Bank capital adequacy and Liquidity Management:

Conventional Vs Islamic bank

Preliminary version Mars 2017

Summary: We consider liquidity creation alternatively in an Islamic banking system and in a conventional one, adapting the Diamond and Dybvig (1983) model to take into account the specifics of Islamic deposit contracts: a contingent payment, a predetermined sharing ratio, a secured but non-remunerated principal in case of early withdrawal. We show that, in the equilibrium without runs, an Islamic banking system would offer deposit contracts that are less favourable to depositors, hold more liquid assets and have a lower equity to deposit ratio than a conventional banking system.

Keywords: Islamic banking, liquidity, deposit contract, equity-to-deposit ratio.

JEL Classification: G21

Introduction

In addition to its role in lowering transaction costs, searching for information and monitoring, a bank is continuously facing liquidity creation and management issues.

Maturity transformation makes banks intrinsically vulnerable, as observed during several bank crises, theoretically analysed and considered in the evolution of regulation[1]. Hence, banks have to structure their assets and liabilities in order to properly manage liquidity risk that materializes when depositors withdraw their money at once or in large amounts. If additional constraints on deposit contracts and thus on liability structure are imposed, it is no doubt an even harder exercise. Such is the case of "Islamic banks", which, by nature, face a number of specific risks in addition to those faced by "conventional" banks.

[1] See for example Freixas & Rochet (2008), Allen & Gale (2007).

Distinct features of Islamic banks compared to conventional banks

Islamic banks	Conventional banks	Changes to Islamic banks risk profile
1- *Shari'ah* compliance	Non-existent	1- *Shari'ah* risk
2- Prohibition of *riba*	Based on interest rates	2- Rate-of-return risk 3- Mark-up benchmark risk
3- Lending facilities must be backed by a physical asset (PLS or salebased)	Facilities are only based on lending money based on interest rates	4- Commodity price risk 5- Increase operational risks for delivering/holding asset
4- Having PLS contracts	Non-existent	6- Equity investment risk 7- Increase operational risk (and asymmetric information)
5- Restrictions in requesting collaterals and penalties	No restrictions imposed	8- Increase credit risk
6- Investment accounts (deposits) are based on a *mudaraba* contract	All deposits are determined by interest rates	9- Displaced commercial risk (DCR)
7- Restrictions on secondary markets and interbank activities	Secondary markets witness continuous innovations	10- Increase liquidity risk

Source: Salem (2013) "Risk Management for Islamic Banks", EDINBURGH GUIDES TO ISLAMIC FINANCE, 2013

In fact, an Islamic banking system is based on profit and loss sharing, on restrictions in debt sales and on the prohibition of interest or "Riba". The requirement to build financial transactions on real transactions restricts in particular the use of financial derivatives, and creates difficulties regarding the cash changeover of bank assets and liquidity risk hedging, see Ahmad (1997) and El-Gamal (2006).

Thus, two types of phenomena, affecting Islamic banks, seem to justify the deepening of banking theory.

On the one hand, the difficulties of liquidity management in Islamic banking systems have been pointed out repeatedly and tend to endure over time. The reports by the Islamic International Rating Agency (2009) and Ernst & Young (2011, 2012) show that the liquidity of Islamic banks tends to decrease but remains excessive, which reduces profitability. The difficulty of achieving interbank "Sharia-compliant" solutions explains the low level of interbank exchanges, difficulties in investing liquidity and refinancing (Hassoun 2003 Islamic Financial Services Board, 2008, Standard & Poor's 2010). Hassoun (2003) also notes that Islamic banks are more active on retail markets than corporate markets, which also explains the over-representation of short term mark-up contracts on the asset side of their balance sheets (Murabaha, Ijarah, Salam).

On the other hand, the subprime crisis has destabilized the conventional financial system, and by contrast, Islamic Financial Institutions appeared to be more "resilient" (IFSB-IDB-IRTI 2010). This resilience can be explained by the role of "safeguard" played by "Sharia-compliance" rules: banks can neither carry highly leveraged exposures nor acquire risky financial structured products, nor grant mortgage (subprime loans). Moreover, they cannot invest in repackaged instruments lacking traceability. ~~Therefore, they are more likely to be robust.~~

The main objective of this article is to highlight the specificities of Islamic banking. We put forward a theoretical representation of Islamic banking building on a basic fundamental model of conventional banking, the Bryant(1980) and Diamond and Dybvig(1983) model, which was extended by Dowd (2000), Gangopadhyay and Singh (2000), Marini (2003), who consider equity capitals as an alternative to deposit insurance in the resolution of banking instability.

We adapt the model in order to take into account the specificities of Islamic banking deposits, and we assume two possible environments for the banker: a conventional environment and an Islamic environment, both competitive, which differ in the possible terms of deposit contracts.

Islamic banks offer schematically two types of deposit contracts (IFSB-IDB-IRTI 2010). Current accounts (qard hassan) are redeemable without notice and the deposited amount is guaranteed but not paid: holders of current accounts do not share the risk of the bank, and they don’t receive any remuneration. Equity investment accounts (Profit-Sharing Investment Accounts, PSIA), as Mudarabah contract are based on the principle of profits sharing: neither the principal nor the remuneration are guaranteed, and holders participate in bank results in proportion to their financial contribution, according to a predetermined sharing ratio. In practice, a number of Islamic banks are trying to "secure" their deposits and smooth out returns on PSIA accounts.

In our theoretical model, we assume, in a very simplistic way, that there is only one type of deposit, one of a hybrid nature. We thus do not distinguish between qard hassan and PSIA within Islamic contracts, but contrast explicitly the characteristics of Islamic contracts (no predefined remuneration but pre-established sharing coefficient, secured principal in case of early withdrawal) with those of conventional contracts (predefined remuneration). In addition, we disregard the moral hazard issues if the "safety net" and we assume that banks operate in an institutional environment without deposit insurance nor a lender of last resort.

Last but not least, to highlight the differences between the convention and Islamic deposit contracts in equilibrium,, we assume that banks have similar investment opportunities in both systems and that depositors have the same preferences, the same constraints in terms of liquidity needs and investment opportunities.

The first section sets up the model. In the second and third sections, we show how a "conventional" banker and an Islamic banker structures the deposit contract and the bank assets. In the fourth section, we compare the two situations: we show that "conventional" deposit contracts are more customer friendly than "Islamic" deposit contracts. The final section concludes with a discussion of further research.

1. ENVIRONMENT:

Our model incorporates the basic characteristics proposed by Bryant (1980) and Diamond and Dybvig (1983) (see also Allen and Gale 2007), but also takes into account equity capital in bank's balance sheet. There are three period. At $t = 0$, the contracts are formulated and deposit and investment decisions are made. At $t = 1$, depositors (consumers) suffer a liquidity shock: they learn if they are early ("type1") consumers and must withdraw their deposits for consumption in $t = 1$, or if they are late ("type2") consumers and can wait up to the next period. It is assumed that the proportion of early consumers is certain and known by bankers, it is noted λ with $(0 \leq \lambda \leq 1)$: there is no aggregate liquidity shock. At $t = 2$, the economy is facing a macroeconomic shock that affects the profitability of investments: two states of the world can come true, "H" for the "high" or good state and "L" for "low" or bad state, with respective probabilities p_H and p_L (with $p_H + p_L = 1$).

There are two assets, short-term asset and long-term asset. The short-term asset can be interpreted as storage technology: any amount invested in t (t = 0, 1) can be fully recovered in t + 1. It is therefore perfectly safe and liquid. The long-term asset (illiquid asset) is an investment portfolio of projects launched in t = 0, maturing in t = 2. The return on long-term assets, $\tilde{R}$, is random, and depends on the state of the world: $\tilde{R} = R_H$ or R_L, with $R_H > R_L$. The long-term asset can be "liquidated" in t = 1 and then yields r. We assume that $R_H > 1 > R_L > r > 0$, i.e. that the long-term asset is risky and illiquid, as early liquidation yields less than completion. We denote $\bar{R}$ the expected return of long asset ($\bar{R} = p_H R_H + p_L R_L$) and assume that $\bar{R} \geq 1$ so that the short-term asset does not dominate the long-term asset.

There are two types of decision makers: bankers and depositors. Bankers are risk neutral. Depositors are risk averse.

At t = 0, a "representative" banker, with a capital endowment K, constitutes a bank. He designs a deposit contract and collect deposits amounting to D. We denote $\delta = K/D$ the equity-to-deposit ratio. The banker invests all resources in the long-term asset (amounting to x) and in the short-term asset (amounting to y). Hence the banker initial budget constraint at t = 0 (Equation 1):

(1) $x + y = D + K \equiv (1 + \delta)D$ (Banker's initial budget constraint)

with $0 \leq x, y \leq D + K$.

We assume that the bank attracts D depositors, each one having an endowment normalized to one. At t = 1, depositors privately learn that they are either "type1" or "type2": "type2" consumers may be "impatient", i.e. withdraw immediately or wait until t = 2. The banker reimburses depositors who come to the bank. We note $U(c_1, c_2)$ the utility function of a depositor, where c_t denotes consumption in period t, and $u(\ )$ is an increasing and concave function, we assume :

$$U(c_1, c_2) = \begin{cases} u(c_1) \text{ if the consumer is "type1"} \\ u(c_2) \text{ if the consumer is "type2"} \end{cases}$$

The expected utility at t = 0 is therefore written: $\lambda\, u(c_1) + (1 - \lambda) u(c_2)$.

At t = 2, the banker collects the fruits of the investment portfolio, reimburses the patient " type 2" consumers who have been waiting and keeps the net resources of the bank.

We will consider two different types of bankers: the "conventional" banker and the "Islamic" banker.

The conventional banker offers non-contingent deposit contracts in the sense that the amounts that can be withdrawn at t = 1 and t = 2 are determined at t = 0. Islamic banker[2] offers a contingent deposit contract, according to sharing conditions defined at t = 0.

2. Equilibrium deposit contracts offered by Conventional banks:

A conventional banker offers a deposit contract that promises, for each unit deposited at t=0, an amount d_1 in t=1 and a d_2 at t=2. Since the banker knows the proportion of "early" consumers, he expects at time t=0 that he will face the following budget constraints:

(2) $\lambda\, d_1\, D \ \leq y$ (*liquidity constraint*)

(3) $(y - \ \lambda\, d_1\, D) + (x\, R_L\ - (1-\lambda)\, d_2\, D) \geq 0$ (*deposit contract feasibility constraint*)

The inequality (2) is the budget constraint at t = 1. The available short term asset (y) should be equal at least to the amount withdrawn by depositors, $\lambda\, \mathrm{d}_1\, D$. Indeed, the banker does not need to liquidate long term assets because he perfectly foresees withdrawals at t=1 and the premature liquidation of long term assets is less profitable than the short term assets (r <1). This equation can be interpreted as a bank's liquidity constraint.

Inequation 3 is the budget constraint at t=2 in case L: the available amount at t=2 in the bad state, i.e. the sum of non-distributed amounts at t=1 and t=2, must be positive or zero. It is a *feasibility constraint of a deposit contract:* if the payment d_2 is possible in the bad state (L) then it is also possible in the good state (H) in t=2.

Moreover, the banker must establish a deposit contract that meets the following conditions:

(4) $\lambda\, u(d_1) + (1-\lambda) u(d_2) \geq u(1)$ (depositors' participation constraint)

(5) $d_2 \ \geq \ d_1$ (patience incentive constraint)

---

[2] The model does not explicitly represents the work of the bank (as in the model of Diamond and Dybvig which serves as a reference). However, it implicitly considers the banker as "entrepreneur", "mudarib", because it brings an expertise in collecting deposits, choosing investments, and allowing applicants to have access to long asset. But he is also represented as "financial", "rab el mal", insofar as it provides funds to establish the bank.

Inequation (4) is the depositors' participation constraint: it requires that the deposit contract be preferable to "autarky" (in which case, consumers can only invest in the short term asset)

Inequation (5) is the patience incentive constraint. If this constraint is not checked, then late consumers have no incentive to wait until t = 2; there is no equilibrium without bank run[3].

Finally, we assume that competition among risk neutral bankers imposes a zero expected profit condition, equation (6), which can be interpreted as a condition of existence of the bank: the banker should receive from the creation of the bank at least as much as if she invested directly K in the long asset. In other words, the net resources of the banker in t = 2 must be equal to the opportunity cost of equity capital.

(6) $y - \lambda\, d_1\, D + x\, \bar{R} - (1-\lambda)\, d_2\, D = \bar{R}K$ (bank existence c*onstraint*)

It should be noted that in the absence of uncertainty about the proportion of early consumers (λ), the banker saturates the liquidity constraint (2), since he earns nothing from keeping excess short-term assets. This translates into equation (7)

(7) $y = \lambda\, d_1 D$

Using (1), we deduce:

(8) $x = (1 - \lambda\, d_1)D + K$

Then, using (1) and (7), the feasibility constraint (3) becomes:

(3') $(1-\lambda)\, d_2 + R_L\, \lambda\, d_1 \leq (1+\delta)R_L$

Similarly, the condition for existence (6) becomes:

(6') $(1-\lambda)\, d_2 + \bar{R}\lambda\, d_1 = \bar{R}$

In such models, consumers are ex-ante identical at t = 0. Also, competing bankers have no incentive to in differentiate the contracts they offer. To attract depositors, they offer a deposit contract that maximizes expected utility of depositors under the constraints of participation of

[3] Late consumers wait patiently until t = 2 if the value they derive, $u(d_2)$, is greater than the utility they get from an early withdrawal and storage up to t=2, $u(d_1)$: $u(d_2) \geq u(d_1)$ if, and only if $d_2 \geq d_1$.

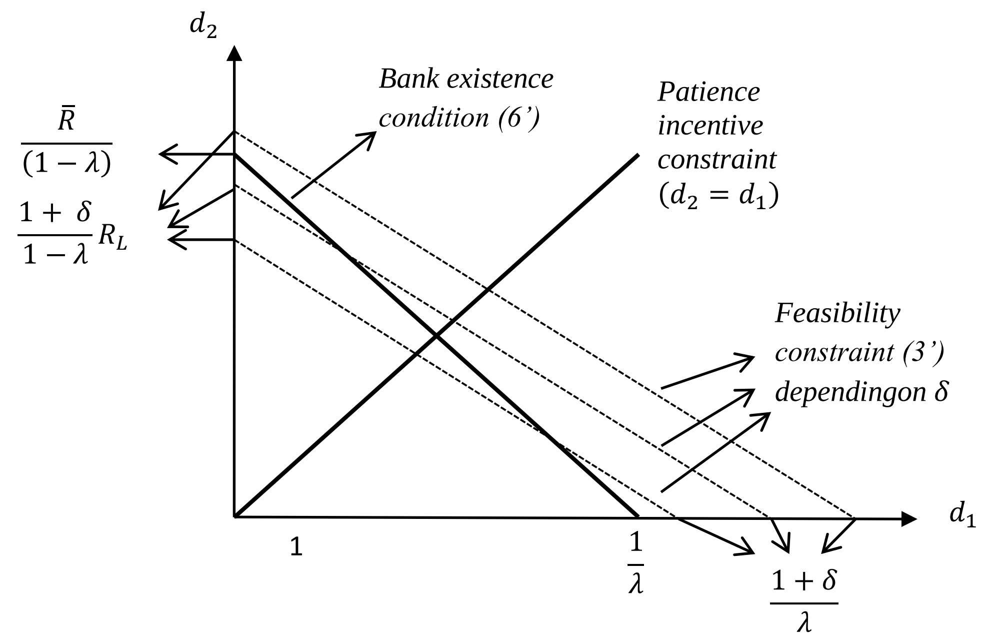
depositors (4) and inciting the patience (5), feasibility (3 ') and existence (6 '). The optimal contract is the solution of the problem:

(9) $$\max_{\{d_1,d_2\}} \lambda\, u(d_1) + (1-\lambda)u(d_2)$$

s.t. (3’), (4), (5), (6’)

Figure 1 illustrates the constraints (3), (5), (6 '). The position of the straight line representing the constraint (3 ') depends on the value of δ. The higher δ (the higher equity capital relative to deposits), the higher the withdrawals the bank can sustain at t = 2, all else being equal, the higher the position of the line representing (3 ').

Figure 1: the constraints



Before considering the alternative configurations, notice that the bank's balance sheet is "fragile" if the maximum withdrawals at t = 1 are higher than the net asset value of the assets, i.e. if $d_1 D > y + rx$ or, using (7) and (8):

(10) $$(\lambda r + 1 - \lambda)d_1 - r > r\delta.$$

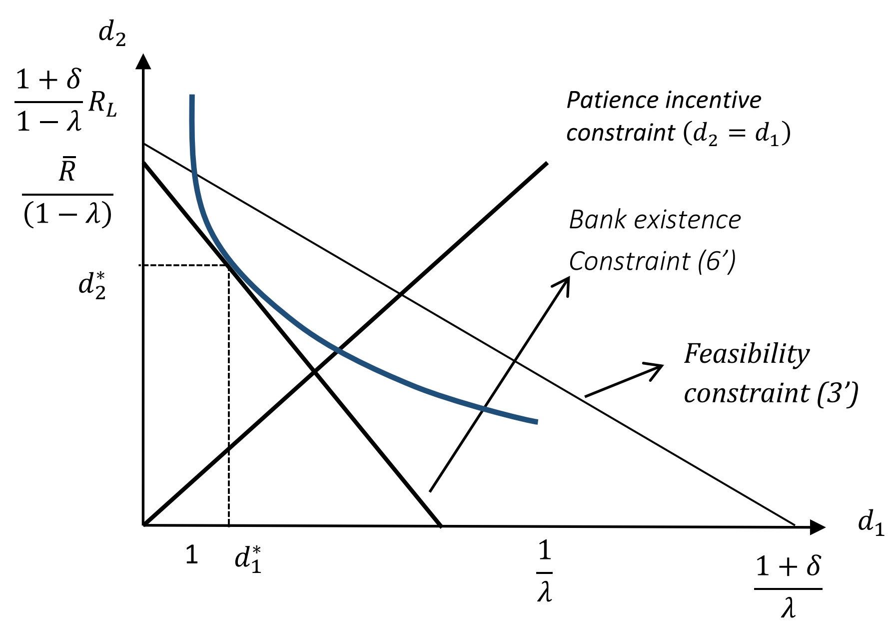
The capital of the bank are invested in the long-term asset (see (8)) and can be used to pay for early withdrawals of “type2” depositors, if their net asset value is sufficient. Otherwise (see (10)), early liquidation of long assets acquired through deposits leads to a loss of value that is not covered by the net asset value of the assets acquired through long capital banking, and self-fulfilling bank runs are then possible.

Case 1: $\delta > \frac{\bar{R}-R_L}{R_L}$.

The line representing (3 ') is in the "high" position (see figure 2). Only the bank existence constraint bites. The bank has sufficient equity capital to meet the feasibility constraint for any deposit contract, particularly any contract that satisfy the patience incentive constraint (depicted by the bisector).

The marginal rate of substitution along the bisector is $\frac{\lambda}{1-\lambda}$. It is less than the slope constraint (6'), which is $\frac{\lambda}{1-\lambda}\bar{R}$. The optimum is thus located above the bisector, so that the incentive constraint is satisfied.

Figure 2 : optimum when $\delta > \frac{\bar{R}-R_L}{R_L}$



If depositors exhibit constant relative risk aversion, we can note $u\ (c) = \ c^{1-\frac{1}{\alpha}}/\left(1-\frac{1}{\alpha}\right)$ with $\frac{1}{\alpha} > 1,$ then the optimal deposit contract has the following characteristics:

(11) $$\begin{cases} d_1^* = \ \bar{R}/\left((1-\ \lambda)\ \bar{R}^{\alpha} + \ \lambda\ \bar{R}\right) > 1 \\ d_2^* = \ \ \bar{R}^{\alpha}\ d_1^*\ >\ d_1^* \end{cases}$$

And the choice of conventional banker of the portfolio is given by:

(12) $$\begin{cases} x^* = K + \frac{(1-\ \lambda)\bar{R}^{\alpha}}{(1-\ \lambda)\ \bar{R}^{\alpha} + \lambda\ \bar{R}} D \\ y^* = \frac{\lambda\ \bar{R}}{(1-\ \lambda)\ \bar{R}^{\alpha} + \lambda\ \bar{R}} D \end{cases}$$

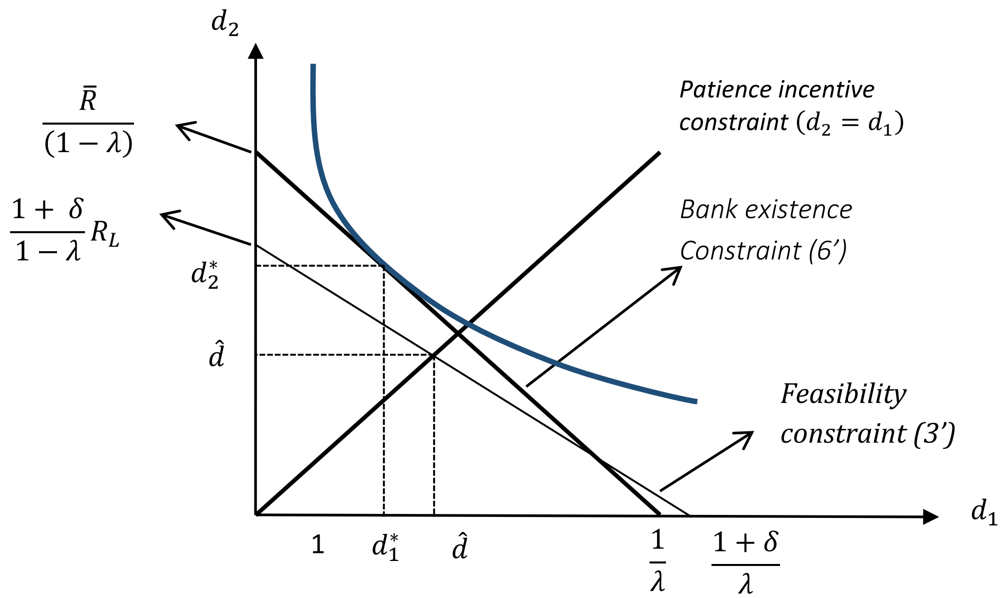
The expected utility of depositors at t = 0, is: $\lambda\ u(d_1^*) + (1-\lambda)u(d_2^*)$ and according to the bank existence condition, the expected profit of banker is $\bar{R}K.$

The constraint (10) with $d_1\ = d_1^*$ shows that the bank is subject to the self-fulfilling runs depositors if: $< \frac{(1-\lambda)\bar{R}}{(1-\lambda)(1+\delta)\bar{R}^{\alpha}+\delta\lambda\bar{R}}$.

Case 2 : $\delta < \frac{\bar{R}-R_L}{R_L\left(1+\bar{R}\frac{\lambda}{1-\lambda}\right)}$:

As shown on figure 3, the line representing (3 ') is in the "low" position. Its intersection with the line representing (6 ') is located below the first bisector. A deposit contract both feasible and satisfying the incentive constraint patience is profitable enough to compensate the banker above the average profitability of the long asset. The previous optimal deposit is not feasible because it violates constraint (3’): in the bad state, the profitability of the long-term asset is too low to cater for promised late withdrawals.

Figure 3: optimal deposit contract when $\delta < \frac{\bar{R}-R_L}{R_L\left(1+\bar{R}\frac{\lambda}{1-\lambda}\right)}$



The slope of the feasibility constraint (3'), $\frac{\lambda}{1-\lambda}R_L$, is lower than the marginal rate of substitution along the first bisector, $\frac{\lambda}{1-\lambda}$. The interior optimum therefore lies below the bisector, so that the incentive constraint is not satisfied.

Thus, a deposit contract that meets both the feasibility constraint and the patience incentive constraint is a corner solution of the optimization problem. If the bank has too little equity, it cannot promise depositors more than $\hat{d}$ regardless of the date of withdrawal, where $\hat{d} = \frac{(1+\delta)R_L}{1+\lambda+\lambda R_L}$.

We note that $\hat{d} > 1$, so that the depositor participation constraint is satisfied , if and only if $\delta > \frac{1-R_L+(1+R_L)\lambda}{R_L}$. Otherwise, the bank has too little capital to provide a non-contingent deposit contract at least as attractive as the short-term asset[4]. The constraint (10) with $d_1 = \hat{d}$ shows that the contract offer $\{\hat{d}, \hat{d}\}$, the bank is subject to the self-fulfilling rushes depositors if: $r < \frac{1-\lambda}{1+\lambda} R_L$.

Case 3: $\frac{\bar{R}-R_L}{R_L\left(1+\bar{R}\frac{\lambda}{1-\lambda}\right)} < \delta < \frac{\bar{R}-R_L}{R_L}$.

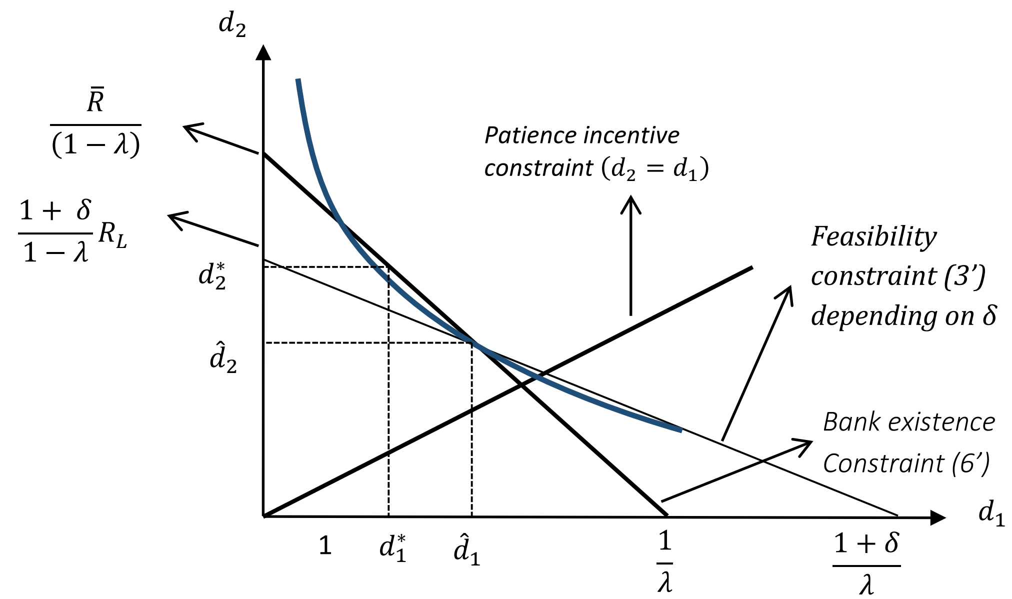
The line representing (3’) is in the "middle" position. Its intersection with the line representing (6') is located above the bisector (see figure 4):

$$\begin{cases} \hat{d}_1 = \dfrac{\bar{R}-(1+\delta)R_L}{\lambda(\bar{R}-R_L)} \\ \hat{d}_2 = \dfrac{\delta R_L \bar{R}}{(1-\lambda)(\bar{R}-R_L)} \end{cases}$$

At this point, the marginal rate of substitution is greater than the slope of the line representing the feasibility constraint (3 ').

[4] It is easy to show that $\frac{1-R_L+(1+R_L)\lambda}{R_L} < \frac{\bar{R}-R_L}{R_L\left(1+\bar{R}\frac{\lambda}{1-\lambda}\right)}$. If $\delta < \frac{1-R_L+(1+R_L)\lambda}{R_L} < \frac{\bar{R}-R_L}{R_L\left(1+\bar{R}\frac{\lambda}{1-\lambda}\right)}$, then the contract $\{d_1 = \hat{d}, d_2 = \hat{d}\}$.does not meet the depositor participation constraint.

Figure 4: optimal deposit contract when $\frac{\bar{R}-R_L}{R_L\left(1+\bar{R}\frac{\lambda}{1-\lambda}\right)} < \delta < \frac{\bar{R}-R_L}{R_L}$



Two sub-cases are possible:

- Case 3.1: the previous contract, $\{d_1^*, d_2^*\}$ defined in equation (11), is feasible, (on figure 4, it would lie to the right of the intersection of the lines representing (3') and (6')), which brings us to the case 1.
- Case 3.2: the previous contract, $\{d_1^*, d_2^*\}$ is not feasible, and the optimal contract corresponds to an corner optimum, at the intersection of straight representative (3') and (6') : $\{\hat{d}_1, \hat{d}_2\}$ (see figure 4).

With $u\ (c) = c^{1-\frac{1}{\alpha}}/\left(1-\frac{1}{\alpha}\right)$, $\{d_1^*, d_2^*\}$ defined in equation (11), is feasible if and only if:

$$\delta \geq \frac{\bar{R}}{R_L}\frac{[(1-\lambda)\bar{R}^{\alpha}+\lambda R_L]}{[(1-\lambda)\bar{R}^{\alpha}+\lambda\bar{R}]} - 1.$$

The non-contingent deposit contract is related to the level of the equity-to-deposit ratio δ. Table 1 summarizes our results.

Table 1: characteristics of the deposit contract according to the level of equity to deposit ratio

$\delta = K/D$ (equity-to-deposit ratio)	Deposit contract	see case	binding constraints
$\delta \geq \frac{\bar{R}}{R_L}\frac{[(1-\lambda)\bar{R}^{\alpha}+\lambda R_L]}{[(1-\lambda)\bar{R}^{\alpha}+\lambda\bar{R}]} - 1$	$d_1^* = \frac{\bar{R}}{\left((1-\lambda)\,\bar{R}^{\alpha} + \lambda\,\bar{R}\right)}$ $d_2^* = \bar{R}^{\alpha}\, d_1^*$	1, 3.1	existence
$\frac{\bar{R}-R_L}{R_L\left(1+\bar{R}\frac{\lambda}{1-\lambda}\right)} < \delta <$ $\frac{\bar{R}}{R_L}\frac{[(1-\lambda)\bar{R}^{\alpha}+\lambda R_L]}{[(1-\lambda)\bar{R}^{\alpha}+\lambda\bar{R}]} - 1$	$\hat{d}_1 = \frac{\bar{R}-(1+\delta)R_L}{\lambda(\bar{R}-R_L)}$ $\hat{d}_2 = \frac{\delta R_L \bar{R}}{(1-\lambda)(\bar{R}-R_L)}$	3.2	existence, feasibility
$\frac{1-R_L+(1+R_L)\lambda}{R_L} < \delta < \frac{\bar{R}-R_L}{R_L\left(1+\bar{R}\frac{\lambda}{1-\lambda}\right)}$	$d_1 = d_2 = \hat{d} = \frac{(1+\delta)R_L}{1+\lambda+\lambda R_L}$	2	feasibility, patience
$\delta < \frac{1-R_L+(1+R_L)\lambda}{R_L}$	No bank	2	participation

So far we have been taking δ as given. Given the amount of equity capital, the relative level of capital obviously depends on the amount of collected deposits.

If competition pushes banks to maximize the utility of depositors, it should lead to an equilibrium without rationing (all depositors find a bank that accepts their deposit). But, taking K as given, accepting more deposits reduces the relative level of capital, which limits the bank's ability to remunerate deposits. A competitive equilibrium is achieved with "small" banks that, given their level of equity K, collect just enough deposits to be able to offer the optimal contract$\{d_1^*, d_2^*\}$. Thus, in equilibrium, we have (see second row of Table 1):

$\delta^{BC} = \frac{\bar{R}}{R_L}\frac{[(1-\lambda)\bar{R}^{\alpha}+\lambda R_L]}{[(1-\lambda)\bar{R}^{\alpha}+\lambda\bar{R}]} - 1 = \frac{(1-\lambda)(\bar{R}-R_L)\bar{R}^{\alpha}}{R_L[(1-\lambda)\bar{R}^{\alpha}+\lambda\bar{R}]}$ and $\{\hat{d}_1, \hat{d}_2\} = \{d_1^*, d_2^*\}$.

3. Equilibrium deposit contracts offered by Islamic banks:

We assume that an Islamic banker offers deposits that have the characteristics of a "*qard hassan"-type* current account, in case of early withdrawal, and of a *PSIA-type account* in case of late withdrawal.

Thus, in case of early withdrawal, deposits are not remunerated, depositors can only withdraw the principal, especially as the result of investments is not yet known at t = 1 and cannot be shared (Equation 13). In case of late withdrawal, depositors receive a share of bank revenues (denoted π_{BI}). . The sharing coefficient, announced by the banker at time t=0, is denoted μ. Then, depositors receive a total of $\mu\,\pi_{BI}$ which corresponds to a unit gross revenue of $\frac{\mu\,\pi_{BI}}{(1-\lambda)D}$ since there are $(1-\lambda)$ late consumers and a total amount of deposit equal to D. This gross revenue must be is greater than 1, as an incentive for type 2 depositors to wait until time t = 2. (13) $\begin{cases} d_1 = 1 \\ d_2 \geq d_1 \end{cases}$

(possible withdrawal at t= 1 of principal and incentive for patience)

Note that these two combined constraints means that we have to force $d_2 \geq 1$. Thus, in a way, Islamic bank is constrained to guarantee deposit principal at t=2 in order to encourage « late» consumers to be patient.

The banker receives the bank's residual income, denoted R_{BI}. (Equation 14)

(14) $$\begin{cases} d_2 = max\left\{1, \frac{\mu\,\pi_{BI}}{(1-\lambda)D}\right\} \\ R_{BI} = min\{(1-\mu)\,\pi_{BI},\, \pi_{BI} - (1-\lambda)D\} \end{cases}$$ (Contingent revenue)

The bank's income is equal to the sum of non-retrieved amounts at t=1, $(y - \lambda D)$, and long-term asset return, $(\tilde{R}x)$:

(15) $$\pi_{BI} = (y - \lambda D) + \tilde{R}x$$

At t=2, a proportion μ of the bank income is distributed to depositors so that the banker retains $(1-\mu)\,\pi_{BI}$, under the patience incentive constraint: the sharing agreement applies only if the bank's income is sufficient to repay the depositors at least the deposited amount. If the bank income is not sufficient to enforce the sharing agreement, the banker must redeem depositors the amount of $(1-\lambda)D$ and can keep the balance. The sharing agreement applies only for "late" consumers who waited until t=2.

Thus, a feasibility constraint occurs for the deposit contract: the bank's income must cover, in the worst case, the deposit amount for "late" consumers.

(16) $(y - \lambda D) + R_L x \geq (1-\lambda)D$ (feasibility constraint)

To guarantee $d_1 = 1$ for early consumers, the Islamic banker invests $y = \lambda D$ in short-term asset and therefore $x = K + (1-\lambda)D$ in long-term asset. The bank income at time t=2 is:

(17) $\pi_{BI} = \tilde{R}x$

The contract feasibility constraint becomes $R_L x \geq (1-\lambda)D$ with $x = K + (1-\lambda)D$, which leads to :

(18) $\delta \geq (1-\lambda)\frac{1-R_L}{R_L}$

The conditions means that the Islamic bank can refund deposits in the worst case scenario only if it has sufficient equity.

The characteristics of the deposit contract and the banker's remuneration are then given by:

(19) $\begin{cases} d_1 = 1 \\ \tilde{d}_2 = max\left\{1, \frac{\mu\tilde{R}}{k}\right\} \\ \tilde{R}_{BI} = min\{(1-\mu)\tilde{R}x, \tilde{R}x - (1-\lambda)D\} \end{cases}$ with $\begin{cases} \tilde{d}_2 = d_{2s} \\ \tilde{R} = R_s \\ \tilde{R}_{BI} = R_{BIs} \end{cases}$ in state s, $s \in \{L, H\}$

When income sharing is not constrained by the warranty of deposit, depositors receive at t=2 a unit compensation $\tilde{d}_2 = \mu\tilde{R}/k$ where $k \equiv \frac{(1-\lambda)}{\delta+(1-\lambda)}$ is the bank leverage at t=2. (proportion of the remaining deposits in total resources).

The remuneration of late depositors is depicted in figure 5. It depends in an increasing and nonlinear fashion on the sharing coefficient:

- If the banker sets a sharing coefficient lower than or equal to a first threshold $\hat{\mu}_H \equiv k/\ R_H$, then late depositors will not be remunerated regardless of the state of nature at time t=2: $d_{2L} = d_{2H} = 1$.

- If the banker sets a sharing coefficient equal to or greater than a second threshold $\hat{\mu}_L \equiv k/\ R_L$, then late depositors receive a state-contingent remuneration at t=2 : $d_{2H} \equiv \frac{\mu R_H}{k} \geq 1$ in state H and $d_{2L} \equiv \frac{\mu R_L}{\text{k}} \geq 1$ in state L.
- If the banker sets a sharing coefficient that lies between the two thresholds ($\hat{\mu}_L > \mu > \hat{\mu}_H$), then late depositors receive at t=2 a remuneration only in state H : $d_{2L} = 1$ and $d_{2H} \equiv \frac{\mu R_H}{k}$.

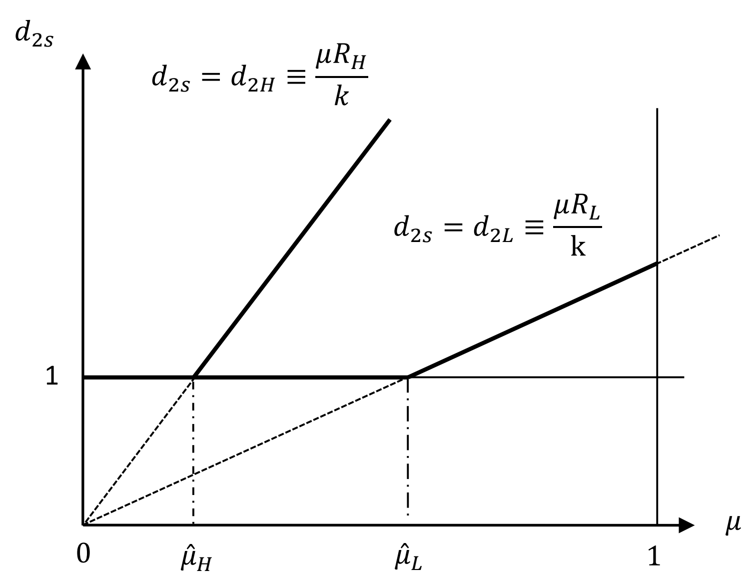
Competition among the Islamic banks induces them to offer deposit contracts with sharing coefficient as favourable as possible to depositors (i.e. with the highest μ). Yet, like for conventional banks, we can consider that this pressure for a high μ is limited by bank existence condition. The Islamic banker founds a bank only if he receives at least as much as if he invested directly in long-term assets (recall the banker is assumed to be risk neutral and a late consumer). In competitive equilibrium, the Islamic banker propose a sharing coefficient that can assure him the same expected income as if he invested directly in long-term assets.

(20) $E\left(\tilde{R}_{BI}\right) = \bar{R}\ K$ *(Islamic bank's existence condition)*

This condition restrains the banker from proposing a partition coefficient μ equal to or higher than the second threshold $\hat{\mu}_L \equiv \frac{k}{R_L}$.

Indeed, for $\mu > \hat{\mu}_L$, at t=2, Depositors would receive $\mu\ x\ R_s$ and the banker would receive $R_{BIs} = (1-\mu)\ x\ R_s$ averaging to $E\left(\tilde{R}_{BI}\right) = (1-\mu)\ x\ \bar{R} < \bar{R}\ K$ which is less than investing in a long-

Figure 5: Deposits fees in Islamic banking



term asset. Income splitting would be “too much” in favour of depositors and too unfavourable for banker who will have no interest in creating the bank.

In competitive equilibrium, the sharing coefficient is between the two thresholds $\hat{\mu}_H \equiv \frac{k}{R_H}$ and $\hat{\mu}_L \equiv \frac{k}{R_L}$. Indeed, in this case, at time t=2:

- In state L, the bank has too little income to enforce the sharing agreement and the banker has to guarantee the deposit amounts: the banker receives $R_L x - (1-\lambda)D$, we will assume that this amount is positive.
- In state H, the bank has enough income to enforce the sharing agreement. The banker receives $(1-\mu)\,x\,R_H$.
- On average, the banker receives $E(\tilde{R}_{BI}) = p_L\,(R_L x - (1-\lambda)D) + p_H(1-\mu)\,x\,R_H$.

The expected income of the banker satisfies the condition of existence of the bank if: : $E(\tilde{R}_{BI}) = \bar{R}\,K$.

Hence the optimal sharing coefficient:

(21) $$\mu = k\frac{\bar{R}-p_L}{p_H\,R_H} = \frac{(1-\lambda)}{\delta+(1-\lambda)}\frac{\bar{R}-p_L}{p_H\,R_H} \equiv \mu^*$$

We can easily check that μ^* lies between $\hat{\mu}_L\ and\ \hat{\mu}_H$.

The deposit agreement proposed by Islamic bankers has then the following characteristics:

(22) $$\begin{cases} d_1 = 1 \\ d_{2\mathrm{L}} = 1 \\ d_{2\mathrm{H}} = \mu^* R_H/k = (\bar{R}-p_L)/p_H \end{cases}$$

This optimal deposit contract is the result of several constraints under which the bank operates: "legal", "competitive" and "existential".

We can note that the competition between banks is essentially based on the sharing coefficient. However, as demonstrated in equation (21), this coefficient is an increasing function of the equity capital ratio δ. Banks therefore arbitrate between the feasibility constraint of the deposit contract, which imposes a minimum capital equity ratio, and the competitive pressure which caps this ratio. In equilibrium, the banks saturate the feasibility constraint (eq 18):

(23) $$\delta^{BI} = (1-\lambda)\frac{1-R_L}{R_L}$$

And, following from (21) and (23), the equilibrium sharing coefficient is equal to:

(24) $\mu^{BI} = \frac{(\bar{R}-p_L)R_L}{p_H\, R_H}$

Notice that it does not impact the equilibrium deposit remunerations (cf. equation 22).

The expected utility of a depositor is given by:

(25) $U_{BI} = \lambda\, u(1) + (1-\lambda)\, \left[p_H\, u\left(\frac{\bar{R}-p_L}{p_H}\right) + p_L\, u(1)\right]$

Finally, the liquidation value of the Islamic bank assets at time t = 1 is: $\lambda D + r(K + (1-\lambda)D)$. But the maximum amount of withdrawals is D. The bank is fragile and is exposed to a run of depositors if it is under-capitalized: $D > \lambda D + r(K + (1-\lambda)D)$, that is $\delta < (1-\lambda)\frac{1-r}{r}$. And we have shown that, in competitive equilibrium, the equity to deposit ratio is $\delta^{BI} = (1-\lambda)\frac{1-R_L}{R_L}$. As $R_L > r$, δ^{BI} is smaller than $(1-\lambda)\frac{1-r}{r}$: the Islamic bank is thus exposed to runs.

4. A comparison between conventional and Islamic deposit contracts:

Our model has outlined that being conventional or Islamic leads to the same equilibrium expected income for the banker, $\bar{R}\, K$. However, this income is reached with substantially different conditions.

In order to compare the welfare of depositors in the two institutional environments, we specify the utility function of depositors:$u\,(c) = c^{1-\frac{1}{\alpha}}/\left(1-\frac{1}{\alpha}\right)$ with $\frac{1}{\alpha} > 1$.

First of all, we can note that, in the bad state of nature (L), the banker income is nil whether he is of the conventional or Islamic type: the feasibility constraint of a deposit contract, in a competitive market, forces banker to “sacrifice” his initial contribution to fulfil the contract when investment income is low.

The difference between equilibrium incomes received in the good state of nature (H) by each type of banker, comes basically from the difference between equity capital levels (see table 4 below). The expected income is always equal to $\bar{R}\; K$, i.e. $\bar{R}\;\delta^{BC}D$ for the conventional banker and $\bar{R}\;\delta^{BI}D$ for the Islamic banker.

Table 2 below recaps the banker’s income in each state of nature.

table 2: Banker income		
	Conventional Bank	Islamic Bank
state L	$R_{BCL} = 0$	$R_{BIL} = 0$
state H	$R_{BCH} = \dfrac{(R_H - R_L)\,\bar{R}^{\alpha+1}(1-\lambda)D}{R_L[(1-\lambda)\bar{R}^{\alpha} + \lambda\bar{R}]}$	$R_{BIH} = \dfrac{\bar{R}(1-R_L)(1-\lambda)D}{p_H R_L}$

- In equilibrium, the deposit contracts is have significantly different features in the two institutional environments (see table 3 below). The guarantee of the deposit amount in the Islamic case amounts to a ban of interests on deposit withdrawn at t=1 ($d_1^{BI} = 1 < d_1^{BC}$), which corresponds to the prohibition of interest in Islamic Finance.
- The remuneration of "patient" depositors is contingent in the Islamic case in accordance with the principle of profit and loss sharing and it is predetermined in the conventional case. This is the most striking difference between the two systems.
- The equilibrium features of the deposit contract are essentially determined by the constraints faced by the banker in the Islamic environment, whereas they result from maximizing depositors' utility in the conventional case.
- Deposits contracts lead to different risk-sharing arrangements, with better consumption smoothing for depositors in the conventional system.

Table 3: Reminder for deposit contracts characteristics	
Conventional Bank	Islamic Bank
(11) $\begin{cases} d_1^{BC} = \dfrac{\bar{R}}{(1-\lambda)\,\bar{R}^{\alpha} + \lambda\,\bar{R}} \\ d_2^{BC} = \dfrac{\bar{R}^{1+\alpha}}{(1-\lambda)\,\bar{R}^{\alpha} + \lambda\,\bar{R}} \end{cases}$	(22) $\begin{cases} d_1^{BI} = 1 \\ d_{2L}^{BI} = 1 \\ d_{2H}^{BI} = \dfrac{\bar{R}-p_L}{1-p_L} \end{cases}$

In competitive equilibrium, equity–to-deposit ratios are also different. They are repeated in table 4. It is easy to show that, under the model assumptions[5], the commercial bank is more capitalized than the Islamic bank in equilibrium: $\delta^{BC} \geq \delta^{BI}$.

[5] The essential assumptions are : $0 \leq \lambda \leq 1$, $0 < \alpha < 1$, $\bar{R} \geq 1 > R_L > 0$

There are naturally two different interpretations for this result. If we consider that the total amount of collected deposits is given, $\delta^{BC} \geq \delta^{BI}$ means that the conventional bank needs more capital than the Islamic bank in equilibrium, which also means that the conventional bank is bigger. On the contrary, if we consider that the initial equity capital brought by the banker is given, $\delta^{BC} \geq \delta^{BI}$ means that conventional bank collects less deposits than Islamic bank at equilibrium, so that it is smaller.

table 4: equity-to-deposit ratio	
Conventional bank	Islamic bank
$\delta^{BC} = \frac{(1-\lambda)(\bar{R}-R_L)\bar{R}^{\alpha}}{R_L[(1-\lambda)\bar{R}^{\alpha}+\lambda\bar{R}]}$	$\delta^{BI} = (1-\lambda)\frac{1-R_L}{R_L}$

The optimal composition of bank assets is also different (table n°5). It is easy to show that, in equilibrium, the Islamic banker chooses a more liquid portfolio than the conventional banker, i.e. a portfolio that contains a larger share of short-term assets : $y^{BI}/[x^{BI}+y^{BI}] > y^{BC}/[x^{BC}+y^{BC}]$.

However, the short-term-assets to-deposits ratio is lower in the Islamic bank's balance sheet. $(y^{BC}/D > y^{BI}/D)$.

table 5 : Asset portfolio characteristics	
Conventional Bank	Islamic Bank
$\begin{cases} x^{BC} = \frac{(1-\lambda)\,\bar{R}^{\alpha+1}}{R_L[(1-\lambda)\bar{R}^{\alpha}+\lambda\bar{R}]}D \\ y^{BC} = \frac{\lambda\bar{R}}{(1-\lambda)\,\bar{R}^{\alpha}+\lambda\,\bar{R}}D \end{cases}$	$\begin{cases} x^{BI} = \frac{1-\lambda}{R_L}D \\ y^{BI} = \lambda\,D \end{cases}$

Last but not least, depositors derive a higher level of utility in equilibrium in the conventional banking system. The conventional deposit contract is more attractive than the Islamic one. The expected utility of a depositor is given by:

$Eu^{BC} = \lambda\, u\,(d_1^{BC}) + (1-\lambda)u\,(d_2^{BC}) = \frac{1}{1-\frac{1}{\alpha}}[(1-\lambda)\,\bar{R}^{\alpha-1}+\lambda]^{1/\alpha}$ in the conventional banking system

$$Eu^{BI} = \lambda u\,(d_1^{BI}) + (1-\lambda)[p_L u\,(d_{2L}^{BI}) + p_H u\,(d_{2H}^{BI})] = \frac{1}{1-\frac{1}{\alpha}}\left[\lambda + (1-\lambda)\left(p_L + \frac{(1-p_L)^{\frac{1}{\alpha}}}{(\bar{R}-p_L)^{\frac{1}{\alpha}-1}}\right)\right]$$

in the Islamic banking system

We show in the appendix that $Eu^{BC} \geq Eu^{BI}$.

5- CONCLUSION

We have proposed a theoretical model of liquidity creation by banks in a competitive system, either of a conventional type or of an Islamic type, and we have compared the equilibrium features in the two environments. To our knowledge, this is the first attempt to extend the Bryant-Diamond-Dybvig model to the specificities of Islamic finance. We determine the optimal deposit contracts and portfolios of bank assets, in the equilibrium without runs, and show that an Islamic banking system would offer deposit contracts that are less favourable to depositors, hold more liquid assets and have a lower equity to deposit ratio than a conventional banking system. These results are mainly due to the more stringent constraints that Islamic finance imposes on the characteristics of deposit contracts.

Two natural extensions of our model would be worth considering. We have compared two distinct institutional environments, whereas real-world banks often operate in a dual system.

We could also consider assuming a random rather than constant proportion of early consumers, and analyse the consequences of this aggregate liquidity risk on bank behaviour, and prudential policy.

REFERENCES :

Ahmed, H., 2002, “A microeconomic model of an Islamic bank”, IRTI Research Paper n°59, Islamic Development Bank.

Ahmed, O., 1997, “Islamic Financial Instruments to Manage Short-term Excess liquidity”, IRTI Research Paper n°41, Islamic Development Bank.

Ali, S. 2006, “Financial Distress and Bank Failure: Lessons from Ihlas Finans Turkey”, *Islamic Economic Studies*, pp. 1-52. Vol. 14, n°1 et 2 (August 2006 & January 2007)

Ali, S. 2012, “State of liquidity management in Islamic Financial Institutions”, IRTI working paper n°1433-06, Islamic Development Bank.

Allen, F. and D. Gale, 2007, *Understanding Financial Crises*, Clarendon Lecture Series in Finance, Oxford University Press.

Archer, S. and R. A. A. Karim, 2006, “On capital structure, risk sharing and capital adequacy in Islamic banks”, *International Journal of Theoretical and Applied Finance*, Vol. 9, n° 3, pp. 269–280.

Archer, S. and R. A. A. Karim, 2009, “Profit sharing investment accounts in Islamic banks: Regulatory problems and possible solutions”, *Journal of Banking Regulation*, vol. 10, pp. 300-306.

Bryant, J., 1980, “A model of reserves, bank runs and deposit insurance”, *Journal of Banking and Finance*, vol. 4, pp. 335–344.

Diamond, D. and P. Dybvig, 1983, “Bank Runs, Deposit Insurance, and Liquidity”, *Journal of Political Economy*, vol. 91, June 1983, pp. 401-419.

Dowd, K., 2000, “Bank capital adequacy versus deposit insurance”, *Journal of Financial Services Research*, vol. 17, pp. 7-15.

El-Gamal, M., 2006, Islamic finance, law economics and practice, Cambridge University Press.

Ernst & Young, 2011, World Islamic Banking Competitiveness Report 2011-12.

Ernst & Young, 2012, World Islamic Banking Competitiveness Report 2012-13.

Freixas, X. and J.-C. Rochet, 2008, Microeconomics of Banking, Second Edition, MIT Press.

Gangopadhyay, S. and G. Singh, 2000, “Avoiding bank run in transition economies: The role of risk neutral capital”, *Journal of Banking and Finance*, vol. 24, pp. 625-642.

Hassoune, A., 2003, « La solvabilité des banques islamique : forces et faiblesses », *Revue d'économie financière*, n°72, pp. 277-297.

Hassoune A., 2008, « La gestion des risques dans les banques islamiques », présentation, Novembre 2008

Holmström, B. and J. Tirole, 1988, “Private and public supply of liquidity”, *Journal of Political Economy*, vol. 106, n°1, pp. 1-40.

IFSB, 2008, Technical note on issues in strengthening liquidity management of institutions offering Islamic financial services: the development of Islamic money markets.

IFSB-IRTI-IDB, 2010, Islamic Finance and Global Stability Report.

Iqbal, A, 2012, « Liquidity risk management: A comparative study between conventional and Islamic Banks of Pakistan”, vol. 12, issue. 5, *Global Journal of Management and Business Research*

Jobst, A. and J. Solé, 2012, “Operative Principles of Islamic Derivatives: Towards a Coherent Theory”, IMF working paper W/12/63, International Monetary Fund.

Marini, F., 2003, “Bank insolvency, deposit insurance and capital adequacy”, Journal of Financial Services Research, vol. 24, pp. 67-78.

Salem, R. (2013), “Risk Management for Islamic Banks”, *Edinburgh Guides To Islamic Finance*

Standard & Poor’s, 2010, Islamic Finance Outlook.

Sundararajan, V., 2008, “Issues in Managing Profit Equalization Reserves and Investment Risk Reserves in Islamic Banks”, Journal of Islamic *Economics, Banking and Finance*, vol.4, pp. 1-11.

APPENDIX: PROOF THAT $\boldsymbol{Eu^{BC} \geq Eu^{BI}}$.

Recall that : $Eu^{BC} = \frac{1}{1-\frac{1}{\alpha}}[(1-\lambda)\,\bar{R}^{\alpha-1} + \lambda]^{1/\alpha}$

and $Eu^{BI} = \frac{1}{1-\frac{1}{\alpha}}\left[\lambda + (1-\lambda)\left(p_L + \frac{(1-p_L)^{\frac{1}{\alpha}}}{(\bar{R}-p_L)^{\frac{1}{\alpha}-1}}\right)\right]$

where: $0 \leq \lambda \leq 1$, $0 < \alpha < 1$, $\bar{R} \geq 1$ and $0 < p_L < 1$.

Rewrite $\left(1-\frac{1}{\alpha}\right)Eu^{BC}$ and $\left(1-\frac{1}{\alpha}\right)Eu^{BI}$ as functions of λ whose parameters depend on p_L (and on $\bar{R}$ but this is not useful for our demonstration), in the following way:

$\left(1-\frac{1}{\alpha}\right)Eu^{BI} = A + \lambda(1-A) \equiv f(\lambda)$ where $A = p_L + \frac{(1-p_L)^{\frac{1}{\alpha}}}{(\bar{R}-p_L)^{\frac{1}{\alpha}-1}} \equiv A(p_L)$

$\left(1-\frac{1}{\alpha}\right)Eu^{BC} = [B + (1-B)\lambda]^{1/\alpha} \equiv g(\lambda)$ where $B = \bar{R}^{\alpha-1}$.

Lemma 1 : $0 < B < 1$

Proof: immediate, because $0 < \alpha < 1$ et $\bar{R} \geq 1$.

Lemma 2 : $B^{\frac{1}{\alpha}} < A(p_L) < 1$.

Proof : $\frac{dA}{dp_L} \equiv A'(p_L) = 1 + \frac{1}{\alpha}\left(\frac{1-p_L}{\bar{R}-p_L}\right)^{\frac{1}{\alpha}-1} + \left(\frac{1}{\alpha}-1\right)\left(\frac{1-p_L}{\bar{R}-p_L}\right)^{\frac{1}{\alpha}}$.

$A'(p_L) > 0$ because $0 < \alpha < 1$, $\bar{R} \geq 1$ et $0 < p_L < 1$.

Thus the function $A(p_L)$ is increasing, thus $A(0) < A(p_L) < A(1)$.

Moreover, $A(0) = \bar{R}^{1-\frac{1}{\alpha}} = B^{\frac{1}{\alpha}}$. And $A(1) = 1$.

Lemma 3 : the function $f(\lambda)$ is linear and increasing, and $A(p_L) < f(\lambda) < 1$.

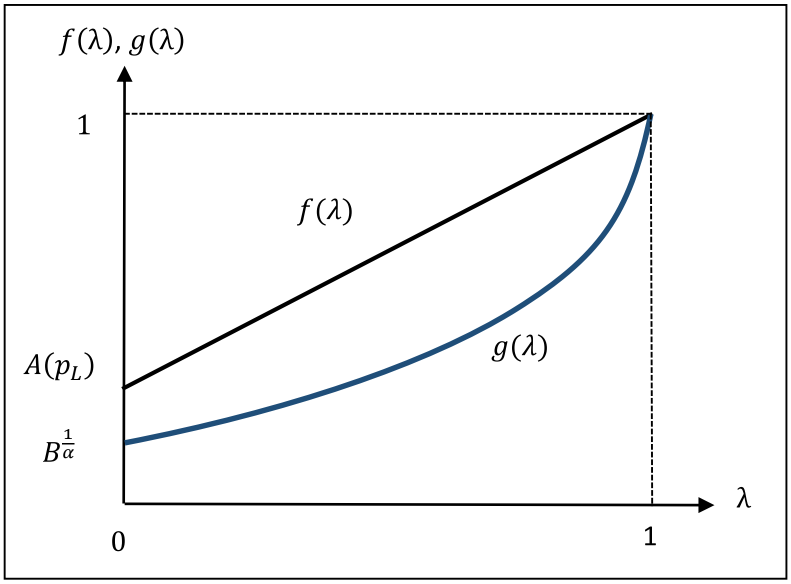
Proof: Obviously$f(\lambda)$ is affine. It is increasing because$0 < A < 1$; from lemmas 1 and 2, it follows immediately that $f(0) = A$ and $f(1) = 1$.

Lemma 4 : The function $g(\lambda)$ is increasing and convex, and $B^{\frac{1}{\alpha}} < g(\lambda) < 1$.

Proof: no particular difficulty.

Proposition : $Eu^{BC} \geq Eu^{BI}$.

Proof (graphical) : according to lemmas 1 to 4, functions $f(\lambda)$ and $g(\lambda)$ can be graphically represented as follows:



Therefore : $\forall\lambda \in [0,1], f(\lambda) \geq g(\lambda)$. Hence: $\left(1-\frac{1}{\alpha}\right)Eu^{BI} \geq \left(1-\frac{1}{\alpha}\right)Eu^{BC}$.

And, since: $0 < \alpha < 1 \Rightarrow 1-\frac{1}{\alpha} < 0$. it follows that $Eu^{BC} \geq Eu^{BI}$. QED.