Skewness Premium and Index Option Returns

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Abstract

This paper presents a simple way to estimate the skewness premium implied in index option returns. Using a methodology proposed by Constantinides/Jackwerth/Savov (2013), we create option-based investment strategies with the same volatility and the same market beta (of one) but different degrees of skewness. The resulting return series can be seen as index returns with controlled skewness. In an empirical analysis for SPX, ESX and DAX options over the period 1995 to 2016, we find a remarkably close association of skewness and average returns that is consistent with a significantly negative skewness premium.

Keywords: Skewness, index options, higher moments, market risk premia

JEL: G12, G14

1. Introduction

The objective of this paper is to estimate the skewness premium in stock market returns in a new way. We use index option strategies to construct a cross-section of return distributions that primarily differ with respect to the degree of skewness. The investment strategies involve the risk-free asset and one index option at a time. The variation of skewness is achieved by using a cross-section of index options of different type (call and put options), moneyness (0.9 to 1.05 in terms of strike to index level) and time to maturity (30, 60, and 90 days). The options are unlevered so that their market exposure (beta) is one, and the portfolio is readjusted after a holding period of one day in order to hold the characteristics of the trading strategy constant over time.

When using *call* options, we have to hold a *long* position for a target beta of one. This means that returns will be convex. In contrast, to achieve the same beta with *put* options requires a *short* position, which implicates concave returns (negative convexity). The degree of convexity (positive or negative) tends to be stronger the deeper out-of-the money the options are and the shorter their time to maturity is. Positive convexity means that high positive index returns are reinforced while extreme negative returns are mitigated, which translates into higher skewness of the return distribution compared to the index. Inversely, negative convexity exacerbates extreme negative index returns and attenuates strongly positive returns, which produces lower skewness compared to the index. The typical dynamics of the smile pattern of index option prices tends to further increase these convexity differences between the call- and put-based strategies. The reason is that rising implied volatilities after a sharp index decrease are detrimental to the short put position while they are advantageous for the long call position.

Although the holding period is only one day, the size of convexity and skewness differences produced by the beta-one strategies is substantial. As other portfolio characteristics are very similar, the return series can be seen as index returns with controlled skewness. Thus, this methodology opens a simple way to estimate the skewness premium.

The idea of creating a cross-section of beta-one option portfolios originates from an innovative contribution of Constantinides et al. (2013). The authors use the option portfolios as test assets in tests of multi-factor pricing models, arguing that the "standard linear factor methodology is applicable because the monthly portfolio returns have low skewness and are close to normal".[1] The low skewness with returns close to normal is emphasized as an important implication of the portfolio construction methodology: "The major advantage of this construction is to lower the variance and skewness of the monthly portfolio returns and render the returns close to normal (about as close to normal as the index return), thereby making applicable the standard linear factor pricing methodology."[2] However, the return series made available by the authors in an online supplement suggest that, in fact, the option portfolios differ considerably in return skewness. The differences are so pronounced that control over skewness appears to be a main characteristic of the method. Therefore, in this paper, we propose to use these portfolio strategies to study the skewness premium, which can be seen as a complement to Constantinides et al. (2013), focusing on an interesting but unexplored aspect of their work. Apart from this focus on the relationship of skewness and expected return, our intended contributions are threefold: (1) to explore the full range of feasible beta-one strategies by allowing index investments as an additional portfolio constituent, (2) to examine the implications of put-call parity for the relationship of call- and put-based strategies and (3) to present a comprehensive empirical analysis with robust and accurate measurement of index option returns.

Our analysis covers the three indices S&P 500 (SPX options), EuroStoxx 50 (ESX options)

[1] Constantinides et al. (2013), p. 229.
[2] Constantinides et al. (2013), p. 230 and similarly on p. 233, 234, 235 and 251.

and DAX (DAX options) for the time periods 1996 to September 2015 for SPX, 2000 to 2016 for ESX and 1995 to 2016 for DAX. In all three markets, we find a strongly negative and remarkably close association of skewness and average returns. Low moneyness options appear to imply a higher premium than high moneyness options. For the portfolios studied in this paper, the Fisher-Pearson coefficient of skewness is very similar to convexity measured by the loading of option returns on squared index returns. Thus, we can interpret our results in terms of a two-stage test of asset pricing models where squared index return is included as a risk factor in the time-series regressions and the squared-return loadings contribute to explaining the cross-sectional return differences.

The relevance of skewness for investor decisions and asset prices has been studied in a large body of prior literature. A preference for positive skewness is observed in lottery experiments (see the critical discussion in Vrecko et al. (2009)). Arditti (1967) shows that a preference for positive skewness follows if the utility function exhibits non-increasing absolute risk aversion in the Arrow-Pratt sense. However, portfolio optimization based on the first three moments of the return distribution is in general only consistent with Expected Utility maximization if a cubic utility function is assumed (for an extension, see Chiu (2010)). Kraus and Litzenberger (1976) derive a market equilibrium model in which systematic skewness is priced. Based on a related asset pricing model, Harvey and Siddique (2000) find an empirical risk premium for systematic skewness of 3.6% per year, which is statistically significant and economically important. In addition to the systematic component of skewness, total skewness of individual stocks also seems to be related to future stock returns (see Conrad et al. (2013)).

In a recent study, Chang et al. (2013) determine the market skewness implied in S&P 500 index option prices and estimate the exposure of a cross-section of stocks with respect to daily changes of implied skewness. The authors find that stocks with a high exposure to the skewness-related risk factor show, on average, low returns: "We find that the average return on the market skewness risk factor portfolio is 0.78% per month, or 9.36% per year, and this return cannot be explained by market beta, the size factor, the book-to-market factor, or the momentum factor."[3]

Another approach to measure the skewness risk premium is to compare realized skewness with options' implied skewness. In this vein, Kozhan et al. (2013) derive option implied skewness from a model-free dynamic strategy which creates a payoff equal to realized market skewness. They find that the strategy is highly exposed to variance risk. When this risk is hedged away, the skewness premium becomes insignificant.

Other studies analyze risk factors which potentially contribute to skewness in returns, in particular volatility risk and jump risk. Andersen et al. (2015) exploit the movements of index option surfaces to study the relation between market risks and risk premia. A main finding is that the dynamics of the option risk premia cannot be explained by the dynamics of the underlying asset prices alone.

Our study is also related to the literature on a potential mispricing of put options because the methodology proposed by Constantinides et al. (2013) produces positive skewness for call-based strategies and negative skewness for put-based strategies. Rubinstein (1994) and Jackwerth and Rubinstein (1996) report that OTM puts are expensive compared to at-the-money (ATM) puts. Jones (2006) confirms that deep-OTM puts on S&P500 index futures are overpriced, generating negative abnormal returns even after taking volatility and jump risk premia into account. In contrast, Broadie et al. (2009) note that very high returns of deep-OTM puts alone are not inconsistent with standard option valuation models because individual option returns are extremely dispersed and highly skewed. Thus, they propose a different test approach based on market-neutral option portfolios. The main finding is that stochastic volatility alone is insufficient to explain returns of

[3]Chang et al. (2013), p. 47.

S&P 500 futures options, but models including estimation risk and jump risk premia are consistent with the data. Chambers et al. (2014) confirm this finding for the extended sample period of 1987-2012. Instead of market-neutral portfolios, the methodology of Constantinides et al. (2013) creates portfolios closely resembling the market, which facilitates the evaluation of abnormal returns in a mean-variance framework.

The paper proceeds as follows. The next section 2 explains the base strategy to create beta-one option portfolios that span a substantial skewness range. Section 3 analyzes the empirical characteristics of these portfolios. Section 4 extends the range of beta-one strategies and demonstrates a way to scale portfolio skewness upwards or downwards. The extended strategies also reveal how put- and call-based strategies are related. Based on this analysis, Section 5 examines the relationship between skewness and average returns in an attempt to estimate the market skewness premium. The last section concludes.

2. Base strategy to generate skewed index returns: delevered options

We implement the methodology proposed by Constantinides et al. (2013) in the following way. Let $\mathcal{C}$ denote the price of an option with strike price X, time to maturity T and underlying index level S. The option delta is the derivative $d\mathcal{C}/dS$. We determine the share ω of wealth invested in the option such that the elasticity of this position with respect to the index is 1:

$$\omega = \frac{1}{\frac{d\mathcal{C}}{dS}\frac{S}{\mathcal{C}}}. \tag{1}$$

The remaining share of $(1-\omega)$ of wealth is invested in the risk-free asset with return r_f. The weight ω will vary over time as it is readjusted to the new elasticity at the beginning of each period.

The portfolio return in period t is

$$r_{pt} = \omega_{t-1} r_{ot} + (1 - \omega_{t-1})\, r_{ft}, \tag{2}$$

where r_o denotes the option return.

The option delta in Eq. (1) depends on the option value function. When estimating delta based on an implied volatility from the Black-Scholes model, we have to take into account that implied volatility changes with index movements. With σ_{imp} as implied volatility, the option delta is given by (see, e.g., Rosenberg (2000)):

$$\begin{aligned} \frac{d\mathcal{C}\left(S, \sigma_{imp}(S)\right)}{dS} &= \frac{\partial \mathcal{C}(S, \sigma_{imp}(S))}{\partial S} + \frac{\partial \mathcal{C}(S, \sigma_{imp}(S))}{\partial \sigma_{imp}} \cdot \frac{d\sigma_{imp}(S)}{dS} \\ &= \Delta_{BS} + \Lambda_{BS} \cdot \frac{d\sigma_{imp}(S)}{dS} \end{aligned} \tag{3}$$

where Δ_{BS} and Λ_{BS} are the option's delta and vega according to the Black-Scholes formula with volatility replaced by implied volatility. The adjustment term in the second summand of Eq. (3) captures the effect of index movements on implied volatility. The impact on implied volatility results from two effects. The first is that implied volatility of index options is typically a decreasing function of moneyness (smile or skew pattern)[4] so that an increase in the index level and the corresponding decrease in moneyness brings about an increase in implied volatility. The second effect is that the skew curve shifts in the opposite direction of index movements. The downward shift in case of a rising index offsets a part of the first effect.

Therefore, in order to obtain an estimate of $d\sigma_{imp}(S)/dS$, we have to consider the structure

[4]We use the terms “smile” and “skew” as synonyms for the strike price pattern of implied volatilities. Although “skew” better describes the downward sloping pattern typically observed in index option markets, “smile” is also commonly used.

and dynamics of the skew curve. A common way to model the implied volatility pattern is to use the cubic regression function:

$$\sigma_{imp}(M) = \beta_0 + \beta_1 M + \beta_2 M^2 + \beta_3 D \cdot M^3 + \varepsilon, \tag{4}$$

where M is time to maturity adjusted moneyness defined as:

$$M = \frac{\ln(Xe^{-rT}/S)}{\sqrt{T}},$$

and β_i, $i = 0, 1, 2, 3$ are regression coefficients, ε is a random error, and D a dummy variable defined as:

$$D = \begin{cases} 0 & , \quad M \leq 0 \\ 1 & , \quad M > 0 \end{cases}.$$

The dummy variable term is included to capture the observation that implied volatility decreases less strongly or even rises at positive moneyness levels.

If the smile pattern does not shift ("sticky moneyness rule"), the second effect described above falls away and the change in implied volatility is fully due to the movement on the smile curve. In this case, for at-the-money options, we obtain:

$$\begin{aligned} \left.\frac{d\sigma_{imp}}{dS}\right|_{M=0} &= \left.\frac{d\sigma_{imp}}{dM}\right|_{M=0} \frac{dM}{dS} \\ &= \beta_1 \left(-\frac{1}{S\sqrt{T}}\right) \end{aligned}$$

where $\beta_1 < 0$ so that the adjustment to Δ_{BS} in Eq. (3) is positive. If the smile pattern shifts in such a way that implied volatility is constant for a given strike X ("sticky strike rule"), the two effects compensate each other and we obtain $d\sigma_{imp}/dS = 0$. If the the smile shifts even more strongly than the sticky strike rule suggests, the second effect overcompensates the first effect and $d\sigma_{imp}/dS$ becomes negative. The empirical evidence for index options suggests that the latter case is typically observed, which is well described by[5]

$$\frac{d\sigma_{imp}(S)}{dS} = 0.5\beta_1 \frac{1}{S\sqrt{T}} = -\frac{0.1}{S\sqrt{T}}. \tag{5}$$

Without this adjustment, the main results remain the same, but the betas of our option portfolios would be less close to the target value of 1.[6]

3. Empirical analysis of the base strategy: delevered option return characteristics

Our sample period extends from Jan. 1996 to Sep. 2015 for SPX options, Jan. 1995 to Dec. 2016 for DAX options and Jan. 2000 to Dec. 2016 for ESX options.[7] On each trading day, we estimate the skew in option prices using the cubic regression model (4). We run the regression separately for each time to maturity. For DAX and ESX options, the estimation is based on all daily transactions at Eurex. For SPX options, we use settlement data provided by Option Metrics. For settlement data, we infer the index level at settlement from put-call parity. The transaction data are synchronized with index future prices by milliseconds to avoid any relevant time mismatch, and the underlying index level is adjusted to account for put-call parity. For details, we refer to Hafner and Wallmeier (2007). As the skew regression model describes the structure of option prices extremely well, we infer option prices for our option portfolios from the estimated smile curves.

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[5]See Wallmeier (2015). For ESX options with a time to maturity in the range of 42 to 67 days or 70 to 95 days, Wallmeier (2015) reports mean β_1 values of -0.1910 and -0.2145, respectively, so that an estimate of $\beta_1 = -0.2$ is regarded as an appropriate choice in Eq. (5).

[6]Constantinides et al. (2013) use the Black-Scholes delta without adjustment and obtain betas reasonably close to 1.

[7]ESX options were launched in January 2000. The sample period for SPX ends in Sep. 2015 because at the time of writing this paper (April 2017), more recent data were not available from OptionMetrics under an academic subscription with annual updates.

Each of our beta-one portfolios is based on only one option at a time. The options differ in terms of time to maturity and moneyness. Following Constantinides et al. (2013), our time to maturity levels are $T \in \{30, 60, 90\}$ days. Constantinides et al. (2013) consider a range of strike to index ratios from 0.9 to 1.1 in steps of 0.25. We fix the upper limit at 1.05 because options with higher moneyness are not always actively traded during the sample period. Thus, in terms of simple moneyness $m = Xe^{-rT}/S$ we consider 7 levels $m \in \{0.9, 0.925, 0.95 0.975, 1.0, 1.025, 1.05\}$. Combining the time to maturity and moneyness levels with the option type $z \in \{call, put\}$, we obtain $3 \cdot 7 \cdot 2 = 42$ different options, each of which is used to create a beta-one strategy as described in Section 2.

More specifically, for a given combination (T_i, m_j, z_k), on day t of the sample period, we identify the option with a time to maturity closest to T_i. Options with time to maturity smaller than 15 days are not considered to avoid particular valuation effects near expiration. Based on the estimated smile regression for the chosen option on day t, we evaluate the option at moneyness m_j and determine the weight ω of the option using Eq. (1) combined with Eqs. (3) and (5). The risk-free asset holding is $(1 - \omega)$. After a one-day holding period, we unwind the position based on the updated moneyness level and the new smile pattern and report the portfolio return. We enter inter a new position each day and thereby obtain a series of daily portfolio returns over the sample period. For the sake of simplicity, in the following we will refer to these particular return series as option returns or, more specifically, call and put option returns.

We inspect scatterplots of option returns versus index returns on a yearly basis in order to detect outliers. Less than 0.2% of the observations are identified as outliers and removed from the analysis. Most of these occur at moneyness levels of 1.025 and 1.05 when trading is relatively infrequent and the coefficient of the cubic element of the smile pattern is estimated with a high standard error.

– Insert Figures 1-3 here –

For SPX options, we illustrate the relationship of daily index and option returns in Figures 1 (T = 30 days), 2 (T = 60 days) and 3 (T = 90 days). Calls are shown in the left panels, puts on the right, and moneyness increases from the top ($m = 0.9$) to the bottom panels ($m = 1.05$). The blue line shows the estimated linear regression of option return on index return, the quadratic regression line is shown in red.

As desired, the linear regression line is almost identical to the 45-degree line which indicates that the strategies were successful in achieving a target beta of 1. The difference in convexity of call option returns and put option returns is clearly visible. As expected, the degree of convexity is largest for deep out-of-the money options. It also tends to be higher for short-dated options.

– Insert Tables 1-3 here –

Tables 1 (SPX), 2 (ESX) and 3 (DAX) show descriptive statistics for the 42 option portfolios. *Beta*, the estimated slope coefficient in the linear index regression model, is always close to 1. This is also true for *Beta1*, the first slope coefficient of the quadratic regression model, while the coefficient of the quadratic term, *Beta2*, is always significantly positive for calls and negative for puts (1% significance level). The average R^2 coefficients of the quadratic model across the 42 portfolios are 96.2%, 96.9% (ESX) and 96.8% (DAX). The volatility (*Vol*) is similar across the 42 portfolios; the differences between the highest and lowest annualized volatility are 1.90 (SPX), 2.42 (ESX) and 1.37 (DAX) percentage points. The last three columns show the Fisher-Pearson coefficient of skewness (*Skew*), the Pearson coefficient of kurtosis (*Kurt*) and the Sharpe ratio (*SR*). Skewness is clearly related to *Beta2,* with mostly positive values for call options and negative values for put options. The most obvious characteristic of the cross-section of Sharpe ratios is that they appear to be considerably higher for put option portfolios compared to call-based portfolios.

– Insert Figure 4 here –

For a better illustration of the close association of skewness and convexity, the left panels of Figure 4 show scatterplots of *Skew* versus *Beta2* across our 42 portfolios for SPX (upper panel), ESX (middle) and DAX (lower panel). The index is positioned at the intersection point of the vertical and horizontal lines. The panels on the right of Figure 4 show how skewness is related to the option characteristics (puts vs. calls; moneyness; time to maturity). As expected, positive and negative skewness is more pronounced for out-of-the-money options. For in-the-money options, the effect of time to maturity (marker 1 for 30 days and 3 for 90 days) is negligible. For out-of-the-money options, skewness becomes more pronounced when the time to maturity is short.

As a first illustration of the cross-section of option returns, in the left panels of Figure 5, we show plots of mean daily option returns versus skewness. The panels on the right of Figure 5 show t-statistics for testing the hypothesis that the expected option return is equal to the expected index return. As the index exposure is the same for all options, this test can be interpreted as a test of abnormal returns with respect to a linear one-factor market model. The horizontal lines indicate the 99% confidence interval around zero. Almost all put option portfolios achieve a significantly positive and almost all call portfolios a significantly negative abnormal return. The striking spread between call and put returns reflects the well-known observation that options are generally more expensive than option pricing models suggest. It is important to keep in mind that our strategy implies long call and, alternatively, short put positions in order to achieve the target beta-one market exposure. Therefore, the put-based strategies *profit* from a high general level of option prices, while the call-based strategies *suffer* from high option prices. The graphs also indicate that there is substantial variation of average returns within each option class (calls and puts).

– Insert Figure 5 here –

4. Extending the range of option-based beta-one strategies: levered skewness and implications of put-call parity

Our beta-one portfolios considered so far consist of a call or put option and the risk-free asset. By adding the underlying index as a third component, we can expand the spectrum of beta-one strategies. Because the index has a beta of one by definition, the relative weights of option and risk-free asset must remain the same as before to ensure an overall beta of one. Therefore, the return r_{p^*t} of the extended portfolios can be written as

$$r_{p^*t} = (1 - b)\, r_{st} + b r_{pt}, \tag{6}$$

where r_{st} is the index return, r_{pt} is the return of our previous portfolio defined in Eq. (2) and b its weight in the new portfolio p^*. As shown in Section 3, r_{pt} is well approximated by the quadratic model $r_{pt} = \alpha_p + r_{st} + \beta_{2,p} r_{st}^2 + \varepsilon_{pt}$, where $\beta_{2,p}$ was used as skewness measure *Beta2*. The quadratic exposure of portfolio p^* is equal to $\beta_{2,p^*} = b\beta_{2,p}$ so that parameter b gives control over portfolio skewness. It can be inflated by choosing $b > 1$, which means that the option component is levered by selling the index short. Eq. (6) shows that the expected portfolio return in excess of the index is levered in the same way: $E[r_{p^*t} - r_{st}] = bE[r_{pt} - r_{st}]$. Thus, the excess return per unit of skewness (measured by β_2) is independent of b:

$$\frac{E[r_{p^*t} - r_{st}]}{\beta_{2,p^*}} = \frac{E[r_{pt} - r_{st}]}{\beta_{2,p}}. \tag{7}$$

The extended p^*-portfolios also reveal that there is a close connection in the pricing of skewness between call and put options, which makes it appear questionable to treat them as separate test assets. In our initial approach, put options had to be sold in order to achieve a positive beta. With the inclusion of index investments, however, a long put combined with the index will also be appropriate. Using put-call parity, we can replicate each combination $(\beta_{2p^*}, E[r_{p^*t} - r_{st}])$ created by put options with call options, and vice versa. To this end, we have to choose the b coefficients for the call-based and put-based portfolios such that r_{p^*t} is the same in both cases:[8]

$$\begin{aligned} &(1 - b_{Call})\, r_s + b_{Call}\omega_{Call} r_{Call} + b_{Call}\,(1 - \omega_{Call})\, r_f \\ &= (1 - b_{Put})\, r_s + b_{Put}\omega_{Put} r_{Put} + b_{Put}\,(1 - \omega_{Put})\, r_f \end{aligned} \tag{8}$$

Inserting ω from Eq. (1) and rearranging gives:

$$\frac{b_{Put}}{\Delta_{Put}} Pr^e_{Put} - \frac{b_{Call}}{\Delta_{Call}} Cr^e_{Call} + (b_{Call} - b_{Put})\, Sr^e_s = 0 \tag{9}$$

where superscript e indicates returns in excess of the risk-free rate. Put-call parity implies $Pr^e_{Put} = Cr^e_{Call} - Sr^e_s = 0$ so that Eq. (9) can be written as:

$$\left(\frac{b_{Put}}{\Delta_{Put}} - \frac{b_{Call}}{\Delta_{Call}}\right) Cr^e_{Call} + \left(b_{Call} - b_{Put} - \frac{b_{Put}}{\Delta_{Put}}\right) Sr^e_s = 0, \tag{10}$$

[8] In the following, we drop the time index for ease of notation.

which holds if b_{Call} and b_{Put} are chosen according to:[9]

$$\frac{b_{Put}}{\Delta_{Put}} = \frac{b_{Call}}{\Delta_{Call}}. \tag{11}$$

Fig. 6, Panel A, illustrates this result for SPX options. The portfolio returns are defined by Eqs. (6) and (2) with $b_{Put} = 1$ and $b_{Call} = \Delta_{Call}/\Delta_{Put}$.[10] As required, the skewness and average portfolio returns of call-based and put-based portfolios match exactly. For $b_{Put} = 1$ and $b_{Call} = -\Delta_{Call}/\Delta_{Put}$, the skewness and mean returns of calls exactly mirror the skewness and mean returns of puts (mean returns in excess of index returns) with reversed signs (see Fig. 6, Panel B). In the general case of $b_{Call} = k \cdot b_{Put}\Delta_{Call}/\Delta_{Put}$ with constant $k \neq 0$, the call option skewness is enlarged or reduced by scaling factor k in such a way that the lines connecting the combinations of skewness and mean return of calls and puts with the same moneyness and time to maturity run through the position of a pure index investment (see Fig. 6, Panel C, for the example $b_{Put} = 1$ and $b_{Call} = -2\Delta_{Call}/\Delta_{Put}$.)

Our original portfolios were defined by $b_{Put} = 1$ and $b_{Call} = 1$ (see Fig. 6 Panel D). In this case, no constant factor k exists that satisfies $b_{Call} = k \cdot b_{Put}\Delta_{Call}/\Delta_{Put}$. The reason is that the call and put deltas vary with volatility and time to maturity. For this reason, the call and put observations in Fig. 6, Panel D are not exactly symmetrical to the index position. However, this symmetry is still a good approximation showing that the call and put portfolios are closely related owing to the implications of put-call parity.

[9] With this choice, the second bracket term in Eq. (10) is zero because $1 + \Delta_{Put} - \Delta_{Call} = 0$ for the Black-Scholes deltas as well as the adjusted deltas in Eq. (3).

[10] The call and put deltas vary with the option's implied volatility and time to maturity so that b_{Call} has to be updated on each day.

– Insert Figure 6 here –

5. Skewness and the cross-section of option returns

We propose three simple models to analyze the association of skewness and the cross-section of option returns. The dependent variable in the first two models is the unlevered option return as defined in Eq. (2) minus the contemporaneous index return. Thus, these models focus on beta-one portfolios that only include one specific option (defined by its type, moneyness and time to maturity) and the risk-free asset. In Model 1, skewness (as measured by *Beta2*) is the sole explanatory variable. In Model 2, we allow for a variation of the slope coefficient in Model 1 with moneyness in order to test whether low moneyness options provide a higher skewness premium than high moneyness options. Technically, this conditional structure is captured by an interaction term of *Beta2*) with moneyness:

$$\text{Model 1:} \qquad r^e_{pt} = \alpha + \beta Beta2_p + \epsilon_{pt} \tag{12}$$

$$\text{Model 2:} \qquad r^e_{pt} = \alpha + \beta Beta2_p + \gamma(Beta2_p : Money_p) + \epsilon_{pt} \tag{13}$$

where r^e_{pt} is the spread between the portfolio return r_{pt} of Eq. (2) and the index return r_{st}, $Beta2_p$ is the skewness measure, $Money_p$ is simple moneyness minus 1 (i.e., $Xe^{-rT}/S - 1$)[11], α, β, γ are regression coefficients and ϵ_{pt} is an error term. We hypothesize that β is negative, consistent with the notion of a negative skewness premium. The intercept α in Model 2 is expected to be zero if skewness and moneyness suffice to explain the cross-sectional differences between option and index returns.

[11] We subtract 1 for convenience in interpreting the regression coefficients.

In Eq. (6), we generalized the set of beta-one portfolios by including index investments as an additional portfolio element. As shown in Section 4, any level of *Beta2* can be achieved by appropriate choice of the weight parameter b. In these transformations, the return premium per unit of skewness, $(r_{pt} - r_{st})/Beta2_p$, is constant. Therefore, it is this ratio that directly reflects the skewness premium. For this reason, in Model 3, for each of the 42 options, we choose weight b in Eq. (6) such that $Beta2_p = 1$. We denote the returns of these portfolios, again in excess of index returns, by r^e_{p*t}. The overall skewness premium is now captured by the intercept. As before, we include moneyness to test whether the skewness premium is conditional on the moneyness of the chosen option:

$$\text{Model 3:} \qquad r^e_{p*t} = \alpha + \beta Money_p + \epsilon_{pt} \tag{14}$$

We apply the two-step GMM method of Hansen (1982) to estimate the models. As our explanatory variables are constant over time, the coefficient estimates of GMM are the same as the coefficients of an OLS regression of average returns on the explanatory variables, and these are the same as the estimates of a pooled OLS regression of cross-sectional and time-series data (see Cochrane (2005)). In estimating the standard errors, however, correcting for residual correlation as in the GMM method is important.

– Insert Table 4 here –

Table 4 reports our estimation results. Panel A includes Models 1 and 2 and Panel B shows Model 3. The t-values based on GMM standard errors are reported in brackets. The R^2 coefficient is the cross-sectional R^2 measure of Jagannathan and Wang (1996), which is also employed by

Lettau and Ludvigson (2001) and Petkova (2006). Annualized premium estimates are reported below the regression results.

The results suggest a significantly negative skewness premium. Its magnitude for SPX in Model 1 is $-5.67 \cdot 10^{-5}$ in daily returns, which corresponds to an annual return of -1.43% for an increase of *Beta2* by 1. Considering that *Beta2* of our initial portfolios ranges from approximately -5 to $+5$, this estimate is economically important. The estimated premium in Model 1 is even higher for ESX (-2.39%) and DAX (-1.68%).

The association of *Beta2* and return depends on moneyness, which can be seen from the significantly positive slope coefficients for the interaction term in Model 2. The returns of low moneyness options imply a higher skewness premium than the returns of high moneyness options. In Model 2, the annualized estimates of the additional skewness premium for moneyness $m = 0.9$ compared to $m = 1.0$ are -1.06% for SPX (-1.20% for $m = 0.9$, -2.26% for $m = 1.0$), -0.67% for ESX (-2.22% for $m = 0.9$, -2.89% for $m = 1.0$) and -0.43% for DAX (-1.58% for $m = 0.9$, -2.01% for $m = 1.0$). Skewness alone explains between 83.6% (SPX) and 93.1% (ESX) of the cross-sectional variation of average returns. In Model 2, the R^2 coefficients increase to 95.7% (SPX), 95.0% (ESX) and 87.9% (DAX).

The results for Model 3 confirm these findings. The estimates are consistent with a strongly negative skewness premium that is particularly pronounced in portfolios with low moneyness options. This means that low moneyness options seem to be attractive for investors who are willing to sell skewness risk and high moneyness options are advisable for investors who want to hold portfolios with positively skewed return distributions. However, in interpreting the coefficients of Models 2 and 3, it is important to keep in mind that the variation of moneyness (0.9 to 1.05) is much smaller than the variation of *Beta2*. Overall, the explanatory power of *Beta2* for mean option returns is much stronger than the explanatory contribution of the interaction with moneyness.

It might be surprising that investors are willing to pay such a high premium for positive skewness in daily returns. An interesting question is whether skewness in daily returns carries over to longer return intervals. For index returns, Neuberger (2012) shows that skewness actually increases from a daily return frequency up to a one-year horizon. With the same persistence of skewness in option returns, it is plausible that even long-term investors might demand a substantial premium for negative skewness. Fig. 7, however, shows a convergence of option returns' skewness towards the level of index skewness (shown as red line) over one year. The option portfolios underlying these graphs are formed according to the base strategy of Sections 2 and 3. The daily returns, which are computed in the same way as before, are aggregated to n-day holding period returns. The skewness measure shown on the y-axis (Fisher-Pearson coefficient) is based on all n-day intervals in the sample period.

The convergence apparent from Fig. 7 stems to a large extent from the put option portfolios which exhibit a peak of negative skewness at a horizon of one to two months. The convergence appears to be particularly pronounced for ESX options while for SPX, the spread of skewness in the cross-section of option portfolios is still substantial at a one-year horizon. In view of the overall convergence pattern, our finding of a high skewness premium suggests a short investment horizon of the marginal investors in these option portfolios. However, a conclusive analysis would have to consider how skewness over different horizons affects investor utility.

– Insert Figure 7 here –

6. Conclusion

Based on a methodology introduced by Constantinides et al. (2013), we create a sample of option-based investment strategies that represent market investments (market exposure of one) with different degrees of skewness. The possibility to control skewness using simple trading strategies allows us to study the skewness premium in a new way. This approach has the advantage that we can modify skewness on the market level and do not have to exploit small differences in systematic skewness of individual stocks. Another advantage is that the range of skewness spanned by our investment strategies is large compared to market skewness itself. For three of the most important index option markets (S&P 500, EuroStoxx 50 and DAX), we provide evidence that is consistent with a significantly negative skewness premium. Skewness alone explains 84% (SPX), 93% (ESX) and 86% (DAX) of the cross-sectional variation of average returns. The premium appears to be highest for low moneyness options. When considering this interaction with moneyness, the R^2 coefficients rise to 96% (SPX), 95% (ESX) and 88% (DAX).

The close relationship between skewness and average returns found in our empirical analysis is not a mere reflection of the skew in option prices (implied volatilities decreasing in moneyness). Apart from the skew, the option returns are also affected by the overall level of option prices, the dynamics of the skew profile and the time-varying skewness of index returns. In further work, the relative importance of these factors could be examined more closely.

The skewness of the strategies' return distributions reflects the sensitivity of the strategies' returns with respect to squared market return. A positive sensitivity means that downward index jumps are mitigated and upward index jumps are enhanced. If we interpret this pattern as reduced jump risk, the skewness premium is, by definition, related to the premium for jump risk. The same holds true for volatility risk because volatility changes are typically associated with index changes. Thus, our analysis is consistent with factor models based on these types of risk. This also highlights that we cannot infer from our results that skewness per se is priced – it might "just" be a reflection of other risk factors that are causal for determining expected returns. But whatever the drivers of market return may be, it seems to be an interesting finding that the average returns of beta-one option-based investment strategies are closely associated with the resulting skewness of realized returns.

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Table 1: Descriptive statistics of daily SPX unlevered option returns from 1996 to 2015.

No.	Option	Type	M	TtM	Beta	Beta1	Beta2	R^2	Vol	Skew	Kurt	SR
1	SPX	c	0.9	30	0.999	0.998	1.162	0.996	0.01240	0.34686	4.32762	0.01757
2	SPX	c	0.9	60	0.994	0.993	1.098	0.994	0.01234	0.32455	4.45856	0.01597
3	SPX	c	0.9	90	0.993	0.992	1.208	0.992	0.01241	0.36987	4.80950	0.01636
4	SPX	p	0.9	30	0.982	0.983	-5.203	0.866	0.01332	-2.29182	15.82051	0.06218
5	SPX	p	0.9	60	1.016	1.017	-3.570	0.920	0.01322	-1.42830	8.79623	0.04398
6	SPX	p	0.9	90	1.016	1.017	-3.331	0.914	0.01332	-1.24709	7.03131	0.03486
7	SPX	c	0.925	30	0.998	0.998	1.474	0.994	0.01242	0.44682	4.43787	0.01590
8	SPX	c	0.925	60	0.994	0.993	1.332	0.992	0.01236	0.40123	4.56976	0.01466
9	SPX	c	0.925	90	0.992	0.992	1.392	0.990	0.01242	0.43140	4.84604	0.01540
10	SPX	p	0.925	30	0.993	0.995	-4.763	0.898	0.01319	-2.04515	13.99925	0.05562
11	SPX	p	0.925	60	1.014	1.015	-3.268	0.939	0.01304	-1.25617	7.60341	0.04125
12	SPX	p	0.925	90	1.015	1.016	-3.026	0.940	0.01310	-1.11689	6.54214	0.03436
13	SPX	c	0.95	30	0.998	0.998	1.886	0.991	0.01245	0.57722	4.60116	0.01379
14	SPX	c	0.95	60	0.994	0.993	1.619	0.989	0.01239	0.49471	4.68612	0.01310
15	SPX	c	0.95	90	0.992	0.992	1.610	0.987	0.01244	0.50214	4.87794	0.01419
16	SPX	p	0.95	30	0.999	1.001	-4.229	0.928	0.01301	-1.68476	10.52444	0.04788
17	SPX	p	0.95	60	1.011	1.011	-2.958	0.955	0.01288	-1.10042	6.71653	0.03805
18	SPX	p	0.95	90	1.012	1.013	-2.722	0.957	0.01293	-0.99606	6.12959	0.03292
19	SPX	c	0.975	30	0.999	0.998	2.443	0.984	0.01252	0.75030	4.82756	0.01148
20	SPX	c	0.975	60	0.995	0.994	1.977	0.983	0.01245	0.60861	4.79336	0.01138
21	SPX	c	0.975	90	0.993	0.992	1.875	0.982	0.01249	0.58519	4.88895	0.01285
22	SPX	p	0.975	30	1.001	1.002	-3.618	0.956	0.01279	-1.31989	7.67800	0.03994
23	SPX	p	0.975	60	1.007	1.008	-2.639	0.968	0.01273	-0.96155	6.11088	0.03453
24	SPX	p	0.975	90	1.009	1.010	-2.426	0.969	0.01280	-0.88217	5.79731	0.03097
25	SPX	c	1	30	1.000	0.999	3.222	0.970	0.01267	0.98524	5.13347	0.01066
26	SPX	c	1	60	0.996	0.995	2.434	0.975	0.01254	0.74929	4.87909	0.01018
27	SPX	c	1	90	0.994	0.993	2.208	0.975	0.01256	0.68611	4.86577	0.01189
28	SPX	p	1	30	1.000	1.001	-2.980	0.975	0.01262	-1.04336	6.36805	0.03173
29	SPX	p	1	60	1.004	1.005	-2.320	0.978	0.01261	-0.83711	5.68492	0.03047
30	SPX	p	1	90	1.006	1.007	-2.144	0.978	0.01269	-0.77869	5.53321	0.02846
31	SPX	c	1.025	30	0.990	0.989	4.324	0.935	0.01285	1.34601	5.94385	0.01599
32	SPX	c	1.025	60	0.991	0.990	3.036	0.958	0.01262	0.94170	5.15767	0.01197
33	SPX	c	1.025	90	0.990	0.989	2.648	0.961	0.01263	0.82433	4.98189	0.01316
34	SPX	p	1.025	30	1.004	1.005	-2.352	0.987	0.01256	-0.81151	5.45184	0.02479
35	SPX	p	1.025	60	1.006	1.006	-2.004	0.986	0.01257	-0.71714	5.22580	0.02576
36	SPX	p	1.025	90	1.007	1.008	-1.881	0.985	0.01266	-0.68003	5.19855	0.02513
37	SPX	c	1.05	30	0.931	0.929	5.658	0.853	0.01281	1.92884	8.90981	0.00801
38	SPX	c	1.05	60	0.966	0.965	3.763	0.926	0.01256	1.21587	6.11685	0.01009
39	SPX	c	1.05	90	0.973	0.972	3.180	0.937	0.01259	1.01443	5.62307	0.01171
40	SPX	p	1.05	30	1.008	1.009	-1.742	0.994	0.01255	-0.59989	4.73239	0.02326
41	SPX	p	1.05	60	1.010	1.011	-1.672	0.992	0.01258	-0.59621	4.77605	0.02382
42	SPX	p	1.05	90	1.011	1.011	-1.611	0.990	0.01266	-0.58022	4.80617	0.02390

Type "c" refers to calls, type "p" to puts. M and TtM denote moneyness and time to maturity. Beta1 and Beta2 are the slope coefficients of a regression of option return on index return and squared index return; R^2 is the R squared of this quadratic regression. Vol, Skew, Kurt and SR denote the volatility, the Fisher-Pearson coefficient of skewness, the Pearson coefficient of kurtosis and the Sharpe ratio, all based on the sample of daily returns of the option-based strategies over the full sample period.

Table 2: Descriptive statistics of daily ESX unlevered option returns from 2000 to 2016.

No.	Option	Type	M	TtM	Beta	Beta1	Beta2	R^2	Vol	Skew	Kurt	SR
1	ESX	c	0.9	30	0.998	1.004	1.473	0.995	0.01374	0.06164	1.05043	-0.01551
2	ESX	c	0.9	60	0.993	1.001	1.613	0.992	0.01381	0.08705	1.06088	-0.01976
3	ESX	c	0.9	90	0.996	1.000	1.522	0.989	0.01422	0.23276	1.57896	-0.02054
4	ESX	p	0.9	30	0.980	0.956	-5.424	0.909	0.01436	-1.97232	11.01887	0.04584
5	ESX	p	0.9	60	1.012	0.990	-4.664	0.933	0.01469	-1.64963	8.66597	0.02232
6	ESX	p	0.9	90	1.010	0.998	-4.003	0.925	0.01505	-1.48228	8.46546	0.01223
7	ESX	c	0.925	30	0.997	1.005	1.804	0.993	0.01374	0.14795	1.03370	-0.01844
8	ESX	c	0.925	60	0.992	1.001	1.891	0.990	0.01382	0.16063	1.08397	-0.02186
9	ESX	c	0.925	90	0.995	1.000	1.763	0.986	0.01424	0.29932	1.61002	-0.02200
10	ESX	p	0.925	30	0.994	0.972	-4.962	0.935	0.01432	-1.76882	9.03504	0.04018
11	ESX	p	0.925	60	1.013	0.993	-4.237	0.949	0.01455	-1.51149	7.45486	0.01894
12	ESX	p	0.925	90	1.009	0.998	-3.690	0.945	0.01487	-1.36159	7.47073	0.01114
13	ESX	c	0.95	30	0.995	1.005	2.224	0.990	0.01376	0.25729	1.04043	-0.02244
14	ESX	c	0.95	60	0.991	1.001	2.215	0.988	0.01383	0.24720	1.13671	-0.02401
15	ESX	c	0.95	90	0.994	1.000	2.036	0.984	0.01426	0.37656	1.67977	-0.02356
16	ESX	p	0.95	30	1.004	0.984	-4.430	0.954	0.01426	-1.57280	7.49511	0.03296
17	ESX	p	0.95	60	1.014	0.996	-3.805	0.962	0.01442	-1.37838	6.44935	0.01439
18	ESX	p	0.95	90	1.008	0.998	-3.347	0.958	0.01473	-1.23696	6.51106	0.00658
19	ESX	c	0.975	30	0.993	1.005	2.756	0.985	0.01379	0.39446	1.09245	-0.02784
20	ESX	c	0.975	60	0.990	1.002	2.598	0.984	0.01386	0.34909	1.21897	-0.02665
21	ESX	c	0.975	90	0.994	1.001	2.349	0.981	0.01429	0.46715	1.80137	-0.02488
22	ESX	p	0.975	30	1.008	0.991	-3.845	0.969	0.01417	-1.38633	6.30792	0.02523
23	ESX	p	0.975	60	1.013	0.997	-3.374	0.974	0.01430	-1.25350	5.62655	0.00992
24	ESX	p	0.975	90	1.008	0.999	-3.005	0.970	0.01460	-1.11713	5.68158	0.00312
25	ESX	c	1	30	0.990	1.005	3.428	0.976	0.01385	0.56577	1.22286	-0.03456
26	ESX	c	1	60	0.988	1.003	3.061	0.978	0.01391	0.47027	1.34066	-0.02954
27	ESX	c	1	90	0.993	1.001	2.724	0.975	0.01434	0.57514	1.97643	-0.02682
28	ESX	p	1	30	1.010	0.995	-3.232	0.980	0.01408	-1.20596	5.32183	0.01686
29	ESX	p	1	60	1.012	0.998	-2.956	0.981	0.01420	-1.13591	4.91522	0.00536
30	ESX	p	1	90	1.007	0.999	-2.672	0.978	0.01451	-1.00589	4.96134	-0.00040
31	ESX	c	1.025	30	0.980	0.998	4.239	0.956	0.01390	0.78693	1.58852	-0.04032
32	ESX	c	1.025	60	0.982	0.999	3.609	0.966	0.01394	0.61848	1.58957	-0.03113
33	ESX	c	1.025	90	0.989	0.998	3.173	0.962	0.01441	0.70259	2.27411	-0.02774
34	ESX	p	1.025	30	1.012	1.000	-2.615	0.988	0.01402	-1.02762	4.40378	0.00770
35	ESX	p	1.025	60	1.013	1.001	-2.549	0.986	0.01416	-1.01959	4.22689	-0.00040
36	ESX	p	1.025	90	1.008	1.001	-2.350	0.982	0.01448	-0.89593	4.25515	-0.00540
37	ESX	c	1.05	30	0.949	0.971	5.031	0.906	0.01393	1.05447	2.38695	-0.04743
38	ESX	c	1.05	60	0.964	0.984	4.196	0.942	0.01391	0.79085	2.07754	-0.03336
39	ESX	c	1.05	90	0.975	0.986	3.674	0.931	0.01448	0.87633	2.78398	-0.02504
40	ESX	p	1.05	30	1.013	1.004	-2.030	0.992	0.01398	-0.86360	3.63983	0.00050
41	ESX	p	1.05	60	1.015	1.004	-2.134	0.989	0.01415	-0.90060	3.57967	-0.00570
42	ESX	p	1.05	90	1.009	1.003	-2.033	0.984	0.01448	-0.79232	3.63917	-0.00948

Type "c" refers to calls, type "p" to puts. M and TtM denote moneyness and time to maturity. Beta1 and Beta2 are the slope coefficients of a regression of option return on index return and squared index return; R^2 is the R squared of this quadratic regression. Vol, Skew, Kurt and SR denote the volatility, the Fisher-Pearson coefficient of skewness, the Pearson coefficient of kurtosis and the Sharpe ratio, all based on the sample of daily returns of the option-based strategies over the full sample period.

Table 3: Descriptive statistics of daily DAX unlevered option returns from 1995 to 2016.

No.	Option	Type	M	TtM	Beta	Beta1	Beta2	R^2	Vol	Skew	Kurt	SR
1	DAX	c	0.9	30	1.007	1.009	1.608	0.995	0.01379	0.37670	4.53401	0.00713
2	DAX	c	0.9	60	1.004	1.006	1.678	0.994	0.01382	0.39402	4.42562	0.00532
3	DAX	c	0.9	90	1.001	1.004	1.728	0.992	0.01395	0.39718	4.48416	0.00374
4	DAX	p	0.9	30	0.937	0.932	-5.098	0.901	0.01373	-1.81769	7.83560	0.05899
5	DAX	p	0.9	60	0.979	0.974	-4.443	0.935	0.01407	-1.42316	4.72272	0.03524
6	DAX	p	0.9	90	0.990	0.984	-4.309	0.926	0.01445	-1.40132	4.81342	0.02182
7	DAX	c	0.925	30	1.008	1.010	1.918	0.993	0.01383	0.47145	4.87955	0.00474
8	DAX	c	0.925	60	1.004	1.007	1.950	0.991	0.01386	0.47820	4.77989	0.00383
9	DAX	c	0.925	90	1.002	1.005	1.974	0.990	0.01398	0.47494	4.82764	0.00291
10	DAX	p	0.925	30	0.956	0.951	-4.705	0.929	0.01374	-1.56929	5.56486	0.05360
11	DAX	p	0.925	60	0.984	0.980	-4.101	0.950	0.01400	-1.27843	3.87012	0.03332
12	DAX	p	0.925	90	0.992	0.986	-3.966	0.951	0.01425	-1.29194	4.19251	0.02276
13	DAX	c	0.95	30	1.010	1.012	2.303	0.990	0.01388	0.58824	5.29501	0.00165
14	DAX	c	0.95	60	1.006	1.008	2.270	0.989	0.01391	0.57594	5.19070	0.00232
15	DAX	c	0.95	90	1.003	1.007	2.263	0.988	0.01403	0.56584	5.23190	0.00172
16	DAX	p	0.95	30	0.971	0.966	-4.263	0.950	0.01376	-1.36229	4.16642	0.04604
17	DAX	p	0.95	60	0.987	0.983	-3.754	0.963	0.01392	-1.16340	3.22956	0.02970
18	DAX	p	0.95	90	0.993	0.987	-3.629	0.964	0.01414	-1.18808	3.61794	0.02041
19	DAX	c	0.975	30	1.011	1.014	2.783	0.985	0.01396	0.73323	5.79552	-0.00214
20	DAX	c	0.975	60	1.007	1.010	2.645	0.985	0.01397	0.69120	5.67645	0.00065
21	DAX	c	0.975	90	1.005	1.009	2.600	0.984	0.01409	0.67261	5.70707	0.00048
22	DAX	p	0.975	30	0.981	0.977	-3.786	0.966	0.01375	-1.18136	3.23911	0.03811
23	DAX	p	0.975	60	0.989	0.986	-3.405	0.972	0.01386	-1.05018	2.73422	0.02607
24	DAX	p	0.975	90	0.994	0.988	-3.309	0.973	0.01406	-1.08428	3.15741	0.01861
25	DAX	c	1	30	1.011	1.015	3.393	0.975	0.01406	0.91668	6.40213	-0.00618
26	DAX	c	1	60	1.008	1.012	3.094	0.978	0.01405	0.82925	6.26761	-0.00081
27	DAX	c	1	90	1.005	1.010	2.998	0.978	0.01416	0.79937	6.28533	-0.00014
28	DAX	p	1	30	0.989	0.986	-3.297	0.978	0.01374	-1.02101	2.59967	0.03044
29	DAX	p	1	60	0.992	0.988	-3.065	0.980	0.01382	-0.94673	2.34662	0.02247
30	DAX	p	1	90	0.995	0.990	-3.001	0.979	0.01401	-0.98826	2.76876	0.01604
31	DAX	c	1.025	30	1.003	1.008	4.164	0.956	0.01416	1.16964	7.38114	-0.00957
32	DAX	c	1.025	60	1.005	1.009	3.642	0.967	0.01412	1.00730	7.12425	-0.00119
33	DAX	c	1.025	90	1.003	1.008	3.483	0.965	0.01426	0.95732	7.01565	-0.00053
34	DAX	p	1.025	30	0.997	0.994	-2.821	0.986	0.01375	-0.87391	2.09070	0.02360
35	DAX	p	1.025	60	0.996	0.993	-2.740	0.985	0.01382	-0.84840	1.98937	0.01882
36	DAX	p	1.025	90	0.998	0.993	-2.723	0.981	0.01402	-0.88804	2.39205	0.01447
37	DAX	c	1.05	30	0.972	0.978	5.021	0.916	0.01410	1.53502	9.52504	-0.02153
38	DAX	c	1.05	60	0.990	0.995	4.275	0.948	0.01411	1.24650	8.62032	-0.00537
39	DAX	c	1.05	90	0.999	0.993	4.233	0.939	0.01413	1.29849	8.52480	0.00059
40	DAX	p	1.05	30	1.002	1.000	-2.365	0.991	0.01377	-0.74042	1.71570	0.01983
41	DAX	p	1.05	60	1.000	0.997	-2.416	0.989	0.01384	-0.75060	1.66928	0.01656
42	DAX	p	1.05	90	1.001	0.997	-2.433	0.983	0.01405	-0.79265	2.02884	0.01436

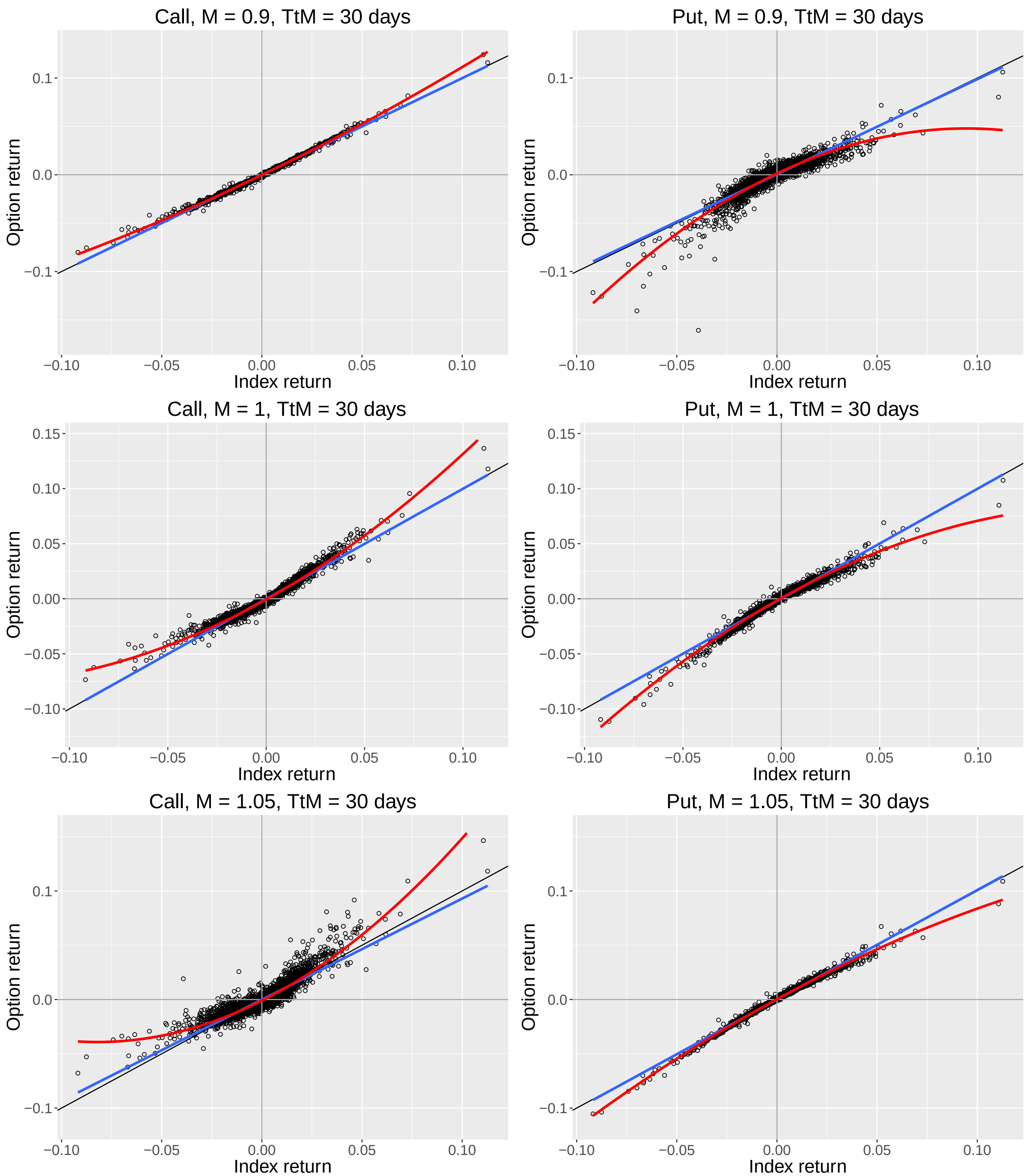
Type “c” refers to calls, type “p” to puts. M and TtM denote moneyness and time to maturity. Beta1 and Beta2 are the slope coefficients of a regression of option return on index return and squared index return; R^2 is the R squared of this quadratic regression. Vol, Skew, Kurt and SR denote the volatility, the Fisher-Pearson coefficient of skewness, the Pearson coefficient of kurtosis and the Sharpe ratio, all based on the sample of daily returns of the option-based strategies over the full sample period.

Table 4: Cross-sectional analysis.

	SPX		ESX		DAX	
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
Interc.	2.74E-05***	8.15E-06	1.44E-05*	2.44E-06	9.41E-06	2.11E-06
	(4.47)	(1.66)	(1.76)	(0.34)	(1.23)	(0.31)
Beta2	-5.67E-05***	-4.76E-05***	-9.48E-05***	-8.81E-05***	-6.67E-05***	-6.28E-05***
	(-19.58)	(-15.17)	(-31.61)	(-27.57)	(-24.70)	(-21.96)
Beta2 x Moneyness		4.22E-04***		2.66E-04***		1.72E-04***
		(7.07)		(4.35)		(3.12)
R^2	0.836	0.957	0.931	0.950	0.864	0.879
Ann. Beta2-Prem.	-1.43%	-1.20%	-2.39%	-2.22%	-1.68%	-1.58%
Ann. Prem. m=0.9		-2.26%		-2.89%		-2.01%
Panel B						
Interc.		-4.39E-05***		-7.84E-05***		-5.54E-05***
		(-19.16)		(-27.75)		(-22.04)
Moneyness		3.18E-04***		2.31E-04***		1.27E-04***
		(7.36)		(4.37)		(2.73)
R^2		0.574		0.227		0.081
Ann. Prem. m=1		-1.11%		-1.98%		-1.40%
Ann. Prem. m=0.9		-1.91%		-2.56%		-1.72%

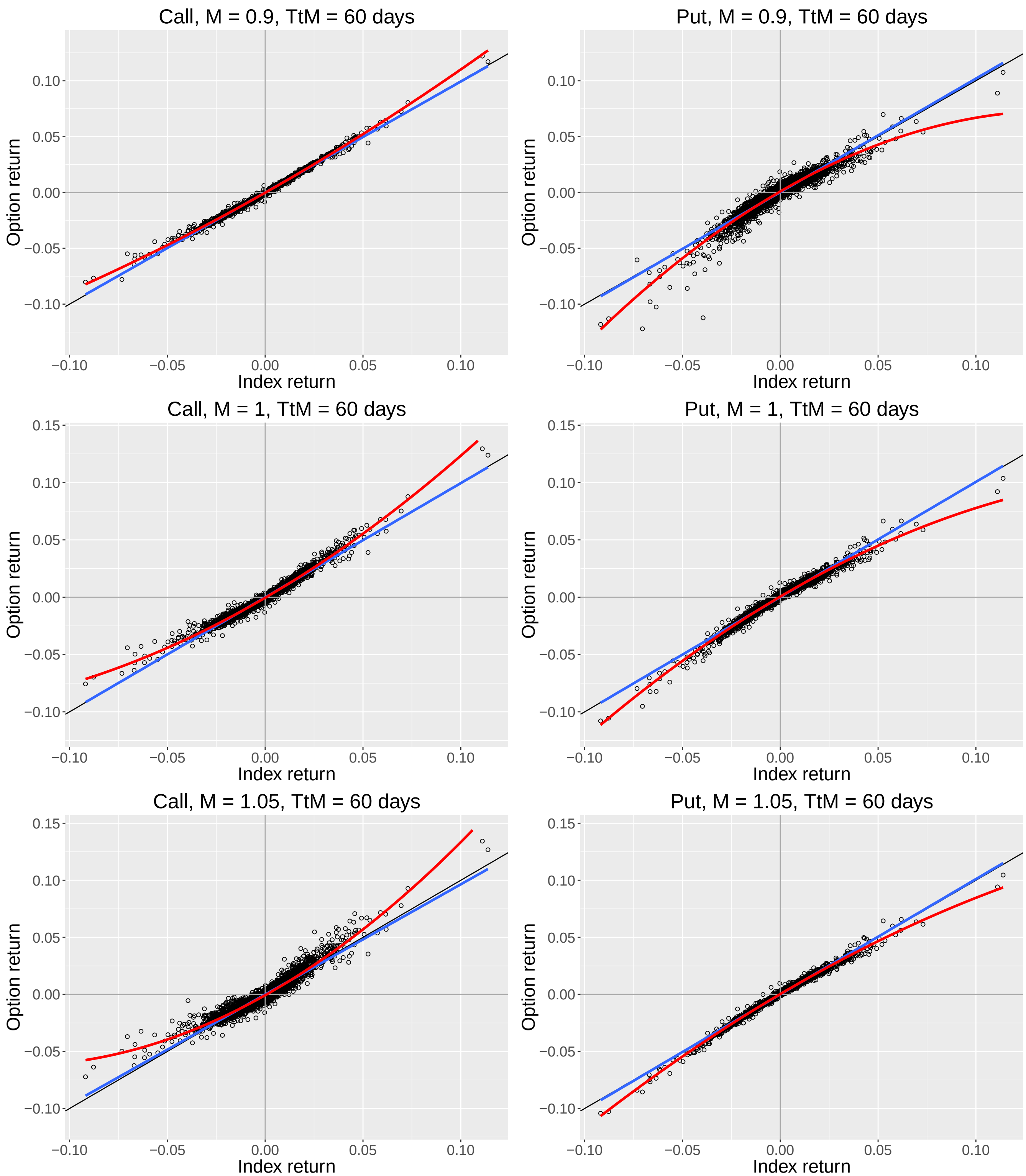
Panel A shows estimation results for Models 1 and 2, Eq. (12) and (13), Panel B shows estimation results for Model 3, Eq. (14). The t-values based on GMM standard errors are reported in brackets. The R^2 coefficient is the cross-sectional R^2 measure of Jagannathan and Wang (1996). “Ann. Prem.” is an annualized premium estimate. “m” is simple moneyness defined as the ratio of discounted strike price and index level. The regressor “Moneyness” is simple moneyness minus 1.

Figure 1: Daily index and unlevered option returns from 1996 to 2015 for SPX options with a time to maturity of 30 days.



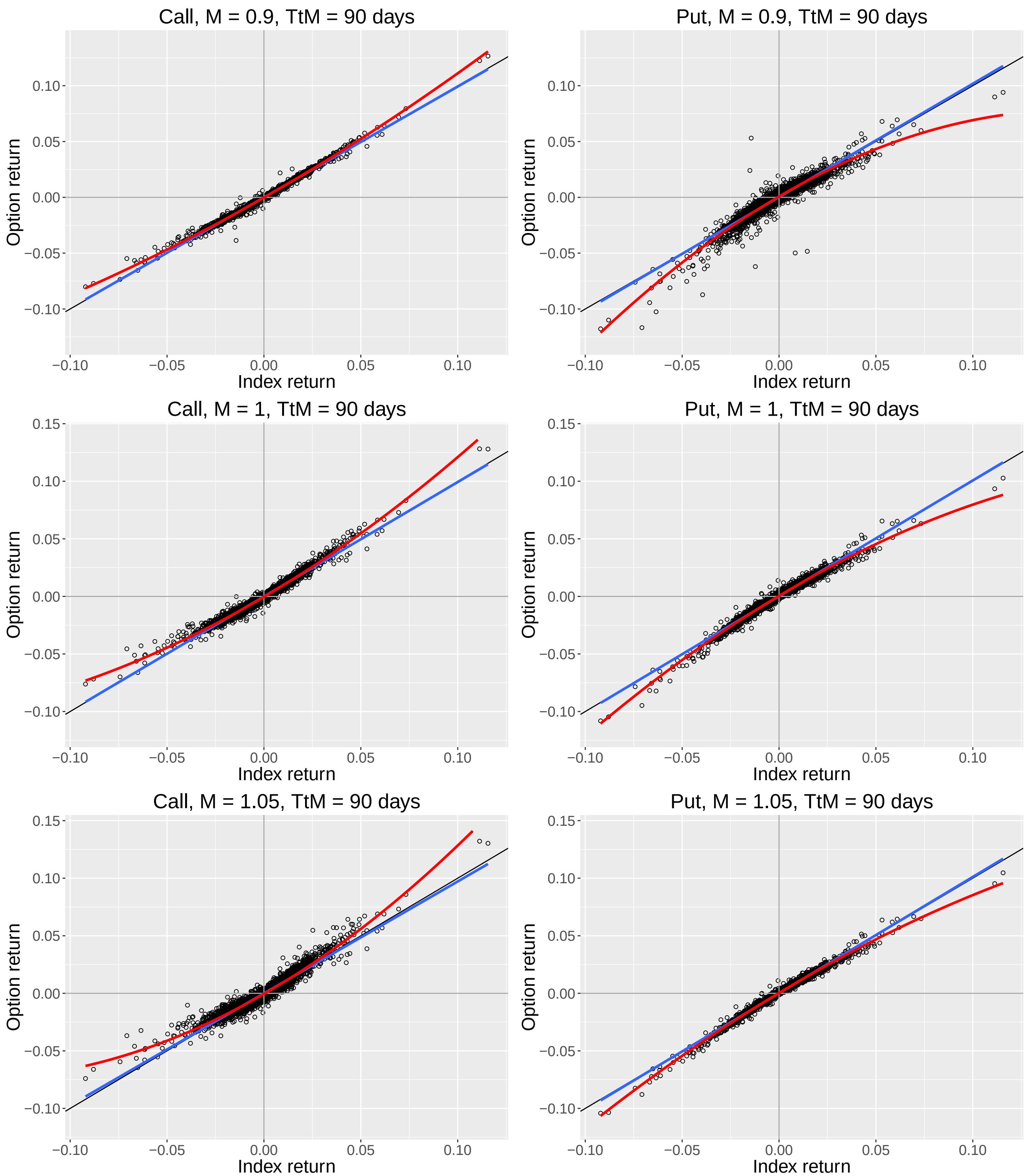
M is moneyness, TtM is time to maturity. The blue line shows the estimated linear regression, the red line the quadratic regression of option return on index return.

Figure 2: Daily index and unlevered option returns from 1996 to 2015 for SPX options with a time to maturity of 60 days.



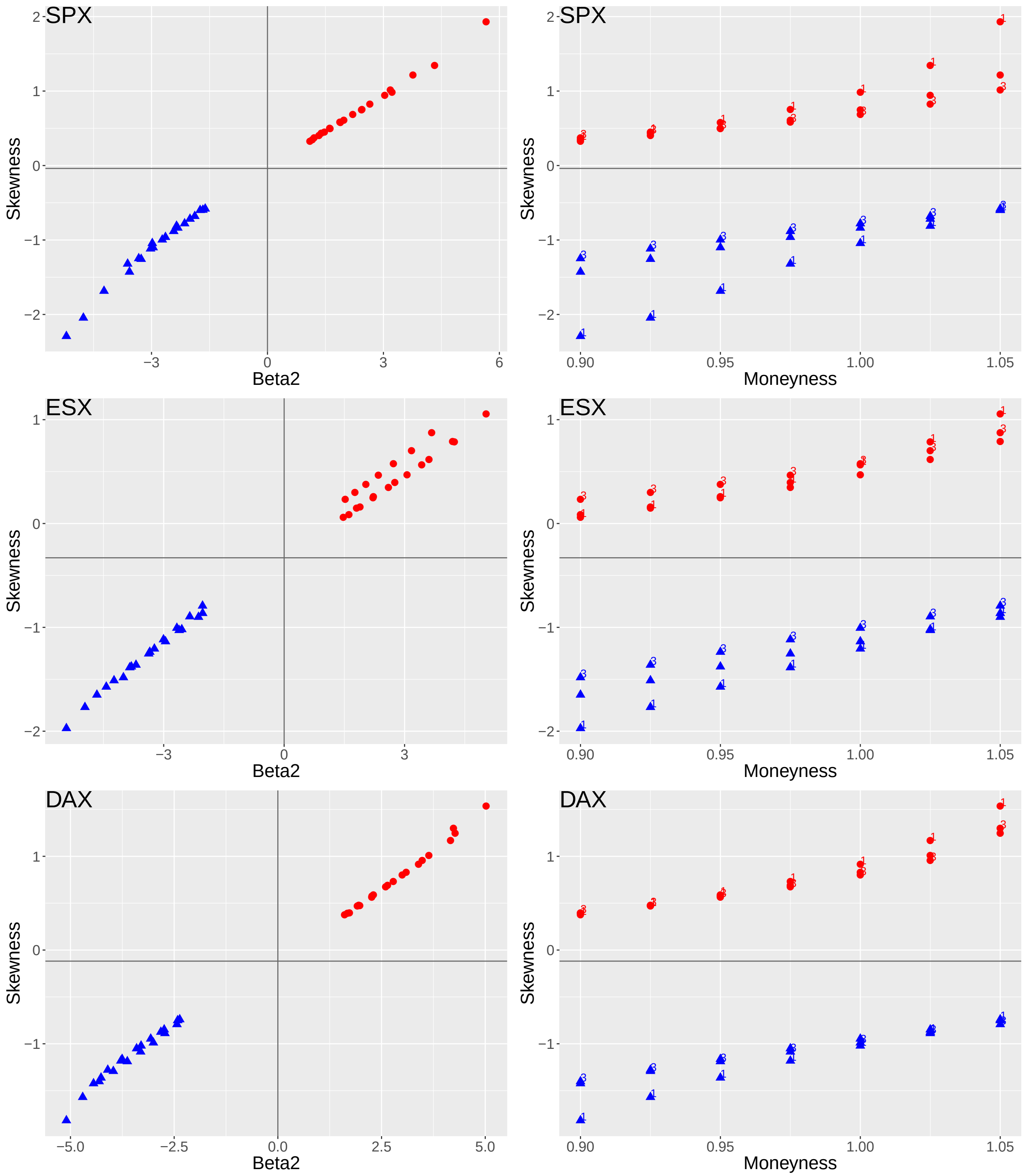
M is moneyness, TtM is time to maturity. The blue line shows the estimated linear regression, the red line the quadratic regression of option return on index return.

Figure 3: Daily index and unlevered option returns from 1996 to 2015 for SPX options with a time to maturity of 90 days.



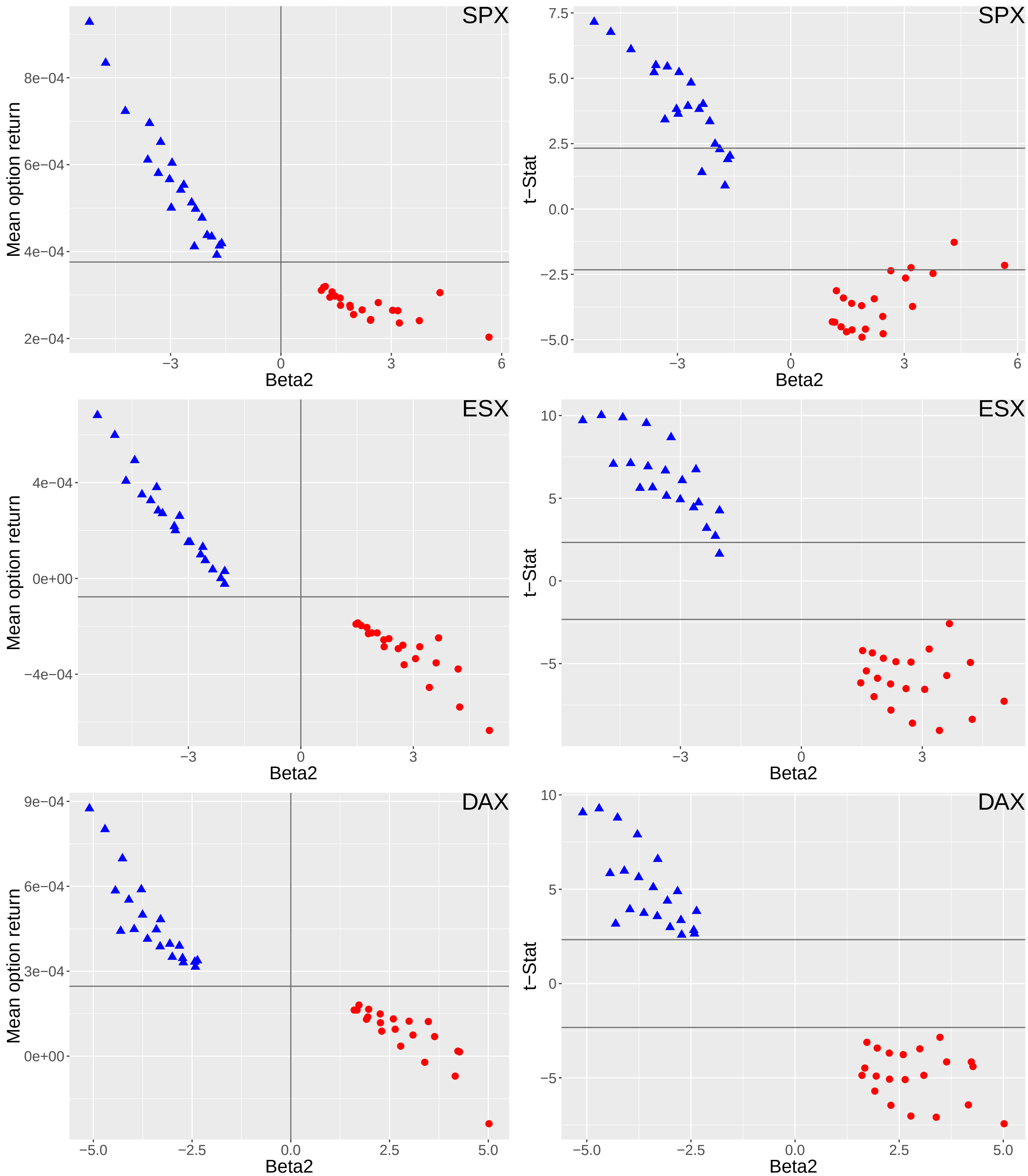
M is moneyness, TtM is time to maturity. The blue line shows the estimated linear regression, the red line the quadratic regression of option return on index return.

Figure 4: Skewness in the cross-section of unlevered option returns.



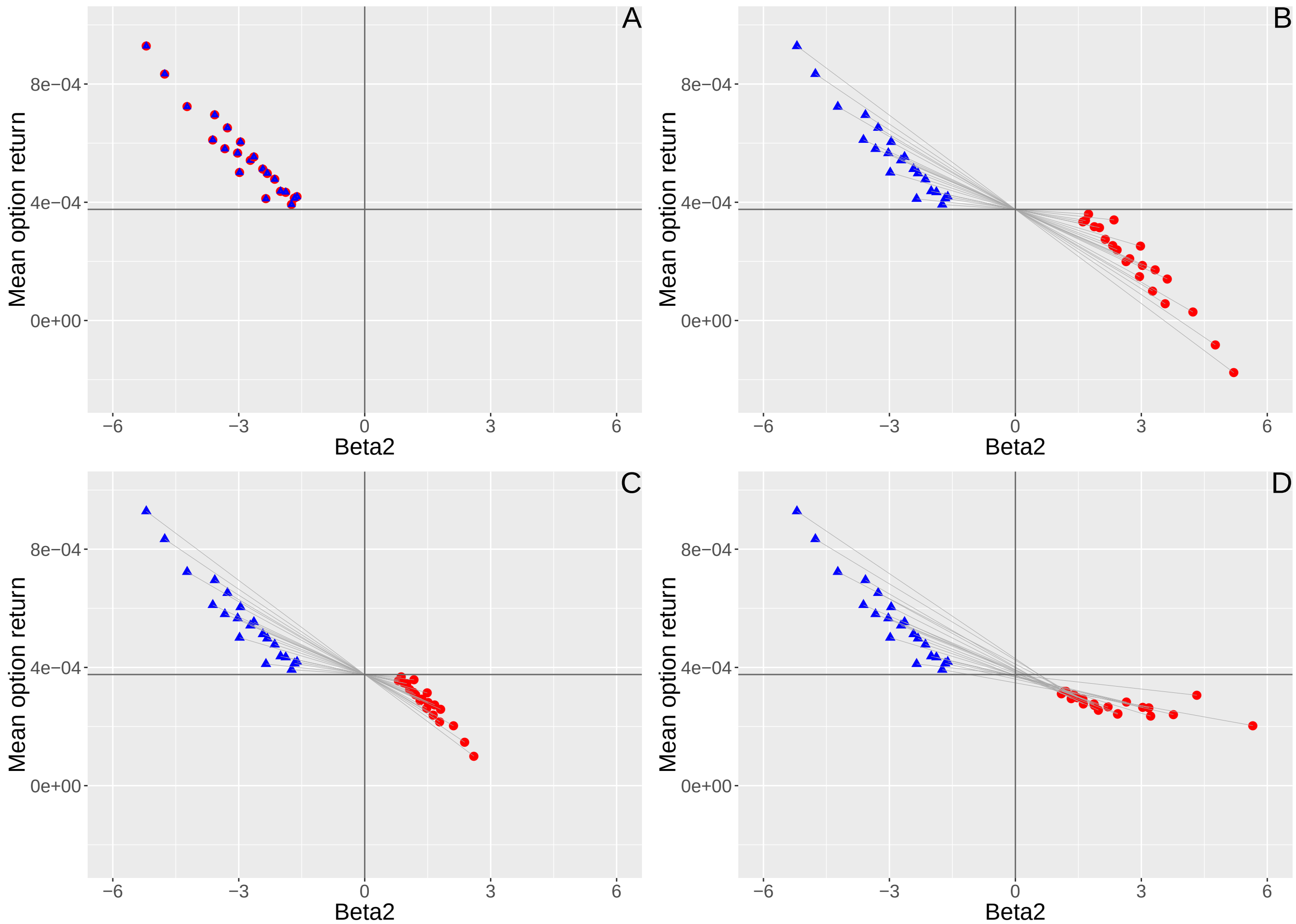
Each graph contains 42 data points for 21 call-based portfolios (red circles) and 21 put-based portfolios (blue triangles). Each portfolio is based on options with a particular moneyness (7 levels from 0.9 to 1.05) and time to maturity (3 levels). In the panels on the right, marker 1 denotes a time to maturity of 30 days, marker 2 of 60 days and marker 3 of 90 days. All portfolios have an index beta of 1. The vertical and horizontal lines indicate index level characteristics. The skewness measures are based on daily returns from 1996 to 2015 for SPX, from 2000 to 2016 for ESX and from 1995 to 2016 for DAX.

Figure 5: Mean unlevered option returns.



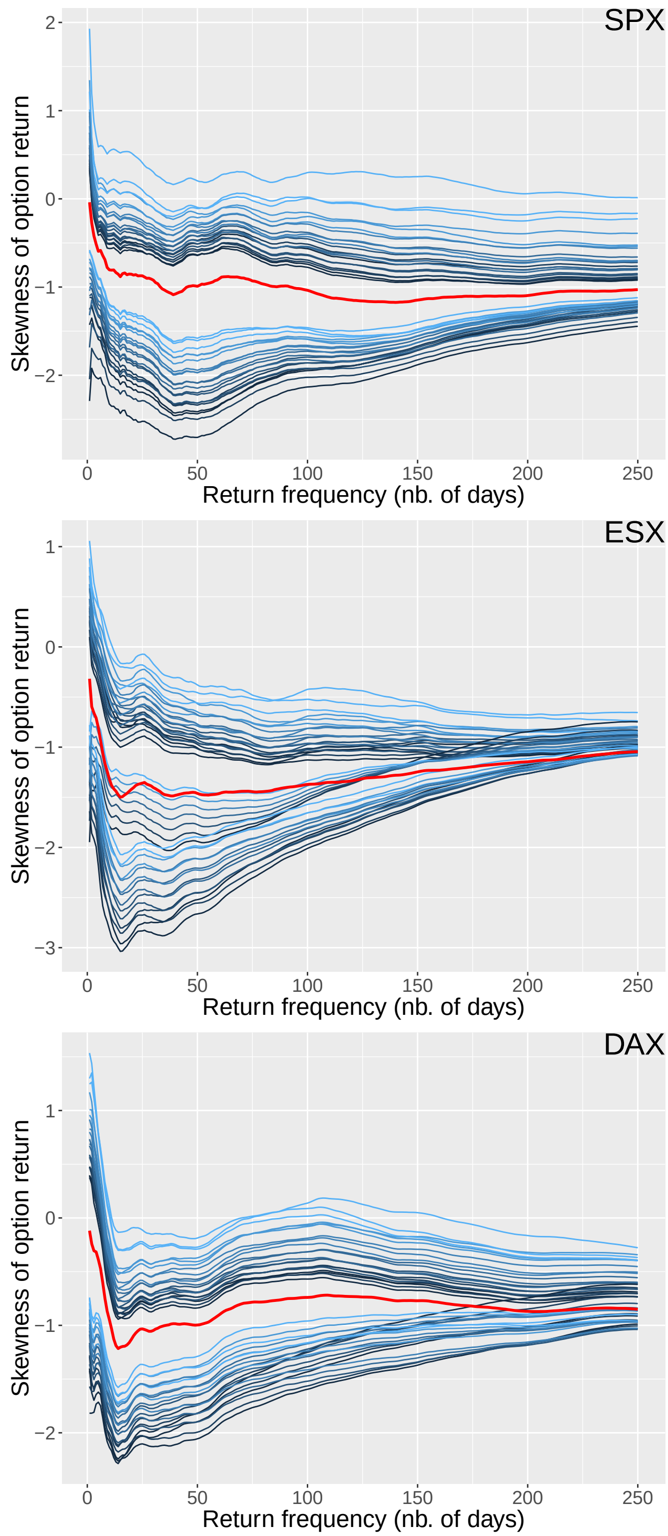
Each graph contains 42 data points for 21 call-based portfolios (red circles) and 21 put-based portfolios (blue triangles). Each portfolio is based on options with a particular moneyness (7 levels from 0.9 to 1.05) and time to maturity (3 levels). All portfolios have an index beta of 1. In the left graphs, the vertical and horizontal lines indicate the index position. In the right graphs, t-Stat is the t-statistic for the mean of unlevered option returns in excess of index returns. The horizontal lines indicate the 1% critical values (two-tailed test). The sample consists of daily returns from 1996 to 2015 for SPX, from 2000 to 2016 for ESX and from 1995 to 2016 for DAX.

Figure 6: Implications of put-call parity for extended SPX beta-one portfolios.



Each graph contains 42 data points for 21 call-based portfolios (red circles) and 21 put-based portfolios (blue triangles). Each portfolio is based on options with a particular moneyness (7 levels from 0.9 to 1.05) and time to maturity (3 levels). All portfolios consist of the respective option, the index and the risk-free asset and have an index beta of 1. The option weights b in Eq. (6) are defined as follows: Panel A: $b_{Put} = 1$ and $b_{Call} = \Delta_{Call}/\Delta_{Put}$; Panel B: $b_{Put} = 1$ and $b_{Call} = -\Delta_{Call}/\Delta_{Put}$; Panel C: $b_{Put} = 1$ and $b_{Call} = -2\Delta_{Call}/\Delta_{Put}$; Panel D: $b_{Put} = 1$ and $b_{Call} = 1$. The position of a pure index investment is indicated by the intersection point of the horizontal and vertical lines. Calls and puts with the same moneyness and time to maturity are connected by lines that run through the index position if the equation $b_{Call} = k \cdot b_{Put}\Delta_{Call}/\Delta_{Put}$ holds for constant $k \neq 1$ on each day of the sample period. In Panel A, the call and put positions are identical. The portfolios in Panel D are the same as in the upper left graph in Fig. 5).

Figure 7: Skewness of option returns for return horizons from one day to one year.



Return skewness over different horizons (number of days n on the x-axis) for 42 option portfolios (blue) and the index (red). The option portfolios are formed according to the base strategy presented in Sections 2 and 3. The portfolios are readjusted daily. The holding period return is the total return over an n-day horizon. The skewness measure (Fisher-Pearson coefficient) is based on all n-day intervals in the sample period. Light blue lines: low moneyness options; dark blue lines: high moneyness options.