The Value of True Liquidity

Working Paper
This version: December 2016

Abstract

This study uncovers the ability of liquid stocks to generate significant higher risk-adjusted portfolio returns than their illiquid counterparts. Using U.S. stocks in the period of 01/1990 to 09/2015, we show that a significant negative illiquidity premium can be obtained when accounting for a high negative correlation between a stocks' illiquidity and its market value of equity. The risk-adjusted orthogonalized illiquidity premium amounts to -0.576% per month and is robust to changes in the portfolio formation setting.

Keywords: *Liquidity*; *asset pricing*
JEL Classification: $G10$, $G11$, $G12$

1 Introduction

The wide literature studying the impact of liquidity on stock returns suggests that illiquid stocks generate higher returns.[1] The difference in returns between illiquid and liquid stocks is associated with a compensation for investors to hold illiquid stocks. Recent studies like that of Brennan, Huh, and Subrahmanyam (2013) and Amihud, Hameed, Kang, and Zhang (2015) confirm the existence of a positive-signed illiquidity premium while Ben-Rephael, Kadan, and Wohl (2015) find that the illiquidity premium diminishes over time. Our findings suggest that the illiquidity premium does not merely diminish over time, but rather changes its sign when accounting for a high correlation between common illiquidity measures. We also show that stocks that we define as being truly liquid exhibit significant higher returns than their truly illiquid counterpart.

Generally, a major issue of dealing with liquidity in stock markets is caused by its multidimensionality. Amihud (2002) points out that liquidity has a number of aspects that cannot be captured in a single measure. Liu (2006) addresses this issue and defines four dimensions of liquidity: trading quantity, trading speed, trading cost and price impact. In line with that, several liquidity measures have been introduced in the past 35 years, each capturing different dimensions of liquidity. Amihud and Mendelson (1986), for example, study the effect of bid-ask spreads on stock returns, therefore modelling the trading cost component. The authors find that excess returns of stocks are an increasing and concave function of their relative bid-ask spread. Brennan and Subrahmanyam (1996) capture the trading cost component by defining the price impact of signed order flow, also known as λ, and show that there exists a concave relation between variable costs of transactions and stock returns.

[1] See, for example, Amihud and Mendelson (1986), Amihud and Mendelson (1989), Amihud, Mendelson, and Wood (1990), Brennan and Subrahmanyam (1996),Easley, Kiefer, O'Hara, and Paperman (1996), Eleswarapu (1997), Datar, Naik, and Radcliffe (1998), Amihud (2002), Chordia, Sahn-Wook, and Subrahmanyam (2009), Duarte and Young (2009), Hasbrouck (2009), Chang, Faff, and Hwang (2010) and Jensen and Moorman (2010) among others.

Easley, Kiefer, O'Hara, and Paperman (1996) introduce another measure of illiquidity, namely the probability of informed trading, PIN. This measure is closely related to the covariance of the number of buy trades and the number of sell trades per day. The proposition includes that the higher the PIN, the higher the information asymmetry and therefore the higher the expected return of the related stock. Easley, Hvidkjaer, and O'Hara (2002) show that PIN is priced in the cross-section of US stocks in the period of 1983-1999. Amihud (2002) introduces a liquidity measure that relates to the price impact of trading quantity (ILLIQ) and therefore to the measure of price impact by Kyle (1985). Amihud (2002), Chordia, Sahn-Wook, and Subrahmanyam (2009) and Li, Sun, and Wang (2014) among others confirm the pricing of ILLIQ in the cross-section of expected stocks returns. Emphasizing the relevance of the trading speed component, Liu (2006) defines the standardized turnover-adjusted number of zero daily trading volumes as a measure of liquidity. The author finds less liquid stocks to be "small, value, low-turnover, high bid-ask spread, and high return-to-volume" stocks. More recent, Brennan, Huh, and Subrahmanyam (2013) utilize a turnover-based version of the Amihud (2002) measure and further decompose it into elements that correspond to positive and negative return days. The results reveal that only the Amihud (2002) measure corresponding to down days is priced in the cross-section of expected returns.

While all of the above mentioned studies identify a positive-signed illiquidity premium, the recent work of Ben-Rephael, Kadan, and Wohl (2015) raises the question whether there is an illiquidity premium in the most recent time periods. Using three different proxies for illiquidity, the authors cannot identify a significant illiquidity premium for 99.5% of the total market value of all publicly listed common stocks in the US.[2]

Based on a market-wide liquidity measure, Pastor and Stambaugh (2003) analyze

[2]For their analysis, the authors use the Amihud (2002) illiquidity measure, a modified version of the Roll (1984) estimator of the effective half bid-ask spread introduced by Hasbrouck (2009) (HR) and, following Goyenko, Holden, and Trzcinka (2009), the ratio of the HR measure and dollar volume.

the effects of a stock's sensitivity on innovations in aggregate liquidity. Similar to the systematic risk of a stock, that is, the covariation between stock and market returns, the authors define the covariation between a stock's individual and market-wide liquidity measure as liquidity risk. Their results show that stocks with higher sensitivities to market liquidity exhibit significantly higher expected returns. Acharya and Pedersen (2005), Sadka (2006), Kamara, Lou, and Sadka (2008) and Korajczyk and Sadka (2008) extend the work of Pastor and Stambaugh (2003) and show that liquidity is a priced risk factor.

Hence, a general distinction in the wide literature that studies the impact of liquidity on stock returns can be made with respect to the measurement of i) a stock's individual liquidity level and ii) the sensitivity of a stock's individual liquidity level to a market-wide liquidity measure (liquidity risk). This study puts emphasis on the former by showing that a small tweak in the portfolio building process that is based on liquidity characteristics substantially affects risk-adjusted portfolio returns.

In our opinion, the major issue that arises when examining the impact of liquidity on stock returns does not concern only the liquidity measure itself, but rather the interdependencies to other dimensions of liquidity. An identification of a "pure" liquidity effect therefore becomes even more difficult. The interdependencies are empirically verifiable by strong correlations among the liquidity measures used for an analysis. Brennan, Huh, and Subrahmanyam (2013) for example report an average cross-section correlation of -0.944 between the logarithm of the Amihud (2002) illiquidity measure and the logarithm of the market capitalization for NYSE and AMEX stocks in the period of January 1983 to December 2009. Given a similar sample period from January 1986 to December 2011, Ben-Rephael, Kadan, and Wohl (2015) find an average correlation between the Amihud (2002) illiquidity measure and the logarithm of the market capitalization of -0.50.[3] Jensen and

[3]Since Ben-Rephael, Kadan, and Wohl (2015) do not use a log-transformation of the Amihud (2002) illiquidity measure, the difference to the correlation reported by Brennan, Huh, and Subrahmanyam

Moorman (2010) also note that substituting well-known liquidity measures such as the Amihud (2002) illiquidity measure by size leads to qualitatively same results when studying the inter-temporal variation in the illiquidity premium. Hence, the Amihud (2002) illiquidity measure scales the well-known size-effect studied by Banz (1981) since illiquid stocks are those with low market value of equity given the same share turnover. This issue is also discussed by Cochrane (2005) and therefore raises the question whether there is any significant benefit of using two highly correlated liquidity measures either in the cross-section of stock returns or in a time-series framework. We show that disregarding this main issue leads to disputable conclusions concerning the pattern of risk-adjusted returns with respect to the illiquidity level of a stock.

In our study, the correlation between the Amihud (2002) illiquidity measure and firm size serves us for calculating the "true" illiquidity level of a stock by orthogonalizing the log-transformed Amihud (2002) illiquidity measure to the logarithm of the market capitalization of a stock at the end of each month. Accordingly, we do not introduce a new liquidity measure in this study, but rather use existing ones to show that the relation between liquidity and stock returns is not that clear as existing literature and theory might suggest.

In fact, our results provide evidence for a "true" illiquidity premium that is significantly negative in the period of January 1990 to September 2015, implying that "truly" liquid stocks exhibit higher returns than their "truly" illiquid counterpart. We consider the significance of our results to be a consequence of the high negative correlation of a stocks illiquidity level and its market value of equity. Most studies do not correct for this issue neither in the cross-section of stock returns nor when building portfolios on liquidity characteristics. We show that capturing the interdependence of different dimensions of liquidity leads to remarkable results concerning the potential of liquid stocks to generate

(2013) might be explained by the skewness of the raw Amihud (2002) measure across stocks. This issue is also discussed in Section 2.1.

abnormal risk-adjusted portfolio excess returns.

To underline this point, we form portfolios two-dimensionally and control for several criteria such as volatility, size, illiquidity and true illiquidity. To evaluate portfolio performance, we use the Carhart (1997) four-factor model and an illiquidity-augmented four-factor model similar to Liu (2006), Marcelo and Quiros (2006) and Amihud (2014). Our main result concerns the pattern of risk-adjusted portfolio returns within volatility and size groups when using either the Amihud (2002) illiquidity measure or our measure of true illiquidity as an additional sorting criteria. Consistent with Chordia, Sahn-Wook, and Subrahmanyam (2009), we find risk-adjusted returns to increase with the Amihud (2002) illiquidity measure within size and volatility groups implying a positive-signed illiquidity premium. However, this pattern collapses and further changes its sign when the strong negative correlation of size and the Amihud (2002) illiquidity measure is neutralized by forming portfolios on size and the orthogonalized illiqudity measure. Similar results are observed when pre-forming portfolios on volatility and when building portfolios three-dimensionally to control for volatility, size and true illiquidity at the same time.

This study proceeds as follows: Section 2 discusses the illiquidity measure used and gives an overview on the portfolio formations utilized for the asset pricing model. Section 3 provides the empirical results and contains several robustness checks. Section 4 concludes the paper.

2 Data and Methodology

2.1 Measuring (true) illiquidity

The dataset is obtained from Bloomberg and includes daily data for 5492 common US stocks in the period of January 1990 to September 2015. To survive in a given month, a stock must at least have 15 days of trading in that month. Furthermore, data for the daily share volume (SVOL), the number of shares outstanding (#SHARE) as well as the market capitalization of a stock (SIZE) must be available. Applying these criteria reduces the number of stocks that are used for the analysis to 4739. At the end of 2014, our sample covers 90% of the total market value of all listed US stocks.

For each stock, we first calculate the illiquidity level using the Amihud (2002) measure. Let $\mathrm{D}_{i,m}$ denote the number of days in month m for which data of stock i are available with $\mathrm{R}_{i,d}$ and $\mathrm{DVOL}_{i,d}$ being the percentage return and traded volume (in million dollars) per day. Thus, the average illiquidity level of stock i in month m is equal to

$$\mathrm{ILLIQ}_{i,m} = \frac{1}{\mathrm{D}_{i,m}} \sum_{d=1}^{\mathrm{D}} \frac{|\mathrm{R}_{i,d}|}{\mathrm{DVOL}_{i,d}}. \tag{1}$$

Numerous studies use ILLIQ as a proxy for illiquidity. While some of these find that ILLIQ has a strong positive relation to the cross-section of expected returns (see, e.g. Amihud (2002), Chordia, Sahn-Wook, and Subrahmanyam (2009), Li, Sun, and Wang (2014)), it is also used in the liquidity-adjusted capital asset pricing model by Acharya and Pedersen (2005). Amihud, Hameed, Kang, and Zhang (2015) as well as Karolyi, Lee, and van Dijk (2012) use ILLIQ to study commonalities in liquidity across countries. In addition, Lesmond (2005) shows that ILLIQ is highly correlated with bid-ask spreads in emerging markets.

Brennan, Huh, and Subrahmanyam (2013) introduce a turnover-based version of the Amihud (2002) measure. Therefore, the daily traded dollar volume ($\mathrm{DVOL}_{i,d}$) is replaced by the daily share turnover ($\mathrm{STO}_{i,d}$) which is the ratio of daily share volume ($\mathrm{SVOL}_{i,d}$) and the total number of shares outstanding ($\#\mathrm{SHARE}_{i,d}$). The monthly illiquidity level of stock i in month m using the turnover-based version can be obtained as

$$\mathrm{ILLIQTO}_{i,m} = \frac{1}{\mathrm{D}_{i,m}} \sum_{d=1}^{D} |\mathrm{R}_{id}| \cdot \frac{\mathrm{SVOL}_{i,d}}{\#\mathrm{SHARE}_{i,d}} = \frac{1}{\mathrm{D}_{i,m}} \sum_{d=1}^{D} \frac{|\mathrm{R}_{i,d}|}{\mathrm{STO}_{i,d}}. \tag{2}$$

Brennan, Huh, and Subrahmanyam (2013) argue that the liquidity premium is captured better when the turnover-based version of the Amihud (2002) measure is used under the condition that firm size effects are accounted for separately. This is due to the fact that the traded dollar volume on day d is the product of firm size and share turnover on that day. This implies ILLIQ and ILLIQTO to be related as follows:

$$\mathrm{ILLIQ}_{i,d} = \frac{|\mathrm{R}_{i,d}|}{\mathrm{DVOL}_{i,d}} = \frac{|\mathrm{R}_{i,d}|}{\mathrm{STO}_{i,d}} \cdot \frac{\mathrm{STO}_{i,d}}{\mathrm{DVOL}_{i,d}} = \mathrm{ILLIQTO}_{i,d} \cdot \frac{1}{\mathrm{SIZE}_{i,d}}. \tag{3}$$

Taking natural logarithms on both sides of Equation 3 leads to

$$\mathrm{lnILLIQ}_{i,d} = \mathrm{lnILLIQTO}_{i,d} - \mathrm{lnSIZE}_{i,d}. \tag{4}$$

Given Equation (4) and a constant share turnover, it is apparent that the Amihud (2002) measure implies an automatic scale of the market capitalization of a stock. Cochrane (2005) picks up this issue and concludes that "smaller stocks which have smaller dollar volume for the same turnover are automatically more illiquid." Our study turns the focus on this key aspect and shows that a clear distinction between these two dimensions of liquidity can be worthwhile to investors.

Table 1 summarizes descriptive statistics and correlations of the liquidity variables introduced above. A stock is excluded from the sample in month m if it has less than

15 days of trading in that month. We also exclude stocks whose value for ILLIQ_m, ILLIQTO_m or the monthly stock return is part of the lower or upper 1% tail of the respective distribution. Applying these criteria leads to an average number of stocks that are used for the analysis of 2334 per month. To further reduce the impact of extreme observations, we calculate log-transformed variables of ILLIQ, ILLIQTO and SIZE, where the latter is the market value of equity at the end of each month.

<<< Table 1 about here >>>

As can be seen in Panel A of Table 1, the log-transformation substantially decreases the heavy skewness of ILLIQ, ILLIQTO and SIZE. It further clarifies a strong negative average correlation of -0.916 between lnILLIQ and lnSIZE reported in Panel B of Table 1. This is in line with the findings of Brennan, Huh, and Subrahmanyam (2013) who report an average correlation of -0.944 for NYSE and AMEX stocks in the period of July 1971 to December 2009. The correlation between lnILLIQTO and lnSIZE (-0.583) is also similar to that found by Brennan, Huh, and Subrahmanyam (2013) (-0.556).

The effects of such a strong negative correlation are substantial. Especially cross-section regressions are affected by multicollinearity in two ways. First, multicollinearity leads to higher estimated standard deviations of the estimated coefficients and therefore lower t-statistics, and second, negative (positive) correlation between two variables results in positively (negatively) correlated coefficient estimates. The majority of studies that relate to the cross-sectional impact of illiquidity on stock returns do not correct for this correlation.[4]

The issue of an automatic scaling of lnILLIQ with lnSIZE does not merely affect cross-sectional regressions in a negative fashion. Suppose that the Carhart (1997) four-

[4]A recent exception can be found in the work of Li, Sun, and Wang (2014) who study the impact of illiquidity on stock returns for the Japanese market in the period of 1975 to 2006.

factor model which is commonly used to evaluate portfolio performance[5] is estimated for portfolios formed on size and illiquidity. Chordia, Sahn-Wook, and Subrahmanyam (2009) use a similar technique and show that, except for the largest size group, the risk-adjusted portfolio return increases with illiquidity in a given size group. Since the authors report a negative correlation of -0.301 between their illiquidity measure and the market capitalization of a stock, the size effect is still present within size groups even after pre-sorting stocks on size. That is, the low (high) illiquidity portfolio in the smallest size group contains of the largest (smallest) stocks within the smallest size group.

To underline this issue, Figure 1 illustrates a portfolio formation strategy that is based on a sorting of lnILLIQ and lnSIZE at the end of September 2015. After pre-sorting stocks by their market value of equity into terciles (break-points are labelled by vertical red lines), the formation of quintile portfolios in lnILLIQ can be understood as horizontal breaks within each size group. Naturally, this type of sorting does not correct for the strong negative correlation and therefore maintains the issue of an automatic scaling of a stocks illiquidity level given its market capitalization. This is especially apparent when, for example, comparing the largest lnSIZE and highest lnILLIQ portfolio with the largest lnSIZE and lowest lnILLIQ portfolio: whereas the former group exhibits very small variation in lnSIZE and mainly concentrates at lnSIZE = 8, the latter group varies considerably more in lnSIZE with a minimum at lnSIZE = 8.7.

<<< Figure 1 about here >>>

The strong negative correlation between lnILLIQ and lnSIZE serves us for calculating the "true" liquidity of a stock by orthogonalizing lnILLIQ to lnSIZE at the end of each month m. It is straightforward to achieve the orthogonalization with an auxiliary regression of the following form:

[5] See, for example, Kosowski, Timmermann, Wermers, and White (2006), Fama and French (2010), Fama and French (2012)

$$\text{lnILLIQ}_{i,m} = \alpha + \beta \cdot \text{lnSIZE}_{i,m} + \text{u}_{i,m} \quad (5)$$

where $\text{u}_{i,m}$ are the errors and $\text{lnSIZE}_{i,m}$ the logarithm of the market value of equity of stock i at the end of month m. The estimated residual term $\hat{\text{u}}_{i,m}$ is denoted by TILLIQ in the following and represents the part of the logarithmized Amihud (2002) measure that is not linearly dependent on the logarithm of the market capitalization of a stock:

$$\hat{\text{u}}_{i,m} = \text{TILLIQ}_{i,m}. \quad (6)$$

Thus, a stock is truly liquid (illiquid) in month m if its value for TILLIQ in that month is negative (positive). This implicates that even stocks with low (high) market capitalization can be liquid (illiquid) and therefore solves the problem of an automatic scaling of ILLIQ with SIZE discussed by Cochrane (2005). Hence, true liquidity enables us to gain even more insight into the liquidity characteristics of a stock since a single measurement and especially judgement of a stock's liquidity either by the Amihud (2002) measure or by the market capitalization can be misleading due to the high negative correlation of these.

The effect of orthogonalizing lnILLIQ to lnSIZE for portfolio building strategies is clarified using the same example as in Figure 1. After pre-sorting stocks by lnSIZE, stocks are ranked in ascending order by their estimated residual term of Equation (5). Figure 2 illustrates this portfolio formation technique.

<<< Figure 2 about here >>>

As is apparent from Figure 2, true liquidity refers to stocks that exhibit low values for lnILLIQ by accounting for the strong negative correlation between lnILLIQ and lnSIZE. Also, the variation of lnSIZE within each lnILLIQ group is considerably increased which should result in more balanced average portfolio market capitalizations.

However, as can be seen in Figure 1 and 2, the joint distribution of lnILLIQ and lnSIZE seems to be non-linear in the sense that the slope of the scatter tends to increase (becomes less negative) with lnSIZE. This issue can be comprehended by comparing the formation of quintile portfolios in the largest lnSIZE group in Figure 2 (green lines) with the slope of the scatter in the largest lnSIZE group. The slope of the scatter is less negative than the estimated slope coefficient of Equation (5).

To solve this issue, we extend the orthogonalization of Equation (5) with two intercept and slope dummies that model the difference in the intercept and slope relative to the largest lnSIZE group. Let g denote the respective lnSIZE group with $g = 1$ being the smallest lnSIZE group. For $g \in 1, 2$, the intercept dummy ID of stock i in month m follows as

$$\mathrm{ID}_{i,m,g} = \begin{cases} 1, \text{ if lnSIZE}_{m,((g-1)/3)} < \mathrm{lnSIZE}_{i,m} \leq \mathrm{lnSIZE}_{m,(g/3)} \\ 0, \text{ else} \end{cases} \tag{7}$$

where $(g/3)$ denotes the respective quantiles of the lnSIZE distribution in each month m. Hence, the slope dummy for $g \in 1, 2$ of stock i in month m is defined as

$$\mathrm{SD}_{i,m,g} = \begin{cases} \mathrm{lnSIZE}_{i,m}, \text{ if ID}_{i,m,g} = 1 \\ 0, \text{ else} \end{cases} \tag{8}$$

Adding both the slope and intercept dummy to the orthogonalization of Equation (5), the modified version can be written as

$$\mathrm{lnILLIQ}_{i,m,g} = \alpha + \beta \cdot \mathrm{lnSIZE}_{i,m} + \gamma \cdot \mathrm{ID}_{i,m,g} + \delta \cdot \mathrm{SD}_{i,m,g} + \mathrm{v}_{i,m,g} \tag{9}$$

for $g \in 1, 2$ and

$$\mathrm{lnILLIQ}_{i,m,g} = \alpha + \beta \cdot \mathrm{lnSIZE}_{i,m} + \mathrm{v}_{i,m,g} \tag{10}$$

for $g = 3$. Similar to Equation (6), we define TILLIQD to be the estimated residual term of Equation (9) and (10) which represents the part of lnILLIQ that is not linearly dependent on the logarithm of the market capitalization of a stock within each lnSIZE group g:

$$\widehat{\mathrm{v}}_{i,m,g} = \mathrm{TILLIQD}_{i,m,g}. \tag{11}$$

Thus, we redefine true liquidity (illiquidity) as being a state in which a stock's estimated residual term TILLIQD of Equation (11) is negative (positive).

Applying the modified orthogonalization on the scatter plot shown in Figure 1 and 2 leads to a portfolio formation illustrated in Figure 3.

<<< Figure 3 about here >>>

As can be seen, the modified orthogonalization allows the portfolio formation to be even more appropriate as it accounts for an individual correlation between lnSIZE and lnILLIQ within each lnSIZE group.

Our definition of true liquidity is related to the turnover-based version of the Amihud (2002) measure introduced by Brennan, Huh, and Subrahmanyam (2013). Consider our initial orthogonalization of Equation (5):

$$\mathrm{lnILLIQ}_{i,m} = \alpha + \beta \cdot \mathrm{lnSIZE}_{i,m} + \mathrm{u}_{i,m}$$

Because of $\mathrm{TILLIQ}_{i,m} = \widehat{\mathrm{u}}_{i,m} = \mathrm{lnILLIQ}_{i,m} - \widehat{\mathrm{lnILLIQ}}_{i,m}$ it follows that

$$\mathrm{TILLIQ}_{i,m} = \mathrm{lnILLIQ}_{i,m} - \widehat{\alpha} - \widehat{\beta} \cdot \mathrm{lnSIZE}_{i,m} \tag{12}$$

where $\widehat{\alpha}$ and $\widehat{\beta}$ are the estimated coefficients from the auxiliary regression in Equation (5). Given a negative correlation between lnSIZE and lnILLIQ and therefore a negative $\widehat{\beta}$, Equation (12) is similar to a re-arranged version of Equation (4):

$$\ln\text{ILLIQTO}_{i,d} = \ln\text{ILLIQ}_{i,d} + \ln\text{SIZE}_{i,d}. \tag{13}$$

Note that Equation (13) only holds on a daily basis. Since we use monthly averages of ILLIQ and ILLIQTO (Equations (1) and (2)), the perfect linear dependence of lnILLIQTO on lnILLIQ and lnSIZE on a daily basis is reflected by a high correlation between lnILLIQTO and TILLIQ on monthly basis. The time-series average of the monthly correlation between lnILLIQTO and TILLIQ across stocks amounts to 0.784 whereas the respective correlation between lnILLIQTO and TILLIQD is 0.747. Table 2 gives an overview on the correlation of all relevant illiquidity measures introduced in this section.

<<< Table 2 about here >>>

As expected, the correlation between lnSIZE and TILLIQ as well as TILLIQD is zero, implying that both TILLIQ and TILLIQD are appropriate portfolio formation criteria when pre-forming portfolios on lnSIZE. On top of that, both orthogonalization methods feature another positive aspect that may not be underrated regarding the advantages of our true liquidity measure: the average correlations of both orthogonalized measures to lnILLIQ and lnILLIQTO reveal that they incorporate both the price impact and turnover component of the Amihud (2002) illiquidity measure without being automatically scaled by lnSIZE. Hence, truly liquid (illiquid) stocks are those that exhibit low (high) values for lnILLIQ and lnILLIQTO at any given level of lnSIZE.

Despite its correlation to lnILLIQ and lnILLIQTO, the orthogonalized liquidity measure is invariant under inflation. As already mentioned by Acharya and Pedersen (2005), ILLIQ is measured in "percent per dollar" which causes the dominator to be biased over time. However, the same applies to SIZE since ILLIQ and SIZE can both be written dependent from the stock price P:

$$\text{ILLIQ}_{i,d} = \frac{|\text{R}_{i,d}|}{\text{DVOL}_{i,d}} = \frac{|\text{R}_{i,d}|}{\text{SVOL}_{i,d} \cdot \text{P}_{i,d}} \tag{14}$$

$$\text{SIZE}_{i,d} = \#\text{SHARE}_{i,d} \cdot \text{P}_{i,d} \tag{15}$$

Consider the price P to be biased by the factor ζ while B denotes the biased version:

$$\text{ILLIQ}^B_{i,d} = \frac{|\text{R}_{i,d}|}{\text{SVOL}_{i,d} \cdot \text{P}_{i,d} \cdot \zeta} = \text{ILLIQ}_{i,d} \cdot \frac{1}{\zeta} \tag{16}$$

$$\text{SIZE}^B_{i,d} = \#\text{SHARE}_{i,d} \cdot \text{P}_{i,d} \cdot \zeta = \text{SIZE}_{i,d} \cdot \zeta \tag{17}$$

Taking natural logarithms leads to:

$$\text{lnILLIQ}^B_{i,d} = \text{lnILLIQ}_{i,d} - \ln(\zeta) \tag{18}$$

$$\text{lnSIZE}^B_{i,d} = \text{lnSIZE}_{i,d} + \ln(\zeta) \tag{19}$$

As can be seen by Equations (18) and (19), inflation appears as a simple linear transformation of lnILLIQ and lnSIZE. Hence,

$$cov(\text{lnILLIQ}^B_{i,d}, \text{lnSIZE}^B_{i,d}) = cov(\text{lnILLIQ}_{i,d}, \text{lnSIZE}_{i,d}) \tag{20}$$

and

$$var(\text{lnSIZE}^B_{i,d}) = var(\text{lnSIZE}_{i,d}). \tag{21}$$

Therefore, the estimated slope coefficients of the orthogonalizations of lnILLIQ^B to lnSIZE^B and lnILLIQ to lnSIZE are equal. This in turn leads to identical residuals in both orthogonalizations despite the difference in means of lnILLIQ^B and lnILLIQ as well as lnSIZE^B and lnSIZE.

2.2 Portfolio building

To examine the role of true liquidity in classical asset pricing models, we form portfolios using several sorting criteria. At the beginning of each month m, portfolios are formed based on their value for a given criteria at the end of month $m-1$. Stocks in each portfolio are then used to calculate an equally-weighted average portfolio return in month m. We apply the following portfolio construction methodologies:

1. Following the recent work of Amihud, Hameed, Kang, and Zhang (2015), we form portfolios that control for volatility in returns and the individual illiquidity level ILLIQ. At the beginning of month m, stocks are ranked by their standard deviation of daily returns (SDRET) of month $m-1$ and sorted in tercile portfolios. Within these tercile portfolios, we form quintile portfolios conditioned on the average illiquidity level ILLIQ of month $m-1$. This leads to 15 portfolios formed at the beginning of month m that control for volatility and illiquidity. Amihud (2002) and Li, Sun, and Wang (2014) among others show that volatility and illiquidity are positively correlated. Due to this fact, one can expect the portfolio including stocks that exhibit high volatility and high illiquidity to have the highest average returns. This expectation gets even stronger when considering the fact that ILLIQ and SIZE are negatively correlated, meaning that the well-known size effect is also apparent when using this type of sorting.

2. We construct portfolios using the standard deviation of daily returns in month $m-1$ and the orthogonalized illiquidity level TILLIQD in month $m-1$. Although this solves the issue of a correlation between lnILLIQ and lnSIZE, to the authors' knowledge this type of sorting has not been used in the literature so far. One can expect a pattern of average returns that is similar to a sorting of SDRET and ILLIQ since higher values for TILLIQD are associated with higher values for ILLIQ

and ILLIQTO, therefore corresponding to higher illiquidity of a stock. Another major advantage of using true illiquidity as a second sorting criteria is the fact that it eliminates the linear dependence of SDRET and lnILLIQ. While the average correlation of SDRET and lnILLIQ amounts to 0.416, the average correlation of SDRET and TILLIQD is -0.024. This implies TILLIQD to be linear independent from lnSIZE and nearly linear independent from SDRET while it still captures an illiquidity effect due to its correlation to lnILLIQ and lnILLIQTO.

3. Following Chordia, Sahn-Wook, and Subrahmanyam (2009), we also use the market value of a stock as a (pre-)sorting variable. Tercile portfolios are formed based on the firm's market capitalization at the end of month $m-1$ followed by a construction of quintile portfolios based on the average illiquidity level ILLIQ of month $m-1$. This methodology corresponds to Figure 1. Given the findings of Chordia, Sahn-Wook, and Subrahmanyam (2009), it can be expected that average returns decrease with firm size and increase with illiquidity.

4. The last portfolio formation used in this study again adjusts for the negative correlation between lnILLIQ and lnSIZE by using true illiquidity TILLIQD instead of ILLIQ. After pre-sorting stocks by their market value at the end of month $m-1$, stocks are ranked in ascending order by their estimated residual term of Equation (9) and (10). Put another way, the largest SIZE and lowest TILLIQD group contains stocks with a high market value of equity and a high share turnover. Given the definition of the turnover-based illiquidity measure introduced by Brennan, Huh, and Subrahmanyam (2013) in Equation (2), it is apparent that, given a constant return, an increase in share turnover leads to smaller values of lnILLIQTO. Due to the high positive correlation between lnILLIQTO and TILLIQD shown in Table 2, a high share turnover corresponds to lower values of TILLIQD. Given this relationship, one might expect the largest SIZE and lowest TILLIQD group to have

lower average returns than the largest SIZE and highest TILLIQD group. Generally speaking, stocks' average returns should decrease with firm size and increase with TILLIQD.

2.3 Constructing a liquidity premium factor

We follow Liu (2006), Marcelo and Quiros (2006) and Amihud, Hameed, Kang, and Zhang (2015) by constructing a zero-investment liquidity premium factor that is defined as the difference in monthly returns of an illiquid and liquid portfolio. For this purpose, we use our first portfolio construction methodology (SDRET and ILLIQ) to calculate the average monthly return of the most illiquid and liquid portfolios across all three volatility groups. The difference in both averages (illiquid minus liquid) is denoted as IML. The methodology used therefore corresponds to that of Amihud, Hameed, Kang, and Zhang (2015) who find a significant positive illiquidity premium of 0.817% per month in global markets and 0.566% in developed markets after controlling for six global and regional risk factors in the period of 1990 to 2011. For the US, the authors report a mean IML of 0.539% and a risk-adjusted IML of 0.299% per month for volume-weighted portfolios.[6]

IML denotes the difference in monthly returns of an illiquid and a liquid portfolio based on a sorting of SDRET and ILLIQ. Given the alternative sorting of SDRET and TILLIQD, we introduce and denote IMLORTH as the corresponding liquidity premium factor which not only corrects for the correlation between lnSIZE and lnILLIQ but also for that of SDRET and lnILLIQ. To study possible differences in the liquidity premium factors IML and IMLORTH, we follow Amihud, Hameed, Kang, and Zhang (2015) and use common risk factors introduced by Fama and French (1993) and Carhart (1997) to calculate risk-adjusted liquidity premia. The four factors, RM, SMB, HML and MOM

[6]Note that these numbers can be found in the online version of the Amihud, Hameed, Kang, and Zhang (2015) paper, see `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2207810`.

are obtained from Kenneth French's homepage.[7] RM denotes the market excess return, SMB is the average return on three small portfolios minus the average return on three big portfolios and HML is the average return on two value portfolios minus the average return on two growth portfolios. Additionally, we use the momentum factor (MOM) introduced by Carhart (1997) which is the average return on two high prior return portfolios minus the average return on two low prior return portfolios.

The risk-adjusted excess returns of the liquidity premia are obtained as the estimated intercept from a regression of the liquidity premium on the risk factors RM, SMB, HML and MOM:

$$\mathrm{IML}_m = \alpha_{IML} + \beta_{RM} \cdot \mathrm{RM}_m + \beta_{SMB} \cdot \mathrm{SMB}_m + \beta_{HML} \cdot \mathrm{HML}_m + \beta_{MOM} \cdot \mathrm{MOM}_m + \varepsilon_m \quad (22)$$

Equation (22) is also used to regress IMLORTH as a dependent variable on the same risk factors. The results of both regressions are reported in Section 3.1 while Section 2.4 examines portfolio statistics and formulates the empirical setup that relates to our main analysis.

2.4 Portfolio statistics and empirical setup

To examine the role of true liquidity in classical asset pricing models, we utilize the portfolio formations discussed in Section 2.2. Table 3 and 4 report averages on monthly stock return, firm size and illiquidity for each of the 15 portfolios that are formed using the first and second portfolio building methodology.[8]

Panel A of Table 3 confirms the expectation of higher average returns for high volatility and high illiquidity stocks since average portfolio returns clearly increase with

[7] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research

[8] The statistics for the third and fourth portfolio formation are reported in Table A.1 and A.2 in the Appendix.

volatility and illiquidity. Panel B of Table 3 also reveals that the average firm size decreases considerably with illiquidity, therefore pointing at the strong negative correlation between ILLIQ and SIZE.

The impact of true liquidity becomes obvious when considering Table 4. As it is the most volatile and most illiquid portfolio in Table 3 that exhibits the highest average return of 2.504% per month, Table 4 shows an inverted pattern of average returns in all volatility groups. The highest average return of 2.280% is obtained in the most volatile and truly liquid portfolio. On top of that, the average market capitalization of this group is more than eight times higher than that of the high illiquidity and high volatility portfolio in Table 3.

Generally, the use of true liquidity leads to a more balanced average firm size distribution within volatility groups. This could be expected since our true liquidity measure is not correlated with lnSIZE. However, the size effect is still present due to the negative correlation of size and volatility which explains the difference in average firm size levels across volatility groups.

Tables A.1 and A.2 in the Appendix contain statistics for portfolios that are presorted on firm size. Table A.1 reveals that an increase in ILLIQ tends to increase average returns only for the smallest size group. In a broader sense, this finding confirms that of Chordia, Sahn-Wook, and Subrahmanyam (2009) and Ben-Rephael, Kadan, and Wohl (2015) who conclude that illiquidity is priced stronger in smaller firms.

In contrast, the pattern of increasing average returns in true liquidity is clearly verified in Table A.2. Similar to the finding of Table 4, average returns increase with true liquidity in each size group. Furthermore, the average firm size distribution for the biggest size groups reveals that true illiquidity is accompanied by a slight increase in average firm size. This might be a result of the negative correlation of lnILLIQTO and lnSIZE reported in Table 2, that is, larger firms size corresponds to lower share turnover (given a constant return).

However, the pattern of average returns may be indicative but it cannot explain the value of true liquidity in terms of classical asset pricing models. We therefore use the portfolio formations discussed in Section 2.2 to estimate the Carhart (1997) four-factor model which is based on the three-factor model proposed by Fama and French (1993). Chordia, Sahn-Wook, and Subrahmanyam (2009) use a similar approach in their time-series regressions to show that risk-adjusted portfolio returns increase with illiquidity and decrease with size.

The three-factor model explains the excess return of an asset with the market excess return RM, the difference between returns on diversified portfolios of small stocks and big stocks, SMB, and HML, the difference between returns of diversified portfolios of high book-to-market stocks and low book-to-market stocks. Carhart (1997) extents the three-factor model by adding a momentum factor (MOM) which is defined as the difference in returns on diversified portfolios of winners and losers of the past year:

$$\mathrm{R}_{p,m} = \alpha_p + \beta_{RM,p} \cdot \mathrm{RM}_m + \beta_{SMB,p} \cdot \mathrm{SMB}_m + \beta_{HML,p} \cdot \mathrm{HML}_m + \beta_{MOM,p} \cdot \mathrm{MOM}_m + \varepsilon_{p,m}. \quad (23)$$

where $\mathrm{R}_{p,m}$ is the excess return of portfolio p in month m. We estimate the Carhart (1997) four-factor model for all four portfolio formations in Section 2.2 and discuss the results in Section 3.2.

3 Empirical Results

3.1 The illiquidity premium factor

Our empirical analysis starts with studying the illiquidity premium factors IML and IMLORTH. Figure 4 shows the development the IML factor over the period of February 1990 to September 2015. The time-series are accompanied by a 12-month moving average.

<<< Figure 4 about here >>>

The 12-month moving average of the plot reveals that IML was mainly negative until the beginning of the 21st century. In terms of ILLIQ, highly liquid stocks possessed higher average returns during the first ten years of the sample period. The negative peak at the end of the 20th century was then followed by a conversion of the sign of the illiquidity premium IML. A similar observation is made by Amihud (2014) who also finds a negative peak in the near 2000 when the dot-com bubble burst. However, during the times of the financial crisis, IML does not show a noteworthy downturn which would imply liquid stocks to exhibit higher returns compared to illiquid stocks.

<<< Figure 5 about here >>>

Using the same scale in returns, Figure 5 shows a different picture. In contrast to IML, the major portion of the IMLORTH time-series remains negative during the whole sample period. While the downturn of IMLORTH in the early 21st century is comparable to that of IML in Figure 4, the minimum of IMLORTH is obtained in April 2009, that is, one month after major US stock indices reached their turning point. Therefore, the development of the IMLORTH time-series during the financial crisis might be interpreted as a confirmation of the “flight to liquidity” hypothesis studied by Amihud, Mendelson, and Wood (1990) and Amihud (2002). Accordingly, an increase in market illiquidity should result in two effects that reinforce each other. First, stock prices decline and expected returns rise for all stocks. Second, capital "flies" from illiquid to liquid stocks. On the one hand, this leads to a further decline in stock prices for illiquid stocks; on the other hand, the decrease in stock prices of liquid stocks is mitigated due to the demand of investors that switched from illiquid to liquid stocks. Thus, Figure 4 and 5 reveal that verifying such an effect considerably depends on the illiquidity measure used.

We extend the graphical analysis by reporting descriptive statistics on both time-series in Panel A of Table 5. Panel B provides correlations of the variables that are used in the risk-adjustment of Equation (22). The respective estimation results are therefore reported in Panel C.

<<< Table 5 about here >>>

Panel A of Table 5 confirms the conclusions based on Figure 4 and 5. The mean of the illiquidity premium IML is positive with a value of 0.589% per month ($t = 3.908$) while the median is 0.570%. This is in line with the findings of Amihud, Hameed, Kang, and Zhang (2015) who report a mean IML of 0.539% for volume-weighted portfolio returns in the US in the period of 1990 to 2011.

In contrast, the mean of the illiquidity premium IMLORTH is highly negative and amounts to -0.635% per month ($t = -4.475$). Also, the graphical impression of stronger negative peaks is confirmed in the negative skewness of the IMLORTH time-series. Panel A of Table 5 therefore strengthens the indication of truly liquid stocks to exhibit higher returns than their truly illiquid counterpart.

Panel C of Table 5 confirms these findings since the risk-adjusted IMLORTH time-series exhibits a negative and significant intercept α that amounts to -0.576% per month. Our findings therefore might indicate a major switch in evaluating the effects of liquidity on stock returns. In fact, using the "standard" liquidity premium IML implies that there is a significant positive illiquidity premium for US stocks in the period of February 1990 to September 2015. This result is fully in line with the recent literature, especially with the findings of Amihud, Hameed, Kang, and Zhang (2015). However, it is apparent that the proposition of illiquid stocks to exhibit higher returns does not hold when accounting for the strong correlation between two common liquidity measures, that is, ILLIQ and SIZE. The economic relevance of these results is further enhanced due to the fact that our true liquidity measure is not only linear independent from lnSIZE but also nearly linear independent from volatility in returns.[9] Hence, the "standard" liquidity premium IML may be biased upward by the well-known size effect studied by Banz (1981) and by the positive correlation of the Amihud (2002) measure to volatility.

We now verify whether the effects of true liquidity are also present when applying the Carhart (1997) four-factor asset pricing model to portfolio formations discussed in Section 2.2.

3.2 True liquidity and asset pricing

3.2.1 Volatility and (true) illiquidity portfolios

Table 6 reports the estimation results of the Carhart (1997) four-factor model for portfolios that control for volatility (SDRET) and illiquidity (ILLIQ).

<<< Table 6 about here >>>

The estimation results confirm the expectation of higher risk-adjusted returns for high volatility and high illiquidity portfolios. Similar to the average portfolio returns reported in Table 3, risk-adjusted returns α show a specific pattern within each volatility group as they rise with ILLIQ. Given the estimation results in Panel A of Table 6, one may

[9]As reported in Section 2.2, the correlation of SDRET and TILLIQD amounts to -0.024 in average.

conclude that portfolio excess returns are not achievable by putting emphasis on low volatility and low illiquidity since the risk-adjusted return of the lowest volatility and lowest illiquidity group is slightly positive but not significant.

However, applying the Carhart (1997) four-factor model on portfolios formed on volatility and true illiquidity leads to remarkable different results. As can be seen in Table 7, the estimated portfolio excess returns α decrease almost monotonically with true illiquidity, implying that truly liquid stocks exhibit the highest excess returns. This pattern sustains across all three volatility groups and therefore confirms the tendency of average returns to increase with true liquidity as shown in Table 4. In fact, the strong pattern yields significant positive excess returns for the low volatility and truly liquid portfolio (0.384% per month). In contrast, the truly illiquid counterpart in the low-volatility group now exhibits a non-significant portfolio excess return of 0.098% per month.

<<< Table 7 about here >>>

To further study the value of true liquidity, we perform several t-tests on the difference in portfolio excess returns within each volatility group and across portfolio formations. Panel A of Table 8 contains the portfolio excess returns in the low-volatility group for portfolios formed on ILLIQ and TILLIQD. The t-statistics in the right column (high-low) relate to a significance test for the difference in the portfolio excess return of portfolio $j = 5$ and portfolio $j = 1$ conditioned on the sorting criteria of ILLIQ or TILLIQD.

<<< Table 8 about here >>>

The results reveal that the difference in portfolio returns across ILLIQ portfolios (high-low) is positive and highly significant for all volatility groups (Panel A to Panel C). In the highest volatility group, the difference amounts to 1.974% per month. Using TILLIQD as sorting criteria leads to markedly different results since higher values for TILLIQD correspond to significantly lower portfolio excess returns. Even in the low volatility group (Panel A), the return difference of high TILLIQD stocks to low TILLIQD stocks is negative and significant at the 5% level. Thus, the results indicate that low volatility and low illiquidity are not necessarily contradictory purposes concerning the ability to create abnormal risk-adjusted portfolio excess returns.

We also perform the t-test on the difference in excess returns across the sorting criteria conditioned on the portfolio index j. As can be seen, true illiquidity significantly increases portfolio excess returns for liquid stocks ($j = 1$) and lowers portfolio excess returns for illiquid stocks ($j = 5$). This effect gets stronger in volatility, that is, true illiquidity significantly increases portfolio excess returns for the most liquid ($j = 1$) and second most liquid stocks ($j = 2$) in the highest volatility group.

3.2.2 Size and (true) illiquidity portfolios

Table 9 reports the estimation results of the Carhart (1997) four-factor model for portfolios that control for size and the Amihud (2002) illiquidity measure.

<<< Table 9 about here >>>

As can be seen, the estimated risk-adjusted portfolio return α declines across size groups which is consistent with the findings of Banz (1981), Chordia, Sahn-Wook, and Subrahmanyam (2009) and Ben-Rephael, Kadan, and Wohl (2015) among others that use size as a first sorting criteria. However, Table 9 reveals that risk-adjusted returns in the largest size group (Panel C) do not increase with illiquidity. This confirms the findings of Chordia, Sahn-Wook, and Subrahmanyam (2009) who also find that risk-adjusted returns do not increase with illiquidity in the largest size group.

The effect of true liquidity for portfolios is clarified using the orthogonalized illiquidity measure TILLIQD as a second sorting criteria. Note that this sorting is the first in this study to provide a clear view on the value of true liquidity since the relevant sorting criteria lnSIZE and TILLIQD are orthogonal to each other.

<<< Table 10 about here >>>

As can be seen in the results table, risk-adjusted returns are higher for truly liquid stocks than for truly illiquid stocks in each size group. Comparing the risk-adjusted returns to those of portfolios formed on SIZE and ILLIQ, it is apparent that truly liquid stocks exhibit higher risk-adjusted returns than stocks whose low illiquidity level refers to a low value of ILLIQ. For example, the four-factor alpha of the smallest size and lowest illiquidity group in Table 9 amounts to 1.073% per month whereas the four-factor alpha of the smallest size and truly liquid portfolio in Table 10 is 1.996% per month.

Similar to the findings of Table 7, risk-adjusted portfolio returns decrease almost monotonically with true illiquidity within each size group. Compared to the sorting of illiquidity and size in Table 9, orthogonalizing lnILLIQ to lnSIZE appears to be a mechanism that provides a clear view on the impact of liquidity on stock returns. This may also be a consequence of the negligible low correlation between true liquidity and volatility that does not distort the results within each size group compared to a sorting of size and ILLIQ.

Table 11 summarizes the t-statistics for tests on the difference in portfolio excess returns within each size group and across portfolio formations.

<<< Table 11 about here >>>

Table 11 shows that the use of TILLQD tends to increase portfolio excess returns for low illiquidity portfolios ($j = 1$) and decrease portfolio excess returns for high illiquidity portfolios ($j = 5$). However, the difference in returns is only significant for the smallest size group in Panel A and for the lowest illiquidity group in Panel C of Table 11.

Summarizing, the estimation results for the illiquidity premium factors in Section 3.1 and the Carhart (1997) four-factor model in this section provide strong evidence for a major switch in evaluating the impact of liquidity on stock returns. The finding of a significant negative illiquidity premium IMLORTH in Section 3.1 is further strengthened by the pattern of risk-adjusted returns in dependence of TILLIQD when pre-sorting portfolios on volatility or size. In fact, risk-adjusted returns decrease significantly with TILLIQD within all volatility and all size groups. In four out of six cases the decrease in risk-adjusted returns is actually monotonic which underlines the relevance of our results.

We now employ several robustness checks to control for the sensitivity of our findings to changes in the portfolio formation technique and examine whether the pattern of risk-adjusted returns in TILLIQD is a temporary phenomenon.

3.3 Robustness checks

3.3.1 Adding a third dimension in the portfolio formation

Estimations of the Carhart (1997) four-factor model in the previous section uncovered the ability of truly liquid stocks to generate significant higher risk-adjusted returns within volatility and size groups. However, the portfolio formations used in Section 3.2 used either volatility or size as a pre-sorting variable. We now turn to the question whether the effects of true liquidity are still observable when pre-sorting stocks on volatility and size. Thus, at the end of month $m - 1$ stocks are ranked in ascending order by their standard deviation of returns in that month and sorted in tercile portfolios. Within these tercile portfolios, we form quintile portfolios based on a stock's market value of equity at the end of month $m - 1$. As a third dimension of sorting, we form quintile portfolios based on TILLIQD at the end of month $m-1$ within each of the 15 portfolios pre-sorted on volatility and size. This results in 75 portfolios where the average number of stocks included in each portfolio amounts to 31 per month.

For convenience, we only report the risk-adjusted portfolio returns α as well as the t-statistics that test for a significant difference in these returns within each volatility group and across portfolio formations. The results are presented in Table 12 and 13.

Several observations are evident. First, there is a significant difference in the risk-adjusted returns of small and large firms within all volatility groups and across all true illiquidity portfolios. The highest difference of 3.396% is observed in the high volatility and low true illiquidity group in Panel C of Table 13. Also, the difference in returns of small and large firms (small-big) tends to decrease with true illiquidity.

Second, the difference in risk-adjusted returns of truly illiquid and truly liquid stocks (high-low) is negative in all volatility and size groups and significant in 10 out of 15 cases. Therefore, the results strongly confirm our previous findings in the sense that truly liquid stocks exhibit significant higher returns than their truly illiquid counterpart even when pre-sorting stocks on volatility and size.

3.3.2 Extending the reference period for portfolio formations

As a second robustness check, we extend the reference period in which the sorting of stocks based on different criteria is conducted with a length of three months. At the beginning of month m, portfolios are formed based on a criterion calculated for months $m-3$, $m-2$ and $m-1$. While the standard deviation of daily returns SDRET and the illiquidity measure ILLIQ are easily calculated for that period, we assume the relevant firm size to be the average of a stock's market capitalization at the end of months $m-3$, $m-2$ and $m-1$. Based on the average size and average illiquidity ILLIQ of the same period, the logarithms of both magnitudes are orthogonalized at the end of month $m-1$

similar to Equations (9) and (10).

The impact of this adjustment is first tested on the illiquidity premium IML and its orthogonalized version IMLORTH. The results further support the previous findings since the risk-adjusted illiquidity premium IML remains positive and amounts to 0.707% per month, being statistically significant at the 1% level. The risk-adjusted illiquidity premium IMLORTH diminishes to -0.300% per month but remains highly significant with a t-statistic of -3.391.

In line with this, the results for the Carhart (1997) four-factor model are very similar to the previous findings in Section 3.2. Truly liquid stocks still exhibit higher risk-adjusted returns than their truly illiquid counterpart in each volatility and size group. Furthermore, truly liquid stocks generate higher risk-adjusted returns than stocks whose low illiquidity level refers to low values of ILLIQ.

3.3.3 Avoiding possible return-reversals

Ang, Hodrick, Xing, and Zhang (2009) show that stocks with recent past high idiosyncratic volatility have low future average returns. To avoid possible short-term return reversals, we shift the reference period for the portfolio formation by skipping one month. Thus, the portfolios are formed at the beginning of month m based on the relevant criteria at the end of month $m - 2$.

Still, the estimation results approve the finding of truly liquid stocks to generate higher risk-adjusted returns than truly illiquid stocks across all volatility and size groups. The same result applies to both the IML and IMLORTH illiquidity premium. The risk-adjusted illiquidity premium IML slightly increases to 0.892% being statistically significant at the 1% level whereas the risk-adjusted illiquidity premium IMLORTH maintains its significant negative sign and exhibits an estimated coefficient of -0.178%.

We further combine the shift in the reference period for the portfolio formation and the skip of one month to avoid possible return reversals. Hence, portfolios are formed at the beginning of month m based on the criteria at the end of month $m - 2$ where the latter is calculated given data of the stock for months $m-4$, $m-3$ and $m-2$. While this procedure is similar to that of Amihud, Hameed, Kang, and Zhang (2015), the estimation results strengthen our findings from above as the pattern of higher risk-adjusted returns for truly liquid stocks maintains throughout all volatility and size groups. However, as can be seen in Panel A of Table A.4 in the Appendix, the mean of the IMLORTH time-series increases to -0.150% per month while the risk-adjusted return of IMLORTH (-0.136% per month) is still significant at the 10% level.

3.3.4 Using an illiquidity-augmented four-factor model

We follow Liu (2006), Marcelo and Quiros (2006) and Amihud (2014) by applying an liquidity-augmented factor model to the portfolio formations discussed in Section 2.2. Therefore, the Carhart (1997) four-factor model of Equation (23) is extended with the illiquidity premium factor IML as an additional explanatory variable:

$$\begin{aligned} \mathrm{R}_{p,m} =& \alpha_p + \beta_{RM,p} \cdot \mathrm{RM}_m + \beta_{SMB,p} \cdot \mathrm{SMB}_m + \beta_{HML,p} \cdot \mathrm{HML}_m + \beta_{MOM,p} \cdot \mathrm{MOM}_m \\ &+ \beta_{IML,p} \cdot \mathrm{IML}_m + \varepsilon_{p,m}. \end{aligned} \tag{24}$$

Amihud (2014) shows that the average β_{IML} coefficient across 25 and 100 portfolios formed on size and book-to-market is not statistically different from zero. However, the author finds the IML factor to be priced in the cross-section of monthly portfolio returns in the period of 1950 to 2012.

The illiquidity-augmented four-factor model of Equation (24) is estimated using our initial portfolio formation setup, that is, portfolios are formed at the beginning of month

m based on their value for a criteria at the end of month $m - 1$. The estimation results of Equation (24) for all portfolios formations are reported in Table A.5 to Table A.10 in the Appendix.

In general, the results reveal that the illiquidity premium factor IML is a significant explanatory variable for portfolio excess returns in 47 out of 60 cases. Comparing the fit of the estimated models to that studied in Section 3.2, the relevance of the IML factor is expressed in higher adjusted R^2 for 48 of out 60 portfolios. The increase in R^2 amounts up to 34%.

Regarding the differences in the risk-adjusted portfolio excess returns α, the t-statistics reported in Table A.7 and A.10 in the Appendix show no relevant changes to the Carhart (1997) four-factor model used in Section 3.2. The pattern of significant higher risk-adjusted returns for truly liquid stocks maintains throughout all volatility and size groups and is therefore reconfirmed.

3.3.5 Substituting true liquidity by the turnover-based Amihud measure

As already discussed in Section 2.1, our measure of true liquidity is strongly related to the turnover-based version of the Amihud (2002) introduced by Brennan, Huh, and Subrahmanyam (2013) which results in an average correlation of 0.747 between TILLIQD and lnILLIQTO as shown in Table 2.

To examine whether the turnover-based version of illiquidity by Brennan, Huh, and Subrahmanyam (2013) provides similar results as our true liquidity measure, we repeat all estimations made in Section 3 and substitute the orthogonalized illiquidity measure TILLIQD by ILLIQTO. The illiquidity premium factor that uses the turnover-based Amihud (2002) measure is denoted by IMLTO. Table A.11 in the Appendix shows the results for the respective risk-adjustment and recaps the results for IMLORTH as well.

Panel B of Table A.11 reveals that the correlation of the IMLTO and IMLORTH time-series amounts to 0.718 which is similar to the average correlation across stocks of 0.747 reported in Table 2. Also, the minimum, maximum and skewness of the IMLTO time-series in Panel A of Table A.11 are very similar to that of the IMLORTH time-series. However, the mean of IMLTO is positive and amounts to 0.137% per month whereas the risk-adjusted IMLTO in Panel C amounts to 0.297% per month and is statistically significant at the 1% level.

We consider this finding to be highly interesting as it implies that the true liquidity measure TILLIQD is not primarily driven by the positive correlation to lnILLIQTO but rather contains of an own component that cannot be explained by this correlation. To emphasize this point, we perform an additional risk-adjustment of the IMLORTH time-series to that already reported in Table 5. Therefore, we use both common risk factors RM, SMB, HML, MOM as well as the illiquidity return premia IML and IMLTO. If the negative mean in IMLORTH can be explained by variation in IML and IMLTO, the estimated intercept α of this regression is not significant different from zero. However, performing such a regression results in an estimated intercept that amounts to -0.250% per month being highly significant ($t = -4.137$) at the 1% level. Thus, the illiquidity premium IMLORTH includes a return premium that is captured neither by the illiquidity premium IML based on the Amihud (2002) illiquidity measure nor by IMLTO that is based on turnover-based version of the Amihud (2002) measure introduced by Brennan, Huh, and Subrahmanyam (2013). This finding further underlines the economic relevance of our true liquidity measure TILLIQD.

3.3.6 Is the impact of true liquidity a temporary phenomenon?

While the previous sections analyzed the impact of true liquidity for portfolio formations in the period of February 1990 to September 2015, we now focus on the question whether the pattern of almost monotonic decreasing portfolio excess returns in TILLIQD may be driven by temporary effects. For this purpose, we estimate the illiquidity-augmented four-factor model of Equation (24) for a rolling five-year window that is shifted by three months for each estimation. Hence, the first estimation period is from February 1990 to January 1995 followed by the second estimation period from May 1990 to April 1995 and so on. This methodology results in 83 estimations for each portfolio.

We focus on portfolios that use volatility as a pre-sorting variable but extend the number of portfolios that are sorted by true liquidity in the second step to 20, resulting in 60 portfolios total.[10] Therefore, we recalculate the illiquidity premium factor IML and estimate the illiquidity-augmented four-factor model of Equation (24) for 83 rolling periods and 60 portfolios.

For each volatility group g with $g \in 1, 2, 3$, we observe 20 estimated risk-adjusted portfolio returns $\alpha_{p,g}$ with $p = 1, \ldots, 20$ for each of the $r = 1, \ldots, 83$ rolling estimation periods. The estimated risk-adjusted portfolio returns $\alpha_{p,g,r}$ are then used to calculate an ordinal trend within each volatility group g and for every rolling estimation period r:

$$\alpha_{p,g,r} = \delta_{g,r} + \gamma_{g,r} \cdot p + \varepsilon_{p,g,r} \tag{25}$$

Thus, the estimation of the slope coefficient γ gives information of whether the distribution of risk-adjusted portfolio returns increases or decreases with TILLIQD within each volatility group at any given estimation period.

The fitted values of the ordinal trend in Equation (25) for the first volatility group are visualized in Figure 6 by using a heat map. An existing trend in risk-adjusted returns given a specific rolling estimation period (x-axis) is then reflected by color changes depending on the portfolio index (y-axis). The higher the divergence in colors the more distinct the difference in risk-adjusted returns across truly liquid and illiquid portfolios.

[10]The change in the number of portfolios ensures a better illustration of the results.

<<< Figure 6 about here >>>

The graphical representation shows that the pattern of increasing portfolio excess returns for low values of TILLIQD is not a temporary phenomenon for the lowest volatility group. Besides the peak in the rolling sample period that corresponds to the end of the 20th century, the color gradients in the heat map shows that there exists a trend in risk-adjusted excess returns across true illiquidity portfolios even in phases that do not correspond to either the dotcom bubble burst or financial crisis. Since the estimated ordinal trend is negative most of the time, the findings made in previous sections are strengthened further.[11]

This impression is confirmed for the second volatility group in Figure 7.

<<< Figure 7 about here >>>

Figure 7 reveals that the pattern of higher risk-adjusted returns for truly liquid portfolios in the second volatility group maintains until the 70th rolling estimation which corresponds to an estimation period from June 2007 to May 2012. A similar though considerably weaker observation can be made for the high volatility group in Figure 8.

<<< Figure 8 about here >>>

Generally speaking, the majority of the 83 rolling estimations of the illiquidity-augmented four-factor model reveal that the pattern of higher risk-adjusted returns for truly liquid portfolios is not a temporary phenomenon. However, Figure 7 and 8 provide weak evidence for a pattern of risk-adjusted returns in TILLIQD in the most recent time periods for mid and high volatile stocks that is in line with the literature.

[11] As the portfolio index 1 (20) corresponds to the truly liquid (illiquid) portfolio, a negative trend in the heat map implies that higher portfolio indexes are accompanied by lower risk-adjusted returns α.

4 Conclusions

This study aims at investigating the value of true liquidity. Given 5492 US stocks in the period of January 1990 to September 2015, we use the strong negative correlation of the log-transformed Amihud (2002) illiquidity measure and the log-transformed market value of equity to show that stocks we define as being truly liquid exhibit significant higher risk-adjusted returns than their truly illiquid counterpart. Based on the difference in returns of truly illiquid and truly liquid stocks, we find the orthogonalized illiquidity premium IMLORTH to be significantly negative in the period under investigation. The risk-adjusted orthogonalized illiquidity premium IMLORTH amounts to -0.576% per month and is robust to changes in the portfolio formation setting.

Following the procedure of Amihud, Hameed, Kang, and Zhang (2015), we identify a significant illiquidity return premium IML that amounts to 0.758% per month which is in line with the findings of Amihud, Hameed, Kang, and Zhang (2015). Therefore, our results suggest that not correcting for a high correlation between the log-transformed Amihud (2002) illiquidity measure and the log-transformed market value of equity hinders the identification of a "true" illiquidity effect. This is especially apparent when considering the issue of strong correlations between illiquidity and size as well as between illiquidity and volatility. A major advantage of our true illiquidity measure is the fact that it solves this issue since the orthogonalized illiquidity measure TILLIQD is linearly independent from lnSIZE and nearly linear independent from volatility.

Applying the Carhart (1997) four-factor model and an illiquidity-augmented four-factor model on several portfolio formations reveal that the orthogonalized illiquidity measure TILLIQD is able to increase the abnormal risk-adjusted portfolio excess return within volatility and size portfolios. To some extent, risk-adjusted returns decrease monotonically with true illiquidity. Furthermore, *t*-statistics that test for a significant increase in risk-adjusted returns across the portfolio formations of ILLIQ and TILLQD within volatility and size groups show that using the orthogonalized illiquidity measure TILLIQD can be advantageous to investors. The rolling estimations in Section 3.3.6 confirm this finding in the sense that the trend in risk-adjusted returns across true illiquidity portfolios within a given volatility or size group is not a temporary phenomenon.

Summarizing, this paper uncovers the ability of liquid stocks to generate significant higher risk-adjusted returns than their illiquid counterparts. The economic relevance of this study is enhanced due to the fact that our findings are broadly inconsistent with that of recent studies, e.g. Amihud, Hameed, Kang, and Zhang (2015), Ben-Rephael, Kadan, and Wohl (2015), Brennan, Huh, and Subrahmanyam (2013), Chordia, Sahn-Wook, and Subrahmanyam (2009) or Liu (2006). In fact, we do observe that illiquid stocks exhibit higher risk-adjusted returns than liquid stocks which is consistent with existing theory. However, this observation is made under the condition that we neglect any correlation structures between two common liquidity measures, that is, ILLIQ and SIZE.

Although our results have strong implications, it should be emphasized that there remains a large field of research for upcoming studies, especially with respect to the individual characteristics of stocks in dependence of their “true” illiquidity level.

References

Acharya, V. V., and L. H. Pedersen (2005): "Asset Pricing with Liquidity Risk," *Journal of Financial Economics*, 77(2), 375 – 410.

Amihud, Y. (2002): "Illiquidity and Stock Returns: Cross-Section and Time-Series Effects," *Journal of Financial Markets*, 5(1), 31 – 56.

——— (2014): "The Pricing of the Illiquidity Factor's Systematic Risk," *Working Paper*.

Amihud, Y., A. Hameed, W. Kang, and H. Zhang (2015): "The illiquidity premium: International evidence," *Journal of Financial Economics*, 117(2), 350 – 368.

Amihud, Y., and H. Mendelson (1986): "Asset Pricing and the Bid-Ask Spread," *Journal of Financial Economics*, 17(2), 223 – 249.

——— (1989): "The Effects of Beta, Bid-Ask Spread, Residual Risk, and Size on Stock Returns," *Journal of Finance*, 44(2), 479 – 486.

Amihud, Y., H. Mendelson, and R. A. Wood (1990): "Liquidity and the 1987 stock market crash," *Journal of Portfolio Management*, 16(3), 65 – 69.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2009): "High idiosyncratic volatility and low returns: International and further U.S. evidence," *Journal of Financial Economics*, 91(1), 1 – 23.

Banz, R. W. (1981): "The relationship between return and market value of common stocks," *Journal of Financial Economics*, 9(1), 3 – 18.

Ben-Rephael, A., O. Kadan, and A. Wohl (2015): "The Diminishing Liquidity Premium," *Journal of Financial and Quantitative Analysis*, 50, 197 – 229.

Brennan, M., S.-W. Huh, and A. Subrahmanyam (2013): "An Analysis of the Amihud Illiquidity Premium," *Review of Asset Pricing Studies*, 3(1), 133 – 176.

Brennan, M. J., and A. Subrahmanyam (1996): "Market Microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns," *Journal of Financial Economics*, 41(3), 441 – 464.

Carhart, M. M. (1997): "On Persistence in Mutual Fund Performance," *Journal of Finance*, 52(1), 57–82.

Chang, Y. Y., R. Faff, and C.-Y. Hwang (2010): "Liquidity and stock returns in Japan: New evidence," *Pacific-Basin Finance Journal*, 18(1), 90 – 115.

Chordia, T., H. Sahn-Wook, and A. Subrahmanyam (2009): "Theory-Based Illiquidity and Asset Pricing," *Review of Financial Studies*, 22(9), 3629 – 3668.

Cochrane, J. H. (2005): "Asset Pricing: Liquidity, Trading, and Asset Prices," in *NBER Reporter*, pp. 1 – 12. NBER.

Datar, V. T., N. Y. Naik, and R. Radcliffe (1998): "Liquidity and stock returns: An alternative test," *Journal of Financial Markets*, 1(2), 203 – 219.

Duarte, J., and L. Young (2009): "Why is PIN priced?," *Journal of Financial Economics*, 91(2), 119 – 138.

Easley, D., S. Hvidkjaer, and M. O'Hara (2002): "Is Information Risk a Determinant of Asset Returns?," *Journal of Finance*, 57(5), 2185 – 2221.

Easley, D., N. M. Kiefer, M. O'Hara, and J. B. Paperman (1996): "Liquidity, Information, and Infrequently Traded Stocks," *Journal of Finance*, 51(4), 1405 – 1436.

Eleswarapu, V. R. (1997): "Cost of Transacting and Expected Returns in the Nasdaq Market.," *Journal of Finance*, 52(5), 2113 – 2127.

Fama, E. F., and K. R. French (1993): "Common risk factors in the returns on stocks and bonds," *Journal of Financial Economics*, 33(1), 3 – 56.

——— (2010): "Luck versus Skill in the Cross-Section of Mutual Fund Returns," *Journal of Finance*, 65(5), 1915 – 1947.

——— (2012): "Size, value, and momentum in international stock returns," *Journal of Financial Economics*, 105(3), 457 – 472.

Goyenko, R. Y., C. W. Holden, and C. A. Trzcinka (2009): "Do liquidity measures measure liquidity?," *Journal of Financial Economics*, 92(2), 153 – 181.

Hasbrouck, J. (2009): "Trading Costs and Returns for U.S. Equities: Estimating Effective Costs from Daily Data," *Journal of Finance*, 64(3), 1445 – 1477.

Jensen, G. R., and T. Moorman (2010): “Inter-temporal variation in the illiquidity premium,” *Journal of Financial Economics*, 98(2), 338 – 358.

Kamara, A., X. Lou, and R. Sadka (2008): “The divergence of liquidity commonality in the cross-section of stocks,” *Journal of Financial Economics*, 89(3), 444 – 466.

Karolyi, G. A., K.-H. Lee, and M. A. van Dijk (2012): “Understanding commonality in liquidity around the world,” *Journal of Financial Economics*, 105(1), 82 – 112.

Korajczyk, R. A., and R. Sadka (2008): “Pricing the commonality across alternative measures of liquidity,” *Journal of Financial Economics*, 87(1), 45 – 72.

Kosowski, R., A. Timmermann, R. Wermers, and H. White (2006): “Can Mutual Fund Stars Really Pick Stocks? New Evidence from a Bootstrap Analysis,” *Journal of Finance*, 61(6), 2551 – 2595.

Kyle, A. S. (1985): “Continuous Auctions and Insider Trading,” *Econometrica*, 53, 1315 – 1335.

Lesmond, D. A. (2005): “Liquidity of emerging markets,” *Journal of Financial Economics*, 77(2), 411 – 452.

Li, B., Q. Sun, and C. Wang (2014): “Liquidity, Liquidity Risk and Stock Returns: Evidence from Japan,” *European Financial Management*, 20(1), 126 – 151.

Liu, W. (2006): “A liquidity-augmented capital asset pricing model,” *Journal of Financial Economics*, 82(3), 631 – 671.

Marcelo, J. L. M., and M. Quiros (2006): “The role of an illiquidity risk factor in asset pricing: Empirical evidence from the Spanish stock market,” *The Quarterly Review of Economics and Finance*, 46(2), 254 – 267.

Pastor, L., and R. F. Stambaugh (2003): “Liquidity Risk and Expected Stock Returns,” *Journal of Political Economy*, 111(3), 642 – 685.

Roll, R. (1984): “A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market,” *The Journal of Finance*, 39(4), 1127 – 1139.

Sadka, R. (2006): “Momentum and Post-Earnings-Announcement Drift Anomalies: The Role of Liquidity Risk,” *Journal of Financial Economics*, 80(2), 309 – 349.

Table 1: Descriptive statistics and correlations

Panel A of this table provides descriptive statistics on the variables that are used for the analysis. Each variable is calculated for each stock in the period of 02/1990-09/2015. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The mean, standard deviation and skewness are then calculated for each month across stocks. Panel B provides an overview of the average correlations. In each month m, the correlation is calculated cross-sectionally followed by an averaging of the correlation over all months in the period of 02/1990-09/2015.

Panel A: Descriptive statistics

Variable	Mean of monthly means	Mean of monthly s.d.	Median of monthly means	Mean of monthly skewness	Min. monthly mean	Max. monthly mean
ILLIQ	$5.403 \cdot 10^{-7}$	$2.107 \cdot 10^{-6}$	$4.303 \cdot 10^{-7}$	8.679	$3.190 \cdot 10^{-8}$	$3.208 \cdot 10^{-6}$
ILLIQTO	26.760	59.212	24.434	5.556	4.486	86.420
SIZE (in $ mio.)	2735.25	7452.064	2695.118	6.844	1249.572	5586.129
lnILLIQ	-18.100	2.621	-17.824	0.271	-20.125	-15.808
lnILLIQTO	1.988	1.246	2.103	0.915	0.795	3.496
lnSIZE	6.344	1.708	6.261	0.239	5.566	7.162

Panel B: Average correlations

Variable	ILLIQ	ILLIQTO	SIZE	lnILLIQ	lnILLIQTO	lnSIZE
ILLIQ	1					
ILLIQTO	0.662	1				
SIZE	-0.086	-0.108	1			
lnILLIQ	0.481	0.565	-0.488	1		
lnILLIQTO	0.495	0.732	-0.208	0.849	1	
lnSIZE	-0.379	-0.339	0.614	-0.916	-0.583	1

Table 2: Average correlations of relevant illiquidity measures

This table provides an overview of average correlations of the illiquidity measures introduced in Section 2. In each month m, the correlation is calculated cross-sectionally followed by an averaging of the correlation over all months in the period of 02/1990-09/2015. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution.

Average correlations					
Variable	lnILLIQ	lnILLIQTO	lnSIZE	TILLIQ	TILLIQD
lnILLIQ	1				
lnILLIQTO	0.849	1			
lnSIZE	-0.916	-0.583	1		
TILLIQ	0.398	0.784	0	1	
TILLIQD	0.381	0.747	0	0.956	1

Table 3: Average values of monthly return, size and illiquidity in the 15 portfolios formed on volatility and illiquidity

This table provides statistics on the average portfolio return, average market capitalization and average illiquidity of portfolios formed on volatility and illiquidity. In each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: average return (in %)

	Illiquidity group				
Volatility group	1 (low ILLIQ)	2	3	4	5 (high ILLIQ)
1 (low)	0.561***	0.655***	0.750***	0.795***	0.909***
	(2.616)	(2.942)	(3.383)	(3.407)	(4.415)
2	0.903***	1.029***	1.137***	1.231***	1.313***
	(2.864)	(3.150)	(3.524)	(3.919)	(4.880)
3 (high)	0.905*	1.239**	1.429***	1.809***	2.504***
	(1.753)	(2.442)	(2.872)	(3.918)	(6.040)

Panel B: average firm size (in $ million)

	Illiquidity group				
Volatility group	1 (low ILLIQ)	2	3	4	5 (high ILLIQ)
1 (low)	17229.633	3883.628	1720.241	841.040	283.263
2	9876.311	1644.281	735.200	398.455	167.267
3 (high)	3397.163	577.557	281.039	155.583	69.285

Panel C: average illiquidity (ILLIQ)

	Illiquidity group				
Volatility group	1 (low ILLIQ)	2	3	4	5 (high ILLIQ)
1 (low)	$1.257 \cdot 10^{-8}$	$4.928 \cdot 10^{-8}$	$1.508 \cdot 10^{-7}$	$6.430 \cdot 10^{-7}$	$6.658 \cdot 10^{-6}$
2	$2.783 \cdot 10^{-8}$	$1.302 \cdot 10^{-7}$	$4.635 \cdot 10^{-7}$	$1.944 \cdot 10^{-6}$	$1.244 \cdot 10^{-5}$
3 (high)	$1.755 \cdot 10^{-7}$	$1.594 \cdot 10^{-6}$	$1.958 \cdot 10^{-5}$	$1.803 \cdot 10^{-5}$	$6.058 \cdot 10^{-5}$

Table 4: Average values of monthly return, size and illiquidity in the 15 portfolios formed on volatility and true illiquidity

This table provides statistics on the average portfolio return, average market capitalization and average illiquidity of portfolios formed on volatility and true illiquidity. In each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: average return (in %)

	True illiquidity group				
Volatility group	1 (truly liquid)	2	3	4	5 (truly illiquid)
1 (low)	0.967***	0.746***	0.681***	0.646***	0.634***
	(4.331)	(3.371)	(3.211)	(3.102)	(3.190)
2	1.688***	1.174***	1.035***	0.944***	0.778***
	(4.528)	(3.675)	(3.404)	(3.283)	(3.184)
3 (high)	2.280***	1.901***	1.473***	1.259***	0.991***
	(3.832)	(3.684)	(3.091)	(3.046)	(2.918)

Panel B: average firm size (in $ million)

	True illiquidity group				
Volatility group	1 (truly liquid)	2	3	4	5 (truly illiquid)
1 (low)	2455.334	4565.017	6058.027	6401.895	4417.545
2	1465.150	2551.254	3335.879	3421.476	2017.397
3 (high)	594.559	1082.481	1134.988	1032.911	628.212

Panel C: average illiquidity (ILLIQ)

	True illiquidity group				
Volatility group	1 (truly liquid)	2	3	4	5 (truly illiquid)
1 (low)	$8.219 \cdot 10^{-7}$	$6.302 \cdot 10^{-7}$	$5.492 \cdot 10^{-7}$	$1.213 \cdot 10^{-6}$	$4.329 \cdot 10^{-6}$
2	$1.330 \cdot 10^{-6}$	$1.201 \cdot 10^{-6}$	$1.796 \cdot 10^{-6}$	$2.777 \cdot 10^{-6}$	$7.948 \cdot 10^{-6}$
3 (high)	$8.593 \cdot 10^{-6}$	$1.109 \cdot 10^{-5}$	$1.438 \cdot 10^{-5}$	$3.155 \cdot 10^{-5}$	$3.421 \cdot 10^{-5}$

Table 5: The illiquidity return premium

This table provides statistics on the variables used for the Carhart (1997) four-factor model and provides the regression results of Equation (22) using both IML and IMLORTH as a dependent variable. At the beginning of each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ (IML) and by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) (IMLORTH). Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Descriptive statistics

Variable	Mean	S.d.	Median	Skewness	Minimum	Maximum
IML	0.589	2.635	0.570	0.185	-6.897	10.752
IMLORTH	-0.635	2.483	-0.344	-1.031	-11.264	5.781
RM	0.616	4.319	1.190	-0.658	-17.230	11.350
SMB	0.169	3.303	0.070	0.814	-16.700	22.320
HML	0.188	3.123	0.010	0.093	-13.110	13.910
MOM	0.613	4.930	0.700	-1.584	-34.580	18.380

Panel B: Correlations

Variable	IML	IMLORTH	RM	SMB	HML	MOM
IML	1					
IMLORTH	0.330	1				
RM	-0.499	-0.577	1			
SMB	0.108	-0.480	0.244	1		
HML	0.338	0.449	-0.249	-0.327	1	
MOM	-0.054	0.253	-0.249	0.052	-0.157	1

Panel C: Estimation results

Variable	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
IML	0.758***	-0.338***	0.276***	0.243***	-0.088***	0.412
	(6.360)	(-11.458)	(7.327)	(5.965)	(-3.535)	
IMLORTH	-0.576***	-0.215***	-0.229***	0.232***	0.112***	0.543
	(-5.818)	(-8.807)	(-7.320)	(6.854)	(5.403)	

Table 6: Four-factor model for portfolios formed on volatility and illiquidity

This table provides the regression results of the Carhart (1997) four-factor model referring to Equation (23). In each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level ILLIQ of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Volatility group 1 (low)

Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (low ILLIQ)	0.004	0.835***	-0.169***	0.361***	0.006	0.842
	(0.049)	(38.264)	(-6.065)	(11.965)	(0.333)	
2	0.036	0.836***	0.064**	0.449***	0.016	0.822
	(0.367)	(34.813)	(2.07)	(13.533)	(0.769)	
3	0.131	0.785***	0.262***	0.473***	0.003	0.827
	(1.375)	(33.287)	(8.682)	(14.515)	(0.157)	
4	0.183*	0.740***	0.466***	0.494***	-0.025	0.844
	(1.952)	(32.018)	(15.777)	(15.443)	(-1.275)	
5 (high ILLIQ)	0.423***	0.589***	0.438***	0.429***	-0.052**	0.739
	(3.887)	(21.899)	(12.749)	(11.552)	(-2.275)	

Panel B: Volatility group 2

Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (low ILLIQ)	0.136	1.178***	0.194***	0.258***	-0.065***	0.888
	(1.247)	(43.661)	(5.619)	(6.918)	(-2.851)	
2	0.249**	1.128***	0.498***	0.368***	-0.110***	0.895
	(2.265)	(41.606)	(14.349)	(9.823)	(-4.792)	
3	0.360***	1.057***	0.668***	0.410***	-0.105***	0.908
	(3.571)	(42.376)	(20.949)	(11.892)	(-4.991)	
4	0.515***	0.941***	0.777***	0.487***	-0.141***	0.887
	(4.719)	(34.885)	(22.529)	(13.059)	(-6.181)	
5 (high ILLIQ)	0.779***	0.744***	0.608***	0.464***	-0.186***	0.804
	(6.315)	(24.42)	(15.614)	(11.025)	(-7.208)	

Panel C: Volatility group 3 (high)

Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (low ILLIQ)	0.039	1.492***	0.955***	-0.406***	-0.225***	0.898
	(0.232)	(35.455)	(17.764)	(-6.977)	(-6.327)	
2	0.372***	1.380***	1.275***	-0.133***	-0.283***	0.927
	(2.629)	(39.428)	(28.499)	(-2.755)	(-9.554)	
3	0.609***	1.263***	1.413***	0.070	-0.342***	0.913
	(4.006)	(33.617)	(29.424)	(1.353)	(-10.765)	
4	1.180***	1.083***	1.231***	0.212***	-0.466***	0.837
	(6.119)	(22.719)	(20.21)	(3.22)	(-11.552)	
5 (high ILLIQ)	2.013***	0.821***	1.037***	0.29***	-0.399***	0.652
	(7.959)	(13.136)	(12.979)	(3.355)	(-7.544)	

Table 7: Four-factor model for portfolios formed on volatility and true illiquidity

This table provides the regression results of the Carhart (1997) four-factor model referring to Equation (23). In each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Volatility group 1 (low)						
True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (truly liquid)	0.384***	0.757***	0.333***	0.433***	-0.033	0.825
	(3.968)	(31.671)	(10.889)	(13.120)	(-1.653)	
2	0.136	0.804***	0.163***	0.470***	-0.002	0.822
	(1.411)	(33.704)	(5.360)	(14.249)	(-0.100)	
3	0.094	0.784***	0.123***	0.453***	-0.002	0.836
	(1.057)	(35.784)	(4.387)	(14.959)	(-0.134)	
4	0.069	0.755***	0.178***	0.457***	-0.006	0.834
	(0.789)	(34.766)	(6.405)	(15.243)	(-0.330)	
5 (truly illiquid)	0.098	0.684***	0.267***	0.393***	-0.009	0.812
	(1.104)	(31.096)	(9.494)	(12.911)	(-0.457)	
Panel B: Volatility group 2						
True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (truly liquid)	0.931***	1.132***	0.773***	0.131***	-0.156***	0.874
	(6.795)	(33.441)	(17.852)	(2.807)	(-5.432)	
2	0.433***	1.075***	0.535***	0.38***	-0.135***	0.896
	(4.061)	(40.83)	(15.881)	(10.434)	(-6.076)	
3	0.290***	1.052***	0.494***	0.465***	-0.121***	0.918
	(3.228)	(47.369)	(17.396)	(15.15)	(-6.428)	
4	0.239***	0.985***	0.466***	0.5***	-0.121***	0.906
	(2.632)	(43.827)	(16.231)	(16.108)	(-6.383)	
5 (truly illiquid)	0.152*	0.804***	0.480***	0.51***	-0.075***	0.887
	(1.790)	(38.267)	(17.865)	(17.573)	(-4.224)	
Panel C: Volatility group 3 (high)						
True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (truly liquid)	1.586***	1.39***	1.419***	-0.290***	-0.566***	0.844
	(6.534)	(23.171)	(18.506)	(-3.497)	(-11.166)	
2	1.105***	1.32***	1.343***	-0.131**	-0.358***	0.906
	(6.74)	(32.58)	(25.928)	(-2.336)	(-10.44)	
3	0.676***	1.267***	1.222***	0.002	-0.311***	0.901
	(4.359)	(33.038)	(24.931)	(0.045)	(-9.594)	
4	0.508***	1.132***	1.068***	0.156***	-0.255***	0.897
	(3.707)	(33.44)	(24.682)	(3.342)	(-8.921)	
5 (truly illiquid)	0.356***	0.929***	0.862***	0.296***	-0.224***	0.859
	(2.694)	(28.47)	(20.664)	(6.567)	(-8.136)	

Table 8: Portfolio alphas and t-statistics within volatility groups

This table provides the t-statistics that test for a significant difference in the risk-adjusted portfolio returns across illiquidity groups j and across portfolio formations. At the beginning of each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ (ILLIQ) and by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) (TILLIQD). Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. The portfolio alphas are obtained as the estimated intercept of Equation (23). ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Volatility group 1 (low)

	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	0.004	0.036	0.131	0.183*	0.423***	0.419***
	(0.049)	(0.367)	(1.375)	(1.952)	(3.887)	(2.989)
TILLIQD	0.384***	0.136	0.094	0.069	0.098	-0.285**
	(3.968)	(1.411)	(1.057)	(0.789)	(1.104)	(-2.171)
$TILLIQD_j$-$ILLIQ_j$	0.379***	0.101	-0.038	-0.113	-0.325**	
	(2.898)	(0.735)	(-0.289)	(-0.883)	(-2.309)	

Panel B: Volatility group 2

	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	0.136	0.249**	0.360***	0.515***	0.779***	0.642***
	(1.247)	(2.265)	(3.571)	(4.719)	(6.315)	(3.901)
TILLIQD	0.931***	0.433***	0.290***	0.239***	0.152*	-0.779***
	(6.795)	(4.061)	(3.228)	(2.632)	(1.790)	(-4.830)
$TILLIQD_j$-$ILLIQ_j$	0.795***	0.184	-0.070	-0.276*	-0.627***	
	(4.536)	(1.204)	(-0.520)	(-1.941)	(-4.184)	

Panel C: Volatility group 3 (high)

	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	0.039	0.372***	0.609***	1.180***	2.013***	1.974***
	(0.232)	(2.629)	(4.006)	(6.119)	(7.959)	(6.474)
TILLIQD	1.586***	1.105***	0.676***	0.508***	0.356***	-1.230***
	(6.534)	(6.740)	(4.359)	(3.707)	(2.694)	(-4.453)
$TILLIQD_j$-$ILLIQ_j$	1.546***	0.733***	0.067	-0.672***	-1.658***	
	(5.216)	(3.382)	(0.310)	(-2.841)	(-5.810)	

Table 9: Four-factor model for portfolios formed on size and illiquidity

This table provides the regression results of the Carhart (1997) four-factor model referring to Equation (23). In each month m, stocks are ranked by their market capitalization at the end of month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level ILLIQ of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-01/2013. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
Panel A: Size group 1 (small)						
1 (low ILLIQ)	1.073***	1.171***	1.126***	0.182***	-0.314***	0.871
	(6.550)	(28.903)	(21.737)	(3.255)	(-9.178)	
2	0.801***	0.982***	1.019***	0.353***	-0.321***	0.863
	(5.490)	(27.240)	(22.111)	(7.082)	(-10.542)	
3	1.111***	0.934***	0.911***	0.406***	-0.323***	0.827
	(7.111)	(24.180)	(18.451)	(7.602)	(-9.882)	
4	1.233***	0.820***	0.863***	0.394***	-0.308***	0.772
	(7.318)	(19.693)	(16.221)	(6.85)	(-8.742)	
5 (high ILLIQ)	1.933***	0.676***	0.823***	0.300***	-0.334***	0.615
	(8.710)	(12.322)	(11.738)	(3.956)	(-7.199)	
Panel B: Size group 2						
Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (low ILLIQ)	0.422***	1.132***	0.810***	0.072**	-0.091***	0.926
	(4.105)	(44.570)	(24.969)	(2.044)	(-4.236)	
2	0.306***	1.076***	0.714***	0.236***	-0.099***	0.927
	(3.296)	(46.821)	(24.323)	(7.420)	(-5.073)	
3	0.283***	1.043***	0.812***	0.328***	-0.140***	0.933
	(3.145)	(46.89)	(28.549)	(10.686)	(-7.419)	
4	0.154	0.990***	0.772***	0.419***	-0.115***	0.913
	(1.583)	(41.272)	(25.161)	(12.627)	(-5.666)	
5 (high ILLIQ)	-0.103	0.861***	0.711***	0.527***	-0.110***	0.883
	(-1.015)	(34.453)	(22.257)	(15.254)	(-5.182)	
Panel C: Size group 3 (big)						
Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (low ILLIQ)	-0.069	1.007***	0.096***	-0.024	0.038***	0.946
	(-1.110)	(65.142)	(4.865)	(-1.146)	(2.936)	
2	0.067	1.063***	0.124***	0.164***	-0.054***	0.922
	(0.838)	(53.681)	(4.91)	(6.007)	(-3.222)	
3	0.003	1.061***	0.321***	0.205***	-0.065***	0.921
	(0.037)	(50.715)	(11.981)	(7.076)	(-3.669)	
4	-0.005	1.042***	0.329***	0.287***	-0.052***	0.924
	(-0.067)	(52.418)	(12.937)	(10.431)	(-3.106)	
5 (high ILLIQ)	-0.158*	1.012***	0.300***	0.377***	-0.090***	0.914
	(-1.889)	(48.829)	(11.338)	(13.165)	(-5.114)	

Table 10: Four-factor model for portfolios formed on size and true illiquidity

This table provides the regression results of the Carhart (1997) four-factor model referring to Equation (23). In each month m, stocks are ranked by their market capitalization at the end of month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Size group 1 (small)

True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (truly liquid)	1.996***	1.130***	1.278***	0.034	-0.527***	0.792
	(8.310)	(19.032)	(16.841)	(0.415)	(-10.490)	
2	1.493***	1.023***	1.081***	0.323***	-0.374***	0.821
	(8.189)	(22.707)	(18.754)	(5.187)	(-9.824)	
3	1.116***	0.945***	0.927***	0.357***	-0.320***	0.836
	(7.272)	(24.919)	(19.118)	(6.815)	(-9.978)	
4	0.897***	0.819***	0.826***	0.483***	-0.222***	0.827
	(6.680)	(24.668)	(19.458)	(10.536)	(-7.892)	
5 (truly illiquid)	0.663***	0.665***	0.633***	0.435***	-0.159***	0.741
	(4.840)	(19.625)	(14.611)	(9.294)	(-5.562)	

Panel B: Size group 2

True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (truly liquid)	0.617***	1.174***	0.980***	-0.049	-0.144***	0.910
	(4.857)	(37.393)	(24.393)	(-1.122)	(-5.409)	
2	0.400***	1.070***	0.832***	0.248***	-0.106***	0.929
	(4.206)	(45.598)	(27.733)	(7.652)	(-5.362)	
3	0.122	1.031***	0.693***	0.383***	-0.114***	0.922
	(1.335)	(45.581)	(23.955)	(12.266)	(-5.944)	
4	0.131	0.973***	0.684***	0.466***	-0.104***	0.913
	(1.422)	(42.668)	(23.452)	(14.781)	(-5.383)	
5 (truly illiquid)	-0.207**	0.852***	0.632***	0.532***	-0.086***	0.866
	(-1.981)	(33.086)	(19.204)	(14.939)	(-3.955)	

Panel C: Size group 3 (big)

True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
1 (truly liquid)	0.156	1.130***	0.513***	-0.177***	0.015	0.897
	(1.367)	(39.947)	(14.169)	(-4.539)	(0.615)	
2	-0.021	1.035***	0.243***	0.226***	-0.035**	0.921
	(-0.263)	(52.545)	(9.653)	(8.314)	(-2.086)	
3	-0.061	1.007***	0.128***	0.315***	-0.051***	0.922
	(-0.808)	(54.202)	(5.372)	(12.254)	(-3.215)	
4	-0.067	0.992***	0.103***	0.330***	-0.090***	0.923
	(-0.894)	(53.897)	(4.398)	(12.994)	(-5.782)	
5 (truly illiquid)	-0.171**	1.021***	0.185***	0.314***	-0.062***	0.924
	(-2.232)	(53.906)	(7.643)	(12.007)	(-3.867)	

Table 11: Portfolio alphas and t-statistics within size groups

This table provides the t-statistics that test for a significant difference in the risk-adjusted portfolio returns across size groups j and across portfolio formations. At the beginning of each month m, stocks are ranked by their market value of equity at the end of month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ (ILLIQ) and by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) (TILLIQD). Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. The portfolio alphas are obtained as the estimated intercept of Equation (23). ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Size group 1 (small)

	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	1.073***	0.801***	1.111***	1.233***	1.933***	0.859***
	(6.550)	(5.490)	(7.111)	(7.318)	(8.710)	(3.115)
TILLIQD	1.996***	1.493***	1.116***	0.897***	0.663***	-1.333***
	(8.310)	(8.189)	(7.272)	(6.680)	(4.840)	(-4.820)
TILLIQD_j-ILLIQ_j	0.923***	0.693***	0.004	-0.335	-1.269***	
	(3.174)	(2.966)	(0.019)	(-1.557)	(-4.867)	

Panel B: Size group 2

	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	0.422***	0.306***	0.283***	0.154	-0.103	-0.524***
	(4.105)	(3.296)	(3.145)	(1.583)	(-1.015)	(-3.638)
TILLIQD	0.617***	0.400***	0.122	0.131	-0.207**	-0.824***
	(4.857)	(4.206)	(1.335)	(1.422)	(-1.981)	(-5.012)
TILLIQD_j-ILLIQ_j	0.196	0.093	-0.161	-0.022	-0.104	
	(1.197)	(0.700)	(-1.253)	(-0.168)	(-0.715)	

Panel C: Size group 3 (big)

	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	-0.069	0.067	0.003	-0.005	-0.158*	-0.089
	(-1.110)	(0.838)	(0.037)	(-0.067)	(-1.889)	(-0.850)
TILLIQD	0.156	-0.021	-0.061	-0.067	-0.171**	-0.328***
	(1.367)	(-0.263)	(-0.808)	(-0.894)	(-2.232)	(-2.377)
TILLIQD_j-ILLIQ_j	0.226*	-0.088	-0.064	-0.061	-0.013	
	(1.732)	(-0.780)	(-0.564)	(-0.558)	(-0.111)	

Table 12: Four-factor alphas for portfolios formed on volatility, size and true illiquidity

This table provides the estimated intercepts α from regression based on the Carhart (1997) four-factor model referring to Equation (23). In each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their market value of equity at the end of month $m-1$ and sorted into five portfolios. Within these fifteen portfolios, stocks are ranked by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Volatility group 1 (low)

	True illiquidity group					
Size group	1 (low)	2	3	4	5 (high)	High-low
1 (small)	0.687***	0.636***	0.368***	0.458***	0.367**	-0.320
	(4.400)	(4.113)	(2.630)	(3.001)	(2.496)	(-1.490)
2	0.401***	0.366***	0.173	0.059	0.092	-0.309
	(2.775)	(2.743)	(1.295)	(0.466)	(0.708)	(-1.592)
3	0.326**	0.046	0.038	0.049	-0.166	-0.492***
	(2.321)	(0.351)	(0.319)	(0.405)	(-1.331)	(-2.619)
4	0.047	-0.035	0.062	-0.023	0.041	-0.006
	(0.372)	(-0.278)	(0.541)	(-0.190)	(0.378)	(-0.036)
5 (big)	-0.036	0.115	-0.011	-0.074	-0.152	-0.116
	(-0.310)	(1.086)	(-0.102)	(-0.694)	(-1.379)	(-0.726)
Small-big	0.723***	0.521***	0.379**	0.532***	0.520***	
	(3.715)	(2.779)	(2.132)	(2.854)	(2.824)	

Panel B: Volatility group 2

	True illiquidity group					
Size group	1 (low)	2	3	4	5 (high)	High-low
1 (small)	2.139***	1.274***	0.815***	0.563***	0.764***	-1.375***
	(7.665)	(5.900)	(4.258)	(3.267)	(5.096)	(-4.340)
2	1.079***	0.718***	0.426***	0.386***	-0.110	-1.188***
	(5.002)	(4.129)	(2.634)	(2.538)	(-0.714)	(-4.489)
3	0.459**	0.317*	0.306*	0.174	-0.030	-0.489**
	(2.414)	(1.987)	(1.914)	(1.176)	(-0.205)	(-2.035)
4	0.375*	0.334*	-0.096	-0.007	-0.126	-0.501**
	(1.952)	(1.931)	(-0.610)	(-0.055)	(-0.936)	(-2.136)
5 (big)	0.361**	-0.102	0.236*	-0.045	-0.128	-0.489**
	(2.071)	(-0.685)	(1.793)	(-0.338)	(-1.078)	(-2.319)
Small-big	1.778***	1.376***	0.579**	0.608***	0.892***	
	(5.404)	(5.246)	(2.492)	(2.796)	(4.665)	

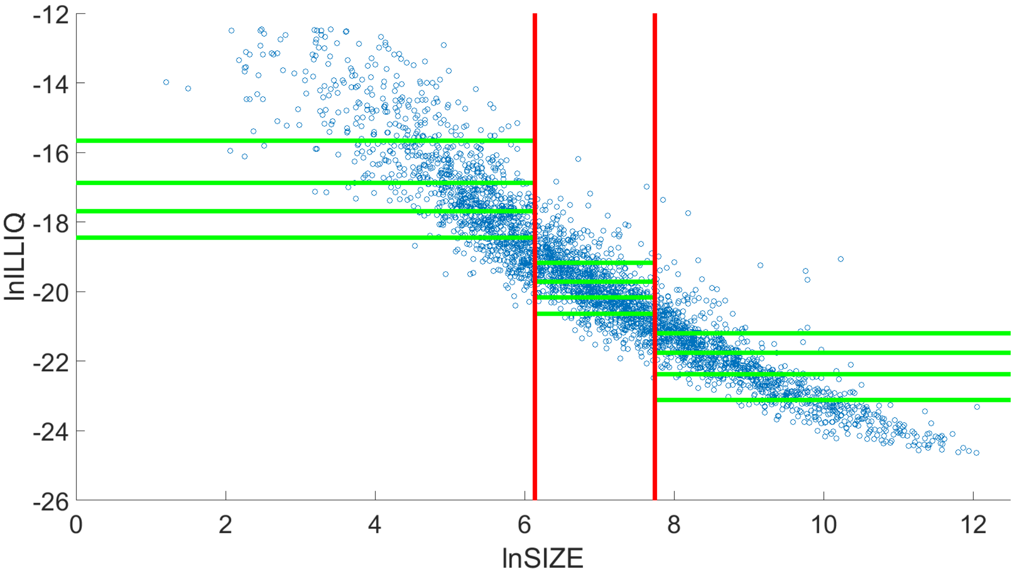
Table 13: Four-factor alphas for portfolios formed on volatility, size and true illiquidity (cont.)

This table provides the estimated intercepts α from regression based on the Carhart (1997) four-factor model referring to Equation (23). In each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their market value of equity at the end of month $m-1$ and sorted into five portfolios. Within these fifteen portfolios, stocks are ranked by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel C: Volatility group 3 (high)

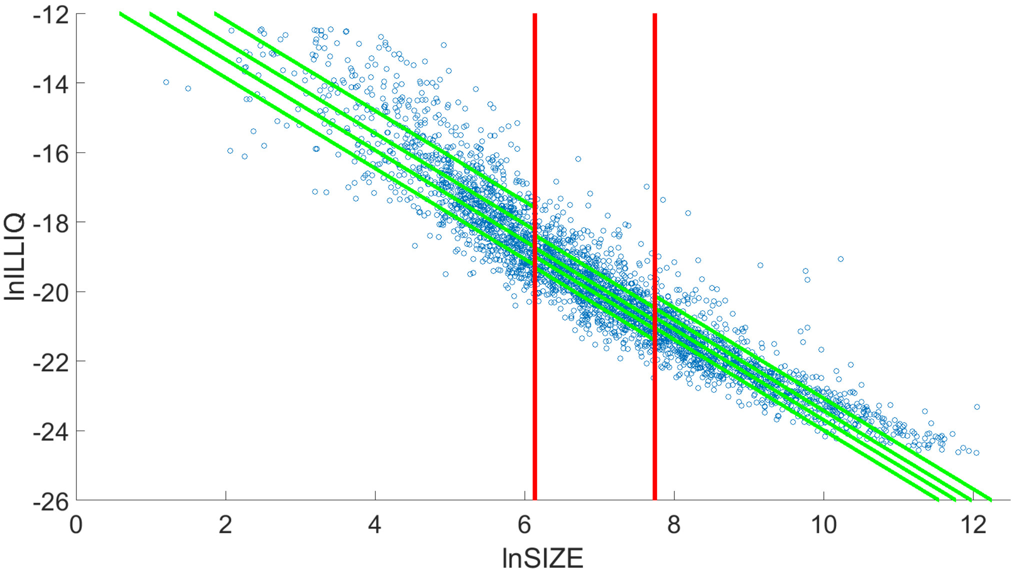
	True illiquidity group					
Size group	1 (low)	2	3	4	5 (high)	High-low
1 (small)	3.465***	3.569***	2.562***	2.154***	1.483***	-1.982***
	(6.293)	(7.464)	(6.356)	(6.466)	(4.751)	(-3.131)
2	1.891***	1.577***	1.241***	0.632**	0.404*	-1.487***
	(5.256)	(5.265)	(4.623)	(2.514)	(1.828)	(-3.523)
3	1.252***	0.758***	0.317	0.325	-0.044	-1.297***
	(3.954)	(3.027)	(1.392)	(1.617)	(-0.216)	(-3.441)
4	0.599**	0.892***	0.165	-0.330*	-0.704***	-1.303***
	(2.348)	(3.672)	(0.745)	(-1.811)	(-3.678)	(-4.086)
5 (big)	0.069	-0.038	-0.113	-0.305	-0.430**	-0.498
	(0.257)	(-0.155)	(-0.474)	(-1.479)	(-2.257)	(-1.518)
Small-big	3.396***	3.607***	2.674***	2.459***	1.913***	
	(5.548)	(6.703)	(5.717)	(6.276)	(5.232)	

Figure 1: Sorting of lnSIZE and lnILLIQ



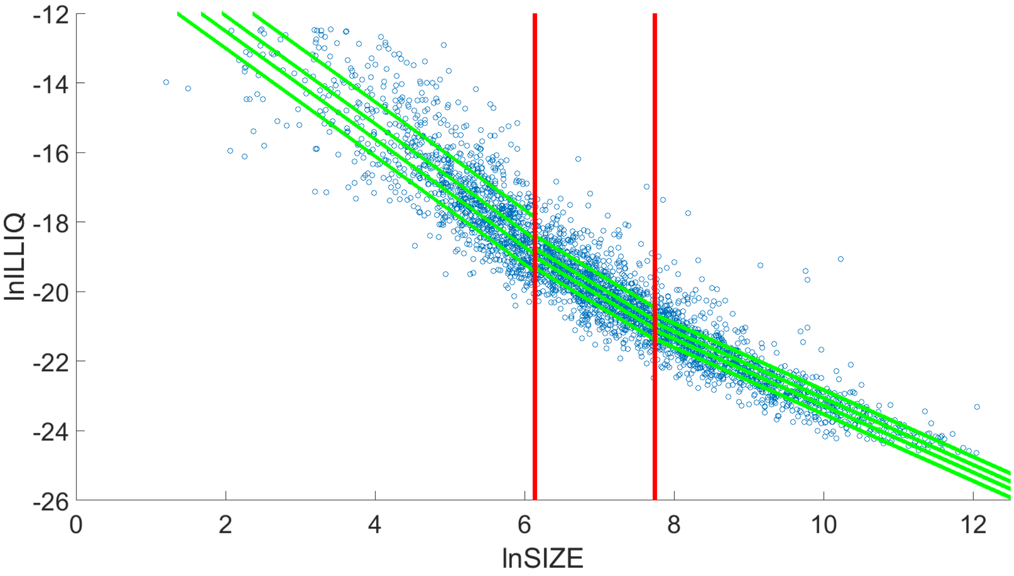
This figure plots the scatter of the log-transformed market value of equitiy, lnSIZE, and the log-transformed Amihud (2002) illiqiudity measure, lnILLIQ, for September 2015. The formation of tercile portfolios is marked by vertical red lines whereas the sorting into quintile portfolios in the second step is determined of using lnILLIQ as a sorting criteria (marked by horizontal green lines). Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution.

Figure 2: Sorting of lnSIZE and TILLIQ



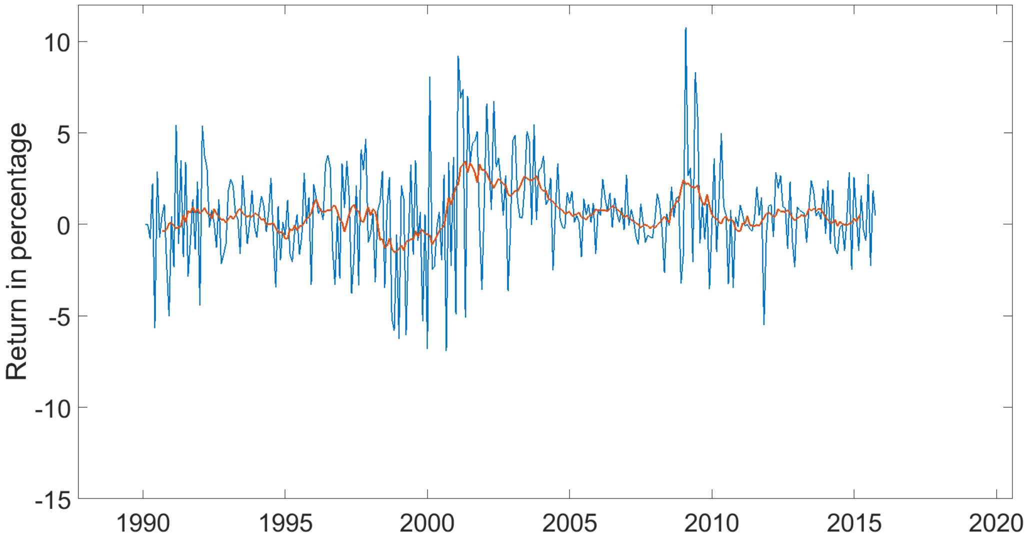
This figure plots the scatter of the log-transformed market value of equitiy, lnSIZE, and the log-transformed Amihud (2002) illiqiudity measure, lnILLIQ, for September 2015. The formation of tercile portfolios is marked by vertical red lines whereas the sorting into quintile portfolios in the second step is determined of using TILLIQ as a sorting criteria (marked by green lines). Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution.

Figure 3: Sorting of lnSIZE and TILLIQD



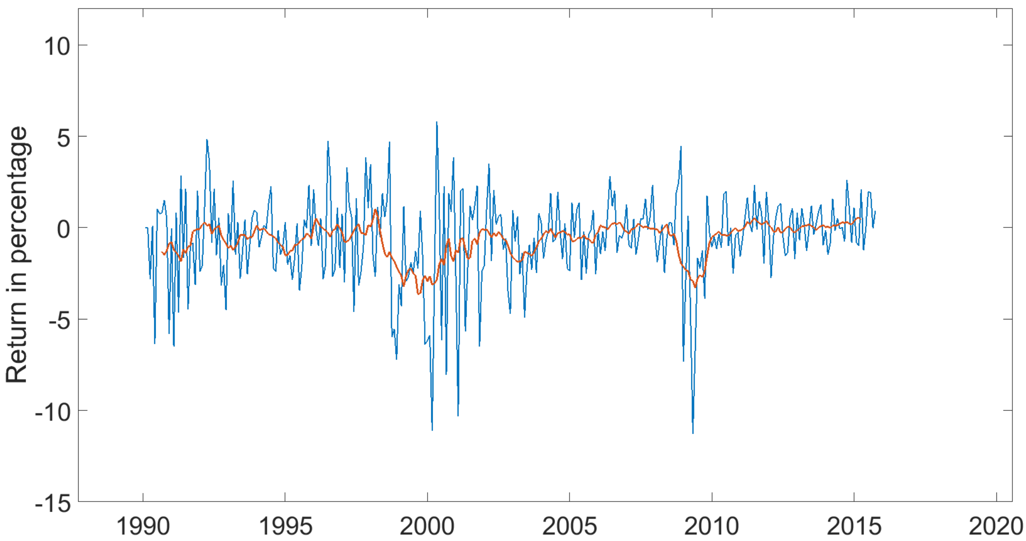
This figure plots the scatter of the log-transformed market value of equitiy, lnSIZE, and the log-transformed Amihud (2002) illiqiudity measure, lnILLIQ, for September 2015. The formation of tercile portfolios is marked by vertical red lines whereas the sorting into quintile portfolios in the second step is determined of using TILLIQD as a sorting criteria (marked by green lines). Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution.

Figure 4: Illiquidity return premium factor IML over time



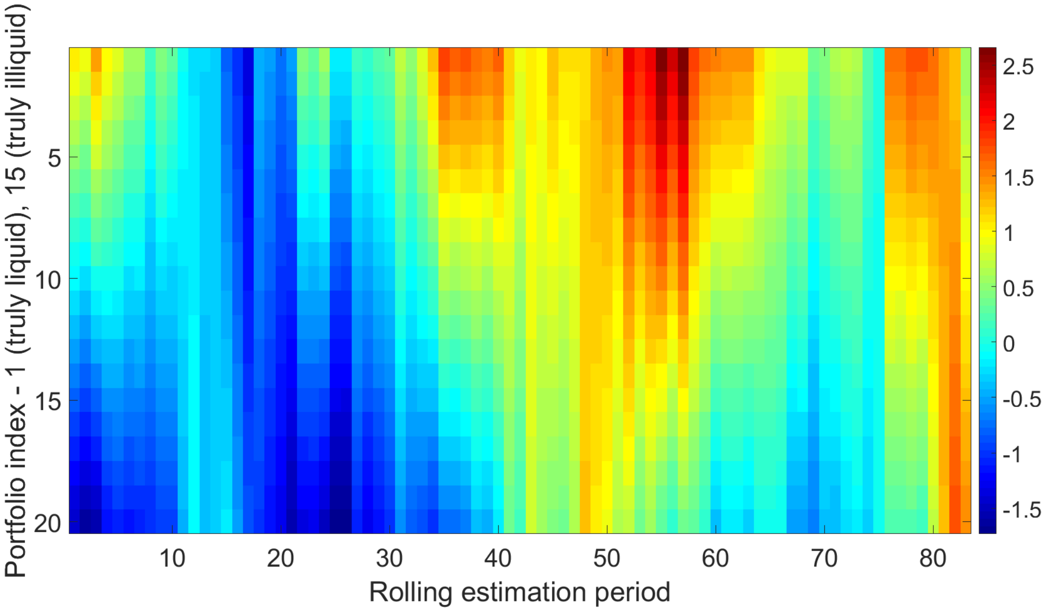
This figure plots the illiquidity return premium IML and its 12-month moving average over the period of 02/1990 to 09/2015. At the beginning of each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ where illiquidity is calculated by the Amihud (2002) illiquidity measure of Equation (1). Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. IML is calculated as the average of monthly returns of the most illiquid and liquid portfolios across all three volatility groups. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution.

Figure 5: Illiquidity return premium factor IMLORTH over time



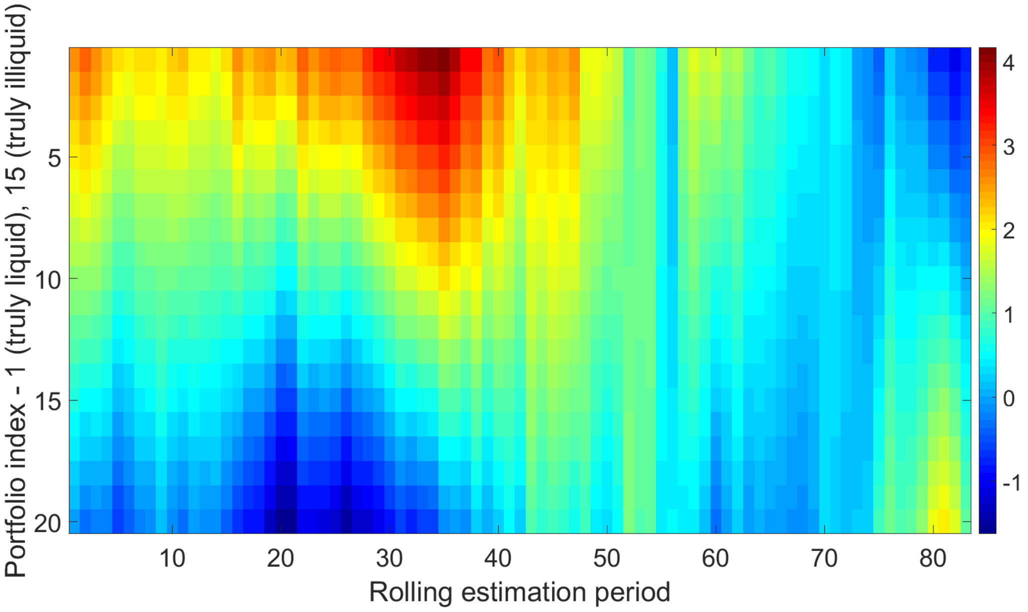
This figure plots the illiquidity return premium IMLORTH and its 12-month moving average over the period of 02/1990 to 09/2015. At the beginning of each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their orthogonalized illiquidity obtained as the estimated residual term of Equation (9) and (10). Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. IMLORTH is calculated as the average of monthly returns of the most illiquid and liquid portfolios across all three volatility groups. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution.

Figure 6: Estimated trend in risk-adjusted portfolio excess returns across true illiquidity portfolios - volatility group 1



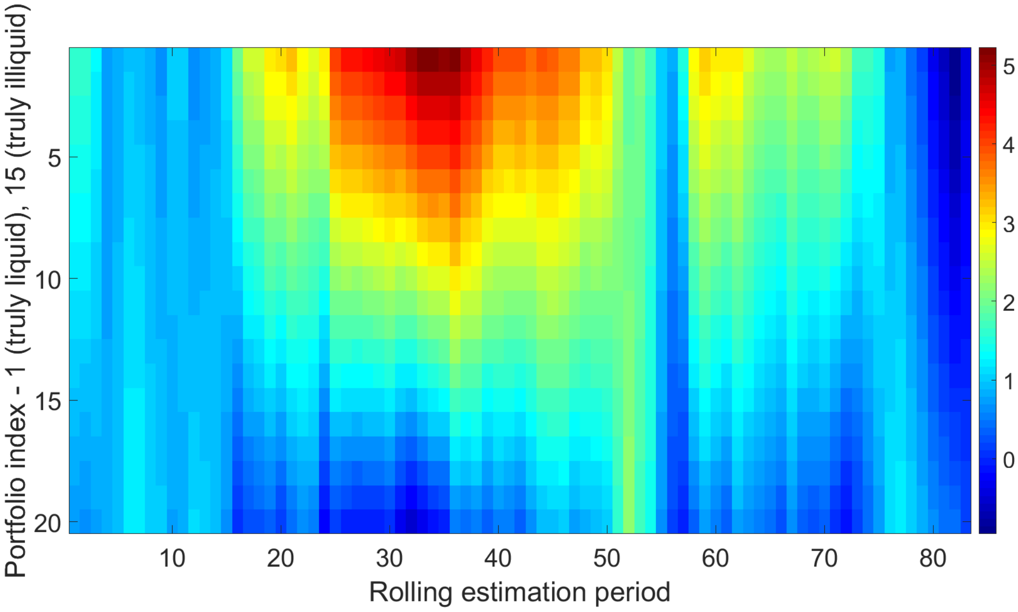
This figure plots the trend in estimated risk-adjusted portfolio excess returns of the illiquidity-augmented four-factor model across true illiquidity portfolios using a rolling sample window of 60 months. The first estimation period is from February 1990 to January 1995 followed by the second estimation period from May 1990 to April 1995 and so on. This methodology results in 83 estimations for each portfolio. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution.

Figure 7: Estimated trend in risk-adjusted portfolio excess returns across true illiquidity portfolios - volatility group 2



This figure plots the trend in estimated risk-adjusted portfolio excess returns of the illiquidity-augmented four-factor model across true illiquidity portfolios using a rolling sample window of 60 months. The first estimation period is from February 1990 to January 1995 followed by the second estimation period from May 1990 to April 1995 and so on. This methodology results in 83 estimations for each portfolio. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution.

Figure 8: Estimated trend in risk-adjusted portfolio excess returns across true illiquidity portfolios - volatility group 3



This figure plots the trend in estimated risk-adjusted portfolio excess returns of the illiquidity-augmented four-factor model across true illiquidity portfolios using a rolling sample window of 60 months. The first estimation period is from February 1990 to January 1995 followed by the second estimation period from May 1990 to April 1995 and so on. This methodology results in 83 estimations for each portfolio. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution.

Appendix

Table A.1: Average values of monthly return, size and illiquidity in the 15 portfolios formed on size and illiquidity

This table provides statistics on the average portfolio return, average market capitalization and average illiquidity of portfolios formed on size and illiquidity. In each month m, stocks are ranked by their market value of equity at the end of month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: average return (in %)

	Illiquidity group				
Size group	1 (low ILLIQ)	2	3	4	5 (high ILLIQ)
1 (small)	1.826***	1.447***	1.719***	1.769***	2.340***
	(4.142)	(3.795)	(4.727)	(5.180)	(6.766)
2	1.213***	1.074***	1.039***	0.902***	0.580**
	(3.330)	(3.225)	(3.091)	(2.833)	(2.028)
3 (big)	0.586**	0.741***	0.710**	0.714**	0.531*
	(2.260)	(2.668)	(2.443)	(2.526)	(1.919)

Panel B: average firm size (in $ million)

	Illiquidity group				
Size group	1 (low ILLIQ)	2	3	4	5 (high ILLIQ)
1 (small)	204.047	160.988	130.462	98.563	57.755
2	890.164	720.98	593.965	501.91	429.109
3 (big)	21811.969	7216.621	3683.124	2502.661	2184.641

Panel C: average illiquidity (ILLIQ)

	Illiquidity group				
Size group	1 (low ILLIQ)	2	3	4	5 (high ILLIQ)
1 (small)	$2.003 \cdot 10^{-6}$	$7.180 \cdot 10^{-6}$	$2.143 \cdot 10^{-5}$	$2.184 \cdot 10^{-5}$	$6.311 \cdot 10^{-5}$
2	$1.574 \cdot 10^{-7}$	$2.689 \cdot 10^{-7}$	$4.892 \cdot 10^{-7}$	$9.466 \cdot 10^{-7}$	$4.681 \cdot 10^{-6}$
3 (big)	$8.643 \cdot 10^{-9}$	$2.489 \cdot 10^{-8}$	$4.275 \cdot 10^{-8}$	$9.621 \cdot 10^{-8}$	$4.478 \cdot 10^{-7}$

Table A.2: Average values of monthly return, size and illiquidity in the 15 portfolios formed on size and true illiquidity

This table provides statistics on the average portfolio return, average market capitalization and average illiquidity of portfolios formed on size and true illiquidity. In each month m, stocks are ranked by their market value of equity at the end of month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: average return (in %)

	True illiquidity group				
Size group	1 (truly liquid)	2	3	4	5 (truly illiquid)
1 (small)	2.592***	2.138***	1.725***	1.496***	1.164***
	(5.093)	(5.134)	(4.705)	(4.784)	(4.469)
2	1.409***	1.181***	0.877***	0.870***	0.472*
	(3.439)	(3.421)	(2.757)	(2.871)	(1.715)
3 (big)	0.915***	0.679**	0.609**	0.569**	0.510*
	(2.650)	(2.470)	(2.338)	(2.192)	(1.903)

Panel B: average firm size (in $ million)

	True illiquidity group				
Size group	1 (truly liquid)	2	3	4	5 (truly illiquid)
1 (small)	133.006	139.546	134.18	128.085	116.452
2	595.316	644.703	660.746	640.701	592.884
3 (big)	5081.113	6412.175	7874.744	9263.826	8691.966

Panel C: average illiquidity (ILLIQ)

	True illiquidity group				
Size group	1 (truly liquid)	2	3	4	5 (truly illiquid)
1 (small)	$1.029 \cdot 10^{-5}$	$1.625 \cdot 10^{-5}$	$1.752 \cdot 10^{-5}$	$3.573 \cdot 10^{-5}$	$3.605 \cdot 10^{-5}$
2	$2.496 \cdot 10^{-7}$	$3.662 \cdot 10^{-7}$	$5.373 \cdot 10^{-7}$	$9.939 \cdot 10^{-7}$	$4.399 \cdot 10^{-6}$
3 (big)	$3.475 \cdot 10^{-8}$	$5.016 \cdot 10^{-8}$	$5.315 \cdot 10^{-8}$	$6.681 \cdot 10^{-8}$	$4.176 \cdot 10^{-7}$

Table A.3: The illiquidity return premium using return-weighted and value-weighted returns

This table provides statistics on the variables used for the Carhart (1997) four-factor model and provides the regression results of Equation (22) using both IML and IMLORTH as a dependent variable. At the beginning of each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ (IML) and by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) (IMLORTH). Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Descriptive statistics

Variable	Mean	S.d.	Median	Skewness	Minimum	Maximum
IML_{EW}	0.589	2.635	0.570	0.185	-6.897	10.752
IML_{RW}	0.589	2.635	0.570	0.185	-6.897	10.752
IML_{VW}	0.589	2.635	0.570	0.185	-6.897	10.752
$IMLORTH_{EW}$	-0.635	2.483	-0.344	-1.031	-11.264	5.781
$IMLORTH_{RW}$	-0.635	2.483	-0.344	-1.031	-11.264	5.781
$IMLORTH_{VW}$	-0.635	2.483	-0.344	-1.031	-11.264	5.781

Panel B: Estimation results

Variable	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
IML_{EW}	0.758***	-0.338***	0.276***	0.243***	-0.088***	0.412
	(6.360)	(-11.458)	(7.327)	(5.965)	(-3.535)	
IML_{RW}	0.758***	-0.338***	0.276***	0.243***	-0.088***	0.412
	(6.360)	(-11.458)	(7.327)	(5.965)	(-3.535)	
IML_{VW}	0.758***	-0.338***	0.276***	0.243***	-0.088***	0.412
	(6.360)	(-11.458)	(7.327)	(5.965)	(-3.535)	
$IMLORTH_{EW}$	-0.576***	-0.215***	-0.229***	0.232***	0.112***	0.543
	(-5.818)	(-8.807)	(-7.320)	(6.854)	(5.403)	
$IMLORTH_{RW}$	-0.576***	-0.215***	-0.229***	0.232***	0.112***	0.543
	(-5.818)	(-8.807)	(-7.320)	(6.854)	(5.403)	
$IMLORTH_{VW}$	-0.576***	-0.215***	-0.229***	0.232***	0.112***	0.543
	(-5.818)	(-8.807)	(-7.320)	(6.854)	(5.403)	

Table A.4: The illiquidity return premium - robustness check

This table provides the regression results of Equation (22) using both IML and IMLORTH as a dependent variable. At the beginning of each month m, stocks are ranked by their standard deviation of daily returns of month $m-3$ to $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level ILLIQ of month $m-3$ to $m-1$ (IML) and by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) (IMLORTH). Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month $m+1$. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 06/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Descriptive statistics

Variable	Mean	S.d.	Median	Skewness	Minimum	Maximum
IML	0.699	2.645	0.631	0.154	-6.803	8.911
IMLORTH	-0.150	2.076	-0.004	-1.038	-12.019	5.163
RM	0.616	4.319	1.190	-0.658	-17.230	11.350
SMB	0.169	3.303	0.070	0.814	-16.700	22.320
HML	0.188	3.123	0.010	0.093	-13.110	13.910
MOM	0.613	4.930	0.700	-1.584	-34.580	18.380

Panel B: Correlations

Variable	IML	IMLORTH	RM	SMB	HML	MOM
IML	1					
IMLORTH	0.475	1				
RM	-0.590	-0.585	1			
SMB	0.057	-0.443	0.250	1		
HML	0.363	0.451	-0.246	-0.332	1	
MOM	0.075	0.318	-0.253	0.054	-0.154	1

Panel C: Estimation results

Variable	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
IML	0.841***	-0.372***	0.252***	0.266***	-0.025	0.478
	(7.384)	(-13.182)	(7.004)	(6.827)	(-1.054)	
IMLORTH	-0.136*	-0.178***	-0.164***	0.213***	0.122***	0.560
	(-1.652)	(-8.767)	(-6.317)	(7.597)	(7.114)	

Table A.5: Illiquidity-augmented four-factor model for portfolios formed on volatility and illiquidity

This table provides the regression results of the five-factor model referring to Equation (24). In each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level ILLIQ of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Volatility group 1 (low)

Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (low ILLIQ)	0.015	0.830***	-0.165***	0.364***	0.005	-0.014	0.841
	(0.155)	(31.722)	(-5.456)	(11.404)	(0.262)	(-0.319)	
2	0.028	0.839***	0.061*	0.446***	0.017	0.011	0.821
	(0.266)	(29.143)	(1.816)	(12.707)	(0.798)	(0.226)	
3	0.099	0.799***	0.250***	0.463***	0.007	0.042	0.827
	(0.976)	(28.287)	(7.642)	(13.427)	(0.336)	(0.916)	
4	0.070	0.791***	0.425***	0.457***	-0.012	0.149***	0.849
	(0.710)	(29.030)	(13.479)	(13.763)	(-0.602)	(3.357)	
5 (high ILLIQ)	-0.020	0.787***	0.277***	0.287***	-0.000	0.585***	0.845
	(-0.228)	(31.737)	(9.650)	(9.507)	(-0.013)	(14.474)	

Panel B: Volatility group 2

Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (low ILLIQ)	0.357***	1.080***	0.274***	0.329***	-0.091***	-0.291***	0.899
	(3.233)	(35.180)	(7.713)	(8.779)	(-4.102)	(-5.819)	
2	0.433***	1.046***	0.565***	0.427***	-0.131***	-0.244***	0.902
	(3.837)	(33.339)	(15.536)	(11.163)	(-5.810)	(-4.759)	
3	0.420***	1.030***	0.689***	0.429***	-0.112***	-0.078	0.909
	(3.915)	(34.586)	(19.979)	(11.799)	(-5.224)	(-1.608)	
4	0.269**	1.050***	0.687***	0.408***	-0.112***	0.325***	0.901
	(2.471)	(34.717)	(19.609)	(11.053)	(-5.156)	(6.579)	
5 (high ILLIQ)	0.210**	0.997***	0.401***	0.282***	-0.120***	0.750***	0.907
	(2.318)	(39.628)	(13.763)	(9.187)	(-6.599)	(18.279)	

Panel C: Volatility group 3 (high)

Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (low ILLIQ)	0.623***	1.232***	1.168***	-0.219***	-0.293***	-0.770***	0.928
	(4.075)	(28.974)	(23.719)	(-4.216)	(-9.564)	(-11.107)	
2	0.550***	1.301***	1.340***	-0.077	-0.303***	-0.234***	0.930
	(3.710)	(31.601)	(28.091)	(-1.524)	(-10.227)	(-3.480)	
3	0.378**	1.366***	1.329***	-0.004	-0.315***	0.305***	0.917
	(2.400)	(31.207)	(26.217)	(-0.071)	(-9.994)	(4.271)	
4	0.541***	1.367***	0.999***	0.007	-0.391***	0.843***	0.881
	(3.082)	(28.009)	(17.666)	(0.125)	(-11.128)	(10.585)	
5 (high ILLIQ)	0.808***	1.358***	0.599***	-0.096	-0.259***	1.590***	0.847
	(4.521)	(27.321)	(10.401)	(-1.585)	(-7.229)	(19.613)	

Table A.6: Illiquidity-augmented four-factor model for portfolios formed on volatility and true illiquidity

This table provides the regression results of the five-factor model referring to Equation (24). In each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Volatility group 1 (low)							
True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (truly liquid)	0.246**	0.818***	0.283***	0.389***	-0.017	0.182***	0.833
	(2.445)	(29.283)	(8.728)	(11.417)	(-0.864)	(3.991)	
2	0.065	0.836***	0.137***	0.447***	0.006	0.094**	0.824
	(0.633)	(29.402)	(4.175)	(12.887)	(0.306)	(2.032)	
3	0.039	0.808***	0.103***	0.435***	0.004	0.071*	0.837
	(0.419)	(30.886)	(3.404)	(13.647)	(0.202)	(1.675)	
4	-0.025	0.797***	0.143***	0.427***	0.005	0.124***	0.838
	(-0.271)	(31.034)	(4.823)	(13.639)	(0.264)	(2.971)	
5 (truly illiquid)	-0.132	0.787***	0.183***	0.319***	0.018	0.304***	0.843
	(-1.527)	(32.641)	(6.557)	(10.837)	(1.053)	(7.740)	
Panel B: Volatility group 2							
True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (truly liquid)	0.865***	1.161***	0.749***	0.110**	-0.148***	0.087	0.874
	(5.938)	(28.675)	(15.959)	(2.232)	(-5.067)	(1.313)	
2	0.408***	1.086***	0.526***	0.372***	-0.132***	0.032	0.896
	(3.594)	(34.397)	(14.376)	(9.655)	(-5.823)	(0.626)	
3	0.274***	1.059***	0.488***	0.460***	-0.119***	0.022	0.918
	(2.856)	(39.772)	(15.816)	(14.149)	(-6.192)	(0.500)	
4	0.161*	1.020***	0.438***	0.475***	-0.112***	0.103**	0.908
	(1.675)	(38.175)	(14.148)	(14.582)	(-5.830)	(2.373)	
5 (truly illiquid)	-0.017	0.879***	0.418***	0.456***	-0.055***	0.223***	0.898
	(-0.201)	(36.740)	(15.081)	(15.615)	(-3.210)	(5.726)	
Panel C: Volatility group 3 (high)							
True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (truly liquid)	1.332***	1.503***	1.326***	-0.371***	-0.537***	0.335***	0.848
	(5.216)	(21.172)	(16.135)	(-4.287)	(-10.498)	(2.893)	
2	1.024***	1.356***	1.313***	-0.157***	-0.348***	0.107	0.906
	(5.872)	(27.985)	(23.397)	(-2.653)	(-9.974)	(1.357)	
3	0.504***	1.344***	1.159***	-0.053	-0.291***	0.228***	0.904
	(3.092)	(29.663)	(22.096)	(-0.957)	(-8.918)	(3.081)	
4	0.191	1.273***	0.953***	0.055	-0.219***	0.418***	0.910
	(1.402)	(33.653)	(21.743)	(1.188)	(-8.018)	(6.777)	
5 (truly illiquid)	-0.139	1.149***	0.682***	0.138***	-0.167***	0.653***	0.908
	(-1.225)	(36.346)	(18.614)	(3.566)	(-7.33)	(12.662)	

Table A.7: Portfolio alphas and t-statistics within volatility groups - illiquidity-augmented four-factor model

This table provides the t-statistics that test for a significant difference in the risk-adjusted portfolio returns across illiquidity groups j and across portfolio formations. At the beginning of each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ (ILLIQ) and by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) (TILLIQD). Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. The portfolio alphas are obtained as the estimated intercept of Equation (23). ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Volatility group 1 (low)

	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	0.015	0.028	0.099	0.070	-0.020	-0.035
	(0.155)	(0.266)	(0.976)	(0.710)	(-0.228)	(-0.27)
TILLIQD	0.246***	0.065	0.039	-0.025	-0.132	-0.378***
	(2.445)	(0.633)	(0.419)	(-0.271)	(-1.527)	(-2.849)
$TILLIQD_j$-$ILLIQ_j$	0.231*	0.037	-0.060	-0.095	-0.112	
	(1.679)	(0.256)	(-0.432)	(-0.702)	(-0.901)	

Panel B: Volatility group 2

	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	0.357***	0.433***	0.420***	0.269**	0.210**	-0.147
	(3.233)	(3.837)	(3.915)	(2.471)	(2.318)	(-1.031)
TILLIQD	0.865***	0.408***	0.274***	0.161*	-0.017	-0.882***
	(5.938)	(3.594)	(2.856)	(1.675)	(-0.201)	(-5.214)
$TILLIQD_j$-$ILLIQ_j$	0.508***	-0.025	-0.146	-0.108	-0.227*	
	(2.779)	(-0.155)	(-1.016)	(-0.744)	(-1.818)	

Panel C: Volatility group 3 (high)

	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	0.623***	0.550***	0.378**	0.541***	0.808***	0.185
	(4.075)	(3.710)	(2.400)	(3.082)	(4.521)	(0.787)
TILLIQD	1.332***	1.024***	0.504***	0.191	-0.139	-1.471***
	(5.216)	(5.872)	(3.092)	(1.402)	(-1.225)	(-5.263)
$TILLIQD_j$-$ILLIQ_j$	0.709**	0.474**	0.126	-0.350	-0.948***	
	(2.381)	(2.073)	(0.556)	(-1.577)	(-4.472)	

Table A.8: Illiquidity-augmented four-factor model for portfolios formed on size and illiquidity

This table provides the regression results of the five-factor model referring to Equation (24). In each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-01/2013. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Size group 1 (small)							
Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (low ILLIQ)	0.837***	1.276***	1.039***	0.106*	-0.287***	0.312***	0.877
	(4.916)	(26.96)	(18.962)	(1.844)	(-8.413)	(4.042)	
2	0.338**	1.188***	0.851***	0.205***	-0.268***	0.610***	0.897
	(2.507)	(31.686)	(19.588)	(4.475)	(-9.908)	(9.980)	
3	0.476***	1.217***	0.680***	0.202***	-0.249***	0.838***	0.898
	(3.712)	(34.156)	(16.470)	(4.652)	(-9.698)	(14.429)	
4	0.449***	1.169***	0.578***	0.143***	-0.217***	1.033***	0.894
	(3.665)	(34.314)	(14.653)	(3.443)	(-8.831)	(18.609)	
5 (high ILLIQ)	0.891***	1.139***	0.444***	-0.034	-0.213***	1.374***	0.824
	(5.579)	(25.653)	(8.629)	(-0.622)	(-6.653)	(18.963)	
Panel B: Size group 2							
Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (low ILLIQ)	0.636***	1.036***	0.889***	0.140***	-0.116***	-0.283***	0.933
	(6.148)	(36.009)	(26.662)	(4.002)	(-5.590)	(-6.032)	
2	0.446***	1.013***	0.765***	0.280***	-0.115***	-0.185***	0.931
	(4.633)	(37.839)	(24.671)	(8.583)	(-5.950)	(-4.225)	
3	0.322***	1.025***	0.826***	0.341***	-0.144***	-0.051	0.933
	(3.359)	(38.525)	(26.779)	(10.495)	(-7.509)	(-1.174)	
4	0.113	1.008***	0.757***	0.406***	-0.110***	0.054	0.913
	(1.095)	(35.103)	(22.752)	(11.578)	(-5.327)	(1.143)	
5 (high ILLIQ)	-0.231**	0.918***	0.665***	0.486***	-0.095***	0.169***	0.887
	(-2.183)	(31.245)	(19.523)	(13.554)	(-4.47)	(3.528)	
Panel C: Size group 3 (big)							
Illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (low ILLIQ)	0.035	0.961***	0.134***	0.009	0.026**	-0.138***	0.949
	(0.547)	(53.672)	(6.476)	(0.414)	(2.034)	(-4.729)	
2	0.245***	0.984***	0.189***	0.221***	-0.075***	-0.234***	0.931
	(3.052)	(44.181)	(7.322)	(8.143)	(-4.647)	(-6.444)	
3	0.176**	0.984***	0.383***	0.260***	-0.085***	-0.228***	0.928
	(2.057)	(41.395)	(13.921)	(8.963)	(-4.961)	(-5.878)	
4	0.135	0.980***	0.380***	0.332***	-0.069***	-0.185***	0.930
	(1.638)	(42.709)	(14.302)	(11.853)	(-4.148)	(-4.956)	
5 (high ILLIQ)	-0.083	0.978***	0.328***	0.401***	-0.098***	-0.100**	0.916
	(-0.936)	(39.746)	(11.501)	(13.364)	(-5.549)	(-2.483)	

Table A.9: Illiquidity-augmented four-factor model for portfolios formed on volatility and true illiquidity

This table provides the regression results of the five-factor model referring to Equation (24). In each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ and sorted into five portfolios. Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for ILLIQ_m or ILLIQTO_m is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-01/2013. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Size group 1 (small)

True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (truly liquid)	1.380***	1.405***	1.054***	-0.163**	-0.455***	0.813***	0.825
	(5.885)	(21.549)	(13.959)	(-2.056)	(-9.691)	(7.652)	
2	0.805***	1.33***	0.830***	0.103*	-0.294***	0.908***	0.883
	(5.141)	(30.567)	(16.467)	(1.931)	(-9.392)	(12.801)	
3	0.507***	1.216***	0.705***	0.162***	-0.249***	0.802***	0.900
	(3.965)	(34.184)	(17.119)	(3.740)	(-9.728)	(13.838)	
4	0.287***	1.09***	0.604***	0.288***	-0.151***	0.805***	0.915
	(2.863)	(39.119)	(18.696)	(8.467)	(-7.502)	(17.703)	
5 (truly illiquid)	0.022	0.950***	0.399***	0.230***	-0.085***	0.845***	0.881
	(0.227)	(34.489)	(12.518)	(6.838)	(-4.276)	(18.818)	

Panel B: Size group 2

True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (truly liquid)	0.801***	1.093***	1.046***	0.010	-0.165***	-0.243***	0.914
	(6.070)	(29.774)	(24.619)	(0.228)	(-6.242)	(-4.053)	
2	0.486***	1.032***	0.864***	0.276***	-0.116***	-0.114**	0.930
	(4.844)	(37.020)	(26.748)	(8.113)	(-5.799)	(-2.502)	
3	0.150	1.019***	0.703***	0.392***	-0.117***	-0.036	0.922
	(1.537)	(37.579)	(22.382)	(11.864)	(-5.987)	(-0.826)	
4	0.162*	0.959***	0.695***	0.476***	-0.107***	-0.040	0.913
	(1.647)	(35.112)	(21.956)	(14.271)	(-5.455)	(-0.908)	
5 (truly illiquid)	-0.311***	0.899***	0.594***	0.498***	-0.074***	0.138***	0.869
	(-2.833)	(29.450)	(16.814)	(13.386)	(-3.366)	(2.770)	

Panel C: Size group 3 (big)

True illiquidity group	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{IML}	Adj. R^2
1 (truly liquid)	0.430***	1.009***	0.612***	-0.090**	-0.017	-0.361***	0.911
	(3.798)	(32.055)	(16.793)	(-2.343)	(-0.752)	(-7.028)	
2	0.125	0.970***	0.296***	0.273***	-0.052***	-0.192***	0.928
	(1.533)	(42.859)	(11.296)	(9.889)	(-3.169)	(-5.211)	
3	0.080	0.945***	0.179***	0.360***	-0.067***	-0.185***	0.929
	(1.042)	(44.327)	(7.244)	(13.839)	(-4.355)	(-5.337)	
4	0.005	0.960***	0.130***	0.353***	-0.098***	-0.095***	0.924
	(0.070)	(43.981)	(5.130)	(13.281)	(-6.256)	(-2.669)	
5 (truly illiquid)	-0.132	1.004***	0.199***	0.327***	-0.067***	-0.052	0.924
	(-1.616)	(44.302)	(7.600)	(11.831)	(-4.076)	(-1.406)	

Table A.10: Portfolio alphas and t-statistics within size groups - illiquidity-augmented four-factor model

This table provides the t-statistics that test for a significant difference in the risk-adjusted portfolio returns across size groups j and across portfolio formations. At the beginning of each month m, stocks are ranked by their market value of equity at the end of month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their average illiquidity level of month $m-1$ (ILLIQ) and by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) (TILLIQD). Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. The portfolio alphas are obtained as the estimated intercept of Equation (23). ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Size group 1 (small)						
	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	0.837***	0.338**	0.476***	0.449***	0.891***	0.055
	(4.916)	(2.507)	(3.712)	(3.665)	(5.579)	(0.233)
TILLIQD	1.380***	0.805***	0.507***	0.287***	0.022	-1.357***
	(5.885)	(5.141)	(3.965)	(2.863)	(0.227)	(-5.333)
TILLIQD$_j$-ILLIQ$_j$	0.543*	0.467**	0.031	-0.162	-0.869***	
	(1.874)	(2.258)	(0.173)	(-1.023)	(-4.622)	
Panel B: Size group 2						
	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	0.636***	0.446***	0.322***	0.113	-0.231**	-0.867***
	(6.148)	(4.633)	(3.359)	(1.095)	(-2.183)	(-5.861)
TILLIQD	0.801***	0.486***	0.150	0.162*	-0.311***	-1.112***
	(6.070)	(4.844)	(1.537)	(1.647)	(-2.833)	(-6.478)
TILLIQD$_j$-ILLIQ$_j$	0.165	0.039	-0.172	0.049	-0.080	
	(0.983)	(0.283)	(-1.257)	(0.342)	(-0.526)	
Panel C: Size group 3 (big)						
	Portfolio j					
Sorting criteria	1 (low)	2	3	4	5 (high)	high-low
ILLIQ	0.035	0.245***	0.176**	0.135	-0.083	-0.118
	(0.547)	(3.052)	(2.057)	(1.638)	(-0.936)	(-1.079)
TILLIQD	0.430***	0.125	0.080	0.005	-0.132	-0.562***
	(3.798)	(1.533)	(1.042)	(0.07)	(-1.616)	(-4.026)
TILLIQD$_j$-ILLIQ$_j$	0.395***	-0.120	-0.096	-0.130	-0.049	
	(3.031)	(-1.048)	(-0.837)	(-1.139)	(-0.406)	

Table A.11: The illiquidity return premium IMLTO

This table provides statistics on the variables used for the Carhart (1997) four-factor model and provides the regression results of Equation (22) using IMLORTH and IMLTO as a dependent variable. At the beginning of each month m, stocks are ranked by their standard deviation of daily returns in month $m-1$ and sorted into three portfolios. Within these three portfolios, stocks are ranked by their orthogonalized illiquidity obtained as the estimated residual term of Equations (9) and (10) (IMLORTH) and by their average illiquidity level ILLIQTO of month $m-1$ (IMLTO). Stocks in each portfolio are then used to calculate an equally-weighted portfolio return in month m. Excluded are stocks with less than 15 days of trading in month m and stocks whose monthly return or value for $ILLIQ_m$ or $ILLIQTO_m$ is part of the lower or upper 1% tail of the respective distribution. The reference period is from 02/1990-09/2015. t-statistic are in parentheses. ***, **, * denote statistical significance on the one, five and ten percent level, respectively.

Panel A: Descriptive statistics

Variable	Mean	S.d.	Median	Skewness	Minimum	Maximum
IMLTO	0.137	2.553	0.356	-0.751	-11.800	5.786
IMLORTH	-0.635	2.483	-0.344	-1.031	-11.264	5.781
RM	0.616	4.319	1.190	-0.658	-17.230	11.350
SMB	0.169	3.303	0.070	0.814	-16.700	22.320
HML	0.188	3.123	0.010	0.093	-13.110	13.910
MOM	0.613	4.930	0.700	-1.584	-34.580	18.380

Panel B: Correlations

Variable	IMLTO	IMLORTH	RM	SMB	HML	MOM
IMLTO	1					
IMLORTH	0.718	1				
RM	-0.605	-0.577	1			
SMB	-0.246	-0.480	0.244	1		
HML	0.492	0.449	-0.249	-0.327	1	
MOM	0.015	0.253	-0.249	0.052	-0.157	1

Panel C: Estimation results

Variable	α	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	Adj. R^2
IMLTO	0.297***	-0.315***	0.002	0.287***	-0.032	0.487
	(2.753)	(-11.824)	(0.047)	(7.781)	(-1.427)	
IMLORTH	-0.576***	-0.215***	-0.229***	0.232***	0.112***	0.543
	(-5.818)	(-8.807)	(-7.320)	(6.854)	(5.403)	