta)\cdot\frac{\left(\overline{F}_s+LOSS_s^{AS,RA}+\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)(1-\rho_s^2)\sigma_{s,x}^2\right)}{R_T}+\gamma\cdot\frac{\left(LOSS_b^{AS,RA}-LOSS_s^{AS,RA}\right)}{R_T} \quad (3.5)$$

$$P_{bid}=\gamma\frac{\overline{F}_s}{R_T}+(1-\gamma)\frac{\left(\overline{F}_b-LOSS_b^{AS,RA}-\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)(1-\rho_b^2)\sigma_{b,x}^2\right)}{R_T}+\delta\frac{\left(LOSS_b^{AS,RA}-LOSS_s^{AS,RA}\right)}{R_T} \quad (3.6)$$

Where the following weights $\gamma=\frac{k}{1-k(1-k)}$; $\delta=\frac{k}{1-k(1-k)}$ where k is defined as in §2.2.

Next, we compute the difference **$P_{ask}-P_{bid}$** and consider the following simplifications:

$$\overline{RV}_b=\rho_b^2\tilde{Z}_{b,t}+(1-\rho_b^2)\cdot(\mu_x^b-\frac{1}{2}\phi\sigma_{b,x}^2)\ ;\ \overline{RV}_s=\rho_s^2\tilde{Z}_{s,t}+(1-\rho_s^2)\cdot(\mu_x^s+\frac{1}{2}\phi\sigma_{s,x}^2)$$

$$ASC_b=\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)(1-\rho_b^2)\sigma_{b,x}^2+LOSS_b^{AS}\ ;\ ASC_s=\frac{1}{2}\phi\sqrt{\frac{B}{OF}}(t-1)(1-\rho_s^2)\sigma_{s,x}^2+LOSS_s^{AS}.$$

We finally obtain the equation of the equilibrium bid- ask spread as formulated in Eq. (7).

Appendix A.4. Proof of Corollaries 1 and 2

According to the results of **Proposition 3**, we have already obtained the optimal bid and ask prices. As we know, the optimal ask minus the optimal bid equals the equilibrium spread. Assuming that **$\rho_b=\rho_s=1$,** $(1-\rho_b^2)(\mu_{b,x}-\frac{1}{2}\phi\sigma_{b,x}^2)$ and $(1-\rho_s^2)(\mu_{s,x}+\frac{1}{2}\phi\sigma_{s,x}^2)$ take the value 0. We thus obtain the simplified version of Eq. (6) which is summarized in **Corollary 1**:

$$\pi'=\frac{\omega_1}{R_T}\cdot\left(\tilde{Z}_{b,t}-\tilde{Z}_{s,t}\right)+\frac{\omega_2}{R_T}\cdot ASC_b+\frac{\omega_3}{R_T}\cdot ASC_s \quad (4.1)$$

With the following weighting factors: $\omega_1=\frac{k(1-k)}{1-k(1-k)}$; $\omega_2=\frac{k^2}{1-k(1-k)}$; $\omega_3=\frac{(1-k)^2}{1-k(1-k)}$

Next we obtain $\omega_1+\omega_2+\omega_3=1$.

To facilitate the demonstration, uninformed buyers and sellers are assumed to suffer from identical adverse selection losses. With this assumption, we get $ASC=ASC_b=ASC_s$. Since the riskless asset value R_T is stable $\forall t\in[0,T]$ we then rewrite (4.1) as follows:

$$\pi' = \frac{k(1-k)}{1-k(1-k)}\left(\tilde{Z}_{b,t} - \tilde{Z}_{s,t}\right) + \frac{(1-2k+2k^2)}{1-k(1-k)}\text{ASC} \tag{4.2}$$

π is a $C^2(\Re)$ function, we calculate the first order derivative of (4.2) given the parameter k :

$$\frac{\partial\pi'}{\partial k} = \frac{(1-2k)}{[1-k(1-k)]^2}\left(\tilde{Z}_{b,t} - \tilde{Z}_{s,t}\right) + \frac{(1-2k)}{[1-k(1-k)]^2}\text{ASC}$$

Under the assumption that uninformed buyers and sellers suffer the same level of adverse selection losses, we find $\frac{\partial\pi'}{\partial k} = 0$ with k=1/2. Since $\frac{\partial^2\pi'}{\partial^2 k} < 0$, π' reaches a maximum for k=1/2 and is equally weighted with $\omega_1 = \omega_2 = \omega_3 = 1/3$. The second order derivative is negative, implying that π' is a concave function of k. π achieves therefore a minimum for the first order conditions k= 0 and k=1 respectively.

In **Corollary 2**, we assume the case for which the precision of noisy signal is totally imperfect *i.e.* $\sqrt{\frac{B}{OF}}$ tends to 0. We then rewrite Eq. (7) in a simplified equation such that:

$$\pi'' = \frac{\omega_1}{R_T}\cdot(\overline{RV}_b'' - \overline{RV}_b'') + \frac{\omega_2}{R_T}\cdot \text{ASC}_b + \frac{\omega_3}{R_T}\cdot \text{ASC}_s \tag{4.3}$$

Where $\overline{RV}_b'' = \mu_{b,x} + \frac{1}{2}\phi\sigma^2_{b,x}$ (*resp.* $\overline{RV}_s'' = \mu_{s,x} + \frac{1}{2}\phi\sigma^2_{s,x}$) is the reservation asset value of the risk averse uninformed buyer (*resp.* seller) and ASC_b and ASC_s are defined as in Eq. (7).

We proceed in a similar manner as the previous case to determine the minimum and maximum value of π''. We also verify that the two results implied by **Corollary 2** remain valid so that π'' achieves a minimum (maximum) for k= 1/2 (k= 0 or k=1) respectively.

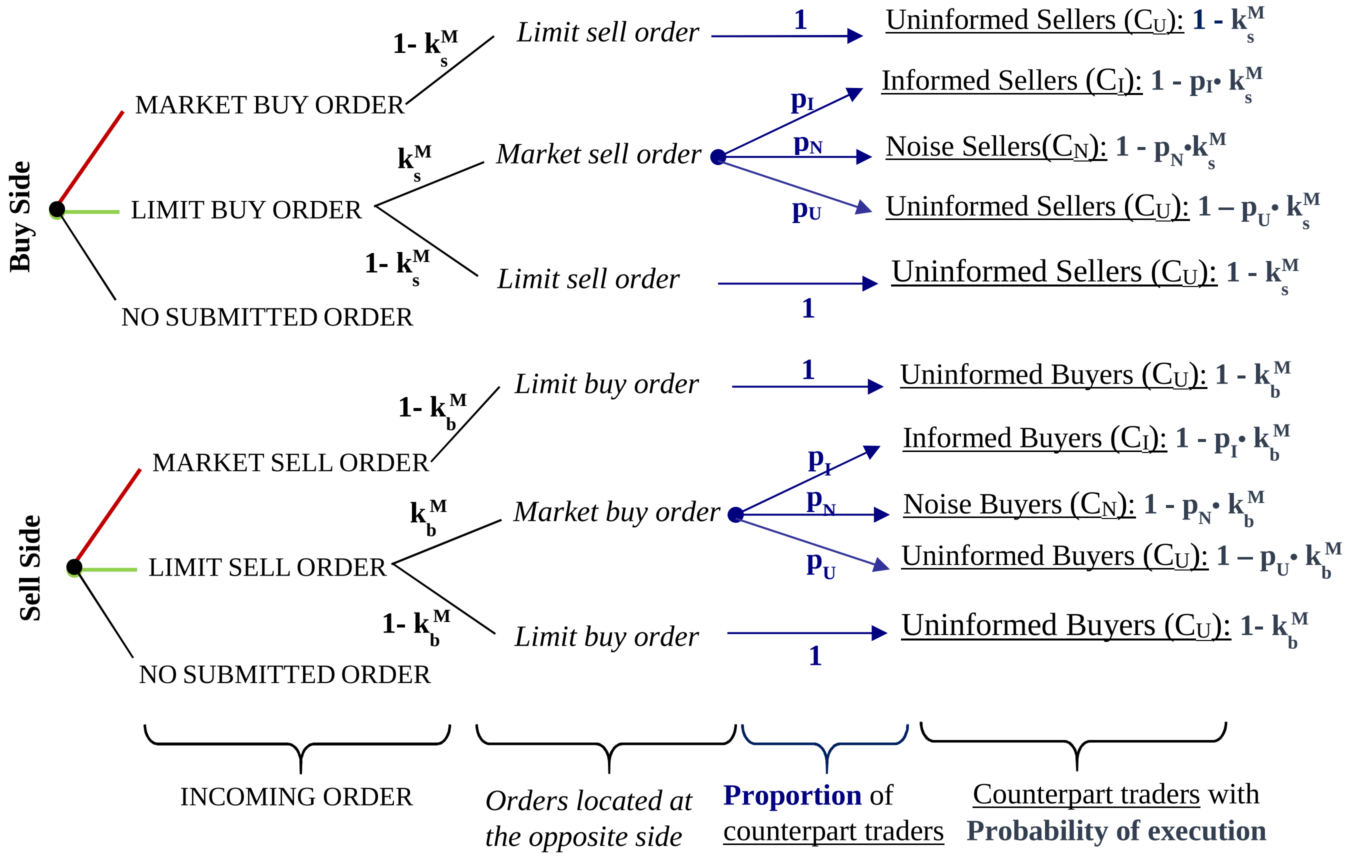

FIG. 1. Order placement: the uninformed trader's decision tree.
The following protocol for the execution of limit and market orders is applied. At Time 2, all market orders are executed whereas limit orders are executed at time t (with t ≥2). At the end of the trading period (*i.e.* time T), the liquidation of the risky asset value occurs.

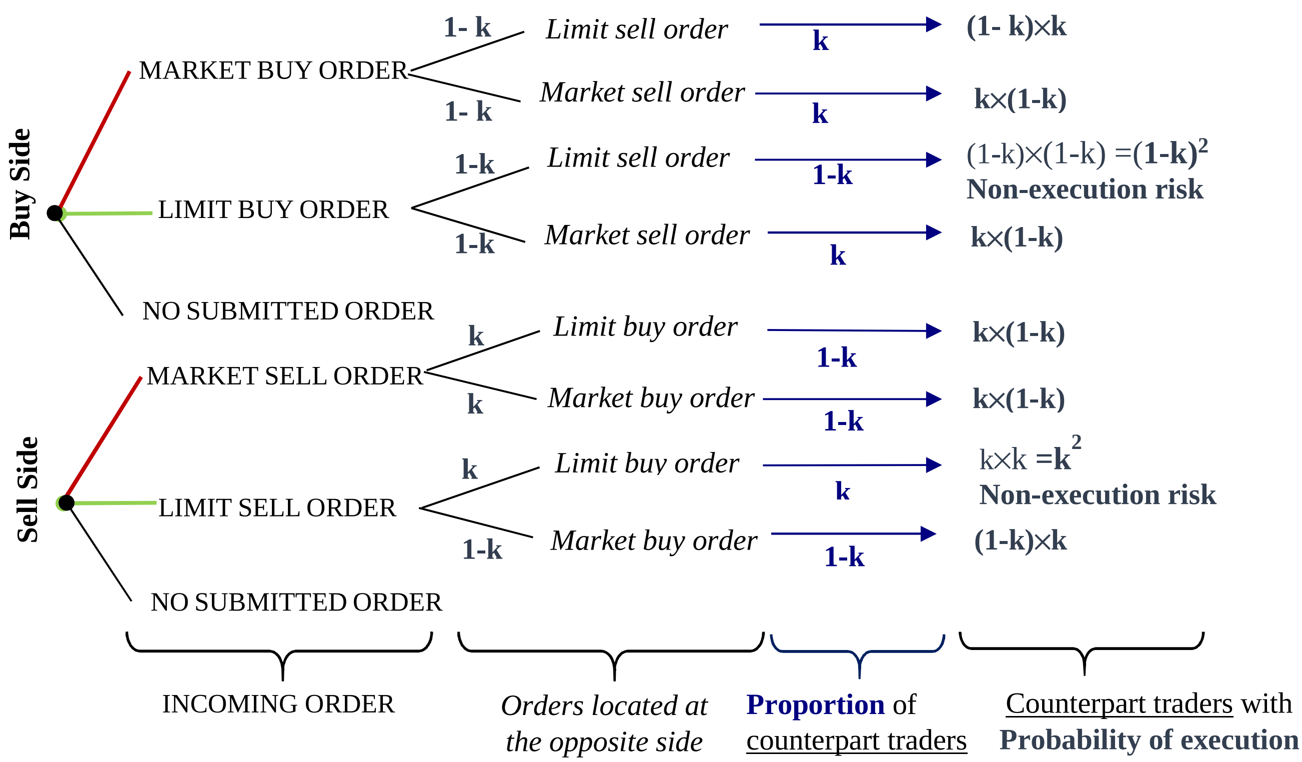

FIG. 2. Order placement (*reduced form of the model*): the uninformed trader's decision tree.
The market protocol for the execution of limit and market orders used in Fig. 1 is applied.

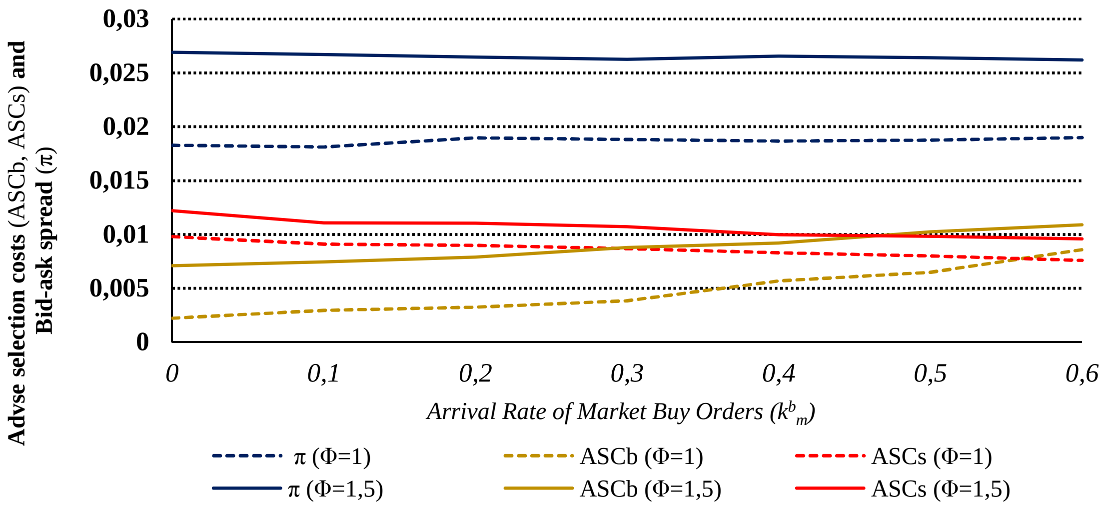

FIG. 3. The behaviour of adverse selection costs and bid-ask spread when only the arrival rate of buy market orders varies (this of market sell orders is 0.5 and remain constant)

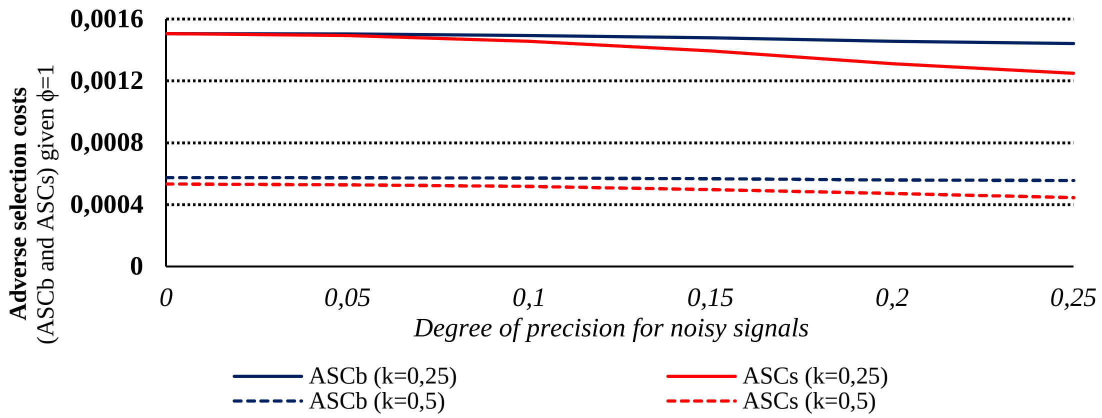

FIG. 4. The behavior of adverse selection costs and bid-ask spread as the precision of noisy buyers' and sellers' signals ($\sqrt{\frac{B}{OF}}\times\rho_b$ and $\sqrt{\frac{B}{OF}}\times\rho_s$) are simultaneously and equally varied

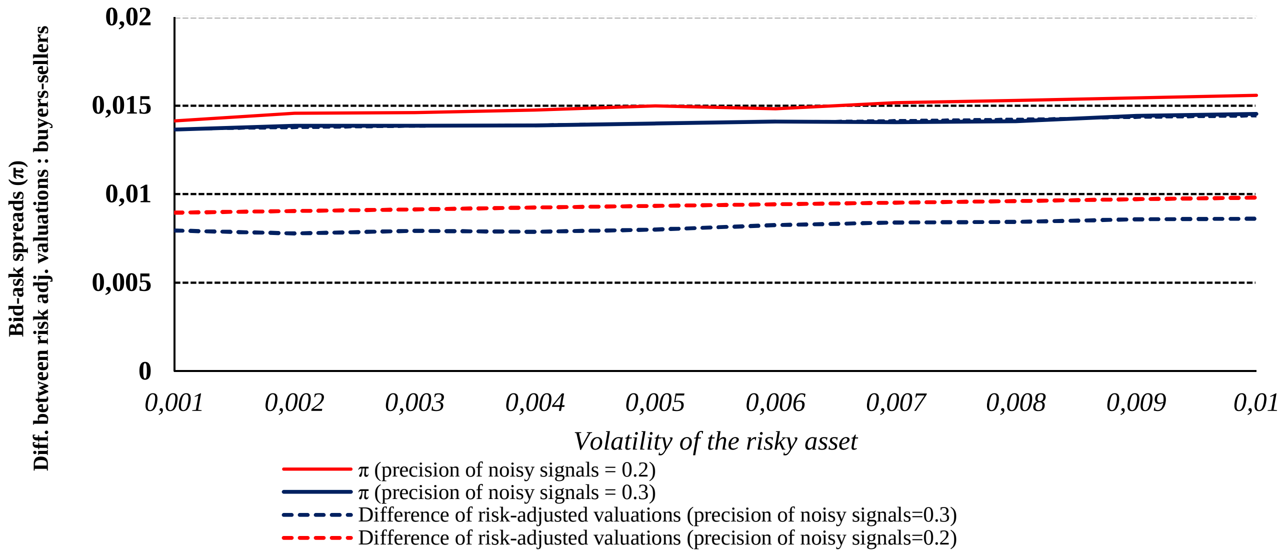

FIG. 5. The behaviour of bid-ask spread and differences in risk adjusted valuations as standard deviation of buyers and sellers' reservation values are equally varied

TABLE 1. Descriptive monthly statistics

The dataset contains the history of the order book concerning each of the 5 EUA December futures, studied during their last year of trading before expiry, from January 3, 2008 to December 31, 2012 (*i.e.* 254 trading days). All variables are calculated on annual basis and their mean (among the five futures contacts) are reported for each month. The proportions of sellers to all traders are defined as:

$$k = \frac{\text{number of trades at the ask} + \text{limit sell orders submitted}}{\text{number of trades at the ask and bid} + \text{limit buy and sell orders submitted}} \times 100\%$$

Panel A: Order book (LOB) liquidity measures

	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	March
Mean PBAS (%)	**0.830**	**0.819**	**0.817**	**0.817**	**0.811**	**0.810**	**0.816**	**0.820**	**0.822**	**0.831**	**0.833**	**0.834**
k % *(mean)*	50.90	51.20	51.62	51.73	51.98	52.01	51.69	51.16	51.02	50.91	50.95	50.93
PBAS% when ***k%<50%***	*0.827*	*0.819*	*0.826*	*0.822*	*0.818*	*0.818*	*0.816*	*0.82*	*0.821*	*0.825*	*0.821*	*0.825*
PBAS% when ***k%>50%***	*0.837*	*0.832*	*0.819*	*0.818*	*0.812*	*0.817*	*0.817*	*0.819*	*0.825*	*0.833*	*0.837*	*0.842*
ω_1**%** *(mean)*	33.67	33.13	30.47	28.30	26.60	26.30	23.37	25.53	28.30	30.87	28.97	25.10
ω_2**%** *(mean)*	30.17	32.23	32.73	33.17	38.73	35.37	37.70	36.00	33.17	33.67	35.17	36.80
ω_3**%** *(mean)*	36.17	34.63	36.80	38.53	34.67	38.33	38.93	38.47	38.53	35.47	35.87	38.10

Panel B: Intraday behavior of the proportional bid-ask spread (*PBAS*%)

	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	March
7:00 to 9:00	*0.835*	*0.832*	*0.818*	*0.819*	*0.811*	*0.818*	*0.82*	*0.828*	*0.83*	*0.836*	*0.839*	*0.841*
9 :00 to 15:00	*0.822*	*0.827*	*0.813*	*0.816*	*0.805*	*0.807*	*0.812*	*0.815*	*0.821*	*0.822*	*0.824*	*0.828*
15:00 to 17:00	*0.834*	*0.823*	*0.823*	*0.821*	*0.813*	*0.82*	*0.815*	*0.824*	*0.829*	*0.834*	*0.838*	*0.839*

TABLE 2. Ordinary least squares regression of *PBAS* on ω_1, ω_2, ω_3 including size and time intervals

We first conduct regressions on the proportional bid-ask spreads (PBAS) according to the model outlined in Eq. (9) using time intervals of a trading day (*Panel A*):

$$\mathrm{PBAS}_t = \overline{\mathrm{RV}}_{b-s} + \sum_{j=1}^{3} \omega_2 \cdot \mathrm{INT}_j \cdot \left(\mathrm{ASC}_{b,t} - \overline{\mathrm{RV}}_{b-s}\right) + \sum_{j=1}^{3} \omega_3 \cdot \mathrm{INT}_j \cdot \left(\mathrm{ASC}_{s,t} - \overline{\mathrm{RV}}_{b-s}\right) + \varepsilon_t$$

We then perform regressions on the proportional bid-ask spreads (PBAS) according to the model presented in Eq. (10) using both time intervals and trade size as control variables (*Panel B*):

$$\mathrm{PBAS}_t = \overline{\mathrm{F}}_{b-s} + \sum_{i=0}^{3}\sum_{j=1}^{3} \omega_2 \cdot \mathrm{SIZE}_i \cdot \mathrm{INT}_j \cdot \left(\mathrm{ASC}_{s,t} - \overline{\mathrm{F}}_{b-s}\right) + \sum_{i=0}^{3}\sum_{j=1}^{3} \omega_3 \cdot \mathrm{SIZE}_i \cdot \mathrm{INT}_j \left(\mathrm{ASC}_{b,t} - \overline{\mathrm{F}}_{b-s}\right) + \varepsilon_t$$

For these two types of regressions, we proceed in four steps. We partition each trading day into three periods as in Eq. (8). Particularly, INT_1 correspond to 7:00 -9:00, INT_2: 9:00 -15:00, INT_3: 15:00- 17:00. Second, we establish three categories: SIZE_1, SIZE_2, SIZE_3 which correspond respectively to a trade size between 1 and 19 contracts, 20 and 49 contracts, and more than 50 contracts. Third, we calculated and classified simulation data according to the parameter values displayed in Eq. (11). Finally, the mean coefficients of $\boldsymbol{\omega_1}$**,** $\boldsymbol{\omega_2}$**,** $\boldsymbol{\omega_3}$, are averaged on a monthly basis, and test their significance with t-statistics (t-stat) to test whether they are significantly different from zero. The last low presents the value of F-test of the regressions.

Panel A: Ordinary least squares regression of *PBAS* on $\boldsymbol{\omega_1}$**,** $\boldsymbol{\omega_2}$**,** $\boldsymbol{\omega_3}$ using time intervals (INT)

		April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jany	Feb.	March	*Simul.*
W₁ Mean Coeff.		**0.0026**	**0.0028**	**0.0022**	**0.0023**	**0.0019**	**0.0018**	**0.0019**	**0.0021**	**0.0023**	**0.0028**	**0.0026**	**0.0021**	***40.2***
	t-stat	*92*	*115*	*74*	*81*	*50*	*45*	*51.4*	*64.9*	*68.3*	*104.5*	*93.9*	*68*	*258*
W₂ Mean Coeff.	**INT₁**	**0.0004**	**0.0023**	**0.0013**	**0.0001**	**0.0012**	**0.0013**	**-0.0004**	**-0.0003**	**-0.0004**	**0.0005**	**0.0009**	**0.0007**	***-37.9***
	t-stat	*6.5*	*89*	*23.6*	*3.2*	*20.1*	*21.1*	*-6.6*	*-5.7*	*-10.2*	*8.5*	*12.1*	*10.3*	*-244*
	INT₂	**0.0002**	**0.0007**	**0.0005**	**0.0001**	**0.0001**	**0.0005**	**0.0002**	**0.001**	**0.0017**	**-0.0003**	**0.0005**	**0.0004**	***-38.4***
	t-stat	*4.7*	*7.3*	*8.8*	*3.4*	*2.9*	*8.7*	*5.3*	*14.9*	*32.4*	*-6.0*	*8.8*	*7.1*	*-235*
	INT₃	**-0.0001**	**-0.0006**	**0.0008**	**-0.0017**	**-0.0016**	**-0.0011**	**-0.002**	**-0.0015**	**-0.0007**	**0.0002**	**-0.0005**	**0.0003**	**-35.7**
	t-stat	*-2.7*	*-9.2*	*-11.6*	*-32.5*	*-30.0*	*-14.1*	*-62.8*	*-28.7*	*-10.9*	*4.2*	*-8.7*	*5.2*	*-222*
W₃ Mean Coeff	**INT₁**	**0.0001**	**-0.0009**	**0.0007**	**0.0013**	**0.0011**	**0.0013**	**0.0003**	**-0.0009**	**0.0007**	**0.0011**	**0.001**	**0.0005**	***-37.5***
	t-stat	*2.7*	*-12.5*	*10.2*	*23.1*	*14.8*	*21.9*	*4.9*	*-16.3*	*12.1*	*14.6*	*13.9*	*8.8*	*-244*
	INT₂	**-0.0003**	**-0.0019**	**0.0002**	**-0.0012**	**0.0006**	**0.0011**	**-0.001**	**0.0007**	**0.0008**	**-0.0005**	**-0.0009**	**-0.001**	***-35.7***
	t-stat	*-5.1*	*-32*	*4.2*	*-17.9*	*9.3*	*15.9*	*-14.3*	*8.6*	*11.0*	*-8.9*	*-12.9*	*-14.1*	*-235*
	INT₃	**0.0001**	**0.0018**	**0.0002**	**0.0018**	**0.0021**	**0.0015**	**0.0029**	**0.0017**	**0.0012**	**0.0014**	**0.0013**	**0.0008**	***-37.5***
	t-stat	*3.2*	*28*	*4.1*	*42.2*	*54.4*	*32.8*	*67.3*	*27.9*	*17.0*	*22.7*	*20.9*	*9.6*	*-222*
F-statistic		**1704**	**3259**	**1580**	**1356**	**1332**	**3258**	**3112**	**3912**	**2290**	**1948**	**2161**	**3652**	***2224***

Panel B: Ordinary least squares regression of *PBAS* on **ω_1, ω_2, ω_3** including time intervals (INT) and trade size (SIZE) as control variables

		April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jany	Feb.	March	*Simul.*
W_1 Avg Coeff.		**0.0055**	**0.0056**	**0.0067**	**0.0069**	**0.0062**	**0.0061**	**0.0062**	**0.0058**	**0.0055**	**0.006**	**0.0063**	**0.0061**	***44.3***
t-stat		*136*	*174*	*247*	*255*	*201*	*198*	*202*	*185*	*167*	*194*	*206*	*196*	*289*
W_2 Avg Coeff..	**INT$_1$. SIZE$_1$**	**0.0013**	**0.0001**	**0.0011**	**-0.0004**	**-0.0002**	**-0.0002**	**-0.0002**	**0.0002**	**0.0007**	**0.001**	**0.0005**	**0.0006**	***-44.4***
	t-stat	*23.6*	*2.3*	*17.8*	*-7.2*	*-3.1*	*-2.9*	*-3.1*	*4.1*	*8.2*	*17.2*	*9.8*	*11.1*	*-219*
	INT2. SIZE$_1$	**0.0005**	**0.0007**	**0.0002**	**0.0001**	**0.0001**	**0.0003**	**-0.0008**	**-0.001**	**0.0023**	**0.001**	**0.0006**	**0.0012**	***-44.5***
	t-stat	*5.3*	*7.9*	*4.1*	*2.8*	*2.5*	*3.7*	*-10.2*	*-17.2*	*32.9*	*17.7*	*10.3*	*22.9*	*-222*
	INT$_3$. SIZE$_1$	**-0.001**	**-0.0002**	**-0.0012**	**-0.0013**	**0.0004**	**-0.0003**	**-0.0002**	**-0.0001**	**-0.0001**	**0.0007**	**0.0001**	**0.0001**	***-43.8***
	t-stat	*-19.2*	*-3.1*	*-22.3*	*-24.3*	*4.3*	*-4.1*	*-3.1*	*-2.8*	*-2.7*	*14.1*	*3.1*	*2.8*	*-209*
W_3 Avg. Coeff	**INT$_1$. SIZE$_1$**	**0.0009**	**-0.001**	**0.0002**	**0.001**	**0.0002**	**0.0006**	**0.0011**	**0.0011**	**0.0015**	**0.0012**	**0.0008**	**0.0013**	***-42.9***
	t-stat	*14.8*	*-15.4*	*3.3*	*15.3*	*4.2*	*11.9*	*20.8*	*20.5*	*28.8*	*20.9*	*8.9*	*23.9*	*-198*
	INT$_2$. SIZE$_1$	**-0.0013**	**0.0005**	**-0.0001**	**-0.001**	**0.0002**	**0.0005**	**-0.0003**	**-0.0004**	**0.0001**	**-0.0009**	**0.0006**	**0.0008**	***-42.4***
	t-stat	*-23.8*	*8.1*	*-1.9*	*-15.6*	*2.2*	*9.2*	*-4.2*	*-5.2*	*2.3*	*-14.9*	*10.9*	*13.2*	*-190*
	INT$_3$. SIZE$_1$	**0.0007**	**0.0006**	**-0.0005**	**0.0007**	**0.0001**	**0.0018**	**0.0001**	**0.0019**	**0.0018**	**0.0015**	**0.0017**	**0.001**	***-42.5***
	t-stat	*12.6*	*11.8*	*-9.8*	*12.9*	*1.9*	*34.2*	*1.9*	*40.5*	*38.1*	*28.2*	*32.2*	*19.8*	*-190*
		April	**May**	**June**	**July**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**	**Jany**	**Feb.**	**March**	***Simul.***
W_1 Avg Coeff.		**0.0055**	**0.0056**	**0.0067**	**0.0069**	**0.0062**	**0.0061**	**0.0062**	**0.0058**	**0.0055**	**0.006**	**0.0063**	**0.0061**	***44.3***
t-stat		*136*	*174*	*247*	*255*	*201*	*198*	*202*	*185*	*167*	*194*	*206*	*196*	*289*
W_2 Avg Coeff..	**INT$_1$. SIZE$_2$**	**0.0015**	**0.0014**	**0.0002**	**0.0009**	**-0.0002**	**-0.0001**	**0.0006**	**0.0007**	**0.0009**	**-0.0008**	**0.0003**	**0.0008**	***-43.9***
	t-stat	*28.2*	*26.2*	*4.3*	*15.7*	*-4.1*	*-2.9*	*11.8*	*12.9*	*14.5*	*-13.2*	*5.7*	*13.2*	*-210*
	INT2. SIZE$_2$	**0.0014**	**0.0002**	**0.0002**	**0.0009**	**0.0003**	**-0.0003**	**0.0014**	**0.0012**	**0.0008**	**0.0007**	**0.0019**	**0.0014**	***-44.8***
	t-stat	*25.9*	*3.1*	*3.0*	*15.6*	*5.7*	*-5.7*	*25.1*	*20.1*	*8.9*	*10.8*	*31.7*	*26.9*	*-230*
	INT$_3$. SIZE$_2$	**-0.0011**	**0.0006**	**0.0001**	**-0.0002**	**0.0007**	**0.0002**	**0.0007**	**-0.0009**	**0.0011**	**0.0009**	**-0.0011**	**-0.001**	***-43.9***
	t-stat	*-21.1*	*9.8*	*2.2*	*-4.3*	*10.6*	*4.1*	*10.8*	*-14.1*	*19.1*	*14.8*	*-19.2*	*-18.1*	*-209*
W_3 Avg. Coeff	**INT$_1$. SIZE$_2$**	**0.002**	**0.0015**	**0.0002**	**-0.0001**	**0.0003**	**-0.0001**	**0.0015**	**0.0003**	**0.0002**	**0.0001**	**0.0009**	**0.0013**	***-42.5***
	t-stat	*34.9*	*27.3*	*4.3*	*-2.8*	*5.7*	*-2.8*	*27.8*	*5.7*	*4.1*	*1.9*	*10.9*	*24.8*	*-191*
	INT$_2$. SIZE$_2$	**-0.0012**	**0.0007**	**-0.0002**	**-0.0013**	**-0.0001**	**0.0004**	**-0.0002**	**-0.001**	**0.0013**	**0.001**	**0.0016**	**-0.0008**	***-42.2***
	t-stat	*-22.3*	*11.9*	*-4.1*	*-22.7*	*-3.0*	*7.2*	*-4.9*	*16.1*	*24.1*	*18.7*	*29.1*	*-13.2*	*-187*
	INT$_3$. SIZE$_2$	**0.0017**	**0.0006**	**0.0005**	**0.0018**	**0.0019**	**0.0002**	**0.0018**	**0.0008**	**0.0010**	**0.0011**	**0.0007**	**0.0015**	***-41.2***
	t-stat	*31.6*	*9.8*	*8.8*	*32.9*	*33.4*	*4.2*	*33.2*	*11.0*	*18.7*	*19.9*	*8.1*	*28.8*	*-189*
		April	**May**	**June**	**July**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**	**Jany**	**Feb.**	**March**	***Simul.***
W_1 Avg Coeff.		**0.0055**	**0.0056**	**0.0067**	**0.0069**	**0.0062**	**0.0061**	**0.0062**	**0.0058**	**0.0055**	**0.006**	**0.0063**	**0.0061**	***44.3***
t-stat		*136*	*174*	*247*	*255*	*201*	*198*	*202*	*185*	*167*	*194*	*206*	*196*	*289*
W_2 Avg Coeff..	**INT$_1$. SIZE$_3$**	**0.0012**	**0.0008**	**-0.0002**	**0.0005**	**-0.0005**	**0.001**	**0.001**	**0.0011**	**0.0004**	**0.0005**	**0.0003**	**0.0009**	***-43.8***
	t-stat	*21.3*	*12.8*	*-4.3*	*9.8*	*-9.6*	*19.8*	*18.9*	*20.9*	*7.2*	*9.5*	*5.7*	*15.7*	*-209*
	INT$_2$. SIZE$_3$	**-0.0014**	**-0.0007**	**0.0004**	**0.0003**	**0.0004**	**0.0001**	**0.0012**	**-0.0009**	**0.0017**	**0.0013**	**0.0012**	**0.0014**	***-43.7***
	t-stat	*-26.9*	*-11.7*	*7.2*	*5.7*	*7.3*	*2.0*	*21.7*	*-13.6*	*31.2*	*24.3*	*21.1*	*26.8*	*-208*
	INT$_3$. SIZE$_3$	**0.0012**	**0.0004**	**0.0001**	**0.0006**	**0.0001**	**-0.0001**	**-0.0003**	**0.0012**	**-0.0011**	**0.0005**	**0.0013**	**0.002**	***-43.4***
	t-stat	*21.3*	*6.8*	*2.8*	*9.3*	*2.8*	*-2.8*	*-5.7*	*21.2*	*-20.3*	*7.5*	*22.0*	*3.9*	*-205*
W_3 Avg. Coeff	**INT$_1$. SIZE$_3$**	**0.0011**	**0.0005**	**-0.0001**	**0.0001**	**-0.001**	**0.0004**	**0.0009**	**0.0008**	**0.0003**	**0.001**	**-0.0005**	**0.0011**	***-41.2***
	t-stat	*20.1*	*9.8*	*-2.9*	*2.6*	*-19.9*	*7.2*	*10.1*	*13.3*	*6.1*	*17.9*	*-9.8*	*20.3*	*-189*
	INT$_2$. SIZE$_3$	**0.0007**	**-0.0003**	**0.0004**	**-0.0011**	**0.0004**	**-0.0014**	**0.0001**	**-0.0004**	**-0.0004**	**-0.0009**	**0.0018**	**0.0001**	***-41.2***
	t-stat	*11.6*	*-5.7*	*7.2*	*-2.9*	*7.2*	*-26.2*	*1.9*	*-7.1*	*-7.4*	*-15.1*	*36.9*	*1.1*	*-188*
	INT$_3$. SIZE$_3$	**0.0001**	**0.0006**	**0.0013**	**0.0001**	**0.0003**	**0.0012**	**0.0008**	**0.0009**	**0.0008**	**0.0007**	**0.0004**	**0.0005**	***-41.2***
	t-stat	*2.6*	*10.8*	*22.8*	*2.9*	*5.0*	*21.2*	*12.6*	*13.2*	*12.7*	*11.9*	*7.1*	*8.8*	*-189*
F-statistic		**2487.2**	**2165.1**	**2052.3**	**1734.1**	**2004.9**	**3536.3**	**3311.2**	**2651.9**	**2799**	**3133.2**	**4219.4**	**4346.4**	*2237.9*

TABLE 3. Estimation of the bid-ask spread components: **ω_1, ω_2, ω_3** according to prior regression using time intervals and trade size

From the coefficients presented in Table 2, we obtain **ω_2** (*resp.* **ω_3**) by multiplying **ω_2** $.SIZE_j.INT_i$ (*resp.* $SIZE_j$ **ω_3**$.INT_i$) with **i=1,2,3** and adding the intercept term **ω_1**.
ASC_b *(resp.* **ASC_s***) % Spread* are the sellers' (*resp.* buyers') expected loss of adverse selection costs expressed as a percentage of the bid-ask spread: PBAS%.
The column *Simul.* contains the simulated adverse selection costs for buyers and sellers calculated from their corresponding coefficients displayed in Table 2.

Panel A: Estimated value of bid-ask spread components according to prior regression using time intervals (see *Panel A* Table 2)

		April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jany	Feb.	March		*Simul.*
Opening Hours (7:00 to 9:00)	**ASC_b**	**0.23**	**0.24**	**0.28**	**0.29**	**0.31**	**0.35**	**0.345**	**0.34**	**0.34**	**0.345**	**0.35**	**0.355**	$ASC_b\rho_b\sigma_b$	***1.3***
	% Spread	*30.3%*	*30.5%*	*33.7%*	*34.5%*	*32.2%*	*37.0%*	*36.6%*	*35.6%*	*35.5%*	*34.7%*	*35.2%*	*36.1%*		
	ASC_s	**0.30**	**0.29**	**0.30**	**0.32**	**0.32**	**0.34**	**0.34**	**0.34**	**0.35**	**0.37**	**0.38**	**0.385**	$ASC_s\rho_s\sigma_s$	***2.2***
	% Spread	*37.0%*	*35.7%*	*35.6%*	*35.1%*	*32.5%*	*36.7%*	*36.0%*	*36.0%*	*37.1%*	*37.7%*	*37.8%*	*38.1%*		
Normal Hours (9:00 to 15:00)	**ASC_b**	**0.235**	**0.24**	**0.27**	**0.29**	**0.31**	**0.36**	**0.35**	**0.35**	**0.36**	**0.34**	**0.355**	**0.36**	$ASC_b\rho_b\sigma_b$	***2.6***
	% Spread	*30.9%*	*31.0%*	*33.9%*	*34.9%*	*32.3%*	*37.5%*	*37.3%*	*35.9%*	*35.2%*	*33.7%*	*36.2%*	*36.3%*		
	ASC_s	**0.27**	**0.26**	**0.30**	**0.31**	**0.30**	**0.31**	**0.325**	**0.35**	**0.36**	**0.365**	**0.37**	**0.37**	$ASC_s\rho_s\sigma_s$	***4.5***
	% Spread	*35.5%*	*34.8%*	*35.2%*	*35.0%*	*32.3%*	*34.5%*	*35.3%*	*35.9%*	*36.0%*	*35.8%*	*36.6%*	*36.5%*		
End of Day Hours (9:00 to 15:00)	**ASC_b**	**0.225**	**0.23**	**0.24**	**0.26**	**0.29**	**0.34**	**0.32**	**0.33**	**0.35**	**0.34**	**0.35**	**0.365**	$ASC_b\rho_b\sigma_b$	***2.7***
	% Spread	*30.3%*	*30.1%*	*29.6%*	*32.1%*	*31.7%*	*33.8%*	*34.2%*	*34.4%*	*35.1%*	*33.6%*	*34.6%*	*35.1%*		
	ASC_s	**0.28**	**0.30**	**0.30**	**0.33**	**0.34**	**0.345**	**0.37**	**0.38**	**0.37**	**0.38**	**0.39**	**0.38**	$ASC_s\rho_s\sigma_s$	***4.5***
	% Spread	*36.0%*	*36.6%*	*39.5%*	*38.5%*	*34.2%*	*34.8%*	*35.5%*	*36.0%*	*36.5%*	*37.9%*	*37.6%*	*37.7%*		

Panel B: Estimated value of bid-ask spread components according to prior regression using time intervals and trade size (see *Panel B* Table 2)

B.1. Small trades (trade size between 1 and 19 contracts)

		April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jany	Feb.	March		*Simul.*
Opening Hours (7:00 to 9:00)	**ASC_b**	**0.22**	**0.23**	**0.26**	**0.25**	**0.24**	**0.26**	**0.31**	**0.32**	**0.31**	**0.33**	**0.32**	**0.34**	$ASC_b\rho_b\sigma_b$	***0.33***
	% Spread	*27.9%*	*28.3%*	*29.8%*	*28.1%*	*27.6%*	*29.8%*	*30.8%*	*32.3%*	*31.1%*	*34.1%*	*33.2%*	*33.9%*		
	ASC_s	**0.29**	**0.27**	**0.27**	**0.30**	**0.27**	**0.25**	**0.31**	**0.29**	**0.315**	**0.355**	**0.34**	**0.35**	$ASC_s\rho_s\sigma_s$	***-0.11***
	% Spread	*31.4%*	*31.9%*	*32.0%*	*31.3%*	*30.3%*	*28.9%*	*30.8%*	*29.1%*	*31.6%*	*35.7%*	*37.4%*	*35.2%*		
Normal Hours (9:00 to 15:00)	**ASC_b**	**0.225**	**0.23**	**0.26**	**0.26**	**0.24**	**0.27**	**0.30**	**0.31**	**0.35**	**0.34**	**0.33**	**0.35**	$ASC_b\rho_b\sigma_b$	***0.34***
	% Spread	*28.1%*	*28.6%*	*29.7%*	*28.7%*	*28.4%*	*29.6%*	*30.5%*	*30.4%*	*36.3%*	*34.3%*	*33.3%*	*34.6%*		
	ASC_s	**0.26**	**0.26**	**0.27**	**0.27**	**0.23**	**0.255**	**0.29**	**0.28**	**0.30**	**0.34**	**0.32**	**0.35**	$ASC_s\rho_s\sigma_s$	***0.43***
	% Spread	*32.4%*	*29.0%*	*31.7%*	*29.1%*	*27.6%*	*29.2%*	*29.6%*	*28.6%*	*30.0%*	*33.1%*	*33.1%*	*35.1%*		
End of Day Hours (9:00 to 15:00)	**ASC_b**	**0.19**	**0.22**	**0.23**	**0.24**	**0.25**	**0.27**	**0.30**	**0.30**	**0.35**	**0.35**	**0.35**	**0.345**	$ASC_b\rho_b\sigma_b$	***0.12***
	% Spread	*29.6%*	*27.9%*	*27.4%*	*31.5%*	*29.6%*	*29.4%*	*30.0%*	*29.2%*	*36.2%*	*35.1%*	*35.7%*	*34.3%*		
	ASC_s	**0.25**	**0.27**	**0.25**	**0.28**	**0.27**	**0.28**	**0.29**	**0.31**	**0.33**	**0.365**	**0.35**	**0.36**	$ASC_s\rho_s\sigma_s$	***1.88***
	% Spread	*33.5%*	*31.2%*	*30.8%*	*35.7%*	*36.2%*	*30.3%*	*29.9%*	*30.1%*	*34.6%*	*36.3%*	*35.8%*	*36.2%*		

B.2. Medium-sized trades (trade size between 20 and 49 contracts)

		April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jany	Feb.	March		*Simul.*
Opening Hours (7:00 to 9:00)	**ASCb**	**0.28**	**0.30**	**0.275**	**0.255**	**0.24**	**0.27**	**0.31**	**0.33**	**0.34**	**0.33**	**0.33**	**0.35**	$ASC_b \rho_b \sigma_b$	***0.43***
	% Spread	*35.0%*	*36.3%*	*33.0%*	*32.3%*	*30.4%*	*34.6%*	*35.7%*	*36.4%*	*36.7%*	*35.8%*	*34.2%*	*35.3%*		
	ASCs	**0.31**	**0.28**	**0.28**	**0.27**	**0.26**	**0.255**	**0.29**	**0.30**	**0.33**	**0.335**	**0.34**	**0.36**	$ASC_s \rho_s \sigma_s$	***-0.20***
	% Spread	*38.7%*	*32.4%*	*33.2%*	*34.1%*	*33.0%*	*32.8%*	*34.1%*	*34.3%*	*36.0%*	*36.2%*	*34.8%*	*36.2%*		
Normal Hours (9:00 to 15:00)	**ASCb**	**0.29**	**0.31**	**0.28**	**0.26**	**0.26**	**0.29**	**0.33**	**0.36**	**0.35**	**0.34**	**0.37**	**0.39**	$ASC_b \rho_b \sigma_b$	***1.35***
	% Spread	*35.8%*	*36.6%*	*33.2%*	*32.6%*	*32.8%*	*35.9%*	*36.9%*	*39.8%*	*38.0%*	*36.7%*	*37.8%*	*39.0%*		
	ASCs	**0.28**	**0.27**	**0.26**	**0.24**	**0.26**	**0.24**	**0.28**	**0.27**	**0.33**	**0.345**	**0.36**	**0.355**	$ASC_s \rho_s \sigma_s$	***0.91***
	% Spread	*35.2%*	*28.5%*	*31.7%*	*31.4%*	*32.9%*	*31.4%*	*32.6%*	*31.8%*	*36.1%*	*37.2%*	*36.8%*	*36.6%*		
End of Day Hours (9:00 to 15:00)	**ASCb**	**0.28**	**0.33**	**0.28**	**0.27**	**0.28**	**0.30**	**0.34**	**0.34**	**0.37**	**0.36**	**0.36**	**0.38**	$ASC_b \rho_b \sigma_b$	***1.77***
	% Spread	*32.9%*	*34.8%*	*32.5%*	*33.4%*	*34.4%*	*37.6%*	*38.6%*	*39.8%*	*39.0%*	*38.5%*	*36.7%*	*38.7%*		
	ASCs	**0.33**	**0.29**	**0.30**	**0.27**	**0.29**	**0.27**	**0.32**	**0.29**	**0.385**	**0.36**	**0.37**	**0.37**	$ASC_s \rho_s \sigma_s$	***3.03***
	% Spread	*39.5%*	*30.5%*	*34.9%*	*33.4%*	*35.5%*	*34.4%*	*36.4%*	*33.9%*	*41.3%*	*38.4%*	*37.8%*	*37.3%*		

B.3. Large trades (trade with a size greater than 50 contracts)

		April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jany	Feb.	March		*Simul.*
Opening Hours (7:00 to 9:00)	**ASCb**	**0.29**	**0.32**	**0.26**	**0.25**	**0.235**	**0.29**	**0.32**	**0.34**	**0.345**	**0.325**	**0.33**	**0.35**	$ASC_b \rho_b \sigma_b$	***0.56***
	% Spread	*35.8%*	*37.2%*	*35.8%*	*32.0%*	*31.0%*	*35.7%*	*36.3%*	*36.8%*	*37.1%*	*35.2%*	*34.2%*	*35.2%*		
	ASCs	**0.30**	**0.28**	**0.26**	**0.28**	**0.26**	**0.27**	**0.30**	**0.32**	**0.33**	**0.37**	**0.36**	**0.38**	$ASC_s \rho_s \sigma_s$	***-0.54***
	% Spread	*36.4%*	*32.8%*	*31.9%*	*33.2%*	*34.5%*	*33.9%*	*34.0%*	*34.2%*	*35.5%*	*37.1%*	*36.3%*	*38.3%*		
Normal Hours (9:00 to 15:00)	**ASCb**	**0.27**	**0.31**	**0.26**	**0.26**	**0.27**	**0.29**	**0.34**	**0.33**	**0.37**	**0.35**	**0.34**	**0.37**	$ASC_b \rho_b \sigma_b$	***3.03***
	% Spread	*32.9%*	*34.1%*	*31.7%*	*34.4%*	*35.9%*	*35.7%*	*39.5%*	*36.2%*	*40.7%*	*36.0%*	*34.8%*	*37.3%*		
	ASCs	**0.31**	**0.28**	**0.28**	**0.25**	**0.28**	**0.25**	**0.30**	**0.31**	**0.32**	**0.36**	**0.39**	**0.38**	$ASC_s \rho_s \sigma_s$	***1.77***
	% Spread	*38.5%*	*30.2%*	*34.2%*	*33.1%*	*37.3%*	*32.1%*	*33.9%*	*33.7%*	*34.9%*	*36.6%*	*39.2%*	*38.0%*		
End of Day Hours (9:00 to 15:00)	**ASCb**	**0.33**	**0.32**	**0.30**	**0.27**	**0.265**	**0.29**	**0.335**	**0.32**	**0.34**	**0.36**	**0.36**	**0.36**	$ASC_b \rho_b \sigma_b$	***3.13***
	% Spread	*37.9%*	*34.4%*	*34.1%*	*34.4%*	*34.9%*	*35.5%*	*38.4%*	*35.6%*	*36.6%*	*36.8%*	*35.4%*	*36.2%*		
	ASCs	**0.31**	**0.29**	**0.30**	**0.27**	**0.29**	**0.27**	**0.33**	**0.325**	**0.33**	**0.37**	**0.40**	**0.385**	$ASC_s \rho_s \sigma_s$	***5.24***
	% Spread	*35.6%*	*30.6%*	*34.1%*	*34.4%*	*37.7%*	*33.5%*	*38.0%*	*34.7%*	*35.2%*	*37.1%*	*39.8%*	*38.2%*		

TABLE 4. Cross-sectional association of estimated adverse selection costs with trade size, market orders and order autocorrelation

Note: We perform ordinary least squares regressions on the proportional bid-ask spreads (PBAS) according to Eq. (12a).

$$PBAS_{i,t} = \overline{RV}_{b-s} + \sum_{i=1}^{3}\sum_{j=1}^{3}\delta_1 INT_j.SIZE_i(ASC_{s,t} - \overline{RV}_{b-s}) + \sum_{i=1}^{3}\sum_{j=1}^{3}\delta_2 INT.SIZE_i\left(ASC_{s,t} - \overline{RV}_{b-s}\right) + \gamma_{1,d}COMP_{t,d} + \varepsilon_{i,t}$$

We then regress the estimates of ASCb and ASCs on four independent variables: MOb, MOs, AUTOCORRb, AUTOCORRb according to Eq. (12b):

$$\begin{cases} ASC_{b,i,t} = \alpha_{1,b} + \beta_{1,b}MO_{b,t} + \beta_{2,b}MO_{s,t} + \beta_{3,b}AUTOCORR_{ask,t} + \beta_{4,b}AUTOCORR_{bid,t} + \mu_{b,t} \\ ASC_{s,i,t} = \alpha_{1,s} + \beta_{1,s}MO_{b,t} + \beta_{2,s}MO_{s,t} + \beta_{3,s}AUTOCORR_{ask,t} + \beta_{4,s}AUTOCORR_{bid,t} + \mu_{s,t} \end{cases}$$

All coefficients are first estimated for each of the five EUA sample futures using the Prais-Winsten method which assumes first-order autocorrelation in disturbance terms based on the Durbin Watson statistic approach.
We then compute the cross-sectional averages of regression coefficients denoted "Mean coeff" that are reported in the following Panels of Table 4. The t-statistic (*t-stat*) test whether this mean coefficient is statistically significant *i.e*. different from zero. * (**) indicate their significance respectively at 0.05 (0.01) level.
The columns with the heading "Nb.signif. correct sign" report the proportion of individual coefficients among the five (EUA sample futures) coefficients obtained which are significant at 0.05 level and have a sign identical to this of the cross-sectional average coefficient "Mean Coeff".

Panel A: Small trades (trade size between 1 and 19 contracts)

	7:00 to 9:00 (INT$_1$)				**9:00 to 15:00 (INT$_2$)**				**15:00 to 17:00 (INT$_3$)**			
	November-April		*May-September*		*November-April*		*May-September*		*November-April*		*May-September*	
DEPENDANT VARIABLE: ASC_b	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)
Intercept	**0.044****	(5/5)	**0.055****	(5/5)	**0.043****	(5/5)	**0.0485****	(5/5)	**0.045****	(5/5)	**0.049****	(5/5)
t-stat	*42.76*		*46.97*		*41.82*		*45.94*		*42.22*		*46.38*	
MO$_b$	**-0.00017**	(0/5)	**-0.0001**	(0/5)	**-0.00015**	(0/5)	**-0.00007**	(0/5)	**-0.00006**	(0/5)	**-0.00003**	(0/5)
t-stat	*-0.98*		*-0.38*		*-0.52*		*-0.2*		*-0.38*		*-0.14*	
MO$_s$	**0.0023****	(5/5)	**0.003****	(5/5)	**0.0016****	(5/5)	**0.0021****	(5/5)	**0.0012****	(4/5)	**0.0016****	(5/5)
t-stat	*10.89*		*12.68*		*9.3*		*10.83*		*4.98*		*5.8*	
AUTOCORR$_b$	**-0.0002**	(0/5)	**-0.0004**	(0/5)	**-0.0041**	(2/5)	**-0.007****	(4/5)	**-0.005***	(3/5)	**-0.011****	(4/5)
t-stat	*-0.08*		*-0.11*		*-1.99*		*-3.12*		*-2.28*		*-4.25*	
AUTOCORR$_s$	**0.0022**	(0/5)	**0.00154**	(0/5)	**0.0037**	(25)	**0.0048***	(3/5)	**0.0047***	(2/5)	**0.009****	(4/5)
t-stat	*0.79*		*0.56*		*1.61*		*2.15*		*2.12*		*3.79*	
DW-statistic	2.03		2.04		2.02		2.02		2.01		2.02	
Adjusted R²	0.64		0.59		0.58		0.61		0.57		0.6	

	7:00 to 9:00 (INT$_1$)				**9:00 to 15:00 (INT$_2$)**				**15:00 to 17:00 (INT$_3$)**			
	November-April		*May-September*		*November-April*		*May-September*		*November-April*		*May-September*	
DEPENDANT VARIABLE: ASC_s	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)
Intercept	**0.051****	(5/5)	**0.050****	(5/5)	**0.0475****	(5/5)	**0.0474****	(5/5)	**0.0468****	(5/5)	**0.0466****	(5/5)
t-stat	*41.65*		*41.61*		*39.57*		*39.53*		*37.8*		*37.16*	
MO$_b$	**0.0017****	(5/5)	**0.0018****	(5/5)	**0.0019****	(5/5)	**0.0021****	(5/5)	**0.0013****	(5/5)	**0.0013****	(5/5)
t-stat	*9.51*		*10.04*		*13.5*		*14.25*		*8.13*		*8.58*	
MO$_s$	**-0.0007****	(4/5)	**-0.0009****	(5/5)	**-0.0006**	(1/5)	**-0.0007***	(2/5)	**-0.0003**	(0/5)	**-0.0004**	(0/5)
t-stat	*-3.26*		*-4.69*		*-1.82*		*-2.61*		*-0.58*		*-0.83*	
AUTOCORR$_b$	**0.0048**	(1/5)	**0.0057***	(2/5)	**0.0013**	(0/5)	**0.0064***	(3/5)	**0.0089****	(3/5)	**0.0134****	(5/5)
t-stat	*1.68*		*2.19*		*0.84*		*2.49*		*2.98*		*7.58*	
AUTOCORR$_s$	**-0.003**	(1/5)	**-0.0012**	(0/5)	**-0.0031**	(1/5)	**-0.0053***	(3/5)	**-0.0078***	(3/5)	**-0.012****	(5/5)
t-stat	*0.99*		*0.39*		*-1.32*		*-2.32*		*-2.24*		*-7.10*	
DW-statistic	2.03		2.04		0.62		2.02		2.01		2.02	
Adjusted R²	0.59		0.57		2.03		0.6		0.6		0.61	

Panel B: Medium-sized trades (trade size between 20 and 49 contracts)

	7:00 to 9:00 (INT_1)				9:00 to 15:00 (INT_2)				15:00 to 17:00 (INT_3)			
DEPENDANT VARIABLE: ASC_b	*November-April*		*May-September*		*November-April*		*May-September*		*November-April*		*May-September*	
	Mean Coef.	(Nb. Signif corr. sign)	Mean Coef.	(Nb. Signif corr. sign)	Mean Coef.	(Nb. Signif corr. sign)	Mean Coef.	(Nb. Signif corr. sign)	Mean Coef.	(Nb. Signif corr. sign)	Mean Coef.	(Nb. Signif corr. sign)
Intercept	0.105**	(5/5)	0.106**	(5/5)	0.099**	(5/5)	0.101**	(5/5)	0.094**	(5/5)	0.095**	(5/5)
t-stat	*52.65*		*52.92*		*49.31*		*50.47*		*49.49*		*49.59*	
MO_b	-0.0011**	(5/5)	-0.0012**	(5/5)	-0.0022**	(5/5)	-0.0023**	(5/5)	-0.0009**	(5/5)	-0.001**	(5/5)
t-stat	*-11.61*		*-12.31*		*-20.43*		*-21.67*		*-9.40*		*-11.09*	
MO_s	0.0025**	(5/5)	0.0027**	(5/5)	0.0024**	(5/5)	0.0026**	(5/5)	0.0026**	(5/5)	0.0027**	(5/5)
t-stat	*24.02*		*26.64*		*25.02*		*27.74*		*27.08*		*30.03*	
$AUTOCORR_b$	-0.0059	(1/5)	-0.0055	(1/5)	-0.008**	(4/5)	-0.009**	(4/5)	-0.008	(3/5)	-0.009**	(4/5)
t-stat	*-1.51*		*-1.61*		*-3.08*		*-3.32*		*-2.53*		*-3.20*	
$AUTOCORR_s$	0.0039	(1/5)	0.0061	(0/5)	0.0089*	(4/5)	0.011**	(4/5)	0.0094*	(4/5)	0.0084**	(4/5)
t-stat	*0.97*		*1.81*		*3.19*		*4.19*		*3.27*		*3.06*	
DW-statistic	2.01		2.02		2.01		2.01		1.99		2.01	
Adjusted R^2	0.69		0.71		0.70		0.72		0.69		0.73	

	7:00 to 9:00 (INT_1)				9:00 to 15:00 (INT_2)				15:00 to 17:00 (INT_3)			
DEPENDANT VARIABLE: ASC_s	*November-April*		*May-September*		*November-April*		*May-September*		*November-April*		*May-September*	
	Mean Coef.	(Nb. Signif corr. sign)	Mean Coef.	(Nb. Signif corr. sign)	Mean Coef.	(Nb. Signif corr. sign)	Mean Coef.	(Nb. Signif corr. sign)	Mean Coef.	(Nb. Signif corr. sign)	Mean Coef.	(Nb. Signif corr. sign)
Intercept	0.0339**	(5/5)	0.0367**	(5/5)	0.0363**	(5/5)	0.0371**	(5/5)	0.0365**	(5/5)	0.0040**	(5/5)
t-stat	*25.28*		*27.72*		*27.65*		*30.32*		*28.32*		*31.05*	
MO_b	-0.0004**	(4/5)	-0.0006**	(5/5)	-0.0005**	(5/5)	-0.0008**	(5/5)	-0.0005**	(5/5)	-0.0008**	(5/5)
t-stat	*-6.36*		*-9.34*		*-10.99*		*-16.15*		*-8.4*		*-12.34*	
MO_s	0.0008**	(5/5)	0.0012**	(5/5)	0.0009**	(5/5)	0.0036**	(5/5)	0.0007**	(5/5)	0.0028**	(5/5)
t-stat	*10.91*		*12.59*		*16.05*		*32.06*		*11.02*		*27.88*	
$AUTOCORR_b$	-0.0015	(0/5)	-0.006*	(3/5)	-0.0033	(2/5)	-0.0112**	(4/5)	-0.007*	(4/5)	-0.008**	(4/5)
t-stat	*-0.82*		*-2.11*		*-1.36*		*-4.13*		*-2.27*		*-2.42*	
$AUTOCORR_s$	0.0012	(0/5)	0.0015	(1/5)	0.001	(0/5)	0.0034	(2/5)	0.0075**	(1/5)	0.0076**	(4/5)
t-stat	*0.46*		*0.57*		*0.52*		*1.41*		*2.26*		*2.39*	
DW-statistic	2.05		2.01		2.02		2.01		1.98		2.01	
Adjusted R^2	0.75		0.73		0.77		0.72		0.73		0.74	

Panel C: Large trades (trade with a size greater than 50 contracts)

DEPENDANT VARIABLE: ASC_b	**7:00 to 9:00 (INT_1)**				**9:00 to 15:00 (INT_2)**				**15:00 to 17:00 (INT_3)**			
	November-April		*May-September*		*November-April*		*May-September*		*November-April*		*May-September*	
	Mean Coef.	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)
Intercept	**0.055****	(5/5)	**0.071****	(5/5)	**0.081****	(5/5)	**0.088****	(5/5)	**0.072****	(5/5)	**0.075****	(5/5)
t-stat	*7.26*		*11.27*		*14.71*		*16.83*		*11.74*		*14.22*	
MO_b	**-0.0003****	(4/5)	**-0.0002***	(3/5)	**-0.0001**	(0/5)	**-0.00017**	(1/5)	**0.00005**	(0/5)	**0.00004**	(0/5)
t-stat	*-3.18*		*-2.36*		*-0.43*		*-1.44*		*0.41*		*0.37*	
MO_s	**-0.0002***	(3/5)	**-0.0004****	(3/5)	**-0.0002***	(3/5)	**-0.0003****	(3/5)	**-0.0001**	(0/5)	**-0.0002**	(1/5)
t-stat	*-2.51*		*-3.34*		*-2.34*		*-3.26*		*-1.72*		*-1.97*	
$AUTOCORR_b$	**0.025**	(0/5)	**-0.022**	(0/5)	**-0.0294***	(3/5)	**0.0071**	(0/5)	**-0.0018**	(0/5)	**0.0013**	(0/5)
t-stat	*1.08*		*-1.00*		*-2.25*		*0.73*		*-0.13*		*0.12*	
$AUTOCORR_s$	**-0.0284***	(2/5)	**-0.021**	(1/5)	**0.0099**	(0/5)	**0.0047**	(0/5)	**0.0094**	(0/5)	**0.0046**	(0/5)
t-stat	*-2.12*		*-1.24*		*0.77*		*0.45*		*0.72*		*0.41*	
DW-statistic	2.06		2.02		2.01		1.98		2.01		2.02	
Adjusted R^2	0.80		0.83		0.70		0.72		0.69		0.68	

DEPENDANT VARIABLE: ASC_s	**7:00 to 9:00 (INT_1)**				**9:00 to 15:00 (INT_2)**				**15:00 to 17:00 (INT_3)**			
	November-April		*May-September*		*November-April*		*May-September*		*November-April*		*May-September*	
	Mean Coef.	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)	**Mean Coef.**	(Nb. Signif corr. sign)
Intercept	**0.049****	(5/5)	**0.047****	(5/5)	**0.036****	(5/5)	**0.047****	(5/5)	**0.036****	(5/5)	**0.059****	(5/5)
t-stat	*8.29*		*7.87*		*6.89*		*8.04*		*6.87*		*10.78*	
MO_b	**-0.0005****	(5/5)	**-0.0004****	(5/5)	**-0.0004****	(4/5)	**-0.0004****	(4/5)	**-0.0003***	(3/5)	**-0.0002**	(1/5)
t-stat	*-7.01*		*-5.34*		*-4.22*		*-4.09*		*-2.54*		*-1.54*	
MO_s	**0.00015**	(1/5)	**0.00014**	(1/5)	**0.00011**	(0/5)	**0.00012**	(0/5)	**0.00003**	(0/5)	**0.00015**	(0/5)
t-stat	*1.49*		*1.59*		*1.12*		*1.21*		*0.49*		*1.25*	
$AUTOCORR_b$	**-0.022***	(3/5)	**-0.019**	(1/5)	**0.0002**	(0/5)	**0.0002**	(0/5)	**-0.0002**	(0/5)	**-0.0001**	(0/5)
t-stat	*-2.28*		*-1.97*		*0.38*		*0.31*		*-0.28*		*-0.10*	
$AUTOCORR_s$	**0.0143**	(1/5)	**0.0103**	(0/5)	**-0.0036**	(0/5)	**0.0124**	(0/5)	**0.0033**	(0/5)	**0.0260**	(1/5)
t-stat	*1.35*		*0.76*		*-0.55*		*1.10*		*0.49*		*1.59*	
DW-statistic	2.01		2.02		2.01		2.01		1.99		2.01	
Adjusted R^2	0.69		0.71		0.70		0.72		0.69		0.73	