Controlling information precision to extract private benefits

Abstract

This article considers an asymmetry of information setting between a CEO (the insider) and the outside investors concerning the cash flow value of a company. The insider tries to maximize the private benefits that she/he is going to divert from the firm. However, she/he does not want to be found out, so they must act in a way that prevents this from happening by choosing in a strategic manner the precision of the signal, that is delivered publicly to outside investors, to evaluate the cash flow value of the company. Hence, private benefits are defined as the difference between the non-manipulated cash flow value and the manipulated cash flow value (by the insider) whilst taking into account a percentage of the cash flow that is owned by the insider as well as costs of manipulation of the signal. Two main contexts are considered: a Bayesian (risky) case and an ambiguity case. In each of them, we study the impact of the manipulation of signal on private benefits and compare the respective outcomes. Regulation is an important tool for reducing them.

JEL classification: G12**,** G31, G32

Keywords: private benefits, disclosure policy, information asymmetry, Bayesian (risk) and ambiguity analysis, cost of manipulation risk and ambiguity, optimization.

Introduction

Asymmetry of information takes place when inside investors (insiders) hold private information about a firm's value that outside investors (outsiders) do not have. Brown and Hillegeist (2007) argue that a firm's disclosure quality affects the level of information asymmetry between insiders and outsiders. Thus, if outsiders cannot properly evaluate their investments, they tend to undervalue the stocks of companies. Lambert and al (2007) link disclosure quality to the cost of capital like, Bhattacharya and al (2012). A firm's visibility also seems to be important (Bushee and al, 2012), and this may be improved through "investor relations" programs enabling outside investors to speak directly with the executives of a firm about their future investment programs and prospects. These interviews may enhance visibility, improve investor following as well as the market price of shares.

Private benefits of control play a central role in corporate governance (Acharya and al, 2011), but they are difficult to observe and to quantify. The use of company money to pay for perquisites is the most visible but it is not the most important way in which corporate resources can be used to the exclusive advantage of the controlling party. Coffee (2001) defines private benefits as all ways in which those in control of a corporation can siphon off benefits for themselves and that are not shared with the other shareholders. De la Bruslerie (2016) assesses that debt may be a monitoring device with respect to private benefits. Yet, Bebchuk and Neeman (2010) stress that the agency problem fundamentally lies between controlling shareholders and outside stockholders. To sum up, the net present value of all these opportunities represents a private benefit of control which is not shared among all the shareholders, but extracted for the sole profit of the controlling party. Thus, a coalition (the CEO and the majority of blockholders or the entrepreneur who combines both roles) can appropriate value for itself only when this value is not totally verifiable by the other investors (Jensen and Meckling, 1976).

Thus if profitability is high enough, majority shareholders will have strong incentives (Bergstresser and Phillipon, 2006) to extract private benefits from the company that they are running. Their aim would be to extract many private benefits as possible from cash flows without the occurrence of any litigation or reputation loss. But to do this efficiently, their objective is to maximize their private benefits whereas complying with the binding condition allows them not to be found out. So, they must very carefully control the information that is released from the firm to the market participants.

In our model, to evaluate a cash flow value of a company, we rely on the representative agent asset pricing model of Epstein and Schneider (2008). Epstein and Schneider consider an agent who receives a signal from the market which includes an aggregate signal, an idiosyncratic signal and a noise signal. The agent evaluates then the cash flow value of the company using Bayes's rule and the variance of each respective signal.

The major difference between what we present and the Epstein and Schneider model is that we have introduced two categories of agents: the insider (e.g. a manager, an entrepreneur) and a representative outsider (i.e. outside investors). And there are asymmetries of information between insiders and outsiders when the insider can manipulate the signal from the market by adding (or hiding) information to (from) it. To be more precise, we consider that the insider cannot manipulate the aggregate signal and the idiosyncratic signal but she/he can manipulate the noise signal. So that when evaluating the cash flow value, the outsider takes the *manipulated* market signal into account while the insider only takes the *non-manipulated* market signal into account.[1] The difference between cash flow values evaluated by insider and outsider[2] is what we call insider *private benefits* whilst taking into account a percentage of the cash flow value owned by the insider as well as cost of manipulation of the noise signal. In effect, manipulating the signal for private benefits is not without cost, so we suppose that the insider has to bear the costs of organizing the invisibility of this practice.

Moreover, we consider the Bayesian (risky) case and an ambiguous case. In the former case, when the insider (or outsider) judges that the quality of a market signal is good enough, she/he evaluates the cash flow value in a risky environment; in which she/he has in mind *only one* variance of the non-manipulated manipulated noise signal. For insider (outsider), the variance of the non-manipulated manipulated market signal is equal to the sum of the variance of the aggregate signal, the variance of the idiosyncratic signal and the variance of the non-manipulated (manipulated) noise signal. Here, we can show that the cash flow value evaluated by either insider or outsider is equal to the mean of cash flow which is fixed minus the risk premium. To extract private benefits, insider manipulates the noise signal through its corresponding variance that will have an impact first on the risk premium and then the cash-flow value. This means that by manipulating the noise signal, insider persuades outsider that the true value of the risk premium is the one while it is not. So she/he must act in a way such as outsiders will never know reality about the true figures of the cash-flow of the firm even at the end of the model period and in the same time, never suspect her/his misbehavior. Manipulating the noise signal through its variance is considering as manipulating risk. This latter leads to some costs for insider which will be called here after “cost of manipulation risk”.

In the ambiguous case, if the quality of the market signal is difficult to judge, then insider or outsider treats market signal as ambiguous. They do not update their beliefs in a standard Bayesian fashion, but behave as if they have several likelihoods in minds when processing signals: they evaluate the cash-flow value having in mind multiple variance of the market signal. (using the worst-case conditional probability). This means that bad news has a greater impact than good news on conditional actions. Indeed, the variance of the market signal belongs to an interval of a lower bound variance and an upper bound variance. As in the Bayesian case, the lower (upper) bound variance of the market signal is equal to the sum of

[1] The non-manipulated market signal is equal to the sum of an aggregate signal, an idiosyncratic signal and a non-manipulated noise signal while the manipulated market signal is equal to the sum of an aggregate signal, an idiosyncratic signal and a manipulated noise signal. So that, the terms “manipulating the market signal” and “manipulating the noise signal” may be used interchangeably.

[2] The following terms may have the same meaning: cash flow value evaluated by the insider (outsider) and non-manipulated (manipulated) cash flow value.

the variance of the aggregate signal, the variance of the idiosyncratic signal and the lower (upper) bound variance of the noise signal. So we also consider that when insider manipulates the (upper bound and lower bound of the) market signal, she/he manipulates (upper bound and lower bound of the) noise signal through its lower (upper) bound variance. Hence, insider evaluates the cash flow value of the company by considering that the variance of the (non-manipulated) noise signal belongs to an interval of a lower bound variance and an upper bound variance of the noise signal. While, for outsider, she/he evaluates the cash flow value of the company by considering the lower bound and upper bound variance of the manipulated noise signal. In this ambiguous case, the cash flow value is equal to the mean of cash flow which is fixed minus risk premium and minus ambiguous premium.[3] In fact, to extract private benefits, insider manipulates the upper bound variance and lower bound variance of the noise signal that will have an impact on both the risk premium and ambiguous premium and then the cash-flow value. Manipulating the signal in the ambiguous case is considering as manipulating both risk (the upper bound and lower bound variance of the noise signal) and ambiguity (the difference between upper bound variance and lower bound variance of the noise signal).[4] This may lead to two kinds of cost for insider: cost of manipulation risk and cost of manipulation ambiguity.

To determine the optimal quantity of private benefits that insider will extract, she/he has to determine the optimal quantity of the noise signal that she/he needs to manipulate through the maximization of a function of private benefits with variance of the noise signal as variable. In the Bayesian setting, insider first determines the optimal variance of the manipulated noise signal; she/he then determines how to send out the noise signal to the outsider. Here, insider manipulates risk: the higher the optimal variance of the manipulated noise signal is (with respect to the variance of the non-manipulated noise signal), the less precise the signal released by the insider is. In the ambiguous setting, as the signal is considered to be unreliable, the variance of the noise signal belongs to an interval and the insider needs to determine the optimal upper and lower bound variance of the (noise) signal. In this case, she/he manipulated both (1) risk and (2) ambiguity: (1) the greater the difference between the optimal upper bound variance of the manipulated noise signal and the upper bound variance of the non-manipulated noise signal (or the greater the difference between the optimal lower bound variance of the manipulated noise signal and the lower bound variance of the non-manipulated noise signal), the more risk increases and the lower quality of the upper bound (the lower bound) noise signal released by the insider is; (2) the greater the difference between the two aforementioned optimal variances of the noise signal, the more ambiguity increases and the lower quality of the noise signal released by the insider is.

Note that our model is not the one that allows insider to swing between the Bayesian structure and the ambiguous one. Indeed, insider depends on the markets' situation and cannot change

[3] We have extended the Epstein and Schneider model to take into account the fact that the coefficient of absolute risk aversion is different from 0.

[4] Recall that by contrast with the Bayesian case in which manipulating the noise signal is considering only as manipulating risk (variance of the noise signal).

it: if the market is under ambiguous situation, then insider cannot change it into the risky situation and vice versa.

In numerical calibrations, we vary parameter (indicator) values[5] in order to measure the impact of manipulating risk and ambiguity to private benefits. The results show that insider can extract more private benefits when the part of the aggregate signal is more important than the one of the idiosyncratic signal, the cost of manipulation the noise signal is not high enough, the fraction of the capital of the company held by the insider are low or the variance of the non-manipulated noise signal and the coefficient of absolute risk aversion are high. In the Bayesian case, manipulating the noise signal may provide positive benefits to the insider up to 12% of the non-manipulated cash-flow value. In the ambiguous case, the insider can extract even more private benefits with respect to the Bayesian case. She/he may extract, as private benefits, up to 40% of the non-manipulated cash-flow value if we take into account only the cost of manipulation risk. This result is explained by the fact that in the ambiguous environment, insider manipulates both risk and ambiguity while she/he has to support only the cost of manipulation risk and not the cost of manipulation ambiguity. When both costs are taken into account, we show that private benefits will fall below those in the Bayesian case. Hence, in the ambiguous environment, taking into consideration the cost of manipulation ambiguity is very important.

In both cases, however, the insider has to be careful about the level of manipulation under penalty of loss because she/he faces a trade-off between extracting private benefits and incurring signal manipulation cost. In developing countries or after a huge crisis like in 2007 and 2008, ambiguity may have developed. But in more normal times the Bayesian case will be encountered and should have to be tackled.

From the above results, regulation is needed to foster the complete disclosure of information particularly in the ambiguous environment, enabling investors to make their decisions.

The paper is organized as follows. Section 1 gives some overview of the related literature on private benefits. Section 2 presents, under information asymmetry, the Bayesian approach and the information ambiguity approach. Both approaches have been designed to maximize the private benefits under the cost constraints. Section 3 is devoted to the simulations and comparisons between the two settings. Section 4 investigates the empirical ways in which limit private benefit extraction to reduce the level of risk and ambiguity on the markets. Section 5 concludes. Some tables are put in the Appendix.

[5] In the Bayesian case, these parameters are the coefficient of absolute risk aversion, the number of assets in the market, the constant cost of manipulation risk, the fraction of the capital of the company held by the insider, the specific risk of a particular asset, the variance of the non-manipulated noise signal. And one more parameter in the ambiguous case is the constant cost of manipulation ambiguity.

1. Private benefits literature

Until now, indirect private benefit measures via the study of controlling block sales value have been thought to give us an idea of the estimation of private benefits: on average a premium value of 14% according to Dyck and Zingales (2004). For sure, the highest private benefits are associated with less developed capital markets where minority shareholder protection is rather weak, but they can also prevail, though to a lesser extent, in more developed financial areas (La Porta and al, 1998). Literature has emphasized the law as the most powerful mechanism by which to curb private benefits. The right granted to outside investors or to minority shareholders to sue management is supposed to limit the discretionary power of the CEO, and thus, to limit private benefit extraction. Reputation is also a powerful source of discipline, and being shamed in the press may be considered as a deterrent mechanism (Zingales, 2000).

Albuquerque and Schroth (2010) show that private benefits increase with the firm's ratio of cash holdings to total assets and decrease with short-term debt to total assets. This evidence supports Jensen's (1986) free cash flow hypothesis. However, private benefits also decrease when the ratio of intangible assets to total assets is low. All these results imply that there is a nontrivial cost for extracting private benefits as well as some economic conditions to meet. First the firm's benefits must be sufficiently high to allow the controlling party to extract the surplus for itseft. Second, extracting private benefits from liquidities or free cash flows may be easier than from asset sales. So we can infer that important cash holdings are a windfall for the controlling shareholders willing to extract private benefits. For sure, costs may occur because it becomes necessary to rig the information system to include these new kinds of self-dealing transactions (Djankov and al, 2008).

Large shareholders (Denis and Denis, 1994) who retain a larger block of equity have less of an incentive to dilute minority shareholders because they internalize more efficiency than they generate. Extracting private benefits when one holds 35% of the shares is very profitable since minority shareholders bear most of the cost generated by the private benefits scheme. Firms with more tangible assets (a percentage of total assets that are fixed) will incur lower private benefits because insiders will have more difficulties diverting resources if assets are easily observable. If a coalition is large enough to win, it should avoid accepting additional shareholders. In other words, a smaller winning coalition is preferable because it has a larger group of shareholders from whom to expropriate.

Theory predicts that where private benefits of control are larger, the controlling coalition should be more reluctant to go public (Marosi and Massoud, 2007). Thus, fewer companies will be listed in countries with high private benefits of control. Moreover, since incumbents are more likely to retain control after they take their company public in countries with high private benefits of control, the percentage of companies widely held should be smaller. An acquirer coming from a country with less investor protection is better able to siphon out corporate resources from a subsidiary than an acquirer coming from a country with very rigid rules. Differences in legal protection between the two countries may explain differences in private benefit extraction levels. The ability of a controlling shareholder to appropriate some of the value generated is limited by the possibility of being sued. The explanatory power of legal rights to give minority shareholders leverage over insiders in firms focusing on the so-called anti-director rights index developed by La Porta and al. (1997) must be high. Countries with better law enforcement should have lower private benefits of control. In fact, governments, by aggressively prosecuting a company, set an example that induces all others to behave in the right way. Thus, there is an incentive to prosecute cases even when the cost of prosecution is higher than the money that is recoverable. Dyck and Zingales (2004) show that countries with a higher degree of tax compliance, have lower private benefits of control.

2. The framework

In this section, we develop the general structure of the model in which we consider two cases (Bayesian and ambiguous respectively) to evaluate the cash flow. In each case, the general formulas of cash flow are derived for a representative agent without considering if she/he is an insider or an outsider; there is no problem of asymmetry of information to begin with. Then, based on an asymmetry of information between insider and outsider, we show how the insider extracts private benefits by manipulating noise (market) signals.

2.1. General Structure of the model

The general structure of the model is the same as the asset pricing model in Epstein and Schneider (2008). Investors hold shares representing claims on the cash flows generated by two different types of companies: a particular firm *i* and all other assets.

There are two dates, labeled 0, 1. At date 1, investors receive a market signal (s) on the amount of cash flows (d) generated by the company i studied. We focus on news about this particular firm i. There are $1/n$ shares of this company outstanding, where each share is a claim to a cash flow. The stochastic process of the cash flow is

$$d = m + \varepsilon^a + \varepsilon^i, \tag{1}$$

m is the mean of the cash flow,

ε^a is an aggregate shock,

ε^i is an idiosyncratic shock that affects only firm i.

Shocks are mutually independent and normally distributed with a mean of zero.

We summarize the payoff on all other assets by a cash flow $\tilde{d} = \tilde{m} + \varepsilon^a + \tilde{\varepsilon}^j$, where $\tilde{m}$ is the mean cash flow and $\tilde{\varepsilon}^j$ is a shock specific to all other assets. There are $n - \frac{1}{n}$ shares outstanding of other assets and each pays $\tilde{d}$. The market portfolio is therefore a claim on $\frac{1}{n}d + \frac{n-1}{n}\tilde{d}$.

In the special case $n = 1$, asset i is the market portfolio, for n large, it can be interpreted as stock in a single company.

News occurrence

The arrival of news about firm i at date 1 is represented in the following signal

$$s = \alpha\varepsilon^a + \varepsilon^i + \varepsilon^s, \quad (2)$$

where s represents the market signal which covers an aggregate signal (ε^a), an idiosyncratic signal (ε^i) and a noise signal (ε^s).

Here the number $\alpha \geq 0$ measures the degree to which the signal is specific to the particular asset on which we focus. For example, suppose that n is large, and hence that d represents future cash flow of a single company. If $\alpha = 1$, then the signal s is simply a noisy estimate of future cash flow d. As such, it partly reflects future aggregate economic conditions ε^a. In contrast, if $\alpha = 0$, then the news is 100% company-specific, that is, while it helps to forecast company cash flow d, the signal is not useful for forecasting the payoff on other assets (that is, $\tilde{d}$).

Like in Epstein and Schneider (2008), in the following, to evaluate the cash flow of company i, we specify two cases. First, if signal s is ambiguous: the variance of the shock ε^s is known to lie within a range $\sigma_s^2 \in \left[\underline{\sigma}_s^2, \overline{\sigma}_s^2\right]$ (multiple likelihoods). The greater the difference between $\underline{\sigma}_s^2$ and $\overline{\sigma}_s^2$, namely ambiguity, the less confident the investor feels about the true information content. The second case is a special case of the first one in which there is a single likelihood, $\underline{\sigma}_s^2 = \overline{\sigma}_s^2$, namely risk, and the investors know precisely how much information the signal contains.[6]

The parameter $\theta = \left(\varepsilon^a + \varepsilon^i, \varepsilon^i\right)$ that agents try to infer from the signal s is two-dimensional because of two shocks (aggregate and idiosyncratic). There is a single normal *prior* for θ in the standard Bayesian fashion and a set of normal likelihoods for s parameterized by σ_s^2 in the ambiguous case. The set of one-step-ahead beliefs about s at date 0 consists of normal laws with a mean of zero and variance ($\alpha^2\sigma_a^2 + \sigma_i^2 + \sigma_s^2$) for $\sigma_s^2 \in \left[\underline{\sigma}_s^2, \overline{\sigma}_s^2\right]$. The set of *posteriors* about θ at date 1 can be calculated using standard rules for updating normal random variables. For a given σ_s^2, let γ denote the regression coefficient:

$$\gamma\left(\sigma_s^2\right) = \frac{\operatorname{cov}\left(s, \varepsilon^a + \varepsilon^i\right)}{\operatorname{var}(s)} = \frac{\alpha\sigma_a^2 + \sigma_i^2}{\alpha^2\sigma_a^2 + \sigma_i^2 + \sigma_s^2}. \quad (3)$$

Given s, the posterior density of $\theta = \left(\varepsilon^a + \varepsilon^i, \varepsilon^i\right)$ is also normal. In particular, the sum $\varepsilon^a + \varepsilon^i$ is normal with mean $\gamma\left(\sigma_s^2\right)s$ and variance $\left(1 - \alpha\gamma\left(\sigma_s^2\right)\right)\sigma_a^2 + \left(1 - \gamma\left(\sigma_s^2\right)\right)\sigma_i^2$ while its covariance with ε^a is ($\left(1 - \alpha\gamma\left(\sigma_s^2\right)\right)\sigma_a^2$. These conditional moments will be useful in the pricing approaches developed below.

[6] In the sub-section 2.2., here after, we suppose that there exist two (types of) agents in our model: an insider and an outsider. To extract private benefits, insider manipulates the noise signal ε^s through the manipulation of its variances $\left(\underline{\sigma}_s^2, \overline{\sigma}_s^2\right)$ that she/he tries to determine the optimal values.

$\gamma(\sigma_s^2)$ may be interpreted as relative information content because it is decreasing function of σ_s^2: a higher value for $\gamma(\sigma_s^2)$ corresponds to a lower value for σ_s^2 and the information is clearer.

To evaluate cash flow of firm *i*, we assume, moreover, that the representative agent (insider or outsider) is risk averse and tries to maximize expected utility. The period utility is given by $u(c) = -e^{-\rho c}$, where ρ is the coefficient of absolute risk aversion investors.

Cash flow value in the Bayesian approach

The equilibrium cash flow for firm at dates 1 in absence of information asymmetry is given by (see Cochrane (2005) or Epstein and Schneider (2008)):

$$q_1(s) = m + \gamma s - \rho\left((1-\alpha\gamma)\sigma_a^2 + (1-\gamma)\frac{1}{n}\sigma_i^2 \right). \tag{4}$$

At date 0, the cash flow value is computed as the expected cash flow at date 1:

$$\begin{aligned} q_0 &= E[d/s] \\ &= m - \rho\left((1-\alpha\gamma)\sigma_a^2 + (1-\gamma)\frac{1}{n}\sigma_i^2 \right). \end{aligned} \tag{5}$$

At both dates, value equals the expected present value minus a risk premium that depends on risk aversion and covariance with the market. At date 0, the expected present value is simply the prior mean dividend *m*. At date 1, it is the posterior mean dividend $m + \gamma s$, as it now depends on the value of signal *s* provided that the signal is informative ($\gamma > 0$). The risk premium depends only on time and not on signal *s*. It consists of two parts, one driven by the variance of the aggregate shock ε^a, and one equal to the variance of the idiosyncratic shock multiplied by $(1-\gamma)/n$. As *n* becomes large, idiosyncratic risk is diversified away and does not matter for prices.

Cash flow value in the ambiguous approach

Because the variance of the signal is not known with precision, it belongs to an interval $\sigma_s^2 \in \left[\bar{\sigma}_s^2, \underline{\sigma}_s^2\right]$ so that we need to evaluate cash flow value under the worst-case conditional probability, which minimizes conditional mean cash flow. The price of cash flow for firm at date 1 is[7]

[7] In our model, the agent is allowed to be risk averse ($\rho \neq 0$). See Epstein and Schneider (2008), for the case when the agent is risk neutral ($\rho = 0$).

$$
\begin{aligned}
q_1(s) &= \min_{\sigma_s^2 \in \left[\bar{\sigma}_s^2, \underline{\sigma}_s^2\right]} E\left[d / s\right] \\
&= \min_{\sigma_s^2 \in \left[\underline{\sigma}_s^2, \bar{\sigma}_s^2\right]} \left\{ m + \gamma s - \rho\left((1-\alpha\gamma)\sigma_a^2 + \frac{1}{n}(1-\gamma)\sigma_i^2 \right) \right\} \\
&= \left(m - \rho\sigma_a^2 - \frac{1}{n}\rho\sigma_i^2 \right) + \min_{\sigma_s^2 \in \left[\underline{\sigma}_s^2, \bar{\sigma}_s^2\right]} \left\{ \gamma\left(s + \rho\alpha\sigma_a^2 + \frac{1}{n}\rho\sigma_i^2 \right) \right\}
\end{aligned} \tag{6}
$$

Indeed, the worst-case conditional probability used to interpret the signal depends on the signal itself (*s*). The term $\gamma\left(s + \rho\alpha\sigma_a^2 + \frac{1}{n}\rho\sigma_i^2 \right)$ is a function of the signal, *s* and of the variance of the signal, σ_s^2. This function has a kink at the point $k = -\rho\left(\alpha\sigma_a^2 + \frac{1}{n}\sigma_i^2 \right)$ for *s*. When the value of the signal is equal to *k*, the function's value is 0 and remains 0 for every σ_s^2. When the value of the signal is superior (inferior) to *k*, the function's value is decreasing (increasing) with σ_s^2. Point *k* determines what "bad news" or "good news" means: if the signal is superior to *k*, then the investor interprets it as good news and they will consider the signal as unreliable and set $\sigma_s^2 = \bar{\sigma}_s^2$; if the signal is inferior to *k*, then the investor interprets it as bad news and they interpret it as very informative: $\sigma_s^2 = \underline{\sigma}_s^2$. The price of cash flow for firm at date 1 may be rewritten as

$$
\begin{aligned}
q_1(s) &= \left(m - \rho\sigma_a^2 - \frac{1}{n}\rho\sigma_i^2 \right) + \begin{cases} \underline{\gamma}\left(s + \rho\alpha\sigma_a^2 + \frac{1}{n}\rho\sigma_i^2 \right) & s \geq k \\ \bar{\gamma}\left(s + \rho\alpha\sigma_a^2 + \frac{1}{n}\rho\sigma_i^2 \right) & s < k \end{cases} \\
&= \left(m - \rho\sigma_a^2 - \frac{1}{n}\rho\sigma_i^2 \right) + \underline{\gamma}\left(s + \rho\alpha\sigma_a^2 + \frac{1}{n}\rho\sigma_i^2 \right) + \left(\bar{\gamma} - \underline{\gamma} \right)\left(s + \rho\alpha\sigma_a^2 + \frac{1}{n}\rho\sigma_i^2 \right) 1_{s<k},
\end{aligned} \tag{7}
$$

with $\underline{\gamma} = \gamma\left(\bar{\sigma}_s^2\right)$ and $\bar{\gamma} = \gamma\left(\underline{\sigma}_s^2\right)$ providing lower and upper bounds on relative information content γ (see, for instance, Epstein and Schneider (2008)).

At date 0, the cash flow value is computed as the expected price at date 1 then evaluated again using the worst-case probability:

$$
\begin{aligned}
q_0(s) &= \min_{\sigma_s^2\in\left[\bar{\sigma}_s^2,\underline{\sigma}_s^2\right]} E\left[q_1(s)\right] \\
&= \min_{\sigma_s^2\in\left[\underline{\sigma}_s^2,\bar{\sigma}_s^2\right]} E\left\{\begin{array}{l}\left(m-\rho\sigma_a^2-\frac{1}{n}\rho\sigma_i^2\right)+\underline{\gamma}\left(s+\rho\alpha\sigma_a^2+\frac{1}{n}\rho\sigma_i^2\right)\\ +\left(\bar{\gamma}-\underline{\gamma}\right)\left(s+\rho\alpha\sigma_a^2+\frac{1}{n}\rho\sigma_i^2\right)1_{s<k}\end{array}\right\} \\
&= m-\rho\left(\sigma_a^2+\frac{1}{n}\sigma_i^2\right)+\rho\underline{\gamma}\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right) \\
&\quad +\left(\bar{\gamma}-\underline{\gamma}\right)\left(\min_{\sigma_s^2\in\left[\underline{\sigma}_s^2,\bar{\sigma}_s^2\right]} E\left\{s1_{s<k}\right\}+\rho\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right)\min_{\sigma_s^2\in\left[\underline{\sigma}_s^2,\bar{\sigma}_s^2\right]} E\left\{1_{s<k}\right\}\right).
\end{aligned}
\tag{8}
$$

Notice that at date 0, the one-step-ahead conditional beliefs of the investor about the signal are normal with mean zero and variance $\alpha^2\sigma_a^2+\sigma_i^2+\sigma_s^2$. Hence, we can write $E\left\{1_{s<k}\right\}$ and calculate $E\left\{s1_{s<k}\right\}$, respectively as

$$
\int_{-\infty}^{k}\frac{1}{\sqrt{\alpha^2\sigma_a^2+\sigma_i^2+\sigma_s^2}\sqrt{2\pi}}e^{-\frac{1}{2}\frac{x^2}{\alpha^2\sigma_a^2+\sigma_i^2+\sigma_s^2}}dx \text{ and } -\frac{\sqrt{\alpha^2\sigma_a^2+\sigma_i^2+\sigma_s^2}}{\sqrt{2\pi}}e^{-\frac{1}{2}\frac{k^2}{\alpha^2\sigma_a^2+\sigma_i^2+\sigma_s^2}}.
$$

For a given value of *k*, we can check that $E\left\{1_{s<k}\right\}$ ($E\left\{s1_{s<k}\right\}$) is increasing (decreasing) functions of σ_s^2. Replacing the above formulas of $E\left\{1_{s<k}\right\}$ and $E\left\{s1_{s<k}\right\}$ into equation (8), $q_0(s)$ can be rewritten as

$$
\begin{aligned}
q_0(s) &= m-\rho\left(\left(1-\underline{\gamma}\alpha\right)\sigma_a^2+\frac{1}{n}(1-\underline{\gamma})\sigma_i^2\right) \\
&\quad -\left\{\left(\bar{\gamma}-\underline{\gamma}\right)\left(\frac{\sqrt{\alpha\sigma_a^2+\sigma_i^2}}{\sqrt{2\pi\underline{\gamma}}}e^{-\frac{1}{2}\frac{k^2\underline{\gamma}}{\alpha\sigma_a^2+\sigma_i^2}}-\rho\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right)\int_{-\infty}^{k}\frac{\sqrt{\bar{\gamma}}}{\sqrt{(\alpha\sigma_a^2+\sigma_i^2)2\pi}}e^{-\frac{1}{2}\frac{x^2\bar{\gamma}}{\alpha\sigma_a^2+\sigma_i^2}}dx\right)\right\},
\end{aligned}
\tag{9}
$$

If we compare the cash flow values at date 0 between the Bayesian case (equation 5) and the ambiguous case (equation 9). This latter exhibits in addition to the risk premium term (second term) the ambiguity premium term (third term) because of the difference between the upper bound and lower bound variance of the noise signal expressed by the term $\left(\bar{\gamma}-\underline{\gamma}\right)$. Intuitively, this third term needs to have two important conditions. First, they are increasing function of ambiguity; it is normal that ambiguity premium increases when ambiguity increases. Second, its value must be positive.

2.2 Private benefits

The difference between our model and the Epstein and Schneider model is that we introduce two categories of agents: insiders (e.g. a manager, an entrepreneur) who know the true value of the cash flow *d*, and outsiders (i.e. minority shareholders) who will never know the true value of *d*. There is an asymmetry of information between insiders and outsiders.

From equation (2), we have that a market signal is equal to the sum of an aggregate signal multiplied by the specific risk of a particular asset (α), an idiosyncratic signal and a noise signal. To extract private benefits, in the Bayesian case (ambiguous case), insider manipulates the (upper and lower bound) market signal or more precisely the (upper and lower bound) noise signal through its (upper and lower bound) variances. When evaluating cash flow value, the outsider takes into account the (upper and lower bound) manipulated noise signal while, the insider only takes into account the (upper and lower bound) non-manipulated noise signal. Consequently, the difference between these two evaluations is what we call the insider's *private benefits,* whilst taking into account a percentage of the cash flow that is owned by the insider as well as cost of manipulation of the signal: cost of manipulation risk in the Bayesian case and cost of manipulation risk and ambiguity in the ambiguous case. Naturally, manipulating the noise signal for private benefits is costly; we suppose that the insider has to bear the cost in organizing the invisibility of this practice. By acting in this ways, insider persuades outsider to believe that the manipulated cash flow value is the non-manipulated one.

Specifically, the insider knows from the beginning what the level of cash flow will be. To extract private benefits, in the Bayesian case (ambiguous case), insider chooses to manipulate the degree of precision of the (upper and lower) market signal revealed to outsiders through the manipulation of the (upper and lower bound) variance of noise signal, a variable(s) that the insider controls. To do so, she/he maximizes her/his *private benefits* (equation 11 here after) at date 0 conditional to the signal received at date 1 in order to obtain the optimal quantity of the (upper and lower bound) variance of the manipulated noise signal. From then, she/he can determine the optimal quantity of the manipulated (upper and lower bound) noise signal which is the difference between the optimal (upper and lower bound) variance of the manipulated noise signal and the (upper and lower bound) variance of the non-manipulated noise signal.

The design of the insider's private benefits is as follows

$$PB_0 = \left\{ \left(q_0^{\text{non-manipulated}} - q_0^{\text{manipulated}} \right) + f q_0^{\text{manipulated}} - c(\gamma) \right\} - f q_0^{\text{non-manipulated}}, \tag{10}$$

where PB_0 is the private benefits of the insider;

$q_0^{\text{non-manipulated}}$ is the price of the cash flow evaluated by the insider at date 0, conditional to the signal received at date 1. This price is evaluated under the non-manipulated market (noise) signal;

$q_0^{\text{manipulated}}$ is the price of the cash flow evaluated by the outsider at date 0, conditional to the signal received at date 1. Conversely, this manipulated cash flow is evaluated under the manipulated market (noise) signal, and is naturally inferior to the non-manipulated one;

f is a fraction of the capital of the company owned by the insider. We have added f to the insider's private benefits because it has an impact on the manipulation of the signal by the insider. Intuitively, the more f increases, the less the insider manipulates signals;

$c(\gamma)$ is the cost of organizing the invisibility of cash flows. The more the signal is manipulated the more the insider has to bear an important cost. Here after, we will distinguish two types of cost: cost for manipulating risk in the Bayesian case and cost for manipulating risk and ambiguity in the ambiguous case.[8]

Equation (10) is the central equation of our article. Private benefits are defined as the difference between what the insider can get if she/he manipulates signals $\left\{\left(q_0^{\text{non-manipulated}} - q_0^{\text{manipulated}}\right) + fq_0^{\text{manipulated}} - c(\gamma)\right\}$ and what she/he gets ($fq_0^{\text{non-manipulated}}$) if she/he does nothing to the signal. This equation can be rearranged as

$$PB_0 = (1-f)\left(q_0^{\text{non-manipulated}} - q_0^{\text{manipulated}}\right) - c(\gamma). \tag{11}$$

Equation (11) shows that the insider cannot monopolize 100% of the difference between the two prices because she/he owns a percentage of the capital of the company (*f*) and she/he has to pay some costs, $c(\gamma)$.

When this is possible, we consider that insider can manipulate the (upper and lower) noise signal until the manipulated cash flow value ($q_0^{\text{manipulated}}$) equals to zero; its value cannot be negative. Naturally, if insider manipulates the market signal then the private benefits, PB_0, are superior to zero then the insider chooses to do it, if not she/he will do nothing.

To enter in detail of $q_0^{\text{non-manipulated}}$, $q_0^{\text{manipulated}}$ and $c(\gamma)$, two cases will be developed hereafter: the classical Bayesian case and the ambiguous case.

Notice that by the model's construction, the mean of the cash flow, *m*, does not have any impact on the insider's private benefits because it vanishes in the subtraction of $q_0^{\text{manipulated}}$ from $q_0^{\text{non-manipulated}}$.

The classical Bayesian case

As we have mentioned before, the insider evaluates the cash flow value by only taking into account the non-manipulated noise signal which is represented by the variance of the non-manipulated noise signal (σ_s^2) and is supposed to be unique. Based on equation (5) in the sub-section 2.1., the expression for $q_0^{\text{non-manipulated}}$ is given as

$$q_0^{\text{non-manipulated}} = m - \rho\left((1-\alpha\gamma)\sigma_a^2 + \frac{1}{n}(1-\gamma)\sigma_i^2\right), \text{ with } \gamma = \gamma\left(\sigma_s^2\right). \tag{12}$$

[8] In parallel with the cost of signal manipulation, one may extend the model to include a penalty function which is a function of the signal (*s*), namely $p(s)$. The latter is interpreted as the reaction of the outsider to signal (*s*): if the insider manipulates the signal too much (*s* too negative) and is inferior to a threshold (comparable to *k*), the insider has to bear a penalty. Hence, the penalty function is a decreasing function of the signal (*s*). As the penalty function plays the same role with respect to the cost function, so we do not propose it in our model.

The outsider evaluates the cash flow value taking into account the manipulated noise signal. The variance of the manipulated noise signal received by the outsider is denoted by $\tilde{\sigma}_s^2$. By construction, we have $\tilde{\sigma}_s^2 > \sigma_s^2$. The value of the cash flow evaluated by the outsider is given as

$$q_0^{\text{manipulated}} = m - \rho\left((1-\alpha\tilde{\gamma})\sigma_a^2 + \frac{1}{n}(1-\tilde{\gamma})\sigma_i^2\right), \text{ with } \tilde{\gamma} = \gamma\left(\tilde{\sigma}_s^2\right). \tag{13}$$

Insider determines how to manipulate the noise signal through the determination of its optimal variance, $\tilde{\sigma}_s^{*2}$. The difference between $\tilde{\sigma}_s^{*2}$ and σ_s^2 is what we call here after the optimal risk manipulation of the insider. Equation (13) shows that, in fact, insider first modifies the risk premium (second term in the right hand side) and then the cash flow value (q_0). Naturally, the real risk premium (the real cash flow value) is higher (lower) than the manipulated one.

Of course, if the insider does not manipulate the noise signal then the variance of the noisy information is equal to zero and $\tilde{\sigma}_s^2$ is equal to σ_s^2 and $q_0^{\text{non-manipulated}} = q_0^{\text{manipulated}}$.

The cost of manipulation risk is supposed to be a quadratic function of signal quality:

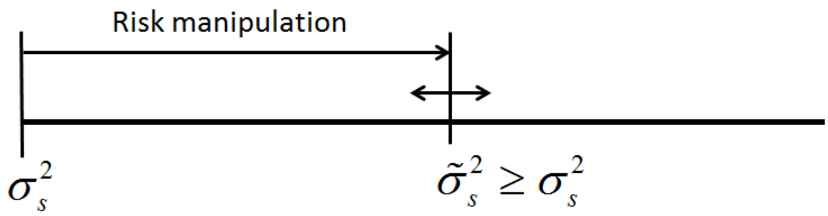
$$c(\gamma) = c(\gamma - \tilde{\gamma})^2 \tag{14}$$

with c a constant term, γ a relative level of information in absence of manipulation by the insider (corresponds to the variance σ_s^2) and $\tilde{\gamma}$ a relative level of information manipulated by the insider ($\tilde{\sigma}_s^2$). The manipulated cost is an increasing function of the difference between γ and $\tilde{\gamma}$. This is, the more that the insider manipulates the signal (the difference between γ and $\tilde{\gamma}$ is high), the more the variance of the manipulated noise signal increases, the higher the cost she/he has to endure.[9]

The above reasoning may be summarized in the following image:

[9] Note that here the cost function ($c(\gamma)$) is used as a function of relative information content (γ). Nevertheless, this cost function may also be used as the function of the variance of signal (σ_s^2) because γ is a decreasing function of σ_s^2.

Image 1: Bayesian case



- Non-manipulated noise signal corresponds to
  - σ_s^2 : Variance of the non-manipulated noise signal
  - $\gamma(\sigma_s^2)$: Relative level of information in absence of manipulation by the insider
  - Non-manipulated risk premium and non manipulated cash flow $(q_0^{\text{non-manipulated}})$
- Manipulated noise signal corresponds to
  - $\tilde{\sigma}_s^2$: Variance of the manipulated noise signal
  - $\gamma(\tilde{\sigma}_s^2)$: Relative level of information manipulated by the insider
  - Manipulated risk premium and manipulated cash flow $(q_0^{\text{manipulated}})$
- Risk manipulation implies a cost $c(\gamma)$

From there, the insider's private benefits at date 0 are given as

$$PB_0 = (1-f)\rho(\gamma-\tilde{\gamma})\left\{\alpha\sigma_a^2 + \frac{1}{n}\sigma_i^2\right\} - c(\gamma-\tilde{\gamma})^2. \tag{15}$$

The insider maximizes her/his private benefits in order to know what the optimal level of signal manipulation will be. The insider's program of optimization is given as

$$\max_{\tilde{\sigma}_s^2}\left(PB_0\right), \tag{16}$$

for a given value of the variance of non-manipulated noise signal σ_s^2.

Using the first derivative of (16), the optimal solution of the insider's optimization program is easily obtained and is given as[10]

[10] Under condition that the second derivative of (16)
$-2\dfrac{\alpha\sigma_a^2+\sigma_i^2}{(\alpha^2\sigma_a^2+\sigma_i^2+\tilde{\sigma}_s^2)^3}\left(\dfrac{3(\alpha\sigma_a^2+\sigma_i^2)c}{\alpha^2\sigma_a^2+\sigma_i^2+\tilde{\sigma}_s^2}+\rho(1-f)\left(\alpha\sigma_a^2+\dfrac{1}{n}\sigma_i^2\right)-\dfrac{2c(\alpha\sigma_a^2+\sigma_i^2)}{\alpha^2\sigma_a^2+\sigma_i^2+\sigma_s^2}\right)$, is non-positive. This condition will be satisfied if we have $\left\{\dfrac{3(\alpha\sigma_a^2+\sigma_i^2)c}{\alpha^2\sigma_a^2+\sigma_i^2+\tilde{\sigma}_s^2}+\rho(1-f)\left(\alpha\sigma_a^2+\dfrac{1}{n}\sigma_i^2\right)\right\}\geq\dfrac{2c(\alpha\sigma_a^2+\sigma_i^2)}{\alpha^2\sigma_a^2+\sigma_i^2+\sigma_s^2}$.

$$\tilde{\sigma}_s^{*2} = \frac{2(\alpha\sigma_a^2+\sigma_i^2)c-(\alpha^2\sigma_a^2+\sigma_i^2)\left(2c\gamma\ -\ \rho(1-f)\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right)\right)}{2c\gamma-\rho(1-f)\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right)}, \tag{17}$$

which may be rewritten under an easier interpretation's form of relative information content γ:

$$\gamma(\tilde{\sigma}_s^{*2}) = \gamma(\sigma_s^2) - \frac{\rho(1-f)\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right)}{2c}. \tag{18}$$

Equation (18) shows that $\gamma(\tilde{\sigma}_s^{*2}) \leq \gamma(\sigma_s^2)$ because the second term (excluding the minus sign) in the right hand side (RHS) is always positive which implies $\tilde{\sigma}_s^{*2} \geq \sigma_s^2$ and $\tilde{\sigma}_s^{*2}$ is an increasing function of σ_s^2. Moreover, when the degree of non-specificity of signal (α) increases, the first term in the RHS decreases (see equation 3), the second term (including the minus sign) also decreases; other parameter values being equal. Over all, the value of the RHS of the equation (18) decreases. The term in the left hand side which also includes α in itself will decrease, but to compensate the decreasing pattern of the second term of the RHS, the optimal variance of manipulated noise signal needs to increase. To sum up, from the equation (17), we can assess that $\tilde{\sigma}_s^{*2}$ is the increasing function of α. The equation (18) also shows that if ρ (*c*, *f*, *n*) increases (increases), then $\tilde{\sigma}_s^{*2}$ increases (decreases). These features of $\tilde{\sigma}_s^{*2}$, by varying parameter values, will be found again and interpreted in our numerical calibration here after.

The ambiguous case

The design of the insider's private benefits is the same as in the Bayesian case. But now the cash flow is evaluated at date 0 under the *ambiguous* signal that the insider will receive at date 1. The insider evaluates the cash flow taking into account the ambiguous noise signal whose variance is supposed to lie within an interval ($\sigma_s^2 \in \left[\underline{\sigma}_s^2, \overline{\sigma}_s^2\right]$) where the lower bound variance of the (non-manipulated) noise signal, $\underline{\sigma}_s^2$ and the upper bound variance of the (non-manipulated) noise signal, $\overline{\sigma}_s^2$ are supposed to be known by the insider. From equation (9), we rewrite the cash flow value computed by the insider at the date 0 as follows

$$\begin{aligned} q_0^{\text{non-manipulated}} &= m - \rho\left((1-\alpha\underline{\gamma})\sigma_a^2 + \frac{1}{n}(1-\underline{\gamma})\sigma_i^2\right) \\ &+ \left(\overline{\gamma}-\underline{\gamma}\right)\left(\begin{array}{l} \int_{-\infty}^{k} \frac{\sqrt{\overline{\gamma}}}{\sqrt{(\alpha\sigma_a^2+\sigma_i^2)2\pi}} e^{-\frac{1}{2}\frac{x^2\overline{\gamma}}{\alpha\sigma_a^2+\sigma_i^2}} dx \\ -\rho\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right)\frac{\sqrt{\alpha\sigma_a^2+\sigma_i^2}}{\sqrt{2\pi\underline{\gamma}}} e^{-\frac{1}{2}\frac{k^2\underline{\gamma}}{\alpha\sigma_a^2+\sigma_i^2}} \end{array}\right), \end{aligned} \tag{19}$$

where $\bar{\gamma} \equiv \gamma\left(\underline{\sigma}_s^2\right) = \dfrac{\alpha\sigma_a^2+\sigma_i^2}{\alpha^2\sigma_a^2+\sigma_i^2+\underline{\sigma}_s^2}$, $\underline{\gamma} \equiv \gamma\left(\bar{\sigma}_s^2\right) = \dfrac{\alpha\sigma_a^2+\sigma_i^2}{\alpha^2\sigma_a^2+\sigma_i^2+\bar{\sigma}_s^2}$.

The signal received by the outsider is also ambiguous and its variance is supposed to lie within a range $\sigma_s^2 \in \left[\underline{\tilde{\sigma}}_s^2, \tilde{\bar{\sigma}}_s^2\right]$ where the lower bound variance of the manipulated noise signal, $\underline{\tilde{\sigma}}_s^2$ and the upper bound variance of the manipulated noise signal, $\tilde{\bar{\sigma}}_s^2$ are controlled and revealed to outsiders by the insider. The outsider evaluates the cash flow price taking into account the upper and lower bound variance of the manipulated noise signal. The cash flow price evaluated by the outsider is given by

$$
\begin{aligned}
q_0^{\text{manipulated}} &= m-\rho\left((1-\alpha\underline{\tilde{\gamma}})\sigma_a^2+\frac{1}{n}(1-\underline{\tilde{\gamma}})\sigma_i^2\right) \\
&\quad +\left(\tilde{\bar{\gamma}}-\underline{\tilde{\gamma}}\right)\left(\begin{array}{l} \displaystyle\int_{-\infty}^{k}\frac{\sqrt{\tilde{\bar{\gamma}}}}{\sqrt{(\alpha\sigma_a^2+\sigma_i^2)2\pi}}e^{-\frac{1}{2}\frac{x^2\tilde{\bar{\gamma}}}{\alpha\sigma_a^2+\sigma_i^2}}dx \\ \displaystyle -\rho\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right)\frac{\sqrt{\alpha\sigma_a^2+\sigma_i^2}}{\sqrt{2\pi\underline{\tilde{\gamma}}}}e^{-\frac{1}{2}\frac{k^2\underline{\tilde{\gamma}}}{\alpha\sigma_a^2+\sigma_i^2}} \end{array}\right),
\end{aligned} \tag{20}
$$

where $\tilde{\bar{\gamma}} \equiv \gamma\left(\underline{\tilde{\sigma}}_s^2\right) = \dfrac{\alpha\sigma_a^2+\sigma_i^2}{\alpha^2\sigma_a^2+\sigma_i^2+\underline{\tilde{\sigma}}_s^2}$, $\underline{\tilde{\gamma}} \equiv \gamma\left(\tilde{\bar{\sigma}}_s^2\right) = \dfrac{\alpha\sigma_a^2+\sigma_i^2}{\alpha^2\sigma_a^2+\sigma_i^2+\tilde{\bar{\sigma}}_s^2}$.

In this case, insider determines how to manipulate the upper and lower noise signal through the determination of its optimal upper bound and lower bound variance of the noise signal ($\tilde{\bar{\sigma}}_s^{*2}$ and $\underline{\tilde{\sigma}}_s^{*2}$, respectively). The differences between $\underline{\tilde{\sigma}}_s^{*2}$ and $\underline{\sigma}_s^2$ and $\tilde{\bar{\sigma}}_s^{*2}$ and $\bar{\sigma}_s^2$ are the lower bound and upper bound optimal risk manipulation of the insider, while the differences between $\underline{\tilde{\sigma}}_s^{*2}$ and $\tilde{\bar{\sigma}}_s^{*2}$ is the optimal manipulation of ambiguity. In manipulating the upper and lower noise signal, equation (20) shows that insider modifies the risk premium (second term) and the ambiguous premium (third term) and then the cash flow value ($q_0^{\text{manipulated}}$). Naturally, the real risk premium or the real ambiguous premium (the real cash flow value) is higher (lower) than the manipulated one.

Next, we distinguish two types of cost:

- Cost for manipulating risk: insider manipulates the lower bound variance of the noise signal ($\underline{\sigma}_s^2$) and/or the upper bound variance of the noise signal ($\bar{\sigma}_s^2$). As in the Bayesian case, we consider that manipulating the two (upper and lower) bounds is costly for the insider. This cost has the following form:

  $$c(\gamma) = c\left[\left(\bar{\gamma}-\tilde{\bar{\gamma}}\right)^2+\left(\underline{\gamma}-\underline{\tilde{\gamma}}\right)^2\right], \tag{21}$$

  where c is a constant term of manipulation risk. The more insider manipulates the lower bound variance of the noise signal (and/or the upper bound variance of the noise signal), the more the difference between $\underset{\sim}{\gamma}$ and $\underline{\gamma}$ ($\tilde{\bar{\gamma}}$ and $\bar{\gamma}$) is important and so it the cost for manipulating risk, $c(\gamma)$.

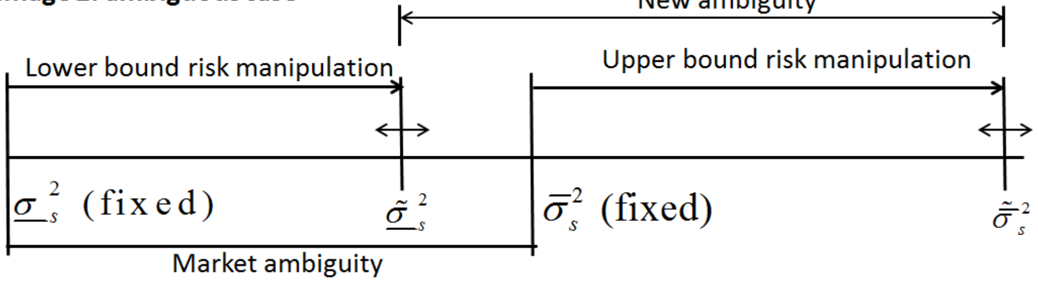
- Cost for manipulating ambiguity: insider manipulates the distance between the lower bound variance of the noise signal and the upper bound variance of the noise signal. So that she/he creates a new ambiguity (comparing to the market ambiguity). Naturally, she/he only has to pay a cost for the difference between the new ambiguity and the market ambiguity, namely "ambiguity manipulation". The more the ambiguity manipulation is high, the more the cost for manipulating ambiguity increases. This cost is supposed to have the following form:

$$a(\gamma) = a\left(\tilde{\bar{\gamma}} - \underset{\sim}{\gamma} - (\bar{\gamma} - \underline{\gamma})\right)^2, \tag{22}$$

where a is a constant term of manipulation ambiguity.

The above reasoning in the ambiguous case may also be summarized in image 2:

Image 2: ambiguous case



- Non-manipulated noise signal corresponds to
  - $\underline{\sigma}_s^2$ and $\bar{\sigma}_s^2$: Lower and upper bound variance of the non-manipulated noise signal
  - $\gamma(\underline{\sigma}_s^2)$ and $\gamma(\bar{\sigma}_s^2)$: Relative lower and upper level of information in absence of manipulation by the insider
  - Non-manipulated risk and ambiguity premium and non manipulated cash flow $(q_0^{\text{non-manipulated}})$
- Manipulated noise signal corresponds to
  - $\underset{\sim}{\sigma}_s^2$ and $\tilde{\bar{\sigma}}_s^2$: Lower and upper bound variance of the manipulated noise signal
  - $\gamma(\underset{\sim}{\sigma}_s^2)$ and $\gamma(\tilde{\bar{\sigma}}_s^2)$: Relative lower and upper level of information manipulated by the insider
  - Manipulated risk and ambiguity premium and manipulated cash flow $(q_0^{\text{manipulated}})$
- (Lower and/or upper bound) risk manipulation implies a cost $c(\gamma)$
- Ambiguity manipulation (= new ambiguity - market ambiguity) also implies a cost $a(\gamma)$

As in the Bayesian case, replacing the above equations in equation (11) gives the insider's private benefits at date 0

$$PB_0 = (1-f)\begin{pmatrix} \rho\left(\underline{\gamma}-\underline{\tilde{\gamma}}\right)\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right) \\ +\begin{pmatrix} \int_{-\infty}^{k}\left(\frac{\left(\bar{\gamma}-\underline{\gamma}\right)\sqrt{\bar{\gamma}}}{\sqrt{(\alpha\sigma_a^2+\sigma_i^2)2\pi}}e^{-\frac{1}{2}\frac{x^2\bar{\gamma}}{\alpha\sigma_a^2+\sigma_i^2}}-\frac{\left(\tilde{\bar{\gamma}}-\underline{\tilde{\gamma}}\right)\sqrt{\tilde{\bar{\gamma}}}}{\sqrt{(\alpha\sigma_a^2+\sigma_i^2)2\pi}}e^{-\frac{1}{2}\frac{x^2\tilde{\bar{\gamma}}}{\alpha\sigma_a^2+\sigma_i^2}}\right)dx \\ -\rho\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right)\left(\frac{\sqrt{\alpha\sigma_a^2+\sigma_i^2}}{\sqrt{2\pi\underline{\gamma}}}e^{-\frac{1}{2}\frac{k^2\underline{\gamma}}{\alpha\sigma_a^2+\sigma_i^2}}-\frac{\sqrt{\alpha\sigma_a^2+\sigma_i^2}}{\sqrt{2\pi\underline{\tilde{\gamma}}}}e^{-\frac{1}{2}\frac{k^2\underline{\tilde{\gamma}}}{\alpha\sigma_a^2+\sigma_i^2}}\right)\end{pmatrix}\end{pmatrix}$$
$$-c\left[\left(\bar{\gamma}-\tilde{\bar{\gamma}}\right)^2+\left(\underline{\gamma}-\underline{\tilde{\gamma}}\right)^2\right]-a\left(\tilde{\bar{\gamma}}-\underline{\tilde{\gamma}}-(\bar{\gamma}-\underline{\gamma})\right)^2 \tag{23}$$

The insider's program of optimization in this case becomes

$$\max_{\tilde{\bar{\sigma}}_s^2 \text{ and } \underline{\tilde{\sigma}}_s^2}\left\{PB_0\right\}, \tag{24}$$

with given $\bar{\sigma}_s^2$ and $\underline{\sigma}_s^2$ as the upper and lower bound variance of the non-manipulated noise signal. Moreover, we suppose that insider can only degrade the signal and cannot improve it, so we add the following constraints: $\tilde{\bar{\sigma}}_s^2 > \bar{\sigma}_s^2 > 0$, $\underline{\tilde{\sigma}}_s^2 > \underline{\sigma}_s^2 > 0$ and $\bar{\sigma}_s^2 > \underline{\sigma}_s^2$ in (24).

Different with the Bayesian case, the above insider's program (24) cannot have the analytical optimal solution because of the integral term, so that we will solve it numerically in the next section.

3. Model calibrations

Using the numerical solution of the insider's program of optimization (equations (16) and (24)), we analyze the impact of the variation of the parameter values on (optimal) variables in the Bayesian case and in the ambiguous case. In the former (latter) case, these parameters are the coefficient of absolute risk aversion, the number of assets in the market, the constant cost of manipulation risk, the fraction of the capital of the company held by the insider, the specific risk of a particular asset, the (upper and lower bound) variance of the non-manipulated noise signal. In the latter case, we have one more parameter which is the constant cost of manipulation ambiguity.

Among the variables, we have the optimal variance of the manipulated noise signal ($\tilde{\sigma}_s^{*2}$) in the Bayesian case and the optimal upper and lower bound variance of the manipulated noise signal ($\tilde{\bar{\sigma}}_s^{*2}$ and $\underline{\tilde{\sigma}}_s^{*2}$) in the ambiguous case. The optimal manipulation of signal is also one of these variables which is represented by

- the optimal risk manipulation in the Bayesian case: the difference between the optimal variance of the manipulated noise signal ($\tilde{\sigma}_s^{*2}$) and the variance of the noise signal (σ_s^2);
- the optimal risk manipulation in the ambiguous case: the difference between the optimal upper and lower bound variance of the manipulated noise signal ($\tilde{\bar{\sigma}}_s^{*2}$ and

$\underset{\sim}{\sigma}^{*2}_{s}$) and the upper and lower bound variance of the noise signal ($\bar{\sigma}^{2}_{s}$ and $\underline{\sigma}^{2}_{s}$), respectively;

and the optimal ambiguity manipulation: the difference between the optimal upper bound and optimal lower bound variance of the manipulated noise signal ($\tilde{\bar{\sigma}}^{*2}_{s}$ and $\underset{\sim}{\sigma}^{*2}_{s}$) minus the difference between the upper and lower bound variance of the non-manipulated noise signal ($\bar{\sigma}^{2}_{s}$ and $\underline{\sigma}^{2}_{s}$).

Others variables are the cash flow value evaluated by insider and by outsider, the non-manipulated and manipulated risk premium, the non-manipulated and manipulated ambiguous premium, the costs for risk and ambiguity manipulation, the private benefits and its ratio with the cash flow value evaluated by insider and by outsider.

From the analyses of these impacts, we know in which situation, insider manipulates more or less the signal to extract private benefits.

To do so, we need to determine our parameter values. The benchmark parameter values are reported in Table 2 (column 3). For the analysis purpose, the parameter values are supposed to vary in a range (based on the benchmark parameter values) in which some values are also picked and reported in Table 2 (column 4). In our calibration, the benchmark parameter values are used until we state otherwise.

This section is divided into three sub-sections. Sub-section 3.1 provides comments about how we determine the parameter values. We investigate then two structures: Bayesian structure (sub-section 3.2) and ambiguous structure (sub-section 3.3). In each structure, we will show that the insider has interest in manipulating signal (risk and ambiguity) to extract private benefits and then compare between them.

3.1. Data and parameter values

To set a benchmark value for the mean of dividend (*m*) and the corresponding variances, we use monthly data on aggregate annual dividend per share of the index S&P 500 from Bloomberg from January 1990 to January 2017 (325 data). Figure 1 shows these data that may be divided into two sub-periods: from the beginning until 2010, data seem more stable than those of the last six year (from 2011 to 2017). So that we compute the mean of dividend and the variance for the whole period and for these two subs periods. The results are reported in Table 1.

Table 1

	Mean of dividend per share	Variance of dividend per share
January 1900 – January 2017	21.562	90.151
January 1900 – December 2010	17.281	27.720
January 2011 – January 2017	33.792	66.648

Taking into account the fact that the more *recent* data has more impact on the data of tomorrow, in our calibration, the benchmark value of the mean of dividend is set to 30.[11] From equation 1, the variance of dividend per share is equal to the sum of the variance of aggregate shock (σ_a^2) and the variance of idiosyncratic shock (σ_i^2). This sum is set to be equal to 70. So that σ_a^2 may vary from 0 to 70, while σ_i^2 is equal to 70 minus σ_a^2. Their benchmark values are set equal to 45 and 25, respectively. It means that the aggregate shock is actually supposed to be more important than the idiosyncratic shock.

As the term ε^s is just a noise signal in the market signal so that its variance should be less important than the variance of dividend per share. In the Bayesian case, we set its benchmark value equal to 20. Moreover, it is reasonable to think that the variance of the (non-manipulated) noise signal in the Bayesian case (20) is in between the upper and lower bound variance of the (non-manipulated) noise signal in the ambiguous case. So that the benchmark values of these upper and lower bound variance are set to be equal to 5 and 35, respectively.

The benchmark number of assets in the market is set equal to 40 because the number of the major indices in the US market is about 40.

Following the empirical works of Johansson-Stenman (2010 and Rabin (2000), among other, who estimate the coefficient of absolute risk aversion, ρ, we set the benchmark value for this coefficient equal to 0.5.

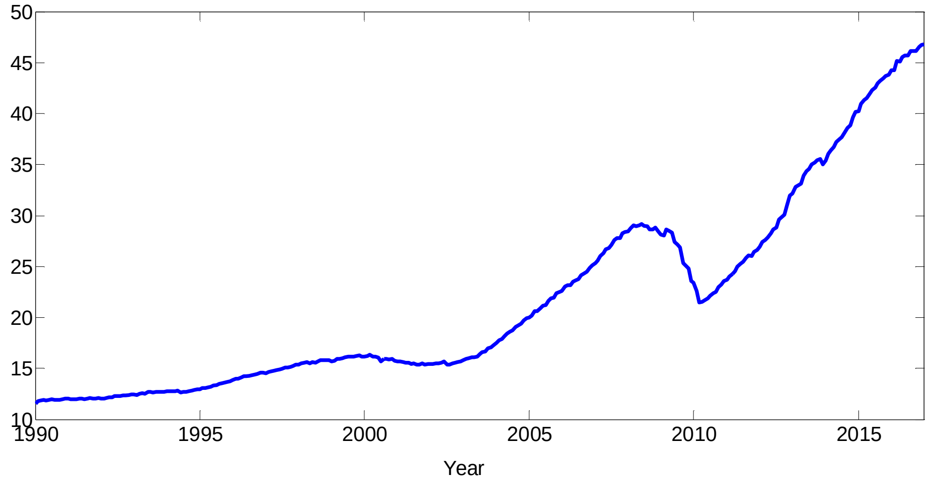


Figure 1. Monthly value on aggregate annual dividend per share of the index S&P 500 from January 1990 to January 2017 (source Bloomberg).

[11] Note that as the mean of dividend (*m*) does not intervene in the insider's private benefits, so we do not need to vary this parameter.

The constant cost of manipulation risk (c) and the constant cost of manipulation ambiguity (a) need to be non-negative and may vary. We calibrate our model and set the benchmark values for these parameters equal to 35 and 0, respectively.[12]

Two remaining parameters have natural limited lower and upper bound value. The news (α) is company specific between zero and 100% and the fraction of its equity held (f) by the insider also vary from zero to one. The benchmark parameter value for α and f are set arbitrarily to 0.8 and 0.3, respectively.[13]

Table 2

Parameter descriptions	**Notations**	**Benchmark parameter values**	**Variations of parameter values**
Cash flow mean	m	30	–
Coefficient of absolute risk aversion	ρ	0.5	0.4 – 0.5 – 0.6
Number of assets in the market	n	40	40 – 400 – 4000
Constant cost of manipulation risk	c	35	15 – 35 – 55
Constant cost of manipulation ambiguity	a	0	0 – 1 – 2
Fraction of the capital of the company held by the insider	f	0.3	0% – 100%
Specific risk of a particular asset	α	0.8	0% – 100%
Variance of the aggregate shock	σ_a^2	45	0 – 70
Variance of the idiosyncratic shock	σ_i^2	25	0 – 70
Variance of the noise signal (Bayesian case)	σ_s^2	20	10 – 20 – 30
Upper bound variance of the noise signal (ambiguous case)	$\overline{\sigma}_s^2$	35	25 – 35 – 45
Lower bound variance of the noise signal (ambiguous case)	$\underline{\sigma}_s^2$	5	–

3.2. The Bayesian structure analysis

Now, we first analyze the relationship between the private benefits and the variance of the manipulated noise signal (equation 15) using benchmark parameter values in Table 2 and by

[12] c and a are constant exogenous terms in the cost of manipulation risk and ambiguity equations (14), (20) and (21), respectively. These terms have an impact on the cost of manipulation risk and ambiguity, respectively. However, they are not themselves the cost of manipulation risk and ambiguity.

We set the same c value for the Bayesian case and for the ambiguous case that allows us to compare between them.

[13] From the benchmark parameter values, we can compute a value for k $\left(=-\rho\left(\alpha\sigma_a^2+\frac{1}{n}\sigma_i^2\right)\right)$ which is equal to -18.3. It means that until -18.3, the signal is considered as reasonable. When the signal is inferior to -18.3, it starts to become "bad news". If insider manipulated too much the signal, it becomes too negative and too suspicious for outsider, then insider may bear penalties. As we have explained before, in our model, we do not add penalty function.

varying the constant cost of manipulation risk (*c*). We show that for each *c*, there exist an optimal variance of the manipulated noise signal and optimal private benefits. We then investigate the impact of the variation of *c* on (optimal) variables such as the optimal manipulation of signals ($\tilde{\sigma}_s^{*2}$ - σ_s^2), the cash flow value evaluated by insider and by outsider, non-manipulated and manipulated risk premium, non-manipulated and manipulated ambiguous premium, costs for risk and ambiguity manipulation, private benefits and its ratio with the cash flow value evaluated by insider and by outsider.

Based on what we have done with *c*, we apply the same analysis procedure for the other parameters: the coefficient of absolute risk aversion (ρ), the fraction of the capital of the company held by the insider (*f*), the specific risk of a particular asset (α), the variance of the non-manipulated noise signal (σ_s^2) and so on.

Varying the constant cost of manipulation risk (*c*)

Figure 2 plots the insider's private benefits in function of the variance of the manipulated noise signal for different values of the constant cost of manipulation risk (*c* = 15 or 35 or 55). As the variance of the (non-manipulated) noise signal, σ_s^2 is set to 20, so that we plot PB_0 in function of different values of the variance of the manipulated noise signal, $\tilde{\sigma}_s^2$ with a starting point at 20. The plot shows that insider has interest in manipulating the noise signal (ε^s) through its variance, $\tilde{\sigma}_s^2$ (risk), to extract private benefits.

When the insider does not manipulate the noise signal; it means that the variance of the non-manipulated noise signal is equal to the variance of the manipulated noise signal (σ_s^2 = $\tilde{\sigma}_s^2$ = 20), then her/his private benefits are equal to zero. If she/he starts to manipulate the signal (so that, $\tilde{\sigma}_s^2 > \sigma_s^2$), the private benefits increase. However, there is an upper threshold for the variance of the manipulated noise signal, $\tilde{\sigma}_s^2$: if she/he manipulates too much, and the variance of the manipulated noise signal becomes superior to this upper threshold, then the insider loses money (negative private benefits). In sum, from the variance of the noise signal (σ_s^2), private benefits increase then decrease; they remain positive until reaching an upper threshold for $\tilde{\sigma}_s^2$. After, they start to become negative due to too much signal manipulation. Thus, an optimal variance of the manipulated noise signal ($\tilde{\sigma}_s^{*2}$) exists, lying between 20 and the upper threshold, for which the insider's private benefits (PB_0^*) are maximal. Otherwise, when the constant cost of manipulation term (*c*) is low, the insider can manipulate the signal more (higher $\tilde{\sigma}_s^{*2}$) and get more private benefits (higher PB_0^*) with respect to the case of higher *c*.

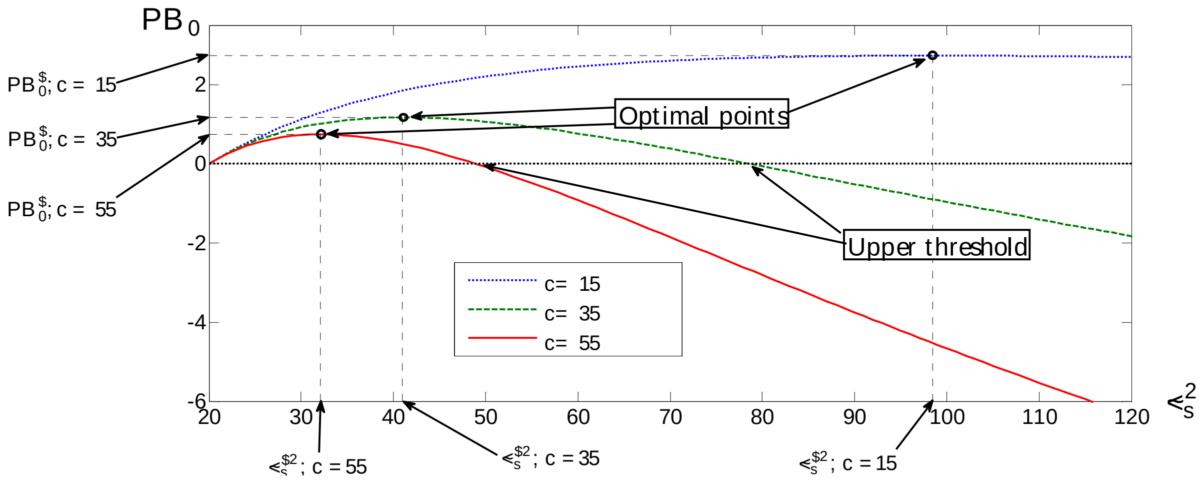


Figure 2. Using benchmark parameter values given in Table 2, we plot the private benefits (15) in function of the variance of the manipulated noise signal ($\tilde{\sigma}_s^2$) for different values of the constant cost of manipulation risk (c = 15, 35, 55).

More results are reported in the Table 3. We find that when c increases from 15 to 55,[14] the optimal variance of the manipulated noise signal, $\tilde{\sigma}_s^{*2}$, decreases and so do the optimal private benefits, PB_0^*. It means that if the cost for risk manipulation is low, insider will manipulate more the noise signal; the optimal variance of the manipulated noise signal is high, so that she/he gets more PB_0^*.

Here, the mechanism is that insider manipulates the noise signal through $\tilde{\sigma}_s^{*2}$ that will have an impact first on the risk premium, and then the cash flow evaluated by outsider and the private benefits. The less the insider manipulates the noise signal, the more $q_0^{\text{manipulated}}$ (q outsider) gets closer to the $q_0^{\text{non-manipulated}}$ (q insider), the more the manipulated risk premium tends toward the (real) risk premium without manipulation (7.676) and the private benefits decrease.

Naturally, the cost for risk manipulation is increasing function of optimal variance of the manipulated noise signal, $\tilde{\sigma}_s^{*2}$. The more the insider manipulates the signal, the more $\tilde{\sigma}_s^{*2}$ increases and the more she/he has to bear costs.

When c equals 15, the optimal private benefits that the insider can extract are 2.7348, which is about 12.25% of the non-manipulated cash flow value. This number is close to the one (14%) provided by the empirical work of Dyck and Zingales (2004).

[14] These values for c are calibrated numerically. Other values for c can be used: if c < 15, then insider will manipulate more the noise signal and gets more private benefits; if c > 55, then insider will manipulate less and gets less private benefits.

Note that the insider needs to determine the optimal quantity based on what she/he will manipulate the noise signal in order to extract private benefits. This optimal quantity, namely the optimal risk manipulation, is equal to the difference between the optimal variance of the manipulated noise signal ($\tilde{\sigma}_s^{*2}$) and the variance of the non-manipulated noise signal (σ_s^2). In our calibration, σ_s^2 is fixed to 20, then the optimal risk manipulation is 78.4500 (= 98.4500 – 20) for c = 15 and so on for the other c.

Table 3. Impact of varying the constant cost of manipulation risk (c)

This table presents variables in function of the constant cost of manipulation risk (c). The optimal variance of the noise signal is calculated numerically using (17) for each value of c (15, 35, 55). The benchmark parameter values are given in Table 2 for the other parameters. Using the optimal variance of the noise signal, we calculate value for the other variables.*

** the optimal variance of the manipulated noise signal ($\tilde{\sigma}_s^{*2}$); the optimal risk manipulation; the cash flow evaluated by insider and by outsider; the manipulated and non-manipulated risk premium; the cost of manipulation risk; the optimal private benefits; the ratio of optimal private benefits/cash flow evaluated by insider.*

c (constant term of cost of manipulation risk)	15	35	55
$\tilde{\sigma}_s^{*2}$ (optimal variance of the manipulated noise signal)	98.450	40.998	32.111
$\tilde{\sigma}_s^{*2} - \sigma_s^2$ (optimal risk manipulation)	78.450	20.998	12.111
$q_0^{\text{non-manipulated}}$ (q insider)	22.324	22.324	22.324
$q_0^{\text{manipulated}}(\tilde{\sigma}_s^{*2})$ (q outsider)	14.525	18.971	20.190
m - $q_0^{\text{non-manipulated}}$ (non-manipulated risk premium)	7.6762	7.6762	7.6762
m - $q_0^{\text{manipulated}}(\tilde{\sigma}_s^{*2})$ (manipulated risk premium)	15.475	11.029	9.8100
$c(\tilde{\sigma}_s^{*2})$ (cost of manipulation risk)	2.7209	1.1732	0.7468
PB_0^* (private benefits)	2.7348	1.1735	0.7469
$PB_0^* / q_0^{\text{non-manipulated}}$	12.250%	5.256%	3.345%

Varying the degree of the specificity of the signal (α)

We apply the same analysis procedure of c to the parameter α: the degree of non-specificity of the signal.

As we have noticed before, α may vary from 0 to 100%. If α equals to 0, it means that in the market signal, there is no aggregate signal; the market signal contains only idiosyncratic signal and a noise signal. If α increases, the part of the aggregate signal increases in the market signal.

Table 4[15] shows that the optimal variance of the manipulated noise signal, $\tilde{\sigma}_s^{*2}$, is an increasing function of the degree of non-specificity of signal (α). This intuitive result is explained by the fact that if α is close to zero, the market signal contains more information specific to the particular firm *i*. Hence, it is more difficult for the insider to manipulate the noise (market) signal, and the optimal variance of the manipulated noise signal is smaller when α is near zero. By contrast, when α is close to one, the market signal contains more aggregate information than the specific one, making that the insider has less difficulty to manipulate the signal. Thus, the optimal variance of the manipulated noise signal is greater and so is the optimal risk manipulation.

It follows that the manipulated risk premium is close to (far from) the non-manipulated (real) one when there are less (more) signal manipulation. The same assessment applies for the cash flow value. The non-manipulated cash flow value evaluated by the insider (non-manipulated risk premium) is naturally higher (lower) than the manipulated cash flow price (manipulated risk premium). The non-manipulated and manipulated cash flow value (non-manipulated and manipulated risk premium) is increasing (decreasing) function of the degree of the specificity of the signal (α). Notice that the risk premium is equal to the difference between the fixed cash flow mean (*m*) and the cash flow value (*q*). The increasing pattern for the (non-manipulated and manipulated) cash flow value with respect to α will be explained now.

First, the non-manipulated cash flow value is affected by the variance of the noise signal (σ_s^2) but not by the optimal variance of the manipulated noise signal $(\tilde{\sigma}_s^{*2})$. It increases with α (see equation 12). This increasing pattern may be explained intuitively by the fact that when α increases, the part of the aggregate signal increases (in the signal) and the part of the specific signal to the firm *i* decreases, it helps less to forecast company cash flow. Hence, the (non-manipulated) risk premium to firm *i* decreases and the non-manipulated cash flow value increases.

Second, the manipulated cash flow evaluated by the outsider is a function of the optimal variance of the manipulated noise signal $(\tilde{\sigma}_s^{*2})$ and of α. From Table 4, we can see, moreover, that $\tilde{\sigma}_s^{*2}$ is an increasing function of α. So that, the manipulated cash flow value may register two effects in the opposite direction. On one side, when the $\tilde{\sigma}_s^{*2}$ increases; the manipulated cash flow value decreases because the risk premium increases (se equation 13). On the other side, as we have explained before for non-manipulated cash flow value, increasing the value of α (at the same time as $\tilde{\sigma}_s^{*2}$) makes that the manipulated cash flow value increases (13). As the increasing effect outweighs the decreasing effect, so that the manipulated cash flow values are an increasing function of α. Otherwise, the increasing gap between the two cash flows also may be explained by the fact that the manipulated cash flow

[15] To save place, Tables 4 to 8 are placed in the Appendix.

price is a decreasing function of the optimal variance of the manipulated noise signal, while this is not the case for its non-manipulated counterpart.

The optimal private benefits are an increasing function of $\tilde{\sigma}_s^{*2}$. This result is intuitive because the more insider manipulates the noise signal, the more $\tilde{\sigma}_s^{*2}$ increases and the more she/he gets private benefits. If α = 1, the insider can monopolize until 7.3% of the non-manipulated cash flow value. If α is close to 0, the private benefits are nearly equal to 0; insider nearly does not manipulate at all the signal because it is very company-specific.

The cost of risk manipulation is an increasing function of α. As we have explained before, the increasing value of α implies more manipulation from the insider, and for sure, the manipulation cost increase.

Note that varying α gives the same results as varying σ_a^2 and σ_i^2. For this reason, we skip to report the results of varying σ_a^2 and σ_i^2. Indeed, we know that σ_a^2 + σ_i^2 = 70, so if σ_a^2 = 0, then σ_i^2 = 70 - σ_a^2 = 70 and so on. It means that if σ_a^2 increases, then σ_i^2 decreases and there is more aggregate signal than idiosyncratic signal in the market signal. This reasoning is the same for α when it increases.

Varying the variance of the (non-manipulated) noise signal (σ_s^2)

Table 5 shows that the optimal variance of the manipulated noise signal ($\tilde{\sigma}_s^{*2}$) is superior to the variance of the non-manipulated noise signal ($\tilde{\sigma}_s^2$). It means that the insider always has interest to manipulate signals to extract private benefits. Moreover, $\tilde{\sigma}_s^{*2}$ is increasing function of $\tilde{\sigma}_s^2$.[16] It means that if the market is initially noisier,[17] the insider needs to manipulate the signal even more heavily in order to extract private benefits. Despite that, optimal private benefits when variance of the (non-manipulated) noise signal is equal to 30 are not superior to those when the variance of the noise signal is equal to 10; they are nearly the same for three variances of the noise signal. However, the ratio private benefits/cash flow non-manipulated is higher for higher $\tilde{\sigma}_s^2$.

In this case, this means that more signal manipulations do not imply more private benefits. This counter-intuitive feature is explained by two effects in opposite directions. The first one is when the insider manipulates more signals, her/his private benefits naturally increase. However this increasing effect is not enough to compensate the decreasing one of private benefits due to a noisier market.

[16] The feature that have been found before in the analysis of equation (18).

[17] Note that the market is noisier if the variance of the noise signal equals to 30 than if the variance of the noise signal equals to 10.

Otherwise, the more insider manipulates the noise signal, (the more the optimal variance of the noise signal increases), the more the manipulated risk premium gets higher so that the manipulated cash flow value evaluated by outsider diminishes. The same way of reasoning applies for the non-manipulated cash flow value evaluated by insider as it is a decreasing function of the variance of the non-manipulated noise signal (12).

Varying the fraction of the capital of the company held by the insider (f)

The intuitive results in Table 6 show that if the insider owns a more important fraction of the capital of the company, she/he will manipulate less the noise signal; the optimal risk manipulation is a decreasing function of f and the private benefits also decrease. Indeed, if insider is the owner of the company (f =100%), then she/he does not manipulate at all the signal ($\tilde{\sigma}_s^{*2} = \sigma_s^2$), while if insider holds zero percentage of the capital of the company, she/he will manipulate the noise signal to take 10.73% of the non-manipulated cash flow.

Varying the coefficient of absolute risk aversion (ρ)

To be more precise here, we need to distinguish two types of coefficient of absolute risk aversion: one for the insider and one other for the outsider. The optimal variance of the manipulated noise signal ($\tilde{\sigma}_s^{*2}$) depend on these two coefficients. If the one for insider increases (decreases), she/he will be more (less) averse to risk and manipulate less (more) the noise signal, $\tilde{\sigma}_s^{*2}$ decreases (increases). By contrast, if the one for outsider increases (decreases), insider will manipulate more (less) the signal, $\tilde{\sigma}_s^{*2}$ increases (decreases).

Without loss of generality, we suppose that the coefficient of absolute risk aversion for insider is the same with the one for outsider. Table 7 shows that the optimal risk manipulation ($\tilde{\sigma}_s^{*2}$) is an increasing function of the coefficient of absolute risk aversion (ρ). This feature can be seen from equation (18). Therefore, the private benefits also increase with ρ: the more insider manipulates the noise signal; the more $\tilde{\sigma}_s^{*2}$ increases and the more the private benefits increase. Indeed, as the non-manipulated risk premium is only a function of ρ so that if ρ increases, the non-manipulated risk premium increases and the cash flow insider decreases. For the manipulated risk premium, thing is a little bit difference because it depends on both ρ and $\tilde{\sigma}_s^{*2}$. As they both increase so that the manipulated risk premium increases even more quickly than the non-manipulated risk premium and the cash flow evaluated by outsider decreases more quickly than the cash flow insider.

Varying the number of assets in the market (n)

By varying the number of assets in the market, Table (8) shows that the more there are assets in the market, the less insider manipulates the noise signal in order to attract more outside investors and the less she/he gets the private benefits. However, it seems that the number of asset in the market does not have a huge impact on the manipulation of risk. It does not matter how many assets exist in the market, insider manipulates nearly the same amount of risk and gets nearly the same number of private benefits.

3.3. The ambiguous structure analysis

As in the Bayesian case analysis, we start by analyzing the relationship (equation 23) between the private benefits and the upper and lower bound variance of the manipulated noise signal by varying the constant cost of manipulation risk (*c*) and using benchmark parameter values in Table 2. For each *c*, there exist an optimal upper and lower bound variance of the manipulated noise signal and an optimal private benefit. We then investigate the impact of the variation of *c* on (optimal) variables such as the optimal upper and lower bound variance of the manipulated noise signal ($\tilde{\bar{\sigma}}_s^{*2}$ and $\underline{\tilde{\sigma}}^{*2}_s$), the optimal upper and lower bound risk manipulation of signals ($\tilde{\bar{\sigma}}_s^{*2}$-$\bar{\sigma}_s^2$ and $\underline{\tilde{\sigma}}^{*2}_s$-$\underline{\sigma}_s^2$), the optimal ambiguity ($\tilde{\bar{\sigma}}_s^{*2}$-$\underline{\tilde{\sigma}}^{*2}_s$), the cash flow value evaluated by insider and by outsider, the non-manipulated and manipulated risk premium, the non-manipulated and manipulated ambiguous premium, the costs for risk and ambiguity manipulation, the private benefits and its ratio with the cash flow value evaluated by insider and by outsider.

Then again, we apply the same analysis procedure for the other parameters. Note that among these latter, there exists one that is not present in the Bayesian case: the constant cost of manipulation ambiguity (*a*).

Varying the constant cost of manipulation risk (*c*)

Figure 3 plots a 3-D figure concerning private benefits in function of the lower bound variance and upper bound variance of the manipulated noise signal when the cost of manipulation risk (*c*) is set to be equal to 15, 35 and 55. The lower (upper) bound variance is allowed to vary from the lower bound variance of the non-manipulated noise signal (the upper bound variance of the non-manipulated noise signal) to 45 (105).[18] The plot shows that the insider has no interest in degrading the lower bound (variance of the) non-manipulated noise signal; we suppose that she/he cannot improve it. Because in the case if she/he does it, the private benefits will not be optimal, and worse, they may become negative when the lower bound signal is manipulated too much. So that in the worst-case conditional probability, it is better for the insider to do nothing with the lower bound signal. It means that the optimal lower bound variance of the manipulated noise signal is equal to the lower bound variance of the non-manipulated noise signal.[19]

By contrast, the insider has interest in manipulating the upper bound noise signal to extract private benefits. This feature may be seen more clearly in Panel B in which we have fixed the lower bound variance of the manipulated noise signal which is equal to the lower bound

[18] Naturally, the insider can only manipulate the signal by degrading it; she/he cannot add betterments. So, she/he cannot improve the (upper and/or lower bound) variance of the non-manipulated market signal: hence, by definition, $\underline{\tilde{\sigma}}_s^2 \geq \underline{\sigma}_s^2$ and $\tilde{\bar{\sigma}}_s^2 \geq \bar{\sigma}_s^2$ and $\bar{\sigma}_s^2 \geq \underline{\sigma}_s^2$.

[19] The same pattern is found for the other parameters.

variance of the non-manipulated noise signal and we vary the upper bound variance ($\tilde{\bar{\sigma}}_s^{*2}$) from 35 to 105 for three values of the constant cost of manipulation risk (15, 35 and 55). Like Figure 2 in the Bayesian case, for each *c*, there exists an optimal upper bound variance of the manipulated noise signal for which the private benefits are maximal. When the insider does not manipulate the noise signal (the upper bound variance of the manipulated noise signal is equal to the upper bound variance of the non-manipulated noise signal (35)), the private benefits are zero. If she/he starts to manipulate the noise signal; $\tilde{\bar{\sigma}}_s^{*2}$ starts to be superior to $\bar{\sigma}_s^2$, the private benefits increase.

However, for higher constant cost of risk manipulation (*c*), there exists an upper threshold for the upper bound variance of the manipulated noise signal, $\tilde{\bar{\sigma}}_s^2$. If she/he manipulates too much the upper bound noise signal, the upper bound variance of the manipulated noise signal becomes superior to this upper threshold, and then the insider loses money (negative private benefits).

It is important to note that in the ambiguous case when the insider manipulates the upper bound and/or the lower bound noise signal through its upper and lower bound variances. It means that she/he manipulates both risk and ambiguity. Indeed, when insider manipulates risk, she/he manipulates the upper bound variance and/or the lower bound variance. By doing so, she/he modifies the real upper and lower bound variance of the noise signal. When insider manipulates ambiguity, she/he manipulates the difference between the upper bound variance of the signal and the lower bound variance of the signal. For example, if the upper bound variance of the manipulated signal increases, other variances being fixed, then so do risk and ambiguity.

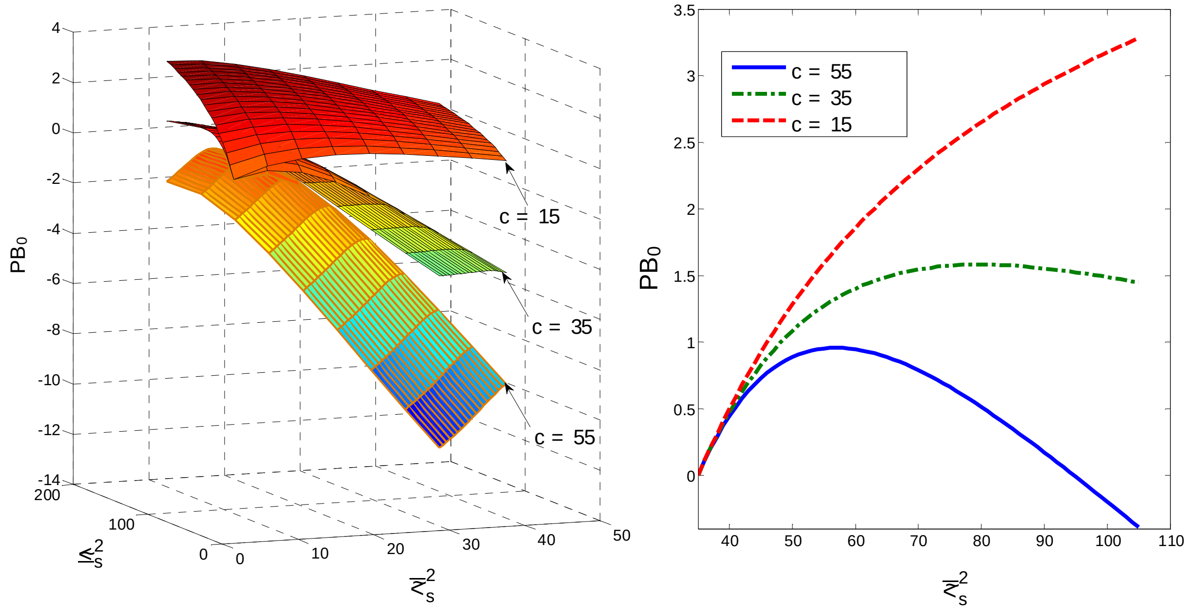


Figure 3. Panel A draws a 3-D figure of the private benefits (23) in function of the lower bound and upper bound variance of the manipulated noise signal for three different values of

the constant cost of manipulation risk (c) (15, 35, 55) and using the benchmark values given in Table 2 for the other parameters. The lower bound variance is allowed to vary from the lower bound variance of the non-manipulated noise signal (5) to 45. The upper bound variance is allowed to vary from the upper bound variance of the non-manipulated noise signal (35) to 105. Panel B plots the private benefits as a function of the upper bound variance for the same values of the constant cost of manipulation risk when the lower bound variance is fixed at 5.

More results are presented in Table 9 in which we report values for different variables in function of three different values of c (15, 35, 55).

Two cases are distinguished. First, when the constant cost of manipulation risk is low (c equals to 15), the insider manipulates the (upper and lower bound) noise signal as she/he wishes until the manipulated cash-flow value ($q_0^{\text{manipulated}}$) equals to zero.[20] However, the insider cannot monopolize 100% of the difference between the two cash-flow values (or more precisely, 100% of the non-manipulated cash-flow value ($q_0^{\text{non-manipulated}}$) because $q_0^{\text{manipulated}}$ =0) because she/he owns 30% of the capital of the company (f = 0.3). Moreover, she/he has to take into account cost of manipulation risk. This result is an interesting one. It is different from the Bayesian framework in the sense that under the constraint of non-negative manipulated cash flow value, there is no limit for the insider to manipulate the signal and take private benefits. The ambiguous environment allows the insider to extract a huge percentage of private benefits with respect to the non-manipulated cash flow value. The ratio of private benefits over non-manipulated cash-flow value equals to 40.54% for c = 15. If we compare with the Bayesian case (Table 3, column c =15), we conclude that in the ambiguous environment, insider may get more private benefits than the risk one (only 12.250%).

However, we need to be careful with the above comparison because in the ambiguous environment, insider manipulates both risk and ambiguity while she/he bears only the cost for manipulating risk; the benchmark constant cost of manipulation ambiguity (a) is equal to 0. In the Bayesian (risk) environment, the insider manipulates only risk and bears the cost of risk manipulation. So, the consideration for cost of manipulating ambiguity is important. It is what we will address here after.

Second, if the constant cost of risk manipulation increase (and equals to 35 or 55), then the optimal upper bound variance decreases with c. It means that there exists an interval for the upper bound variance of the manipulated noise signal in which private benefits increase and then decrease. Out of this interval, the insider's private benefits are decreasing and negative. In these cases, insider still manipulates the noise signal but she/he cannot do as she/he wishes

[20] This result is explained by the fact that when c is very low, it is optimal for insider to do nothing with the lower bound variance. However, she/he manipulates the upper bound variance as she/he wants because her/his private benefits are an increasing function of the upper bound variance of the manipulated noise signal.

as in the case of lower c (=15). The increasing value for c implies that the insider has to be careful and manipulate less the noise signal if she/he does not want to lose money.

In sum, when the cost of manipulation risk increases, insider needs to be more careful and manipulates less the noise signal, so that $\tilde{\bar{\sigma}}_s^{*2}$ decreases, the manipulated risk premium decreases and tends toward the (real) non-manipulated risk premium, the manipulated cash flow increases toward the non-manipulated manipulated cash flow and the private benefits decrease.

Table 9. Impact of varying the constant cost of manipulation risk (c)

This table presents variables in function of the constant cost of manipulation risk* (c). *We numerically maximize equation 24 to obtain the optimal upper and lower bound variance of the noise signal for each value of c (15, 35, 55). The benchmark parameter values are given in Table 2 for the other parameters. Using the optimal upper and lower bound variance of the noise signal, we calculate value for the other variables.*

** the optimal upper and lower bound variance of the manipulated noise signal (* $\tilde{\bar{\sigma}}_s^{*2}$ *and* $\underline{\tilde{\sigma}}_s^{*2}$*), the optimal upper and lower bound risk manipulation of signals (* $\tilde{\bar{\sigma}}_s^{*2}-\bar{\sigma}_s^2$ *and* $\underline{\tilde{\sigma}}_s^{*2}-\underline{\sigma}_s^2$*), the optimal ambiguity (* $\tilde{\bar{\sigma}}_s^{*2}-\underline{\tilde{\sigma}}_s^{*2}$*), the cash flow value evaluated by insider and by outsider, the non-manipulated and manipulated risk premium, the non-manipulated and manipulated ambiguous premium, the cost for risk and ambiguity manipulation, the private benefits and its ratio with the cash flow value evaluated by insider.*

The ambiguous case (a = 0)			
c (constant cost of manipulation risk)	15	35	55
$\tilde{\bar{\sigma}}_s^{*2}$ (optimal upper bound variance)	735.35	77.463	56.021
$\underline{\tilde{\sigma}}_s^{*2}$ (optimal lower bound variance)	5.0000	5.0000	5.0000
$\tilde{\bar{\sigma}}_s^{*2}-\bar{\sigma}_s^2$ (optimal upper bound risk manipulation)	700.35	42.463	21.021
$\underline{\tilde{\sigma}}_s^{*2}-\underline{\sigma}_s^2$ (optimal lower bound risk manipulation)	0.0000	0.0000	0.0000
$q_0^{\text{non-manipulated}}$	19.622	19.622	19.622
$q_0^{\text{manipulated}}(\tilde{\bar{\sigma}}_s^{*2},\underline{\tilde{\sigma}}_s^{*2})$	0.0000	15.057	16.996
Non-manipulated risk premium	10.233	10.233	10.233
Manipulated risk premium	21.341	14.302	12.641
Optimal ambiguity manipulation ($\tilde{\bar{\sigma}}_s^{*2}-\underline{\tilde{\sigma}}_s^{*2}-(\bar{\sigma}_s^2-\underline{\sigma}_s^2)$)	700.35	42.463	21.021
Non-manipulated ambiguous premium	0.1450	0.1450	0.1450
Manipulated ambiguous premium	8.2867	0.6410	0.3630
$a(\tilde{\bar{\sigma}}_s^{*2},\underline{\tilde{\sigma}}_s^{*2})$ (cost of manipulation ambiguity)	0.0000	0.0000	0.0000
$c(\tilde{\bar{\sigma}}_s^{*2},\underline{\tilde{\sigma}}_s^{*2})$ (cost of manipulation risk)	7.0780	1.7280	0.9510
PB_0^* (private benefits)	7.9554	1.4670	0.8870

$PB_0^* / q_0^{\text{non-manipulated}}$	40.54%	7.50%	4.50%

Varying the constant cost of manipulation ambiguity (a)

Table 10[21] shows that when the cost of manipulation ambiguity increase, insider manipulates less the noise signal so that the optimal upper bound variance of the manipulated noise signal decreases: manipulated risk premium and manipulated ambiguity premium decrease, and so do the private benefits.

Comparing to Table 3, (column c = 35) in the Bayesian case, we find that when the insider does not have to bear a cost for manipulating ambiguity (a = 0) then the private benefits are superior in the ambiguous case (1.467) than the Bayesian case (1.174). If the insider has to support a low cost for manipulating ambiguity (a = 1), the private benefits in the ambiguous case (1.151) are nearly the same with those in the Bayesian case. Just only when the cost for manipulation ambiguity is higher (a = 2) that the private benefits are inferior in the ambiguity case. Hence, the role of the cost for manipulating ambiguity is very important.

Impact of varying the degree of non-specificity of signal (α)

Table 11 shows that the optimal upper bound variance, $\tilde{\bar{\sigma}}_s^{*2}$, is an increasing function of the degree of non-specificity of signal (α). The explanations of the movement of the variables such as manipulated risk premium, manipulated ambiguity premium, manipulated cash flow and the private benefits here with respect to $\tilde{\bar{\sigma}}_s^{*2}$ and α are the same with the Bayesian case. Indeed, the risk premium and the ambiguity premium are function of $\tilde{\bar{\sigma}}_s^{*2}$ and α. As they increases with $\tilde{\bar{\sigma}}_s^{*2}$ but decrease with α and because the decreasing pattern is more important than the increasing one so that they are decreasing function of $\tilde{\bar{\sigma}}_s^{*2}$ and α. It follows that the manipulated cash flow increases but more slowly than the non-manipulated one, so the private benefits increase with $\tilde{\bar{\sigma}}_s^{*2}$ and α.

Impact of varying the upper bound variance of the non-manipulated noise signal ($\bar{\sigma}_s^2$)

Table 12 shows that if the market itself becomes more risky and ambiguous, insider will add even more risk and ambiguity to extract private benefits. The other variables follow: if insider manipulates more the noise signal then the manipulated risk premium and ambiguous premium increases, the manipulated cash flow decreases, the cost for manipulating risk increases and the private benefits increase.

Varying the lower bound variance of the non-manipulated noise signal ($\underline{\sigma}_s^2$) provides the same results with varying the upper bound variance, $\bar{\sigma}_s^2$. So we skip to give its results here.

Impact of varying the fraction of the capital of the company held by the insider (f)

[21] To save place, we report Tables 10 to 15 in the Appendix.

As in the Bayesian case, Table 13 shows that insider will manipulate less the noise signal (risk and ambiguity) if she/he possess a higher fraction of the capital of the company. However, in the ambiguous case, when insider owns zero fraction of the capital of the company, she/he will attempt to manipulate a lot more the noise signal to extract private income (40.54%). By contrast, if she/he is the owner of the company, she/he will do nothing to the noise signal.

When varying other parameters, the results are found qualitatively the same for both the Bayesian case and the ambiguous case. Table 14 shows that when the coefficient of absolute risk aversion (ρ) increases, insider will manipulate more risk and ambiguity and gets more private benefits. In Table 15, the number of assets in the market (*n*) does not have a lot of impact on the optimal risk and ambiguity manipulation; they stay stationary even when *n* increases. If *n* increases, insider will manipulate slightly less the noise signal then her/his private benefits slightly decrease.

Overall, insider manipulates at the same time risk and ambiguity in the ambiguous case while in the Bayesian case, she/he manipulates only risk. The result of this is that in the ambiguous case, if insider has to bear only a cost for manipulating risk, her/his private benefits are always higher than those in the Bayesian environment. It means that in the ambiguous environment, a cost for manipulating ambiguity are needed to be add and the outside investors must be more careful here.

4. Private benefits: From the point of view of regulation

Quantifying the level of private benefits extraction is important, because it allows us to measure their impact on the wealth of insiders, and gives us reasons to fight against this practice, since they are significant. Here, we have shown that insiders will try (if it is possible for them) not only to manipulate the precision of the signal (volatility level) but also the ambiguity of the signal (the range of possible volatilities). Our simulations show that private benefits reaching about 10% of the initial cash flow value are attainable, and ambiguity may allow insiders to go even further. Regulation is then needed where the manipulation of cash flow is supposed to take place more easily. Our model points out some important parameters for the regulatory bodies to consider.

- *A request for an increase in the quality of information disclosure by firms*

This refers, in our model, to parameters σ_s^2 or γ the non-manipulated variance and the quality of the signal. As σ_s^2 decreases, the quality of the signal increases and becomes more informative. Lundholm and Myers (2002) find that current stock returns reflect more information about future earnings when disclosure quality is higher. More informative disclosures reduce the total set of information about future cash flows that can be privately discovered about a firm. To proxy disclosure quality (Dechow et al, 2010), one can use the absolute value of the difference between the firm's actual per share earnings, and the IBES consensus analyst forecast (scaled by price), or correlation between annual stock returns and annual earnings measured over the previous years (a high correlation represents low levels of firm-investor asymmetry).

In fact, firms have incentives to increase disclosure quality prior to raising capital in order to reduce the level of information asymmetry, and hence the cost of capital. The percentage of shares owned by institutional shareholders is also important because it forces disclosure (Boone and White, 2015). Analysts are another monitoring device, and their publication, and number, covering the future prospects of the firm are important signals over a twelve-month period (Zhang, 2006). Thus, when higher disclosure quality exists, private benefit extraction should be lower. Still, some firms choose to be delisted (Crocci and Del Giudice, 2014), and turn to opacity (Marosi and Massoud, 2007). Then, they are no longer compelled to comply with the disclosure rules demanded by public investors. Some others choose to add to the complexity of the firm in order to be insulated from the market discipline. The disclosure policy of firms should be at the heart of the regulatory body's concerns.

- *The legal system and a higher protection of minority shareholders*

Here we are concerned with the penalty cost P and the manipulation cost C in our model. In that line, la Porta and al (2002) conjecture that by "limiting expropriation, the law raises the price that securities fetch in the market place". Legal changes may influence practices (Atanasov and al, 2010), and limit some kind of tunneling. La Porta and al (1997, 1998) have highlighted the impact of different laws across countries, and stressed that they imply different behaviours. Investors' protection seems to be determinant to avoid self-dealing practices. Shareholders' activism is to be developed. Once again, regulatory bodies should protect minority shareholders' rights as much as they can. The major penalty for misbehavior is a loss of reputation. In some cases, the insider may be dismissed. Thus, it might be very difficult for the fraudulent insider to find a new job. Moreover, as Liu (2016) argues, corruption culture may act as a selection mechanism by attracting and selecting individuals with similar corruption attitudes to the firm. As financial fraud is more and more pervasive (Zingales, 2015), we need a turnaround of legislation concerning fraud detection, whistle blowers, and heavy penalties to stop misbehaviors or at least to restrict them.

Promoting corporate governance (Shleifer and Vishny, 1997), as soft law inside the firm may also augment the manipulation cost for the insider. A public corporation is often viewed as an organization run by a CEO and monitored by a board of directors on behalf of shareholders. We know that CEOs are self-interested, and therefore, not automatically faithful servants to the shareholders (Djankov and al, 2008). It could be argued that board of directors must care about the future. The existence of a second block holder may limit or mitigate the behavior of the controlling one. But one should recognize that manipulation costs may be lower if firms are hoarding cash, or if firms handle a large fraction of intangible assets. In these conditions it is easier to extract private benefits, thus regulation should have a special look at these kinds of firms, especially those hoarding cash abroad in tax havens.

- *Reinforcing legal rules in front of extreme events*

These regulations should address the consequences of ambiguity, e. g $\sigma_s^2 \in \left[\underline{\sigma}_s^2, \overline{\sigma}_s^2\right]$ in our model. Dimmock and al (2016) find empirically as theory predicts, ambiguity is negatively related to stock market participation, but conditional on stock ownership, and it is positively associated to under-diversification. People invest only in a few stocks in which they have ~~a~~ specific knowledge, or information. They show that greater participation by unsophisticated investors results in a lower risk premium. Easley et O'Hara (2009) also show that ambiguity aversion can lead to nonparticipation on the financial market because the naïve investor is heavily influenced by the worst possible state. But the main result of their analysis is that regulation, particularly the regulation of unlikely events, can moderate the effects of ambiguity, thereby increasing participation in financial markets. Welfare gains may follow because legal rules designed to limit "worst case" outcomes (bankruptcy and bails-out, for example) can succeed in fostering participation when more traditional market remedies, such as disclosure will fail. Because ambiguity aversion is driven by extreme negative outcomes (market crashes) and possible correlation of cash flows to macro-uncertainty, effective regulation need only to concentrate on these "left tail" events. That was the case during the crisis in 2007-2008 when the TARP program was put in place to bail-out banks. These remarks reinforce the role that the legal system can play in markets.

5 Conclusion

Our aim is to maximize the entrepreneur's wealth provided that her/his strategy allows her/his not to be discovered by outside investors. In fact, we have to maximize her/his private benefits knowing she/he is a block holder and bears extracting and penalty costs. So, in the Bayesian case, she/he has to choose very carefully between two parameters: the level of disclosed cash flow, and the degree of precision of signal (the higher the precision the lower the possible manipulation). In the ambiguous case, insider can choose the signal's ambiguity: the range of the possible values of the signal's precision (the higher the ambiguity the higher the possible manipulation).

Conversely to a lot of previous studies, we are concerned with the cash flow of the firm and the chosen precision of the signal given to outside shareholders. We show that idiosyncratic information given to investors is important. With the help of two parameters, the chosen and manipulated precision of the signal, and the level of uncovered cash flow, we are able to derive the total amount earned by the entrepreneur stemming from non-manipulated cash flow, their toehold, as well as manipulation and penalty costs.

A further version of our model could include some dynamics, over a period of time. As a matter of fact, some learning process may be at stake this time and the behavior of the outsiders may improve, constraining the insider to steal fewer private benefits. We leave this avenue open for future research.

Finally, a lot of studies (Albukerque and Schroth, 2010; Dyck and Zingales, 2004) indicate that the extraction of private benefits is a very real phenomenon that can be consistently measured, even in developed countries and on mature markets. Protecting outside investors against expropriation by insiders is therefore relevant. Our model contributes to identifying the main channel by which diverted funds are extracted by entrenched insiders: information asymmetry engendering information manipulation. To mitigate the incentive that insiders may have to produce additional ambiguity, disclosure rules must be reinforced, and to make insiders fully accountable for the information that they disclose to outside investors.

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